Global Constraints
Exam

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The Sum constraint

We will denote by $X(C)$ the set of variables on which a constraint C is defined, and by d the largest size of the domain.

Interval consistency (defined by Pascal van Hentenryck and sometimes called bound consistency) is derived from a relaxation of arc consistency for continuous domains. It is based on an approximation of finite domains by finite sets of successive integers. More precisely, if D is a domain, Interval consistency works on D^*, the interval of integers $[\min(D), \max(D)]$ where $\min(D)$ and $\max(D)$ denote respectively the minimum and maximum values in D.

A constraint C is interval–consistent if the following properties hold:

1. For all x_i in $X(C)$, $\min(D(x_i)) \leq \max(D(x_i))$
2. For all x_i in $X(C)$, C is arc-consistent when $D^*(x_i)$ is restricted to $\{\min(D(x_i)\}$ and $D(x_j)$ is replaced by $D^*(x_j)$ for all $i \neq j$
3. For all x_i in $X(C)$, C is arc-consistent when $D^*(x_i)$ is restricted to $\{\max(D(x_i)\}$ and $D(x_j)$ is replaced by $D^*(x_j)$ for all $i \neq j$.

Questions:

1. Consider the constraint $x+y \geq z$. Give the conditions of a filtering algorithm (FA) associated with this constraint which establishes interval consistency. Give the time complexity of your FA. Explain for which modifications this FA should be called.
2. Consider the constraints $x + y \geq z$ and $x + y \leq z$. Consider the domains: $D(x) = \{1, 4\}, D(y) = \{1, 4\}$ and $D(z) = \{3, 6\}$. Show that these two constraints are interval consistent. Show that the conjunction of these two constraints is also interval consistent.
3. Consider the constraint $x+y = z$. Give the conditions of a filtering algorithm (FA) associated with this constraint which establishes interval consistency. Give the time complexity of your FA. Explain for which modifications this FA should be called.
4. Give an example of domains of x,y, and z, for which $x + y = z$ is interval consistent but not consistent (Hint: use 2.)

5. Give an example of domains of x,y, and z, for which $x + y = z$ is consistent and interval consistent but not arc consistent.
6. In your opinion is it possible to give an algorithm establishing arc consistency for this constraint whose complexity is not a power of d?
7. Consider the constraint $\sum_i x_i = y$. Give the conditions of a filtering algorithm (FA) associated with this constraint which establishes interval consistency. Give the time complexity of your FA. Explain for which modifications this FA should be called.
8. In your opinion, is it difficult to establish arc consistency for the constraint $\sum_i x_i = y$ when the domains of the variables are interval of integers?
9. Same question for the constraints $\sum_i x_i \geq y$ and $\sum_i x_i \leq y$?
10. In your opinion, is it difficult to establish arc consistency for the constraint $\sum_i x_i = y$ when the domains of the variables are set of discrete values?
11. Same question for the constraints $\sum_i x_i \geq y$ and $\sum_i x_i \leq y$?
12. In your opinion, is it difficult to establish arc consistency for the constraint $\sum_i \alpha_i x_i = y$ where α_i are given constant, when the domains of the variables are interval of integers?