## Personal data

Personal data: name: ALESSANDRO LANGUASCO, born in Imperia, Italy, on 12/23/1966; citizenship: Italian. Email address: alessandro.languasco@unipd.it - Webpage: https://www.math.unipd.it/~languasc
Scientific ID-codes: Mathematical Reviews; Zentralblatt; Orcid ID; Scopus Author ID; Web of Science Researcher ID; Researchgate page; Google Scholar profile.
List of papers: Complete list of papers; Complete list of papers with abstracts.
Position: From 10/01/2006: Associate Professor of Mathematical Analysis, Dipartimento di Matematica "Tullio Levi-Civita", University of Padova (Italy).
Previously: from 08/26/1998 to 10/01/2006: Assistent Professor of Mathematical Analysis, Dipartimento di Matematica, University of Padova (Italy).

Full professorship habilitation: From 06/30/2020: "Abilitazione scientifica nazionale", ASN - National Scientific qualification as full professor in the Italian higher education system, Italy - for full professor positions in Mathematical Analysis.

## Language skills:

- Italian: native language;
- English: C1 (QCER-CEFR) level; C1 level badge;
- French: I am able to read and understand scientific papers.


## Awards:

2003: Distinguished Award, Hardy-Ramanujan Society.

## Studies

## Studies and degrees:

- 1989: Laurea in Matematica (Master in Mathematics), mark: 110/110 cum laude, University of Genova (Italy). Thesis in Computational Number Theory entitled "Codici a chiave pubblica ed algoritmi di primalità" (Public key codes and primality algorithms).
- 1994: Ph.D. in Mathematics, Administrative headquarters: University of Torino (Italy). Title conferred by the Università di Roma "La Sapienza", 21/05/1998. Dissertation in Analytic Number Theory entitled "La congettura di Goldbach" (On the Goldbach conjecture).
- 1998: Assistant Professor in Mathematical Analysis, from 08/26/1998.
- 2006: Associate Professor in Mathematical Analysis, from 10/01/2006.
- 2020: Abilitazione scientifica nazionale ASN - National Scientific qualification as full professor in the Italian higher education system, Italy - for full professor positions in Mathematical Analysis; from 06/30/2020.


## International activities

## Scientific committees:

Here I list the scientific committees I was member of (the lists of seminars and congresses are in another part of this document).

- From 2005 I am a teacher and a tutor for the Erasmus Mundus Master "ALGANT" (ALgebra, Geometry And Number Theory) organised by the Universities of Bordeaux (France), Paris Sud (Paris 11, France), Leiden (Netherlands), Milano and Padova (Italy). For the analog Ph.D program are also involved the following Universities: Chennai (India), Stellenbosch (South Africa), Montreal Concordia (Canada).
- Member of the Scientific Committee of the exposition "Numeri. Tutto quello che conta, da zero a infinito" (Numbers: all that counts, from zero to infinity), by C. Bartocci (University of Genova, Italy) and L. Civalleri, Palazzo delle Esposizioni, Rome, Italy, 10/16/2014-05/31/2015.
- Member of the program committee (PC) of the conference "Number Theory Methods in Cryptology" (NuTMiC), International conference Number Theory Methods in Cryptology, September 11-13, 2017, Warsaw University, Poland,
proceedings published in link.
- Member of the organizing committee of the congress: "La Decifris incontra Torino"; Politecnico di Torino (Italy), 14/10/2019.


## Organisation and Higher Teaching Activities; Committees

- from november 2007 to may 2011, I represented the mathematicians on the board of the School in Statistics, University of Padova (Italy).
- in 2007 the CARIPARO foundation granted a Ph.D. studentship in Mathematics on a research topic I suggested.
- in july 2009 I become a member of the "Commissione Assegni di Ricerca" (Research grants committee) for Mathematics, University of Padova (Italy), academic year 2009-10.
- in 2010 I developed the online version of the "Precorso di Matematica" (Pre-course in Mathematics) for the School in Statistics of the University of Padova (Italy); I used the WeBWork software.
- from february 2009 to april 2011 I was a member of the "Commissione Nuovo Dipartimento" (New department committee) of the "Dipartimento di Matematica Pura e Applicata", University of Padova (Italy).
- from 2009 to 2011 I was a member of the "Commissione Pagine Web" (Webpages committee) of the "Dipartimento di Matematica Pura e Applicata", University of Padova (Italy).
- from january 2012 to may 2013 I was the coordinator of the "Commissione Comunicazione Esterna" (External communications committee) of the "Dipartimento di Matematica Pura Applicata", University of Padova (Italy); I was member of the same committee until 2014.
- from january 2008 until now I was a member of the Teaching Commitee of the Ph.D. School in Mathematics of the University of Padova (Italy).


