

# Introduction to Harmonic Analysis on Semisimple Groups

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**Timetable:** 24 hrs. First lecture on Tuesday May 7th, 2024, 14:00 (dates already fixed, see on <https://dottorato.math.unipd.it/calendar>), Torre Archimede, Room 2BC30.

**Course requirements:** The prerequisites are reduced to the minimum:

- standard notions from first and second year courses in analysis
- elementary linear algebra

All other needed concepts will be illustrated in the course.

**Examination and grading:** Lectures will be complemented with exercise sheets which may be handed in for grading. Alternatively, the exam may consist in an oral examination where the student is supposed to deliver a lecture on a chosen argument.

**SSD:** MAT02/03/05

## **Aim:**

Classically, harmonic analysis deals with the expansion of functions of one or more real variables as series or integrals of simple harmonics. A natural setting of the theory is that of locally compact commutative topological groups. Applications range from number theory to differential equations. Noncommutative harmonic analysis on Lie groups is more recent and was initially forged for the needs of invariant theory and quantum mechanics. It studies possibly infinite-dimensional representations of a Lie group, the special functions on the group afforded by the matrix coefficients, and the expansion of functions on the group in terms of these. The course is meant as an introduction to the representation theory and harmonic analysis on semisimple Lie groups. The introduction will succinctly survey the commutative theory and some of its applications. Next, theory for compact groups will be dealt in greater detail, up to the Peter-Weyl theorem. Finally, the course will concentrate on infinite-dimensional representations of semisimple Lie groups, representations of its Lie algebra, and characters of these. The basic example will be the group  $SL(2, \mathbb{R})$ .

## **Course contents:**

1. Introduction
2. Compact groups
3. Unitary representations of locally compact groups
4. Parabolic induction, principal series and characters
5. Representations of the Lie algebra
6. Plancherel formula

7. Invariant eigendistributions
8. Harmonic analysis on the Schwartz space

**Bibliography:**

1. An introduction to harmonic analysis on semisimple Lie groups” V.S. Varadarajan.
2. “Representation theory of semisimple Lie groups” A.W. Knap.