Doctoral Program in Mathematical Sciences
Department of Mathematics “Tullio Levi-Civita”
University of Padova

Doctoral Program in Mathematical Sciences

Catalogue of the courses 2020

Updated May 19, 2020

INTRODUCTION

This Catalogue contains the list of courses offered to the Graduate Students in Mathematical Sciences for the year 2019-2020.
The courses in this Catalogue are of two types.

1. Courses offered by the Graduate School. This offer includes courses taught by internationally recognized external researchers. Since these courses might be not offered again in the near future, we emphasize the importance for all graduate students to follow them.
2. Some courses selected from those offered by the Graduate School in Information Engineering of the University of Padova, by the Master in Mathematics, and by other institutions, that we consider of potential interest for the students in Mathematics.

We underline the importance for all students to follow courses, with the goal of **broadening their culture in Mathematics**, as well as developing their knowledge in their own area of interest.

REQUIREMENTS FOR GRADUATE STUDENTS

Within the **first two years of enrollment (a half of these requirements must be fulfilled within the first year)** all students are required to follow and **pass the exam** of

- **at least 2 among the courses called "Courses of the Doctoral Program"** in this catalogue;
- other courses for a total commitment **of at least 56 additional hours**;
- at least one activity on soft skills.

Students are warmly encouraged to take more courses than the minimum required by these rules, and to commit themselves to follow regularly these courses. At the end of each course the teacher will inform the Coordinator and the Secretary on the activities of the course and of the registered students.

Students **must register** to all courses of the Graduate School that they want to attend, independently of their intention to take the exam or not. We recommend to register as early as possible: the Graduate School may cancel a course if the number of registered students is too low. If necessary, the registration to a Course may be canceled.

Courses for Master of Science in “Mathematics”

Students have the possibility to attend some courses of the Master of Science in Mathematics and get credits for the mandatory 56 hours.
The recommendation of these courses must be made by the Supervisor and the amount of credits is decided by the Executive Board.

Courses attended in other Institutions

Students are allowed to take Ph.D. courses offered by PhD Programs of other Universities or in Summer Schools. Acquisition of credits will be subject to approval of the Executive Board.

Seminars

All students must attend the **Colloquia of the Department** and participate in the Graduate Seminar (“**Seminario Dottorato**”). They are also encouraged to attend the seminars of their research group.

HOW TO REGISTER AND UNREGISTER TO COURSES

The registration to a Course must be done online.
Students can access the **online registration form** on the website of the Doctoral Course http://dottorato.math.unipd.it/ (select the link Courses Registration), or directly at the address http://dottorato.math.unipd.it/registration/.
In order to register, fill the registration form with all required data, and validate with the command “Register”. The system will send a confirmation email message to the address indicated in the registration form; please save this message, as it will be needed in case of cancellation.

Registration to a course implies the commitment to follow the course.

Requests of **cancellation** to a course must be submitted in a timely manner, and **at least one month before the course** (except for courses that begin in October and November) using the link indicated in the confirmation email message.

REQUIREMENTS FOR PARTICIPANTS NOT ENROLLED IN THE GRADUATE SCHOOL OF MATHEMATICS

The courses in this catalogue, although part of activities of the Graduate School in Mathematics, are open to all students, graduate students, researchers of this and other Universities.
For reasons of organization, external participants are required to **communicate their intention** (loretta.dallacosta@unipd.it) to take a course at least two months before its starting date if the course is scheduled in January 2020 or later, and as soon as possible for courses that take place until December 2019.
In order to **register,** follow the procedure described in the preceding paragraph.
Possible **cancellation** to courses must also be notified.

Courses of the Doctoral Program

1. Prof. Giancarlo Benettin, Prof. Carlangelo Liverani
   Introduction to Ergodic Theory **DP-1**
2. Prof. Bruno Chiarellotto, Prof. Matteo Longo
   The Arithmetic of Elliptic Curves **DP-2**
3. Prof. Francesco Esposito
   Introduction to Quantum Groups **DP-4**
4. Prof. Markus Fischer
   Stochastic Differential Equations and Applications **DP-5**
5. Prof. Luca Martinazzi
   Degree theory and applications to geometry and analysis **DP-7**

Courses of the “Computational Mathematics” area

1. Prof. Immanuel Bomze
   Introduction to Conic Optimization **MC-1**
2. Prof. Alessandra Buratto
   Introduction to differential games **MC-2**
3. Prof. Antoine Jacquier
   A smooth tour around rough models in finance. (From data to stochastics to machine learning) **MC-3**
4. Prof. Sergei Levendorskii
   Fourier-Laplace Transform and Wiener-Hopf Factorization in Finance, Economics and Insurance **MC-4**
5. Prof. Gianmarco Manzini
   Introduction to the Virtual Element Method and to numerical methods for PDEs on unstructured polytopal meshes **MC-6**
6. Dott.ssa Elena Sartori
   Modeling interacting agents in social sciences **MC-7**
7. Dr. Francesco Tudisco
   Eigenvector methods for learning from data on networks **MC-9**
8. Prof. Tiziano Vargiolu
   Topics in Stochastic Analysis **MC-10**

Courses of the “Mathematics” area

1. Prof. Andrei Agrachev, Prof. Davide Barilari
   Introduction to Subriemannian geometry
   THE COURSE IS POSTPONED TO THE NEXT YEAR

Courses offered within the Masters’s Degree in Mathematics

Soft Skills

1. Maths information: retrieving, managing, evaluating, publishing **SS-1**
2. Brains meet Digital Enterprises **SS-2**
3. Entrepreneurship and Technology-based Startups **SS-3**

Other courses suggested to the students

1. Other suggested Courses **EC-1**

Courses in collaboration with the Doctoral School on "Information Engineering"

1. Dr. Juan José Alcaraz Espín
   Introduction to Reinforcement Learning **DEI-1**
2. Prof. Giorgio Maria Di Nunzio
   Bayesian Machine Learning **DEI-3**
3. Prof. Deniz Gunduz
   Introduction to Information Theory **DEI-5**
4. Prof. Lorenzo Finesso
   Statistical Methods **DEI-7**
5. Prof. Fabio Marcuzzi
   Computational Inverse Problems **DEI-8**
6. Prof. Gianluigi Pillonetto
   Applied Functional Analysis and Machine Learning **DEI-9**
7. Prof. Domenico Salvagnin
   Heuristics for Mathematical Optimization **DEI-10**
8. Prof. Andrea Serrani
   Control of Multivariable Systems: A Geometric Approach **DEI-11**
9. Dr. Gian Antonio Susto
   Elements of Deep Learning **DEI-12**

Courses of the Doctoral Program

introduction to Ergodic Theory

Prof. Benettin Giancarlo[1], Prof. Carlangelo Liverani[2]

[1] *Dipartimento di Matematica "Tullio Levi-Civita", Universitá di Padova*
Email: benettin@math.unipd.it

[2] *Dipartimento di Matematica, Universitá Tor Vergata, Roma*
Email: liverani@mat.uniroma2.it

Timetable: 16+8 hrs. First lecture on October 23, 2019, 11:00 (dates already fixed, see calendar), Torre Archimede, Room 2BC/30.

Course requirements:

Examination and grading:

SSD: MAT/07

Aim:

Course contents:

Part I - G. Benettin (16 h)

Introduction; dynamical systems with measure; elementary examples; the Liouville measure for Hamiltonian systems; isomorphism and classification.
General results: the Birkhoff-Kinchin ergodic theorem; the Poincarè return theorem.
Excursus: the physical roots of ergodic theory; some basic ideas of Boltzmann and Gibbs.
The notion of ergodicity; examples. The notion of mixing; examples. The ergodic decomposition (hints).
The Kolmogorov-Sinai entropy: notion, main results, examples.
Possible additional topics, if there is time: the spectral approach to ergodic theory.

Part II - C. Liverani (8 h)

Abstract: Fluctuations around the average are of fundamental physical relevance (starting with the proof of the existence of atoms in Einstein's 1905 seminal paper). Such fluctuations can appear in space averages (when many degrees of freedom are present) or in time averages (ergodic averages), or in both at the same time (hydrodynamics).

I will discuss the case of ergodic averages by analysing some simple non-trivial examples. This will allow to illustrate some surprising and fundamental differences between regular and chaotic motions. In addition, I will explain in which exact sense chaotic and random motions are similar.

The Arithmetic of Elliptic Curves

Prof. Bruno Chiarellotto[1], Prof. Matteo Longo[2]

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[2]Dipartimento di Matematica ”Tullio Levi-Civita”, Universitá di Padova
Email: mlongo@math.unipd.it

Timetable: 24 hrs. First lecture on January 23, 2020, 09:00 (dates already fixed, see calendar), Room 2BC/30, Torre Archimede

Course requirements:

Examination and grading:

SSD: MAT/02-03

Aim:
I propose to give an introduction to some of the previous topics and, if time permits, to explain what are some of the known results on this conjecture. Each of the topic can be very technical (especially Wiles's proof). Instead of explaining complete proofs, which would be clearly out of the scope of this course, I would like to propose an overview of the topics and the main results, giving some hint, when possible, on the techniques which are used to attack the problems.

The plan of the lectures (24h) is as follows:

1. Introduction to the arithmetic theory of elliptic curves (6h).
2. The Mordell-Weil Theorem (2h).
3. The L-function of an elliptic curve (2h)
4. Wiles Modularity Theorem, modular forms and Fermat Last Theorem (6h).
5. The Birch and Swinnerton-Dyer Conjecture (BSD) (4h).
6. Results on the BSD Conjecture: Heegner points and the work of Kolyvagin (4h).