## Committees:

- I was a member of several Committees for the final examination of Masters in Statistics and Mathematics for the University of Padova (Italy).
- November 2006: Ph.D. School in Mathematics, University of Padova (Italy): member of the admission Committee;
- January 2007: Ph.D. School in Mathematics, University of Torino (Italy): member of the Committee for the evaluation of the Ph.D. thesis of Dr. Stefano Barbero.
- November 2007: Ph.D. School in Mathematics, University of Padova (Italy): member of the admission Committee;
- July 2009: School of Statistics, University of Padova (Italy): member of the hiring Committee for the tutoring positions, academic year 2009-2010.
- 2009: University of Padova (Italy): member of the Committee to evaluate the Research Grants in Mathematics.
- November 2010: Ph.D. School in Mathematics, University of Trento (Italy): member of the Committee for the evaluation of the Ph.D. thesis of Dr. Luca Goldoni.
- 2014: member of the committee for the Associate professorship confirmation in Mathematical Analysis (07/11/2008, University of Padova); nominated by the Italian minister of University: 12/17/2013.
- 2016: member of the committee for the INDAM research grant "Ing. Giorgio Schirillo".
- April 2017: Ph.D. School in Mathematics, Universities of Ferrara, Modena, Parma, Reggio Emilia (Italy): member of the Committee for the evaluation of the Ph.D. thesis of Dr. Marco Cantarini.
- March 2019: Ph.D. School in Mathematics, Universities of Ferrara, Modena, Parma, Reggio Emilia (Italy): member of the Committee for the evaluation of the Ph.D. thesis of Dr. Mattia Cafferata.


## Teaching

Promoted courses: "Crittografia", later named as "Cryptography", and finally inserted as the first part of "Cybersecurity and Cryptography: principles and practices".
I promoted this course in 2003 to insert these topics coming from Algebraic, Elementary, Analytic and Computational Number Theory into the teaching offer of the University of Padova. From that moment on, with the only exception of a sabbatical year, I was the only teacher and representative of this course. From 2005 the course is taught in English and it was inserted in several international programs hosted by the Schools of Mathematics, Computer Sciences, Engineering of the University of Padova. So far, more than six hundreds students coming from different countries and having
different backgrounds (mathematics, computer science, computer engineering, telecommunications) had the chance to learn the scientific basis of a discipline that is an important part of the contemporary way of living. As a support for this course I wrote two books, both in a collaboration with A. Zaccagnini: "Introduzione alla Crittografia" (Introduction to Cryptography) [3] and "Manuale di Crittografia" (A Cryptography manual) [4]; both written in Italian and printed by Hoepli Editrice (Milano).
I also lead a group to show the same topics to the high school students of the Veneto region (Progetto Lauree Scientifiche per il Veneto); this work is collected in another book, written with A. Zaccagnini, entitled "Crittografia" (Cryptography) [2], and printed by CLEUP (Padova).

