Lie Groups and Symmetry

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Timetable: 24 hrs. First lecture on Oct 13, 2020, 11:00 (dates already fixed, see calendar), Torre Archimede, Room 1BC/45.

Course requirements: (very) basic knowledge in differential geometry. The course is addressed to all students.

Examination and grading: oral examination on the topics covered during the course

SSD: MAT/

Aim: The course aims at providing an introduction to the theory of Lie groups and their actions, which is a topic of broad interest but almost completely absent from the courses of our Laurea Magistrale. After covering the fundamentals of the subject, the course will provide some examples of use of Lie groups in the study of ODE with symmetry.

Course contents:

Synopsis: Lie groups and their differential—and group—structure (left and right trivializations, Lie algebra of a Lie group, exponential map, maximal tori, (co)adjoint action, structure of compact Lie groups). The classical matrix groups and their properties. Differentiable actions of Lie groups on manifolds, quotient spaces (for proper actions), invariant vector fields. Reduction of invariant vector fields. Applications to ODEs with symmetry (reduction and reconstruction; integrability).

References

- 1. A. Baker, Matrix groups. An introduction to Lie group theory. (Springer, 2002)
- 2. J. Lee, Introduction to Smooth manifolds. 2nd edition. (Springer, 2013)
- 3. T. Bröcker and T. tom Dieck, Representations of compact Lie groups. (Springer 1985)
- 4. R. Cushman, J.J. Duistermaat and J. Śnyaticki, Geometry of Nonholonomically Constrained Systems. (World Scientific, 2010).