Course contents:
Number Theory is a wide branch of pure mathematics which studies a number of different problems having to do (in a vague sense) with arithmetic proprieties of numbers such as the fleld of rational numbers. For example, one of the main fascinating problems in number theory is to describe the structure of the Galois group $\mathrm{Gal}(\bar{\mathbb{Q}}/\mathbb{Q})$ of the algebraic closure of $\mathbb{Q}$. It is quite common in number theory to study a problem by means of a great number of different techniques, going from the complex analysis to representation theory, from dynamic systems to algebraic geometry, from functional analysis to abstract algebra. In the previous example, if one looks at representations $\rho : \mathrm{Gal}(\bar{\mathbb{Q}}/\mathbb{Q}) \to \mathrm{GL}_2(\mathbb{C})$, one finds in a natural way that an interesting subset of these is made out of complex analytic functions (called modular forms) having a large number of symmetries, which can be studies by complex analytic method on one side, and by means of techniques from algebraic geometry on the other side (modular forms also appear in

other field of mathematics, and in recent years also in physics). It is the mix of these techniques which makes the subject difficult and, at the same time, fashinating. Let us remark that two of the Millennium Problems are about Number Theory: The Riemann Hypothesis and the Birch and Swinnerton-Dyer conjecture. The purpose of this course is to give a brief introduction to one of the two Millennium Problems alluded to before: the Birch and Swinnerton-Dyer conjecture. To explain the problem, one considers an elliptic curve given by an affine equation

$$E : y^2 = f(x)$$

with $f(x)$ a non-singular cubic polynomial with coefficients in $\mathbb{Z}$. When we add the point at infinity (i.e. we consider this curve in the projective plane) we obtain a projective non-singular curve, whose points$E(F)$ over anyfinite field extension F of the field of rational numbers can be equipped with a structure of finitely generated abelian group (Mordell-Weil Theorem). In particular, we can write

$$\text{E}(\mathbb{Q}) \simeq \mathbb{X}^r \oplus E(\mathbb{Q})_{tors}$$

where $E(\mathbb{Q})_{tors}$ is a finite group. The integer $r \geq 0$ appearing above is the *algebraic rank* of E. On the other hand, one can construct a function $s \mapsto L(E/\mathbb{Q}, s)$ of the complex variable s as a convergent (for the real part of s greater that 3/2) product of factors, one for each prime number p, such that each of these factor counts the number of points (with coordinates in the field with p elements) of the curve obtained by reducing the coefficients of $f(x)$ modulo p (note the similarity with similar products for the Riemann Zeta function). Thanks to the work of Wiles (leading the the proof of Fermat Last Theorem) one knows that this complex function can be extended to all $\mathbb{C}$ to an entire function, having a functional equation with center of symmetry $s = 1$. The Birch and Swinnerton-Dyer Conjecture states that the algebraic rank of $E(\mathbb{Q})$ is equal to its *analytic rank*, namely, the order of vanishing of $L(E/\mathbb{Q}; s)$ at $s = 1$. (One also has a more precise description of the leading value in terms of fine arithmetic invariants of E.) This is a deep conjecture because relates object of completely diffrent nature, algebraic and analytic.

Introduction to Quantum Groups

Prof. Francesco Esposito[1]

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Email: esposito@math.unipd.it

Timetable: 24 hrs. First lecture on April 15, 2020, 11:00 (dates already fixed, see calendar) Torre Archimede, Room 2BC/30.

Course requirements:

Examination and grading:

SSD: MAT/02-03

Aim:
Quantum groups arose at first in connection with problems in statistical mechanics and are closely related to conformal field theory. Moreover, they have applications to many different areas of mathematics, e.g. knot theory and topology, the study of the absolute Galois group of $\mathbb{Q}$, representation theory of algebraic groups in characteristic p, Poisson-Lie groups, the theory of (q)- special functions. They have also served as a rich source of examples in non-commutative geometry. For these reasons, the theory of quantum groups may be of interest to mathematicians with expertise in any of the following: algebra, number theory, geometry, topology, mathematical physics, analysis. The intent of this course is to offer an introduction to quantum groups and survey its diverse applications. Being aimed at a general audience, its prerequisites are limited to the basics of linear algebra and elementary notions of topology. Any other needed notion will be introduced through examples or formal definition.

Course contents: (Tentative)

1. History and Motivations, applications. Basic notions of representation theory.
2. The Lie algebra $\mathfrak{sl}_2$ and its representations (characteristic zero and modular). The universal enveloping algebra $U(\mathfrak{sl}_2)$ of $\mathfrak{sl}_2$.
3. The quantized enveloping algebra $Uq(\mathfrak{sl}_2)$ for q generic and q a root of unity.
4. Semisimple Lie algebras and universal enveloping algebras.
5. Quantized universal enveloping algebras. PBW theorem. Specializations and its center
6. Representations of algebras and Hopf algebras. Braided tensor categories. Applications to knot theory.
7. Knizhnik-Zamolodchikov equations.
8. Quasi-Hopf algebras and applications to the absolute Galois group of $\mathbb{Q}$.
9. Representations of semisimple Lie algebras in characteristic zero. Classification of finite-dimensional irreducible representations. Category $\mathcal{O}$.
10. Representations of Quantized universal enveloping algebras for generic q.
11. Peter-Weyl theorem. R-matrices and quantized function algebras.

Stochastic Differential Equations and Applications

Prof. Markus Fischer[1]

[1] *Dipartimento di Matematica "Tullio Levi-Civita", Universitá di Padova*
Email:fischer@math.unipd.it

Timetable: 24 hrs. First lecture on May 5, 2020, 11:00 (dates already fixed, see calendar), Torre Archimede, Room 2BC/30.

Course requirements:
Basics of real analysis and measure-theoretic probability. Familiarity with continuous-time stochastic processes is advantageous, but not required.

Examination and grading:

SSD: MAT/06

Aim:
Introduce fundamental results and techniques in the theory of stochastic differential equations (SDEs) and illustrate their connections with ordinary and certain classes of partial differential equations. Some applications to optimal control, many-particle systems, and their mutual interplay will be treated.

Scope: Focus on SDEs driven by finite-dimensional Wiener processes.

Course contents:

1. 1. Itô calculus and variants of Itô's formula. SDEs driven by Brownian motion (Wiener processes). Questions of existence and uniqueness in weak and strong solutions, results by Yamada-Watanabe. Strong solutions for non-globally Lipschitz coefficients, weak solutions through the Girsanov transformation.
2. Markov processes, martingale problems, and weak convergence of measures. Martingale problem formulation for SDEs, including models with jumps. Existence-uniqueness duality. Euler-Maruyama approximation scheme.
3. Kolmogorov backward and Kolmogorov (Fokker-Planck) forward equation. Differentiation of SDEs with respect to initial conditions and probabilistic solution of linear parabolic PDEs. Feynman-Kac and Bismut-Elworthy-Li formulae. Regularization by noise for ODEs. 1 / 2
4. Stochastic optimal control, dynamic programming and Hamilton-Jacobi- Bellman equation. Control through change-of-measure and backward SDEs. Variational representation of relative entropy and large deviations from the small noise limit.
5. McKean-Vlasov SDEs, nonlinear Kolmogorov equation, and propagation of chaos for mean field systems.
6. If time permits and depending on participants' interests, introduction to optimal control of McKean-Vlasov SDEs or to mean field games.

References:

[1] G. Da Prato. Introduction to Stochastic Analysis and Malliavin Calculus.Edizioni della Normale, Pisa, 3rd ed., 2014.

[2] W.H. Fleming and M. Soner. Controlled Markov Processes and Viscosity Solutions. Springer, New York, 2nd ed., 2006.

[3] I. Karatzas and S.E. Shreve. Brownian Motion and Stochastic Calculus. Volume 113 of Graduate Texts in Mathematics. Springer, New York, 2nd ed., 1998.

Degree theory and applications to geometry and analysis

Prof. Luca Martinazzi[1]

[1] *Dipartimento di Matematica "Tullio Levi-Civita", Universitá di Padova*
Email:martinaz@math.unipd.it

Timetable: 24 hrs. First lecture on October 7, 2019, 14:00 (dates already fixed see calendar), Torre Archimede, Room 2BC/30.

Course requirements: Analysis and basic differential geometry.

Examination and grading: oral exam.

SSD: MAT/05, MAT/03

Aim: Give the basic notions and some beautiful applications of degree theory in analysis and geometry.

Course contents:
The course starts with an elementary introduction to submanifolds in $\mathbb{R}^n$ and to the differentiable functions among them, in order to present the notion of Brouwer degree of a function and its fundamental properties (particularly the homotopy invariance). With elementary but non-trivial methods we use the Brouwer degree to prove:

- The fundamental theorem of algebra
- The fixed-point theorem of Brouwer
- The sphere theorem (there are no non-vanishing tangent vector fields on an even-dimensional sphere)
- The separation theorem of Jordan in $\mathbb{R}^n$.

We then introduce the equivalent definition of de Rahm degree in terms of differential forms, used to prove a first version of the theorem of Gauss-Bonnet. Later we will work on tangent vector fields of a submanifold of $\mathbb{R}^n$, introducing the notion of index of a vector field and proving the famous theorem of Poincaré-Hopf. This uses Morse theory, which we will also discuss in a self-contained way. Using the Poincaré-Hopf theorem we then give a complete proof of the Gauss-Bonnet theorem for submanifolds of $\mathbb{R}^n$ of arbitrary dimension compact and without boundary. This is an elegant proof, normally not seen in standard courses of differential geometry.

We also discuss the relation between the Brouwer degree in $\mathbb{R}^n$, the winding number and the Cauchy formula from complex analysis.

In the last part of the course we develop the notion of Leray-Schauder degree and obtain the Caccioppoli-Schauder fixed-point theorem in Banach spaces, an infinite-dimensional analog of the fixed-point theorem of Brouwer. This will be used to prove Peano's theorem on the existence of solutions to ODEs with continuous field. We will also use the Leray-Schauder degree to prove the existence of solutions to some nonlinear elliptic equations.

Bibliography:

1. D. Outerelo, J.M. Ruiz, Mapping degree theory, AMS 2009.
2. J. Milnor, Topology from the differentiable viewpoint, Princeton 1965

Courses of the “Computational Mathematics” area

Conic, especially copositive optimization

Prof. Immanuel Bomze[1]

[1] Dept. Applied Mathematics and Statistics, University of Vienna
Email: immanuel.bomze@univie.ac.at

Timetable: 8 hrs. THE COURSE IS TEMPORARILY SUSPENDED. THE TIMETABLE OF THE LECTURES WILL BE COMMUNICATED AS SOON AS POSSIBLE

Course requirements:

Examination and grading:

SSD: MAT/09

Aim:

Course contents:

Quite many combinatorial and some important non-convex continuous optimization problems admit a conic representation, where the complexity of solving non-convex programs is shifted towards the complexity of sheer feasibility (i.e., membership of the cone which is assumed to be a proper convex one), while structural constraints and the objective are all linear. The resulting problem is therefore a convex one, and still equivalent to some NP-hard problems with inefficient local solutions despite the fact that in the conic formulation, all local solutions are global.

Using characterizations of copositivity, one arrives at various approximations. However, not all of these are tractable with current technology. In this course, we will address some approaches on which tractable SDP- or LP-approximations, and also branch-and-bound schemes, may be based.

Introduction to differential games

Prof. Alessandra Buratto[1]

[1] *Dipartimento di Matematica "Tullio Levi-Civita", Universitá di Padova*
Email:buratto@math.unipd.it

Timetable: 20 hrs. First lecture on February 4, 2020, 11:00 (dates already fixed, see calendar), Torre Archimede, Room 2BC/30.