## Main teacher; B.A. and Master courses:

- a.y. 1999/2000: "Analisi Matematica Uno" (Calculus 1), (modulo A), B.A. in Computer Science, School in Mathematics, University of Padova (Italy).
- from a.y. 2001/2002 to a.y. 2003/2004: "Matematica B (Algebra Lineare, Geometria e Calcolo Differenziale in più variabili)", (Linear algebra and differential calculus in several variables), B.A. in Computer Science Engineering, School in Engineering, University of Padova (Italy).
- a.y. 2002/2003, 2003/2004: "Un'introduzione alla Teoria dei Numeri e applicazioni alla Crittografia" (An introduction to Computational Number Theory and Cryptography) for the Master in Applied Mathematics, School of Engineering, University of Padova (Italy).
- a.y. 2003/2004: "Teoria dei Numeri B (Teoria della funzione $\zeta$ di Riemann ed applicazioni)" (Theory of the Riemann $\zeta$-function and applications) Master in Mathematics, School in Mathematics, University of Padova (Italy).
- a.y. 2004/2005: "Metodi Matematici per la Statistica" (Mathematical methods for Statistics), Master in Statistics, School in Statistics, University of Padova (Italy).
- a.y. 2006/2007-2007/2008: "Istituzioni di Analisi Matematica 1" (Calculus 1), B.A. in Statistics, School in Statistics, University of Padova (Italy).
- a.y. 2005/2006-2006/2007 and from a.y. 2008/2009 to a.y. 2011/2012: "Istituzioni di Analisi Matematica 2" (Calculus 2), B.A. in Statistics, School in Statistics, University of Padova (Italy).
- from a.y. 2003/2004 to a.y. 2011/2012; from a.y. 2013/2014 to a.y. 2019/2020: "Crittografia" (Cryptography), Master in Computer Science, Master in Mathematics, ALGANT program, School in Mathematics, University of Padova; from 2005-06 the course is taught in English. From 2017/2018 the course it is also offered for the Master ICT for internet and multimedia, School in Engineering, University of Padova (Italy). From 2020-2021 it became the first part of the course "Cybersecurity and Cryptography: principles and practices", Master in Cybersecurity, School in Sciences, University of Padova (Italy).
- from a.y. 2020/2021 to a.y. 2021/2022: "Fondamenti di Analisi Matematica 2" (Calculus 2), B.A. in Mechanical Engineering, B.A. in Aerospace Engineering, School in Engineering, University of Padova (Italy).
- from a.y. 2013/2014 to a.y. 2019/2020 and a.y. 2022/2023: "Analisi Matematica Uno" (Calculus 1), B.A. in Chemical Engineering, School in Engineering, University of Padova (Italy).
- from a.y. 2020/2021 to a.y. 2022/2023: "Cybersecurity and Cryptography: principles and practices" (in English), Master in Cybersecurity, School in Sciences, University of Padova (Italy). The course it is also offered for the Master in Computer Science, Master in Mathematics, ALGANT program, School in Mathematics, University of Padova; Master ICT for internet and multimedia, Master in Computer Engineering, School in Engineering, University of Padova (Italy).


## Main teacher; Ph.D. courses:

- a.y. 1998/1999: "Introduzione alla funzione $\zeta$ di Riemann" (An introduction to the Riemann $\zeta$-function) for the Ph.D. school in Mathematics, University of Padova (Italy).
- a.y. 1998/1999: "Introduzione alla teoria dei numeri computazionale ed alla crittografia" (An introduction to Computational Number Theory and Cryptography) for the Ph.D. school in Mathematics, University of Genova, (Italy).
- a.y. 2001/2002 and 2004/2005: "Analisi Funzionale" (Functional Analysis) in collaboration with Prof. G. Treu (Univ. di Padova) for the Ph.D. school in Statistics, University of Padova (Italy).
- a.y. 2005/2006: "Introduzione alla funzione $\zeta$ di Riemann" (An introduction to the Riemann $\zeta$-function) for the Ph.D. school in Mathematics, University of Padova (Italy).
- a.y. 2007/2008: "Funzioni $L$ di Dirichlet e Teoria dei Crivelli" (Dirichlet $L$-functions and Sieve Theory) for the Ph.D. school in Mathematics, University of Padova (Italy).
- a.y. 2015/2016: "Diophantine problems with prime numbers", may-june 2016. for the Ph.D. school in Mathematics, Universities of Ferrara and Parma (Italy).


## Teaching collaborations:

- a.y. 1996/1997 and 1997/1998: "Geometria e Calcolo Numerico" (Geometry and Numerical Analysis), School of Engineering, University of Genova, (Italy).
- from a.y. 1998/1999 to a.y. 2000/2001: "Matematica Generale" (General Mathematics), School of Statistics, University of Padova (Italy).
- from a.y. 2001/2002 to a.y. 2004/2005: "Istituzioni di Analisi Matematica I e II", (Calculus 1 and 2) School of Statistics, University of Padova (Italy).
- a.y. 2005/2006: "Istituzioni di Analisi Matematica I" (Calculus 1), School of Statistics, University of Padova (Italy).
- a.y. 2005/2006 and a.y. 2008/2009: "Metodi Matematici per la Statistica" (Mathematical Methods for Statistics), School of Statistics, University of Padova (Italy).
- a.y. 2009/2010-2011/2012: "Analisi Matematica" (Mathematical Analysis), School of Statistics, University of Padova (Italy).


