Stochastic optimal transport and applications in mathematical finance

Prof. Beatrice Acciaio, ETH Zurich¹

¹ Department of Mathematics, ETH Zurich Email: beatrice.acciaio@math.ethz.ch

Timetable: 12 hrs. First lecture on April 4th, 2023, 10:30 (dates already fixed, see Calendar of Activities at https://dottorato.math.unipd.it/calendar), Torre Archimede, Room 2BC30.

Course requirements: Probability and Stochastic Calculus (basic)

Examination and grading: oral examination on the topics covered during the course

SSD: MAT/06, SECS-S/06

Aim: This course aims at introducing the required basis on optimal transport, to then focus on recent developments of stochastic transport with applications to mathematical finance. In particular we will discuss: weak transport, martingale transport and model-independent finance, causal and adapted transport and model uncertainty.

Course content: We will motivate the introduction of different kinds of optimal transport in order to deal with several problems in mathematical finance. Specifically, to price and hedge in a model-independent setting, to gauge the distance between financial models, to account for model uncertainty. We will see how results from classical transport theory modify to account for a generalization of the cost or the introduction of constraints. We will appreciate how tools from optimal transport find wide applications in mathematical finance and stochastic analysis. Special attention will be devoted to the constraint of causality, that takes into account the flow of information arriving in time, and turns out to be the suitable one in order to account for model uncertainty in stochastic optimization.

The organization of the course will be as follows:

- Classical optimal transport: recall of main concepts and results (existence, duality, cyclical-monotonicity).
- Weak optimal transport: introduction of the problem, expositions of main results, application to robust pricing in fixed-income markets, analysis of special cases: entropic transport, barycentric transport.
- Martingale optimal transport: introduction of the problem, expositions of main results, model-independent pricing and hedging, Skorokhod Embedding problem.
- Causal and adapted optimal transport: introduction of the problem, expositions of main results, stability in mathematical finance, applications to: filtration enlargement, equilibrium problems, quantification of arbitrage.