Course requirements: Basic notions of Differential equations and Optimal control

Examination and grading: Homework assignments and final test

SSD: SECS-S/06

Aim: Differential games are very much motivated by applications where different agents interact exhibiting an inter-temporal aspect. Applications of differential games have proven to be a suitable methodology to study the behaviour of players (decision-makers) and to predict the outcome of such situations in many areas including engineering, economics, military, management science, biology and political science.
This course aims to provide the students with some basic concepts and results in the theory of differential games, as well as some applications in Economics and Management Science

Course contents:

Part 1: General theory

- Recall of basic concepts of game theory, equilibrium (Nash ...)
- Dynamic games: formalization of a differential game
- Simultaneous and competitive differential games (Nash Equilibrium)
- Hierarchic differential games (Stackelberg equilibrium)
- Time consistency and perfectness
- Differential games with special structures (linear quadratic games, linear state games)

Part 2: Applications in Economics and Management Science

- Advertising and Promotion in a Marketing Channel.
- Consignment contracts with cooperative programs and price discount mechanisms in a dynamic supply chain
- Competition between national brands and private labels in a vertical channel structure.

References:

- Basar T., and Olsder G.J., Dynamic Noncooperative Game Theory Classics in Applied Mathematics.. SIAM 2 Ed., 1999.

A smooth tour around rough models in finance (From data to stochastics to machine learning)

Prof. Antoine Jacquier[1]

[1] *Imperial College, Londra*
Email: a.jacquier@imperial.ac.uk

Timetable: 16 hrs. THE COURSE IS TEMPORARILY SUSPENDED. THE TIMETABLE OF THE LECTURES WILL BE COMMUNICATED AS SOON AS POSSIBLE

Course requirements: Probability and Stochastic Calculus

Examination and grading: oral examination on the topics covered during the course

SSD: MAT/06, SECS-S/06

Aim: Aim: the course aims at introducing the recent theory on rough volatility models, namely stochastic volatility models in finance driven by the fractional Brownian motion. This class of models will naturally arise by looking at market data and at the end of the course the PhD student will have full control of advanced tools in stochastic calculus which are crucial in modern finance.

Course contents:
A quick glance at time series in market data (Equities, Currencies, Commodities, Rates...) leaves no doubt that volatility is not deterministic over time, but stochastic. However, the classical Markovian setup, upon which a whole area of mathematical finance was built, was recently torn apart when Gatheral-Jaisson-Rosenbaum showed that the instantaneous volatility is not so well behaved and instead features memory and more erratic path behaviour. Rough volatility was born. This new paradigm does not come for free, though, and new tools and further analyses are needed in order to put forward the benefits of this new approach. The goal of this course is to explain how Rough Volatility naturally comes out of the data, and to study the new techniques required to use it as a tool for financial modelling. We shall endeavour to strike a balance between theoretical tools and practical examples, and between existing results and open problems. The contents shall span, with more or less emphasis on each topic, the following:

1. Estimating roughness from data. Constructing a rough volatility model.
2. Constructing a model consistent between the historical and the pricing measure: joint calibration of SPX and VIX options.
3. Pricing options in rough volatility models: from Hybrid Monte Carlo to Deep learning

The first item is anchored in fairly classical Statistics and Probability, while the second deals with Stochastic analysis. The last item draws upon recent literature connecting Path-dependent PDEs, Backward SDEs and Deep Learning technology. Prior knowledge in all areas is not required, but good Probability/Stochastic analysis background is essential.

Fourier-Laplace transform and Wiener-Hopf factorization in Finance, Economics and Insurance

Prof. Sergei Levendorskii[1]

[1] *Calico Science Consulting. Austin, TX*
Email:

Timetable: 12 hrs. First lecture on June 22, 2020, 14:00 (date already fixed, see calendar), Torre Archimede, Room 2BC/30.

Course requirements: Probability and Stochastic Calculus

Examination and grading: oral examination on the topics covered during the course

SSD: MAT/06, SECS-S/06

Aim: The course aims at introducing recent fast and robust pricing techniques for exotic derivatives (such as Bermudan and American) in general Lévy models.

Course contents:
The Fourier-Laplace transform and Wiener-Hopf factorization are ubiquitous in Mathematics, Physics, Engineering, Probability, Statistics, Insurance and Finance. Recently, several difficult problems in Game Theory and Economics were solved using the Wiener-Hopf factorization techniques. From the analytical viewpoint, problems considered in the course can be reduced to a sequence of calculations, each involving either the Fourier (or inverse Fourier) transform of a given function, or the convolution of two given functions. In turn, each of these operations can be performed numerically with high efficiency using the standard fast Fourier and Hilbert transforms and fast convolution. We introduce new more efficient versions of the fast Fourier and Hilbert transforms. The second general topic of the course is the new general methodology for efficient evaluation of integrals with integrands analytic in regions around the lines of integration, examples being numerical Fourier-Laplace inversion, calculation of the Wiener-Hopf factors and high transcendental functions. We introduce three families of conformal deformations of the contour of integration in the Fourier inversion formula and the corresponding changes of variables, which lead to much faster and more accurate calculations. The third general topic of the course is the EPV (expected present value operators) method. The strength of the EPV method stems from the interaction of the probabilistic and analytical techniques. In the standard analytical approach to solution of boundary problems, the operators are interpreted as the expectation operators. This allows one to relatively easily evaluate complicated expectations and solve optimal stopping problems with non-standard payoffs. All topics in the course and additional topics will be covered in S. Boyarchenko, M. Boyarchenko, N. Boyarchenko, and S. Levendorskij. Spectral Methods in Finance, Economics and Insurance. Springer, New York, 2020, the monograph in preparation for Springer due to be finished in this Fall. The full lists of references for the lectures and a more detailed contents' list will be given during the first lecture.

- Lecture 1. Lévy models
- Lecture 2. Evaluation of probability distributions and pricing European options in Lévy models
- Lecture 3. Simplified trapezoid rule, Fast Fourier Transform and its variations
- Lecture 4. Conformal acceleration techniques
- Lecture 5. Barrier options with discrete monitoring and Bermudan options. Calculations in the state space
- Lecture 6. Barrier options with discrete monitoring and Bermudan options. Calculations in the dual space
- Lecture 7. Wiener-Hopf factorization
- Lecture 8. Contingent claims with continuous monitoring, boundary value problems and Wiener-Hopf factorization
- Lecture 9. Options with continuous monitoring, cont-d
- Lecture 10. Affine models
- Lecture 11. American options with infinite time horizon
- Lecture 12. American options with finite time horizon

Introduction to the Virtual Element Method and to numerical methods for PDEs on unstructured polytopal meshes

Prof. Gianmarco Manzini[1]

[1] CNR-IMATI, Pavia
Email: marco.manzini@imati.cnr.it

Timetable: 16 hrs. First lecture on January 21, 2020, 14:30 (dates already fixed see calendar), Torre Archimede, Room 2BC/30.

Course requirements: basic notions of numerical analysis and numerical methods for partial differential equations (Finite Elements, Finite Volumes, Finite Differences).

Examination and grading: brief presentation on a course-pertinent subject and oral examination on the topics covered during the course.

SSD: MAT/08 - Numerical Analysis

Aim: The course aims at introducing the fundamental ideas and results on numerical methods for solving partial differential equations of elliptic and parabolic type, with special emphasis on the Virtual Element method.

Course contents:

- **Week 1, Lecture 1: introduction to numerical methods for partial differential equations of elliptic types on unstructured meshes:**
  - Polygonal Finite Element method (PFEM);
  - Mimetic Finite Difference (MFD) method;
  - Virtual Element method (VEM);
  - other variants (wG, HHO, HDG, etc).
- **Week 1, Lecture 2: the conforming VEM; construction of the basic method, convergence analysis and implementation**
- **Week 2, Lecture 3: the nonconforming formulation; construction of the basic method, convergence analysis and implementation**
- **Week 2, Lectures 4: enhanced and serendipity formulations of the virtual element method**
- **Week 3, Lectures 5 and 6: the mixed formulation of the virtual element method**
- **Week 4, Lectures 7 and 8: virtual de Rham sequences and ap- plications to electromagnetism and Stokes**

Modeling interacting agents in social sciences

Dott.ssa Elena Sartori[1]

[1] Dipartimento di Matematica "Tullio Levi-Civita", Universitá di Padova
Email: esartori@math.unipd.it

Timetable: 12 hrs. First lecture on December 4, 2019, 10:30 (dates already fixed, see calendar), Torre Archimede, Room 2BC/30.

Course requirements: Basic knowledge of probability theory.

Examination and grading: Oral exam

SSD: MAT06; SECS-S06

Aim: In this course we consider a class of stochastic models describing the collective behavior of a large system of interacting agents, where the interaction is of the mean-field type and the state space is finite. We analyze some of the main approaches proposed during the last 20 years in literature, paying particular attention to the techniques used for the case. From an applicative viewpoint modeling microscopic interaction between individuals allows to describe macroscopic phenomena such as, for instance, polarization of ideas or adoption rates of a new technology seen as the result of conformism, imitation or peer pressure. In the last decades in the context of social sciences, the first proposed models were inspired by statistical mechanics; in that setting each agent could update his state at most one per time following some probabilistic rates of transition with sequential updating, thus defining a Markov chain. The introduction of a random utility function defining agents rationality led to models closer to reality. What these models do not account for is strategic interaction: when deciding, agents forecast the action of others . This is crucial since the single agent payoff, in fact, depends explicitly on some function of the actions of others. This new approach leads to move to a game-theoretic framework, where the emerging dynamics is a controlled Markov process.

Course contents:

1. overview of models inspired by statistical mechanics describing the macroscopic limit of a large system of interacting agents;
2. introduction of the so called Random Utility Model and its time evolution, focusing on some techniques used to analytically tackle it, e.g. weak convergence of stochastic processes (convergence of generators);
3. basic tools of game theory (normal form games, Nash equilibrium, best response map, ...);
4. basic notions in control theory (optimal control problems, dynamic programming, HJB equations, ...);
5. an example of mean field game for spin systems;
6. formalization of a finite-state mean-field game.

References:

- Steven N. Durlauf and H. Peyton Young (2004) “Social Dynamics”, MIT Press Books.
- Engelbert Dockner, Steffen Jergensen, Ngo VanLong and Gerhard Sorger (2000) “Differential Games in Economics and Management Science”, Cambridge University Press.
- Alain Haurie, Jacek B. Krawczyk and Georges Zaccour (2012) “Games and Dynamic Games”, World Scientific Books.

Eigenvector methods for learning from data on networks

Dr. Francesco Tudisco[1]

[1] GSSI – Gran Sasso Science Institute – L'Aquila (AQ)
Web: http://personal.strath.ac.uk/f.tudisco/
Email: francesco.tudisco@gssi.it

Timetable: 12 hrs. THE COURSE IS TEMPORARILY SUSPENDED. THE TIMETABLE OF THE LECTURES WILL BE COMMUNICATED AS SOON AS POSSIBLE

Course requirements: Standard background in numerical linear algebra, numerical and mathematical analysis.