## Advised Ph.D. students:

- Valentina Settimi, "On some additive problems with primes and powers of a fixed integer".
- Co-advisor of Antonella Rossi, Marco Cantarini and Alessandro Gambini (advisor: A. Zaccagnini).


## Advised Master thesis:

I was the advisor of no. 35 thesis in Number Theory regarding both theoretical and computational aspects. In particular they are no. 25 Master thesis and no. 10 B.A. thesis.
The topics cover theoretical aspects in Elementary and Analytic Number Theory (Prime Number Theorem, Goldbach conjecture, large sieve, twin primes problem) and applications (cryptographic protocols, homomorphic cryptography, elliptic curve cryptography, primality algorithms, factorisation algorithms). In particular, I lead two students of mine in studying the breakthrough results of Maynard on the finite distance between consecutive prime numbers.

## Popularisation of Mathematics

## Books:

- 2004: with A. Zaccagnini, University of Parma (Italy), "Introduzione alla Crittografia", [3], Hoepli.
- 2006: with A. Zaccagnini, University of Parma (Italy), "Crittografia", [2], CLEUP.
- 2015: with A. Zaccagnini, University of Parma (Italy), "Manuale di Crittografia", [4], Hoepli.
- 2017: "Analisi Matematica 1", [1], Hoepli.


## Popularisation of Mathematics, conferences:

- 1998: conference for "Mathesis" entitled "Una breve introduzione alla crittografia" (A short introduction to Cryptography).
- 2001: Interview about "Steganografia e Crittografia" (Steganography and Cryptography), in "Il sommergibile" by V. Masotti, Radio Svizzera Italiana, 10/02/2001;
- 2001: Interview about "Trasmissioni e codici cifrati" (Secret codes and communications), "GR-Scienza" by S. Sciancalepore, BluSat radio network, 10/18/2001;
- 2005-2007: Progetto Lauree Scientifiche, coordination of the project entitled "Crittografia" (Cryptography) for four High Schools in the Veneto region.
- 2009: Interview by U. Rondi documentary movie entitled "Caccia ai numeri primi" (Hunting for prime numbers) for the RAI (Italian public television). Interview realised during the congress "Advances in Number Theory and Geometry", Verbania, 2009.
- 2010: Conference entitled "Comunicazione sicura nell'era di Internet" (Secure communication in the Internet age), for the conference series "Eppur si Muove".
- 2012: Conference entitled "Dai messaggi cifrati di Cesare alla comunicazione sicura nell'era di Internet" (From Caesar's secret messages to the secure communication in the Internet age), for the conference series "Caffè \& Scienza".
- 2014: Member of the Scientific Committee of the exposition "Numeri. Tutto quello che conta, da zero a infinito" (Numbers: all that counts, from zero to infinity), by C. Bartocci and L. Civalleri, Palazzo delle Esposizioni, Rome, Italy, 16/10/2014-31/05/2015. I wrote chapter 11 entitled "Numeri primi" of the exposition's book and the corresponding panels, see [45].
- 2016: "Ma la matematica non è un'opinione" (Math is not an opinion), Interview for "Il Bo", University of Padova newspaper.
- 2018: Interview by S. Camisasca for Agorà (Avvenire newspaper cultural page), published on 02-02-2018; about the role of Cryptography in our society.
- 2018: Interview by S. Camisasca for Agorà (Avvenire newspaper cultural page), published on 04-04-2018; about Ramanujan and Hardy.
- 2019: Interview by S. Camisasca for Avvenire (an Italian newspaper), published on 02-20-2019; about the influence of mathematical thinking in our society.
- 2020: Interview about the life and the work of Ramanujan and Hardy in the framework of "La scienza al cinema" [Science in the movies] (a series of movies on scientific topics organised by the Piano Lauree Scientifiche in Padova); and comments about the movie "The Man Who Knew Infinity" of M. Brown.
- 2022: (Feb. 25th) Conference entitled "Alla ricerca dei numeri primi" (Looking for prime numbers) for the Mathesis association; Locandina.


## Journal collaborations

## Membro del comitato editoriale:

- Former member of the Editorial Board of Open Mathematics, ISSN 2391-5455, from 09/01/2016 to 02/01/2024.
- Member of the Editorial Board of Indian Journal of Mathematics, ISSN 0019-5324, from 01/01/2019.
- Member of the Editorial Board of Journal of Approximation Software, from 01/01/2024.
- 2003-2005: Managing editor "Rendiconti del Seminario Matematico dell'University of Padova".


