

Nonlinear methods for linear equations: the low-regularity theory

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Timetable: 16 hrs First lecture on Tuesday March 5th, 2023, 10:30 (dates already fixed, see on <https://dottorato.math.unipd.it/calendar>), Torre Archimede, Room 2BC30..

Course requirements: Basic knowledge of classical functional spaces, without PDE requirements.

Examination and grading: The exam will be oral and tailored on the students' interests.

SSD: MAT/05

Aim: Introduce some classical and modern methods to study regularity properties of solutions to the Laplace equation, focusing on nonvariational techniques based mostly on the maximum principle.

Course contents:

- Introduction and motivations: the importance of the regularity theory for elliptic equations;
- Review of maximum principles and applications;
- Weak-Harnack inequalities via Aleksandrov-Bakel'man-Pucci techniques;
- Harnack inequalities and Hölder a priori estimates;
- The notion of viscosity solution;
- The Bernstein technique to obtain a priori gradient estimates;
- Hölder/Lipschitz regularity estimates via doubling variables: the Ishii-Lions method;
- Lipschitz regularity estimates via doubling variables: the weak Bernstein method.

Bibliography:

1. L. Caffarelli and X. Cabré, Fully nonlinear elliptic equations, American Mathematical Society, Providence, RI, 1995.
2. X. Fernández-Real and X. Ros-Oton, Regularity theory for elliptic PDE, Zurich Lectures Notes in Advanced Mathematics, European Mathematical Society, 2022.
3. L. C. Evans, Partial Differential Equations, American Mathematical Society, Providence, RI, 2010.

4. D. Gilbarg and N.S. Trudinger, Elliptic partial differential equations of second order, Springer-Verlag, Berlin, 2001.
5. Q. Han and F. Lin, Elliptic partial differential equations, second edition, Courant Lecture Notes in Mathematics, American Mathematical Society, Providence, RI, 2011.