Examination and grading: Oral presentation or written essay

SSD: MAT/08 Numerical Analysis; INF/01 ComputerScience; MAT/05 Mathematical Analysis

Aim: Provide an introduction to spectral methods for unsupervised and semisupervised learning tasks and centrality on networks based on eigenvectors of matrices, tensors, and multihomogeneous mappings.

Course contents: Graphs are fundamental tools for learning from data and for the analysis of complex systems. In fact, we often state questions and develop analysis in terms of nodes and edges. For example, we can describe a social system by modeling individuals as nodes and social interactions as edges between pair of nodes. Similarly, we can model a biochemical reaction by assigning a node to each protein and edges between them to model physical contacts of high specificity, so-called protein-protein interactions.

Due to the broad scope of this modeling paradigm, the analysis of systems or datasets as networks has enjoyed a tremendous success over the last decade and many mathematical models and numerical methods for handling learning problems on networks have been developed, based on eigenvectors and singular vectors of matrix, tensors (or hypermatrices) and, more generally, multihomogeneous mappings.

The course will give an introduction to spectral methods for learning from network data from a numerical linear algebra perspective by discussing the following topics in various detail:

- Graph Laplacian, spectral partitioning, semisupervised learning
- p-Laplacian and nonlinear spectral clustering
- Lovász extension and homogeneous mappings
- Eigenvector centrality and centrality based on matrix functions
- Centrality in higher order networks (time-varying, multilayer, hypergraphs)
- Nonlinear Perron–Frobenius theory and nonlinear power methods

References:

1. E. Estrada and P. Knight *A first course in network theory*, Oxford University Press, 2015.
2. J. Gallier *Spectral Theory of Unsigned and Signed Graphs. Applications to Graph Clustering: a Survey*, arXiv:1601.04692, 2016.

Topics in Stochastic Analysis

Prof. Tiziano Vargiolu[1]

[1]Università di Padova
Dipartimento di Matematica "Tullio Levi-Civita"
Email: vargiolu@math.unipd.it

Calendario: 8 hrs. First lecture on October 9, 2019, 14:30 (dates already fixed, see the calendar), Torre Archimede, Room 2BC/30.
A preliminary meeting with the Students will be held on October 7, 2019, 10:00, Torre Archimede, Room 2BC/30.

Prerequisiti: A previous knowledge of the basics of continuous time stochastic analysis with standard Brownian motion, i.e. stochastic integrals, Itô formula and stochastic differential equations, as given for example in the master course "Analisi Stocastica".

Tipologia di esame: Seminar

SSD: MAT/06

Programma: The program will be fixed with the audience according to its interests. Some examples could be:

- continuous time stochastic control;
- Levy processes;
- numerical methods;
- stochastic control.

Courses of the “Mathematics” area

Sub-Riemannian Geometry

Prof. Andrei Agrachev[1], Prof. Davide Barilari[2]

[1]Scuola Internazionale Superiore di Studi Avanzati (SISSA), Trieste
Email: agrachev@sissa.it

[2]Dipartimento di Matematica "Tullio Levi-Civita", Universitá di Padova
Email:barilari@math.unipd.it

Timetable: THE COURSE IS POSTPONED TO THE NEXT YEAR

Course requirements: basic differential geometry

Examination and grading:

SSD: MAT/05

Aim: Sub-Riemannian geometry is the geometry of a world with nonholonomic constraints. In such a world, one can move, send and receive information only in certain admissible directions but eventually one can reach every position from any other. In the last two decades sub-Riemannian geometry has emerged as an independent research domain impacting on several areas of pure and applied mathematics, with applications to many areas such as quantum control, image reconstructions, robotics and PDEs.

The fist part of the course is mainly an introduction to the subject towards theory and examples coming from applications such as mechanics. The second part focuses on more advanced questions, providing students to recent progress in the field and open questions.

Course contents:
Part 1
Vector fields, flow and Lie brackets. Frobenius and Chow-Rashevskii theorem. Applications to rolling spheres and isoperimetric problems.
Sub-Riemannian distance. Metric completeness. Pontryagin extremals : Symplectic geometry and Hamiltonian formalism. Normal and abnormal extremals. Examples in dimension 3.
Part 2
The second part will focus on different questions around abnormal extremal and length-minimizers, with discussions on recent advances and open questions.

References:
A Comprehensive Introduction to Sub-Riemannian Geometry, Cambridge Studies in Advanced Mathematics, Cambridge University Press, 2019

Introduction to Hyperbolic Conservation Laws

Prof. Fabio Ancona[1], Prof. Massimiliano Daniele Rosini[2]

[1] *Dipartimento di Matematica, Universit'ị Padova*
Email: ancona@math.unipd.it

[2] *Dipartimento di Matematica e Informatica, Universitá di Ferrara*
Email: massimilianodaniele.rosini@unife.it

Timetable: 16 hrs. First lecture on May 27, 2020, 11:00 (dates already fixed, see calendar), Torre Archimede, Room 2BC/30.

Course requirements: very basic notions of ODE and PDE theory

Examination and grading: seminar

SSD: MAT/05 - Mathematical Analysis

Aim: the course aims at providing an introduction to:
- fundamental features of the theory of hyperbolic conservation laws in one space variable;
- topics in recent research on traffic flow models and networks for this class of first order non-linear PDEs.

The course shall be of particular interest for students in Mathematical Analysis, Mathematical Physics, Numerical Analysis, especially if interested in fluid dynamics models.

Course contents:
Part 1
Introduction to the general theory of entropy weak solutions of conservation laws. Discontinuous distributional solutions. Riemann problem. Wave front-tracking algorithm.
Part 2
Conservation laws with discontinuous flux and with point constraints. Analysis of traffic flow models via vanishing viscosity and many particle approximations (micro-macro limit).

References:

- A. Bressan, Hyperbolic Systems of Conservation Laws, The One-Dimensional Cauchy Problem, Oxford.
- C.M.Dafermos, Hyperbolic Conservation Laws in Continuum Physics, Fourth, ed. Springer Verlag.
- M. Garavello, K. Han, and B, Piccoli, Models for vehicular traffic on networks, AIMS, 2016.
- Massimiliano D. Rosini, Macroscopic Models for Vehicular Flows and Crowd Dynamics: Theory and Applications, Springer, 2013.

Convex integration: from isometric embeddings to Euler and Navier Stokes equations

Sara Daneri[1]

[1] *Gran Sasso Science Institute, L'Aquila, Italy*
Email: sara.daneri@gssi.it

Timetable: 12 hrs. THE COURSE IS TEMPORARILY SUSPENDED. THE TIMETABLE OF THE LECTURES WILL BE COMMUNICATED AS SOON AS POSSIBLE

Course requirements: Very basic notions of ODE and PDE theory and of differential geometry (Riemannian manifold,length of curves).

Examination and grading: Seminar

SSD: MAT/05 - Mathematical Analysis

Aim and Course contents:
Convex integration, first introduced by Nash to prove nonuniqueness of C^1 isometric embeddings of Riemannian manifolds, turned out in the last ten years (starting from De Lellis and Székelyhidi) to be a very powerful tool to show nonuniqueness and flexibility of solutions (h-principle) in problems of fluid mechanics.
Aim of the course is to explain different applications of the technique of convex integration to some of these problems. In particular, we will focus on the solution of major problems like the Onsager's conjecture on the existence of dissipative Hölder solutions to the Euler equations and the recent proof of nonuniqueness for weak (non Leray) solutions to the 3D Navier Stokes equations.
The first application of convex integration, namely that to the nonuniqueness of C^1 isometric embeddings of Riemannian manifolds, will also be covered. The course should be particularly interesting for students in Mathematical Analysis, Differential Geometry and Mathematical Physics, in particular those interested in Fluid Mechanics.

Refereces:

- L. Székelyhidi "From isometric embeddings to turbulence" Lecture Notes avaliable online.
- S. Daneri, "Convex integration: from isometric embeddings to Euler and Navier stokes equations", Lecture notes which will be given during the course.

PDEs and Hörmander vector fields

Dott. Federica Dragoni[1]

[1] *Cardiff University*
Email: DragoniF@cardiff.ac.uk

Timetable: 8 hrs. COURSE CANCELLED

Course requirements: Sobolev spaces, PDEs.

Examination and grading: Oral exam on 1 topic chosen by the student

SSD: MAT/05

Aim: We will give an overview of PDEs associated to Hörmander vector fields. We will highlight some of the challenges and give some ideas to overcome them. We will also present some regularity results for PDEs in this setting (both order I and order II).

Course contents:

1. Vector fields and horizontal derivatives: the sub-Gradient and the sub-Laplacian.
2. Taylor's Theorem.
3. I-order PDEs: non coercive Hamiltonian and Hölder regularity.
4. II-order PDEs:
   (a) Weak solutions and intrinsic Sobolev spaces
   (b) Hypoelliptic operators, classical solutions and Hölder regularity.
5. Rescaling and horizontal derivatives: asymptotic expansion.

Intersection Theory in Algebraic Geometry

Tom Graber[1]

[1] *The Division of Physics, Mathematics and Astronomy, California Institute of Technology*
Email: graber@caltech.edu

Timetable: 16 hrs, First lecture on October 8, 2019, 09:00 (dates already fixed, see calendar), Torre Archimede, Room 2BC/30.

Course requirements: Students should have basic knowledge of algebraic geometry and commutative algebra. It will also be helpful (although not logically necessary) to be familiar with properties of topological homology and cohomology of manifolds.

Examination and grading:

SSD: MAT/03

Aim: The goal is to develop basic concepts and techniques of intersection theory and to see how they can be applied to problems in algebraic geometry.

Course contents: Some topics that we will definitely cover will be algebraic cycles, rational equivalence, Chern classes, and intersection products. Depending on student interests, additional topics could include the Grothendieck–Riemann–Roch Theorem, Chow rings of homogeneous spaces and relation to enumerative geometry, equivariant Chow groups, or others.

Refereces:
Good references for the course would be *Intersection Theory* by Fulton and/or *3264 and All That* by Harris and Eisenbud.

Morrey-Campanato Spaces and classical operators

Prof. Alexey Karapetyants[1]

[1] Institute for Mathematics, Mechanic and Computer Sciences, Southern Federal University, Russia
Email: karapetyants@gmail.com

Timetable: 12 hrs. First lecture on November 6, 2019, 09:00 (dates already fixed, see calendar), Torre Archimede, Room 2BC/30.

Course requirements: Basic knowledge of Functional analysis, Real analysis and Linear operator theory

Examination and grading:

SSD: MAT/05

Aim: Morrey spaces are widely used in applications to regularity properties of solutions to PDE's. We overview known and recently obtained results on Morrey-Campanato spaces with respect to the properties of the spaces themselves, and also, we overview the study of classical operators of harmonic analysis in these spaces. We also proceed with some generalizations and modifications.