## Reviewer:

From 1997: Reviewer for "Mathematical Reviews" for the topics: 11M (Zeta and $L$-functions: analytic theory), 11N (Multiplicative number theory), 11P (Additive number theory; partitions). I wrote 65 reviews (until March 19, 2024).

## Referee for the journals (in alphabetical order):

1) Acta Arithmetica;
2) Acta Mathematica Hungarica;
3) Analysis, Geometry and Number Theory;
4) Annali della Scuola Normale Superiore di Pisa;
5) Applicable Algebra in Engineering, Communication and Computing;
6) Applied Mathematics E-Notes;
7) Atti della Accademia Peloritana dei Pericolanti - Classe di Scienze Fisiche, Matematiche e Naturali;
8) Bollettino dell'Unione Matematica Italiana;
9) Bulletin of the Allahabad Mathematical Society;
10) Canadian Mathematical Bulletin;
11) Communications in Algebra;
12) Complex Variables and Elliptic Equations;
13) Czechoslovak Mathematical Journal;
14) Electronic Research Archive;
15) Functiones et Approximatio, Commentarii Mathematici;
16) Frontiers of Mathematics;
17) Indian Journal of Mathematics;
18) Integers;
19) International Journal of Number Theory;
20) Journal of Algebra and its Applications;
21) Journal of Algebra, Number Theory and Applications;
22) Journal of Inequalities and Applications;
23) Journal of Mathematical Analysis and Applications;
24) Journal of Number Theory;
25) Mathematics of Computation;
26) Mathematica Slovaca;
27) Mathematika;
28) Missouri Journal of Mathematical Sciences;
29) Monatshëfte für Mathematik;
30) Open Mathematics;
31) Quarterly Journal of Mathematics;
32) Periodica Mathematica Hungarica;
33) Publicationes Mathematicae Debrecen;
34) Rendiconti del Circolo Matematico di Palermo;
35) Rendiconti del Seminario Matematico dell'Università di Padova;
36) Rendiconti del Seminario Matematico dell'Università di

Torino;
37) Rendiconti per gli studi Economici Quantitativi dell'Università di Venezia;
38) Research in Number Theory;
39) Rivista di Matematica della Università di Parma;
40) Rocky Mountain Journal of Mathematics;
41) Taiwanese Journal of Mathematics;
42) The American Mathematical Monthly;
43) The Ramanujan Journal.

## Scientific conferences and congresses; seminars

- January 1995: Incontro Italiano di Teoria dei Numeri, Rome, Italy (speaker).
- July 1997: Arithmetical Theory of Elliptic Curves, CIME Course, Cetraro (Cs), Italy.
- March 1999: Matematica e Cultura, Venice, Italy.
- July 2002: Analytic Number Theory, CIME Course, Cetraro (Cs), Italy.
- July 2003: Journées Arithmétiques 2003, Graz, Austria.
- November 2003: Secondo Incontro Italiano di Teoria dei Numeri, Parma, Italy, (speaker).
- July 2005: Journées Arithmétiques 2005, Marseille, France.
- May 2006: Italian-Polish Number Theory Days, Poznań, Polonia (invited speaker).
- July 2006: Special Session in Number Theory of the SIMAI-SMAI-SMF-UMI meeting, Torino, Italy.
- July 2007: Journées Arithmétiques 2007, Edimburgo, Regno Unito, (speaker).
- September 2007: Arithmetic Geometry, CIME Course, Cetraro (Cs), Italy.
- May 2008: Analytic Number Theory Workshop, Parma, Italy, (invited speaker).
- September 2008: A p-adic differential equations: a conference in honor of Gilles Christol, Bressanone (Italy).
- April 2009: Advances in Number Theory and Geometry, Verbania (Italy).
- May 2009: La Teoria dei Numeri, Università di Rome, Italy, Tre, Rome, Italy, (Italy), (invited speaker).
- March 2010: International Italy-India Conference on Diophantine and Analytic Number Theory, Scuola Normale Superiore, Pisa (Italy), (invited speaker).
- August-September 2010: Analytic and Combinatorial Number Theory, ICM satellite conference, Institute of Mathematical Sciences, Chennai (India), (invited speaker).
- October 2010: Number Theory and its applications, An International Conference Dedicated to Kálmán Győry, Attila Pethő, János Pintz, András Sárközy, Institute of Mathematics, University of Debrecen, Hungary, (invited speaker).
- February 2011: From p-adic differential equations to arithmetic algebraic geometry, on the occasion of Francesco Baldassarri's 60th birthday, 3-5 February 2011, Padova, Italy.
- August 2011: Paul Turán Memorial Conference, 22-26 August 2011, Budapest, Hungary.
- September 2011: Congresso UMI, 12-16 Settembre 2011, Bologna, Italy (section chairman).
- July 2013: Paul Erdôs Memorial Conference, 01-05 Luglio 2013, Budapest, Hungary.
- September 2015: Terzo Incontro Italiano di Teoria dei Numeri, Scuola Normale Superiore, Pisa (Italy), (invited speaker), proceedings [38].
- November 2016, Workshop di Teoria dei Numeri, Dipartimento di Matematica, University of Torino (Italy), (invited speaker).