Course contents:
Classical Morrey spaces: Definitions and basic facts, Interpolation results, Embeddings in Morrey spaces, Holder' s inequality, Interpolation. Vanishing Morrey spaces, Local/Integral Morrey spaces. Approximations in Morrey spaces. (4-6 hours).
Campanato spaces. Definitions and basic facts, Interpolation results, Different characterizations of Campanato spaces. (2 hours).
Generalizations of Morrey-Campanato spaces. Variable exponents, Orlich-Morrey-Campanato, Miscellaneous, Different underlying spaces. (2 hours).
Operators in Morrey-Campanato spaces: Maximal operator, Calderon-Zygmund operator, Riesz potential. Hardy operator (boundedness conditions and some important examples). (2 hours).

Bibliography: (the full list of sources will be given to the students in a separate file before the beginning of the course with recommendations).

[1] C. B. Morrey. On the solutions of quasi linear elliptic partial differential equations, Trans. Amer. Math. Soc. 43 (1938), 126–166.

[2] Campanato, Sergio. Proprietà di hölderianità di alcune classi di funzioni, Ann. Scuola Norm. Sup. Pisa (3), 17: 175–188 (1963).

[3] Giaquinta, Mariano. Multiple integrals in the calculus of variations and nonlinear elliptic systems, Annals of Mathematics Studies, 105, Princeton University Press, (1983).

[4] Wen Yuan, Winfried Sickel and Dachun Yang. Morrey and Campanato Meet Besov, Lizorkin and Triebel. Lecture Notes in Mathematics. Springer Heidelberg Dordrecht London New York. 2005.

[5] Luboš Pick, Alois Kufner, Oldřich John and Svatopluk Fucík. Function Spaces. De Gruyter Series in Nonlinear Analysis and Applications 14/1. 2013

[6] Colin Bennett and Robert Sharpley. Interpolation of Operators, Volume 129. Academic Press. 1988.

[7] V.Burenkov. Recent progress in studying the boundedness of classical operators of real analysis in general morrey-type spaces. -I,II. Eurasian Mathematical Journal. 2012.

[8] Humberto Rafeiro, Natasha Samko and Stefan Samko. Morrey-Campanato Spaces: an Overview. Operator Theory: Advances and Applications, Vol. 228, 293–323, 2013.

Introduction of Shimura Varieties

Prof. Elena Mantovan[1]

[1] Department of Mathematics, California Institute of Technology
Email: mantovan@caltech.edu

Timetable: 16 hrs, First lecture on October 25, 2019, 10:30 (dates already fixed, see calendar), Torre Archimede, Room 2BC/30.

Course requirements: basic topology and basic geometry.

Examination and grading:

SSD: MAT/03

Aim:

Course contents:
We will give an introduction to the theory of Shimura varieties, and to the arithmetic theory of automorphic forms, focusing on examples. Among the aims of this course, it is to explain the role of the arithmetic theory of Shimura varieties in the classical Langlands program, that is in the study of the connections between automorphic forms and Galois representations.
Topics will include:

1. A first example: elliptic curves, modular curves, modular forms, 2-dimensional Galois representations. Eichler–Shimura theorem.
2. Complex theory: Double coset spaces, Hermitian symmetric domains, bounded realizations, holomorphic automorphic forms.
3. Arithmetic theory: abelian varieties, complex multiplication, and moduli. Siegel varieties, PEL-type Shimura varieties. Canonical models, algebraic automorphic forms. Shimura–Taniyama theorem.
4. Simple examples: Unitary Shimura varieties of signature (n,1). Lubin–Tate spaces. Harris–Taylor models.
5. Good reduction: Abelian varieties over finite fields, Barsotti–Tate groups, Serre–Tate theory. Geometry of the special fibers of Shimura varieties: Newton polygons strata, Ekedahl–Oort strata, Oort leaves.

Refereces:

1. Deligne, Travaux de Shimura, Sèminaire Bourbaki, 389, 1971.
2. Genestier and B.C. Ngó, Lectures on Shimura varieties, 2006.
3. J. Milne, Introduction to Shimura varieties, Clay Mathematics Proceedings, Volume 4, 2005.
4. M. Rapoport, A guide to the reduction modulo p of Shimura varieties, Astérisque 298, 2005.

Introduction to Floer Homology

Marco Mazzucchelli[1]

[1] CNRS, École Normale Supérieure de Lyon, UMPA, France
Email: marco.mazzucchelli@ens-lyon.fr

Timetable: 12 hrs. THE COURSE IS TEMPORARILY SUSPENDED. THE TIMETABLE OF THE LECTURES WILL BE COMMUNICATED AS SOON AS POSSIBLE

Course requirements: The students attending this course are required to know the basics of functional analysis (Banach and Hilbert spaces), differential geometry and topology (manifolds, vector fields, differential forms, vector bundles, Riemannian metrics, critical points of a smooth map), and some symplectic geometry (symplectic forms, Hamiltonian vector fields).

Examination and grading:

SSD: MAT/

Aim:

Course contents (tentative):

- **Lecture 1:** Crash course in algebraic topology: singular homology and co-homology, De Rham cohomology.
- **Lecture 2:** The Morse homology theorem.
- **Lecture 3:** Variational principle for Hamiltonian periodic orbits, action spectrum, the Conley-Zehnder index.
- **Lecture 4:** Construction of the Floer homology groups for aspherical manifolds I.
- **Lecture 5:** Construction of the Floer homology groups for aspherical manifolds II, proof of the Arnold conjecture on the fixed points of generic Hamiltonian diffeomorphisms.
- **Lecture 6:** Bott's iteration formula for the Conley-Zehnder index, proof of the Conley conjecture on the periodic points of generic Hamiltonian diffeomorphisms of aspherical manifolds. Bonus arguments: Floer homology for monotone manifolds, products in Floer homology, spectral invariants, symplectic homology, etc.

Refereces:

- M. Audin, M. Damian, Morse theory and Floer homology, Universitext. Springer, London; EDP Sciences, Les Ulis, 2014. xiv+596 pp. ISBN: 978-1-4471-5495-2; 978-1-4471-5496-9; 978-2-7598-0704-8.
- A. Banyaga, D. Hurtubise, Lectures on Morse Homology, Lectures on Morse homology. Kluwer Texts in the Mathematical Sciences, 29. Kluwer Academic Publishers Group, Dordrecht, 2004. x+324 pp. ISBN: 1-4020-2695-1.

- D. Salamon, Lectures on Floer homology, Symplectic geometry and topology (Park City, UT, 1997), 143-229, IAS/Park City Math. Ser., 7, Amer. Math. Soc., Providence, RI, 1999.
- D. Salamon, E. Zehnder, Morse theory for periodic solutions of Hamiltonian systems and the Maslov index, Comm. Pure Appl. Math., 45 (1992), 1303-1360
- M. Schwarz, Morse homology, Progress in Mathematics, 111. Birkhauser Verlag, Basel, 1993. x+235 pp. ISBN: 3-7643-2904-1

Positivity Principles and decay of Solutions in Semilinear Elliptic Problems

Vitaly Moroz[1]

[1]Swansea University, UK
Email: v.moroz@swansea.ac.uk

Timetable: 8 hrs, First lecture on September 14, 2020*. A tentative timetable will follow.

Course requirements: Students should have a basic knowledge in partial differential equations.

Examination and grading:

SSD: MAT/05

Aim: The purpose of the course is to introduce two core but not widely known ideas of the linear elliptic theory, namely Allegretto–Piepenbrink positivity principle and Phragmén-Lindelöf comparison principle, and to show how these two fundamental principles provide a powerful tool in the analysis of the structure of positive solutions for large classes of semilinear elliptic equations. The course will consist of the core part, delivered in 5 lectures during the Mini-courses in Mathematical Analysis 2020, and additional 3 lectures containing advanced material.

Course contents:

Mini-course material – 5 lectures
Lecture 1: Allegretto–Piepenbrink positivity principle for linear Schrödinger operators and some corollaries: optimal and improved Hardy inequalities, Barta type inequality, torsion function estimate.
Lecture 2: Phragmén-Lindelöf comparison principles for linear Schrödinger operators, large and small positive solutions, admissible decay for sub–and super–solutions; concept of a weak and strong perturbation potentials.
Lecture 3: Nonlinear Liouville theorems for semilinear elliptic equations in unbounded domains, Serrin's critical exponent(s), fast and slow decay solutions.
Lecture 4: classification of singularities of semilinear elliptic equations, Keller–Osserman bound, removable singularities.
Lecture 5: boundary blow-up solutions of semilinear elliptic equations, global Keller–Osserman bound, classification of boundary blow-up solutions.

Advanced material – 3 lectures
Lecture 1: Nonlinear elliptic equations with nonlocal interactions: physical background, attractive and repulsive interactions. Choquard type equations.
Lecture 2: Riesz potentials and their basic properties. Decay estimates and localization principle for the Riesz potentials.

*Please, note that the first part of the course will be included in the Workshop "Minicorsi di Analisi Matematica"

Lecture 3: Nonlocal Allegretto–Piepenbrink positivity principle. Liouville theorems and admissible asymptotic decay of positive solutions to Choquard equations.

Refereces:

1.] S. Agmon, On positivity and decay of solutions of second order elliptic equations on Riemannian manifolds, Methods of functional analysis and theory of elliptic equations (Naples, 1982), Liguori, Naples, 1983, pp. 19–52.
2. C. Bandle, V. Moroz, and W. Reichel, 'Boundary blowup' type sub-solutions to semilinear elliptic equations with Hardy potential, J. Lond. Math. Soc. (2) 77 (2008), no. 2, 503–523.
3. V. Kondratiev, V. Liskevich, and V. Moroz, Positive solutions to superlinear second-order divergence type elliptic equations in cone-like domains, Ann. Inst. H. Poincaré Anal. Non Linéaire 22 (2005), no. 1, 25–43.
4. V. Liskevich, S. Lyakhova, and V. Moroz, Positive solutions to nonlinear p-Laplace equations with Hardy potential in exterior domains, J. Differential Equations 232 (2007), no. 1, 212–252.
5. XX, Positive solutions to singular semilinear elliptic equations with critical potential on cone-like domains, Adv.Differential Equations 11 (2006), no. 4, 361–398.
6. M. Marcus, V. J. Mizel, and Y. Pinchover, On the best constant for Hardy's inequality in R^n, Trans. Amer. Math. Soc. 350 (1998), no. 8, 3237–3255.
7. M. Marcus and V. Moroz, Moderate solutions of semilinear elliptic equations with Hardy potential under minimal restrictions on the potential, Ann. Sc. Norm. Super. Pisa Cl. Sci. (5) 18 (2018), no. 1, 39–64.
8. V. Moroz and J. Van Schaftingen, Nonexistence and optimal decay of supersolutions to Choquard equations in exterior domains, J. Diffrential Equations 254 (2013), no. 8, 3089–3145.

Topics in the representation theory of infinite-dimensional Lie algebras

Prof. Ivan Penkov [1]

[1] *Jacobs University Bremen, Germania*
Email: i.penkov@jacobs-university.de

INdAM Visiting Professor

Timetable: 16 hrs. First lecture October 24, 2019, 14:00 (dates already fixed, see calendar), Torre Archimede, Room 2BC/30.

Course requirements: solid knowledge of linear algebra and knowledge of Lie theory at beginner's level.

Examination and grading: oral examination on the topics covered during the course

SSD: MAT/02

Aim: The course aims at providing an introduction to the theory of locally finite Lie algebras and their representations. After a brief introduction into Lie algebra theory, the course will concentrate on the structure and representation theories of the three simple finitary infinite-dimensional Lie algebras $\mathfrak{sl}_\infty$, $\mathfrak{o}_\infty$, $\mathfrak{sp}_\infty$. If time permits, an application to the boson-fermion correspondence will be presented.

Course contents:

Synopsis: Finite-dimensional simple Lie algebras and their representations (3 hours), the Lie algebras $\mathfrak{sl}_\infty$, $\mathfrak{o}_\infty$, $\mathfrak{sp}_\infty$ - introduction (1 hour), Cartan, Borel and parabolic subalgebras (3 hours), weight representations (2 hours), simple modules with a highest weight and without highest weight (2 hours), bounded weight modules (1 hour), tensor modules and boson-fermion correspondence (4 hours).

References:

1. I. Dimitrov, I. Penkov, Weight modules of direct limit Lie algebras, IMRN 1999, no. 5, 223-249.
2. E. Dan-Cohen, I. Penkov and N. Snyder, Cartan subalgebras of root-reductive Lie algebras, Journ. of Algebra, 308(2007), 583-611.
3. I. Dimitrov, I. Penkov, Locally semisimple and maximal subalgebras of the finitary Lie algebras $\mathfrak{gl}_\infty$, $\mathfrak{sl}_\infty$, $\mathfrak{o}_\infty$ and $\mathfrak{sp}_\infty$, Journal of Algebra 322 (2009), 2069-2081.
4. E. Dan-Cohen, I. Penkov, Parabolic and Levi subalgebras of finitary Lie algebras, IMRN, Article ID rnp169, doi:10.1093/imrn/rnp169.

5. E. Dan-Cohen, I. Penkov and V. Serganova, A Koszul category of representations of finitary Lie algebras, Advances of Mathematics 289 (2016), 250-278
6. I. Frenkel, I. Penkov and V. Serganova, A categorification of the Boson-Fermion correspondence via representation theory of sl(1), Comm. Math. Phys., 341:3 (2016), 911-931

Contramodules and their applications to tilting theory and Enochs' conjecture

Prof. Leonid Positselki[1]

[1] *Russian Academy of Sciences*
Email: posic@mccme.ru

Timetable: 16 hrs. First lecture on November 21, 2019, 11:00 (dates already fixed, see calendar), Torre Archimede, Room 2BC/30.

Course requirements: knowledge of basic concepts of category theory and homological algebra, such as abelian categories and their derived categories, will be largely presumed.

Examination and grading:

SSD:

Aim:

Course contents:
Contramodules are module-like algebraic structures with infinite summation operations subject to natural axioms. For any infinitely generated n-tilting (or infinity-tilting) module, the heart of the related tilting t-structure is the category of contramodules over the topological ring of endomorphisms of the tilting module. The course will start with a discussion of comodules and contramodules over coalgebras and proceed to the tilting-cotilting correspondence, contramodules over topological rings, topologically semisimple and topologically perfect topological rings, and a discussion of the contramodule-based approach to the Enochs conjecture about covers and direct limits in module categories.

Introduction to Optimal Control Theory

Monica Motta[1], Franco Rampazzo[2]

[1] Dipartimento di Matematica "Tullio Levi-Civita", Universitá di Padova
Email: motta@math.unipd.it

[2] Dipartimento di Matematica "Tullio Levi-Civita", Universitá di Padova
Email: rampazzo@math.unipd.it

Timetable: 16 hrs. First lecture on Febuary 11, 2020, 09:00 (dates already fixed, see calendar), Torre Archimede, Room 2BC/30.

Course requirements: Calculus for many variables and basic tools of Lebesgue measure theory. The needed functional analysis will be recalled during classes, so it not a prerequisite.*

Examination and grading: The final exams will consists either of a stan- dard oral questioning on the main parts of the program or of a shortened recognition of the program togetrher with the dissertation on a research pa- per previously studied by the student.

SSD: MAT/05

Aim:
This course aims to provide the student with some basic tools of Optimal Control Theory. The latter generalizes Calculus of Variations to the case when the state trajectories are subject to differential equations with control parameters. Besides being a crucial for many mathematical subjects (e.g. Differential Geometry, Hamilton-Jacobi Pde's, Mean Field Games, Differential games) Optimal Control Theory is quite motivated by applications like Aerospace Engeneering, Medicine, Economics, Ecology.

Course contents:
Nonlinear Ordinary Differential Equations on Euclidean spaces and on manifolds. Variational and adjiont equations. Control Ordinary Differential Equations. Optimal control problems and existence of minima. Necessary conditions (Pontryagin Maximum Principle) for local minima: the intrinsic set-separation and its expression in terms differential equations. It time permits: characteristics of first order PDEs.

*Students who aim to a more detailed preparation to the course may follow the lectures by Prof. Rampazzo within the math undergraduate course "Analisi Superiore". Contact him if interested.

An introduction to Supergravity in 11-dimensions

Dr. Andrea Santi[1]

[1] Email: asanti.math@gmail.com

Timetable: 16hrs. First lectur on May 21, 2020, 14:00 (dates already fixed, see calendar), Torre Archimede, Room 2BC/30.

Course requirements: linear algebra, group theory and very basic knowledge in differential geometry. I do not assume any knowledge of physics: all relevant definitions will be given along.

Examination and grading: written homework.

SSD: MAT/02-03, MAT/07

Aim: The purpose of the course is to give a gentle introduction to the geometric and Lie algebraic aspects of 11-dimensional supergravity and to motivate the study of its supersymmetric backgrounds.

Course contents: We will start with the fundamental objects of Riemannian and Lorentzian geometry, leading to the classical Einstein's and Maxwell's equations. We will discuss the basics of G-structures and Riemannian geometry, with a particular attention on the connection defining the so-called Killing transport and then consider special PDEs of geometric origin on spinor fields on Riemannian manifolds. In the second part of the course, we introduce 11-dimensional supergravity, the most important theory where Einstein's General Relativity is combined with supersymmetry. We will discuss the construction of a Lie superalgebra generated from spinor fields satisfying a certain PDE and the homogeneity theorem, which states that supergravity backgrounds preserving more than half of the supersymmetry are homogeneous. Considerable emphasis will be placed on clarifying the relevant definitions, along with examples.

Course outline:

1. Differential calculus on manifolds
   (Tensor fields, Lie derivative, linear connections, curvature, Riemannian and Lorentzian metrics,Levi-Civita connection, Ricci and scalar curvatures, Einstein's equations, Maxwell's equations)
2. G-structures
   (Basic definitions and examples, symmetries of the integrable model, the automorphism group of a G-structure and Sternberg's Theorem, Poincaré algebra)
3. Riemannian geometry
   (Killing vector fields and Killing transport, space forms, Killing algebras as filtered deformations)
4. Spin geometry
   (Clifford algebras, basics of spin geometry, spinor fields satisfying special PDEs, a toy

model:extended Killing algebras)

5. Supersymmetry and Detour on Lie superalgebra theory
(physical motivation of supersymmetry, Kac's classification of simple Lie superalgebras, supertranslation algebra and Poincaré superalgebra)

6. Supergravity in 11-dimensions
(Basic definitions, supergravity Killing spinors, examples: maximally supersymmetric backgrounds and brane solutions, homogeneity theorem, Killing superalgebras, Killing supertransport, highly supersymmetric backgrounds).

Courses offered within the Master's Degree in Mathematics

The Master Degree (Laurea Magistrale) in Mathematics of this Department offers many courses on a wide range of topics, in Italian or in English. The PhD students are encouraged to follow the parts of such courses they think are useful to complete their basic knowledge in Mathematics. In some cases this activity can receive credits from the Doctoral school, upon recommendation of the supervisor of the student. Since the courses at the Master level are usually less intense than those devoted to graduate students, the number of hours given as credits by our Doctorate will be less than the total duration of the course. Some examples of courses that receive such credits, unless the student already has the material in his background, are the following.

Topology 2

Prof. Andrea D'Agnolo
Università di Padova, Dipartimento di Matematica
Email: dagnolo@math.unipd.it

Period: 1st semester

Contents and other information:

https://didattica.unipd.it/off/2019/LM/SC/SC1172/001PD/SC03111819/N0

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Calculus of Variations

Prof. Roberto Monti
Università di Padova, Dipartimento di Matematica
Email: monti@math.unipd.it

Period: 2nd semester

Contents and other information:

https://didattica.unipd.it/off/2019/LM/SC/SC1172/010PD/SCP3050978/N0

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Homology and Cohomology

Prof. Bruno Chiarellotto
Università di Padova, Dipartimento di Matematica
Email: chiarbru@math.unipd.it

Period: 2nd semester

Contents and other information:

https://didattica.unipd.it/off/2019/LM/SC/SC1172/001PD/SC02111817/N0

Soft Skills

Doctoral Program in Mathematical Sciences

a.a. 2019/2020

SOFT SKILLS

Maths information: retrieving, managing, evaluating, publishing

Abstract: This course deals with the bibliographic databases and the resources provided by the University of Padova; citation databases and metrics for research evaluation; open access publishing and the submission of PhD theses and research data in UniPd institutional repositories.