## Seminars:

- "Il teorema di Bombieri-Vinogradov e sue estensioni, I, II, III", (The Bombieri-Vinogradov theorem and its generalisations, I, II ,III) University of Genova (Italy), 1993.
- "Crivello pesato e Teorema di Chen, I,II" (The weighted sieve and Chen's theorem, I II), University of Genova (Italy), 1994.
- "Alcuni risultati sulla congettura di Goldbach" (Some results on the Goldbach conjecture), Incontro Italiano di Teoria dei Numeri, Third University of Roma (Italy), 1995.
- "Una (breve) introduzione alla crittografia" (A short introduction to Cryptography), Associazione Mathesis, University
of Padova (Italy), 1998.
- "Approssimazione diofantea e algoritmo LLL, I,II,III" (Diophantine approximation and the LLL algorithm, I, II, III), University of Padova (Italy), 2001.
- "Sull'insieme eccezionale in intervalli corti di due problemi additivi con numeri primi" (On the exceptional set in short intervals for two additive problems with primes), Secondo Incontro Italiano di Teoria dei Numeri, University of Parma (Italy), 2003.
- "Piccole differenze tra primi consecutivi (dopo Goldston, Motohashi, Pintz, Yildirim)" (Small differences between consecutive primes (after Goldston, Motohashi, Pintz, Yildirim)), University of Genova (Italy), 08.06.2005.
- "On the sum of a prime and a $k$-free number", Italian-Polish Number Theory Days, Poznań, Poland, 18.05.2006.
- "Numeri primi e Crittografia" (Prime numbers and Cryptography), University of Modena (Italy), 04.10.2006.
- "On the sum of two primes and $k$ powers of two", Univ. Genova, 15.05.2007 - Univ. Parma 18.05.2007 - Journées Aritmétiques 2007, Edimburgh, UK, 02.07.2007.
- "Alcuni Attacchi a RSA" (Some attacks to RSA), University of Ferrara (Italy), 23.05.2007.
- "On the constant in the Mertens product for arithmetic progressions: Numerical values", University of Parma (Italy), 16.05.2008.
- "Sul problema di Goldbach-Linnik" (On the Goldbach-Linnik problem), University of Roma Tre (Italy), 29.05.2009.
- "On the Montgomery-Hooley theorem in short intervals", March 2010: Scuola Normale Superiore, Pisa (Italy).
- "On the average number of Goldbach representation of an integer", August 2010: Institute of Mathematical Sciences, Chennai (India); October 2010: University of Debrecen, Debrecen (Hungary).
- "Una formula esplicita per i numeri di Goldbach" (An explicit formula for the Goldbach problem), September 2011: University of Bologna (Italy).
- "RSA: firma digitale e attacchi" (RSA: digital signatures and attacks), November 2014, Progetto CAM, University of Padova (Italy).
- "On some exponential sums over prime powers and applications", September 2015: Terzo Incontro Italiano di Teoria dei Numeri, Scuola Normale Superiore, Pisa (Italy), proceedings [38].
- "Breve storia del Teorema dei Numeri Primi" (A short history about the Prime Number Theorem), Maggio 2016: Associazione Mathesis, University of Padova (Italy).
- "Diophantine problems with prime numbers", Maggio-Giugno 2016: Ph.D. School in Mathematics , University of Parma (Italy).
- "Formule esplicite per problemi additivi con numeri primi" (An explicit formula some additive problems with primes), Workshop di Teoria dei Numeri, University of Torino (Italy), November 2016.
- March 2021: "Calcolo efficiente della costante di Euler-Kronecker per campi ciclotomici (e problemi collegati)", (Efficient computation of the Euler-Kronecker constants for cyclotomic fields (and related problems)), the Number Theory Online conference, invited speaker.
- March 2021: "On computing $\frac{L^{\prime}}{L}(1, \chi)$ and related problems", Nancy-Metz online Théorie des Nombres Seminaire, invited speaker.
- October 2021: "On computing $\frac{L^{\prime}}{L}(1, \chi)$ ", 5th Number Theory Meeting, Torino (Italy), invited speaker.
- February 2022: "Alla ricerca dei numeri primi" (Looking for prime numbers), Associazione Mathesis, University of Padova (Italy), 2022.
- 25 April 2023, ore 11.00-12.00: ONLINE talk entitled: "Computing $\frac{L^{\prime}}{L}(1, \chi)$ using special functions, their reflection formulae and the Fast Fourier Transform". For the "Lithe and fast algorithmic number theory" (LFANT) series, INRIA and Institut de Mathématiques de Bordeaux, the Mathematics Institute of CNRS, Université de Bordeaux and Bordeaux INP, link.