Language: The Course will be held in Italian or in English according to the participants

Timetable: 4 hrs – March 2, 2020, 09:00 (2 hrs), March 9, 2020, 09:00 (2 hrs), Room 2BC/30

SOFT SKILLS


DIGITAL meet
22-27 ottobre 2019
SCOPRI | USA | CREA | SOGNA
800 ANNI 1222-2022
UNIVERSITÀ DEGLI STUDI DI PADOVA
HUMAN INSPIRED TECHNOLOGIES
RESEARCH CENTRE
OCT. 22
BOTANIC GARDEN | AUDITORIUM - SALA DELLE COLONNE
FROM 18:30
BRAINS MEET DIGITAL ENTERPRISES
PHD
BRAIN, MIND AND COMPUTER SCIENCE,
INGEGNERIA AEROSPAZIALE
INGEGNERIA DELL'INFORMAZIONE
MATEMATICA
PSYCHOLOGICAL SCIENCE
SCIENZE DELL'INGEGNERIA CIVILE, AMBIENTALE E DELL'ARCHITETTURA
STORIA, CRITICA E CONSERVAZIONE DEI BENI CULTURALI
HIT|DM YOUNG RESEARCHER AWARD 2019 (II EDITION)
RIVER COLLEGE VIRTUAL REALITY
VR|AR THE VIRTUAL EXPERIENCE RECEPTION
BFT Be ahead
IRS Ingegneria Ricerca Sistemi
ADVANCED ERGONOMIC SOLUTIONS
VITRUVIAN VR
RIFF
GOLDEN SPONSOR
DIPARTIMENTO DI PSICOLOGIA GENERALE DPG
DPSS Dipartimento di Psicologia dello Sviluppo e della Socializzazione
DIPARTIMENTO MATEMATICA
DIPARTIMENTO DI INGEGNERIA DELL'INFORMAZIONE
dBC
ICEA
CISAS G. COLOMBO
ICT4SSL

Entrepreneurship and Technology-based Startups

Course Area: Transversal Skills

Credits: 5

Instructors: Prof. Moreno Muffatto, Ing. Francesco Ferrati, Dipartimento di Ingegneria Industriale, Università di Padova
e-mail: moreno.muffatto@unipd.it, francesco.ferrati@unipd.it

From the idea to the market

- From a research project to an entrepreneurial project
- Recognize and evaluate an entrepreneurial opportunity
- Market dimension, customers profiles and value proposition
- Development of the product/service concept
- Go-to-Market strategies

Intellectual Property Rights

- Types of IPR (patent, copyright, trademark)
- The structure of a patent application (description, claims, etc)
- Getting a patent: the patenting process (step by step)
- When to file a patent application: priority date, Patent Cooperation Treaty (PCT)
- Where to protect an invention
- Different IPR strategies

The team and the early decisions

- The creation of the founders' team
- Types and characteristics of founders' teams
- Founders' decisions and their consequences
- Frequent mistakes and suggestions deriving from experience

The economic and financial aspects of a startup

- The fundamental economic and financial operations of a technology-based startup
- The structures of the financial statements
- Income Statement, Balance Sheet, Cash Flow
- Evaluation of the value of the company
- Sources and cost of capital

Funding a startup

- Different sources of funds: Angel Investors and Venture Capital
- Investment companies and funds: how they work
- How and what investors evaluate
- The investment agreements between investors and startups
- New ventures' funding options

References:

- Noam Wasserman (2013) The Founder's Dilemmas: Anticipating and Avoiding the Pitfalls That Can Sink a Startup, Princeton University Press.
- Thomas R. Ittelson (2009), Financial Statements: A Step-by-Step Guide to Understanding and Creating Financial Reports, Career Press.
- Hall, J., & Hofer, C. W. (1993). Venture capitalists' decision criteria in new venture evaluation. Journal of Business Venturing, 8(1), 25-42.

Schedule and room: please, see Class Schedule

Enrollment:
To attend the course registration is compulsory by using the Moodle platform of the PhD Course (in order to enter the Moodle platform click on "dettagli" of the course at the page http://www.cdii.dii.unipd.it/corsi). Once you are registered, if you cannot attend the course, please inform the lecturer.

Examination and grading: Attendance is required for at least 70% of the lecture hours (i.e. 14 hours). Final evaluation will be based on the discussion of a case study of a technology-based startup.

Other courses suggested to the students

The students are encouraged to follow also courses outside Padova if they are useful for their training to research, in accordance with their supervisor. Parts of such course can be counted in fulfilment of their duties, provided the student passes an exam. The number of hours recognised as credits will be decided by the Coordinator after hearing the supervisor. Some examples of courses that receive such credits are the following.

Courses in collaboration with the Doctoral School on "Information Engineering"

for complete Catalogue and class schedule see on

https://phd.dei.unipd.it/course-catalogues/

Introduction to Reinforcement Learning

Dr. Juan José Alcaraz Espín[1]

[1] *Technical University of Cartagena, Spain*
email: juan.alcaraz@upct.es

Timetable: see on https://phd.dei.unipd.it/course-catalogues/

Course requirements: Basics of linear algebra, probability theory, Python scripting

Examination and grading: The grading will be based on the students' solutions to the proposed assignments.

SSD: Information Engineering

Aim: The course will provide an introduction to the field of reinforcement learning, covering its mathematical foundations and the description of the most relevant algorithms. The main concepts and techniques will be illustrated with Python code and application examples in telecommunications and other related areas. The students will acquire hands-on experience with the proposed assignments in which they will have to implement Python code for solving several challenges and exercises. The course will start with the basic concepts of learning in sequential decision problems, formalized in the multi-armed bandit (MAB) problem and its variants. Then, the Markov decision processes (MDPs), which generalize the MAB problem, will be introduced. The objective of reinforcement learning (RL) is to find approximate solutions to MDPs. The main RL approaches will be presented incrementally: 1) tabular methods, which are capable of addressing relatively small problems, 2) value function approximation, which allows scaling up previous algorithms to larger problems, and 3) policy gradient algorithms which follow a different scaling approach and can be used in combination with value function approximation (Actor-Critic methods).

Course contents:
Unit 1. Introduction to Reinforcement Learning
Unit 2. Multi-Armed Bandits: Stochastic Bandits, Boltzmann Exploration, UCB algorithms, Thompsom Sampling, Contextual Bandits.
Unit 3. Markov Decision Processes: Stochastic Shortest Path problems. Policy Iteration. Value Iteration. MDPs with discount.
Unit 4. Tabular Methods: Monte Carlo Method, Temporal Difference, Off-policy algorithms, Planning at decision time.
Unit 5. Value Function Approximation (VFA) Methods: Linear VFA, Monte Carlo with VFA, TD methods with VFA.
Unit 6. Policy Gradient Algorithms: Score functions, Policy Gradient Theorem, Monte Carlo Policy Gradient, Actor-Critic Policy Gradient.
Unit 7 (Optional) Evolutionary Algorithms

References:

1. Reinforcement Learning: An Introduction, Second Edition, Richard S. Sutton and Andrew G. Barto, MIT Press, Cambridge, MA, 2018.
2. Approximate Dynamic Programming: Solving the Curses of Dimensionality, Second Edition, Warren B. Powell, Wiley, 2011.
3. Dynamic Programming and Optimal Control Vol I and Vol II, 4th Edition, Dimitri P. Bertsekas, Athena Scientific, 2012.
4. Algorithms for Reinforcement Learning, Csaba Szepesvári, Morgan and Claypool, 2010.
5. Reinforcement Learning and Optimal Control, Dimitri P. Bertsekas, Athenea Scientific, 2019.
6. Markov Decision Processes: Discrete Stochastic Dynamic Programming, Martin L. Puterman, Wiley, 2006.

Bayesian Machine Learning

Giorgio Maria Di Nunzio[1]

[1] *Department of Information Engineering*
Email: dinunzio@dei.unipd.it

Timetable: see on https://phd.dei.unipd.it/course-catalogues/

Course requirements: Basics of Probability Theory. Basics of R Programming.

Examination and grading: Homework assignments and final project.

SSD: Information Engineering

Aim: The course will introduce fundamental topics in Bayesian reasoning and how they apply to machine learning problems. In this course, we will present pros and cons of Bayesian approaches and we will develop a graphical tool to analyse the assumptions of these approaches in classical machine learning problems such as classification and regression.

Course contents:

Introduction of classical machine learning problems.

1. Mathematical framework
2. Supervised and unsupervised learning

Bayesian decision theory

1. Two-category classification
2. Minimum-error-rate classification
3. Bayes decision theory
4. Decision surfaces

Estimation

1. Maximum Likelihood Estimation
2. Expectation Maximization
3. Maximum A Posteriori
4. Bayesian approach

Graphical models

1. Bayesian networks
2. Two-dimensional visualization

Evaluation

1. Measures of accuracy

References:

1. J. Kruschke, Doing Bayesian Data Analysis: A Tutorial Introduction With R and Bugs, Academic Press 2010
2. Christopher M. Bishop, Pattern Recognition and Machine Learning (Information Science and Statistics), Springer 2007
3. Richard O. Duda, Peter E. Hart, David G. Stork, Pattern Classification (2nd Edition), Wiley-Interscience, 2000
4. Yaser S. Abu-Mostafa, Malik Magdon-Ismail, Hsuan-Tien Lin, Learning from Data, AML-Book, 2012 (supporting material available at http://amlbook.com/support.html)
5. David J. C. MacKay, Information Theory, Inference and Learning Algorithms, Cambridge University Press, 2003 (freely available and supporting material at http://www.inference.phy.cam.ac.uk/m
6. David Barber, Bayesian Reasoning and Machine Learning, Cambridge University Press, 2012 (freely available at http://web4.cs.ucl.ac.uk/staff/D.Barber/pmwiki/pmwiki.php?n=
7. Kevin P. Murphy, Machine Learning: A Probabilistic Perspective, MIT Press, 2012 (supporting material http://www.cs.ubc.ca/ murphyk/MLbook/)

Introduction to Information Theory

Deniz Gunduz[1]

[1] *Department of Information Engineering*
Email: d.gunduz@imperial.ac.uk

Timetable: see on https://phd.dei.unipd.it/course-catalogues/

Course requirements: Basics of Probability Theory. Basics of R Programming.

Examination and grading: Homework assignments and final project.

SSD: Information Engineering

Important note: the first half of the course will be held by Prof. Gunduz in two, four-hour lectures at the University of Padova (see schedule below). The remaining lectures will be offered by Prof. Gunduz remotely using the videoconferencing room. Date and time of these remaining lectures will be agreed between the instructor and attendees during the first two lectures.

Aim: The aim of this course is to introduce basic information theoretic concepts to students. We will start by introducing entropy, divergence, and, mutual information, and their mathematical properties. The rest of the course will be dedicated to illustrating engineering applications of these seemingly abstract quantities. We will see that entropy corresponds to the ultimate limit in data compression, divergence provides the best error exponent in hypothesis testing (i.e., binary classification), and mutual information sets the limit of how much data one can transmit reliably over a noisy communication channel.

Course contents:

Information measures

1. Entropy, divergence, mutual information
2. Properties of information measures (chain rule, data processing inequality, convexity)

em Lossless data compression

1. Asymptotic equipartition property (AEP)
2. Kraft inequality, Huffman coding and its optimalityTwo-category classification

Information theory and learning

1. Method of types, universal source coding, large deviations: Sanov's theorem
2. Hypothesis testing, Stein's lemma, Chernoff exponent h

Channel coding

1. Channel capacity theorem, achievability, joint AEP
2. Converse to channel coding theorem, feedback capacity, Joint source-channel coding

References:

1. R. B. Ash, Information Theory, Dover, 1990.
2. T. M. Cover and J. A. Thomas, Elements of Information Theory, Wiley, 1991.
3. R. G. Gallager, Information Theory and Reliable Communication, Wiley, 1968.

Statistical Methods

Dr. Lorenzo Finesso[1]

[1] email: lorenzo.finesso@unipd.it

Timetable: see on https://phd.dei.unipd.it/course-catalogues/

Course requirements: familiarity with basic linear algebra and probability.

Examination and grading: Homework assignments

SSD: Information Engineering

Aim: The course will present a small selection of statistical techniques which are widespread in applications. The unifying power of the information theoretic point of view will be stressed.

Course contents:

- *Background material.* The noiseless source coding theorem will be quickly reviewed in order to introduce the basic notions of entropy and I-divergence. (a.k.a. relative entropy, Kullback-Leibler distance) between two probability measures.
- *Divergence minimization problems.* Three I-divergence minimization problems will be posed and, via examples, they will be connected with basic methods of statistical inference: ML (maximum likelihood), ME (maximum entropy), and EM (expectation-maximization).
- *Multivariate analysis methods.* The three standard multivariate methods, PCA (principal component analysis), Factor Analysis, and CCA (canonical correlations analysis) will be reviewed and their connection with divergence minimization discussed. Applications of PCA to least squares (PCR principal component regression, PLS Partial least squares). Approximate matrix factorization and PCA, with a brief detour on the approximate Nonnegative Matrix Factorization (NMF) problem. The necessary linear algebra will be reviewed.
- *EM methods.* The Expectation-Maximization method will be introduced as an algorithm for the computation of the Maximum Likelihood (ML) estimator with partial observations (incomplete data) and interpreted as an alternating divergence minimization algorithm à la Csiszár Tusnády.
- *Applications to stochastic processes.* Introduction to HMM (Hidden Markov Models). Maximum likelihood estimation for HMM via the EM method. If time allows: derivation of the Burg spectral estimation method as solution of a Maximum Entropy problem.

References:
A set of lecture notes and a complete list of references will be posted on the web site of the course.

Computational Inverse Problems

Prof. Fabio Marcuzzi[1]

[1] *Dipartimento di Matematica Tullio Levi-Civita, Univ. Padova*
e-mail: marcuzzi@math.unipd.it

Timetable: https://phd.dei.unipd.it/course-catalogues/

Course requirements:
- basic notions of linear algebra and, possibly, numerical linear algebra.
- the examples and homework will be in Python (the transition from Matlab to Python is effortless).

Examination and grading: Homework assignments and final test.

SSD: INF/01

Aim: We study numerical methods that are of fundamental importance in computational inverse problems. Real application examples will be given for distributed parameter systems in continuum mechanics. Computer implementation performance issues will be considered as well.

Course contents:

- definition of inverse problems, basic examples and numerical difficulties.
- numerical methods for QR and SVD and their application to the square-root implementation in PCA, east-squares, model reduction and Kalman filtering; recursive least-squares;
- regularization methods;
- numerical algorithms for nonlinear parameter estimation: Gauss-Newton, Levenberg-Marquardt;
- examples with distributed parameter systems in continuum mechanics; HPC implementations

References:

1. F.Marcuzzi ”Analisi dei dati mediante modelli matematici”, http://www.math.unipd.it/ marcuzzi/MNAD.html

Applied Functional Analysis and Machine Learning

Prof. Gianluigi Pillonetto[1]

[1] *Department of Information Engineering, Univ. Padova*
e-mail: giapi@dei.unipd.it

Timetable: https://phd.dei.unipd.it/course-catalogues/

Course requirements: The classical theory of functions of real variable: limits and continuity, differentiation and Riemann integration, infinite series and uniform convergence. The arithmetic of complex numbers and the basic properties of the complex exponential function. Some elementary set theory. A bit of linear algebra.

Examination and grading: Homework assignments and final test.

SSD: Information Engineering

Aim: The course is intended to give a survey of the basic aspects of functional analysis, machine learning, regularization theory and inverse problems.

Course contents:

Review of some notions on metric spaces and Lebesgue integration: Metric spaces. Open sets, closed sets, neighborhoods. Convergence, Cauchy sequences, completeness. Completion of metric spaces. Review of the Lebesgue integration theory. Lebesgue spaces.

Banach and Hilbert spaces: Finite dimensional normed spaces and subspaces. Compactness and finite dimension. Bounded linear operators. Linear functionals. The finite dimensional case. Normed spaces of operators and the dual space. Weak topologies. Inner product spaces and Hilbert spaces. Orthogonal complements and direct sums. Orthonormal sets and sequences. Representation of functionals on Hilbert spaces.

Compact linear operators on normed spaces and their spectrum: Spectral properties of bounded linear operators. Compact linear operators on normed spaces. Spectral properties of compact linear operators. Spectral properties of bounded self-adjoint operators, positive operators, opera- tors defined by a kernel. Mercer Kernels and Mercer theorem.

Reproducing kernel Hilbert spaces, inverse problems and regularization theory: Representer theorem. Reproducing Kernel Hilbert Spaces (RKHS): definition and basic properties. Examples of RKHS. Function estimation problems in RKHS. Tikhonov regularization. Primal and dual formulation of loss functions. Regularization networks. Consistency/generalization and relationship with Vapnik's theory and the concept of V-gamma dimension. Support vector regression and classification.

References:

1. W. Rudin. Real and Complex Analysis, McGraw Hill, 2006
2. C.E. Rasmussen and C.K.I. Williams. Gaussian Processes for Machine Learning. The MIT Press, 2006
3. H. Brezis, Functional analysis, Sobolev spaces and partial differential equations, Springer 2010

Heuristics for Mathematical Optimization

Prof. Domenico Salvagnin[1]

[1] *Department of Information Engineering, Padova*
email: dominiqs@gmail.com

Timetable: https://phd.dei.unipd.it/course-catalogues/

Course requirements:
- Moderate programming skills (on a language of choice)
- Basics in linear/integer programming.

Examination and grading: Final programming project.

SSD: Information Engineering

Aim: Make the students familiar with the most common mathematical heuristic approaches to solve mathematical/combinatorial optimization problems. This includes general strategies like local search, genetic algorithms and heuristics based on mathematical models.

Course contents:
- Mathematical optimization problems (intro).
- Heuristics vs exact methods for optimization (intro).
- General principle of heuristic design (diversification, intensification, randomization).
- Local search-based approaches.
- Genetic/population based approaches.
- The subMIP paradigm.
- Applications to selected combinatorial optimization problems: TSP, QAP, facility location, scheduling.

References:

1. Gendreau, Potvin "Handbook of Metaheuristics", 2010
2. Marti, Pardalos, Resende "Handbook of Heuristics", 2018

Control of Multivariable Systems: A Geometric Approach

Prof. Andrea Serrani[1]

[1] *The Ohio State University, USA*
e-mail: serrani.1@osu.edu

Timetable: https://phd.dei.unipd.it/course-catalogues/

Course requirements: A basic course in linear system theory and proficiency in linear algebra are required. Working knowledge in MATLAB/SIMULINK is needed for the design examples.

Examination and grading: Homework assignments and/or take-home final examination.

SSD: Information Engineering

Aim: The goal of the course is to introduce the geometric theory of linear multivariable systems as a fundamental tool for the solution of relevant control problems, including disturbance rejection, non-interaction, fault detection and isolation, and tracking and regulation. Attention will be devoted to routines available in MATLAB for numerical implementation of the control algorithms presented in class. Design examples on a realistic model of an aerospace system will be introduced.

Course contents:

1. **Background:** Subspaces, maps, factor spaces, projections.
2. **Systems Theory:** Controllability, observability, compensator design.
3. **Disturbance Decoupling:** Controlled invariance, controllability subspaces. Duality: Conditioned invariance, unknown-input observers.
4. **Eigenvalue Assignment under Invariance Constraints:** Multivariable zeros. Zero dynamics.
5. **Non-interacting Control:** Synthesis via dynamic extension. Duality: Fault detection and isolation.
6. **Tracking and Regulation:** Right-inversion. The regulator problem.

References:

1. W.M. Wonham, "Linear Multivariable Control: A Geometric Approach," Springer-Verlag; Supplementary notes.

Elements of Deep Learning

Prof. Gian Antonio Susto[1]

[1] Department of Information Engineering, Univ. Padova
e-mail: gianantonio.susto@dei.unipd.it

Timetable: https://phd.dei.unipd.it/course-catalogues/

Course requirements: Basics of Machine Learning and Python Programming.

Examination and grading: Final project.

SSD: Information Engineering

Aim: The course will serve as an introduction to Deep Learning (DL) for students who already have a basic knowledge of Machine Learning. The course will move from the fundamental architectures (e.g. CNN and RNN) to hot topics in Deep Learning research.

Course contents:

- Introduction to Deep Learning: context, historical perspective, differences with respect to classic Machine Learning.
- Feedforward Neural Networks (stochastic gradient descent and optimization).
- Convolutional Neural Networks.
- Neural Networks for Sequence Learning.
- Elements of Deep Natural Language Processing.
- Elements of Deep Reinforcement Learning.
- Unsupervised Learning: Generative Adversarial Neural Networks and Autoencoders.
- Laboratory sessions in Colab.
- Hot topics in current research.

References:

1. Arjovsky, M., Chintala, S., Bottou, L. (2017). Wasserstein GAN. CoRR, abs/1701.07875.
2. Bahdanau, D., Cho, K., Bengio, Y. (2014). Neural Machine Translation by Jointly Learning to Align and Translate. CoRR, abs/1409.0473.
3. I. Goodfellow, Y. Bengio, A. Courville 'Deep Learning', MIT Press, 2016
4. Goodfellow, I.J., Pouget-Abadie, J., Mirza, M., Xu, B., Warde-Farley, D., Ozair, S., Courville, A.C., Bengio, Y. (2014). Generative Adversarial Nets. NIPS.
5. Hochreiter, S., Schmidhuber, J. (1997). Long Short-Term Memory. Neural computation, 9 8, 1735-80.
6. Kalchbrenner, N., Grefenstette, E., Blunsom, P. (2014). A Convolutional Neural Network for Modelling Sentences. ACL.

7. Krizhevsky, A., Sutskever, I., Hinton, G.E. (2012). ImageNet Classification with Deep Convolutional Neural Networks. Commun. ACM, 60, 84-90.
8. LeCun, Y. (1998). Gradient-based Learning Applied to Document Recognition.