## Research

## Awards:

2003: Distinguished Award, Hardy-Ramanujan Society.

## Research grants earned:

- March 1991 - November 1994: Ph.D. studentship, University of Torino, (Italy).
- March 1996 - February 1997: C.N.R. grant "ricerca" n. 201.01.121.
- March 1997 - June 1997: C.N.R. grant "ricerca" n. 201.01.123.
- Luglio 1997 - August 1998: Post-doctoral grant, University of Genova, (Italy).


## Participation to Research projects:

- PRIN 2000, MM01118441_001, Funzioni L e numeri primi, University of Genova (Italy);
- PRIN 2002, 2002018334_001, Funzioni L e problemi Diofantei Additivi, University of Genova (Italy);
- PRIN 2004, 2004010549_001, Funzioni L e problemi Diofantei Additivi, University of Genova (Italy);
- PRIN 2006, 2006018391_004, Geometria aritmetica : teorie p-adiche e motivi, University of Padova (Italy);
- PRIN 2008, 2008LMSMTY_005, Metodi differenziali p-adici e motivi, University of Padova (Italy);
- CARIPARO 2008-2009, "Eccellenza", Differential Methods in Arithmetic, Geometry and Algebra, University of Padova (Italy);
- PRIN 2010-2011, 20105LL47Y_001, Geometria algebrica aritmetica e teoria dei numeri, University of Padova (Italy);
- PRIN 2015, 2015XBNXYC_002, Number Theory and Arithmetic Geometry, University of Padova (Italy);
- PRIN 2017, 2017JTLHJR_002, Geometric, algebraic and analytic methods in arithmetic, University of Padova (Italy);
- TRACE4EU 2023-2025; member at the University of Padova (Italy) Unit for the EU funded TRACE4EU Project. Topic IP DIGITAL-2022-DEPLOY-02-EBSI-SERVICES. Digital Europe Programme.


## My works cited in the Online Encyclopedia of Integer Sequences (OEIS):

Founded in 1964 by N.J.A. Sloane, the OEIS contains a large number of data about integer sequences; for a detailed descriptions of its aims see their welcome page. Some research contributions of mine were inserted in the encyclopedia. They are listed here:

1) $\mathrm{A} 000466: a(n)=4 n^{2}-1$;
2) A002375: From Goldbach conjecture: number of decompositions of 2 n into an unordered sum of two odd primes;
3) A064533: Decimal expansion of Landau-Ramanujan constant;
4) A073010: Decimal expansion of $\pi / \sqrt{27}$;
5) A135311: A greedy sequence of prime offsets;
6) A161529: Decimal expansion of negative of constant $M(3,1)$ arising in Mertens and Meissel-Mertens constants for sums over arithmetic progression;
7) A187549: Arises in a Diophantine problem with one prime, two squares of primes and s powers of two;
8) A227158: Second-order term in the asymptotic expansion of $B(x)$, the count of numbers up to $x$ which are the sum of two squares;
9) A309520: Primes $p$ for which $h_{1}(p) / G(p)$ has a record value;
10) A335576: Decimal expansion of Mertens constant $C(5,2)$;
11) A336798: Decimal expansion of Mertens constant
$C(5,3)$;
12) A336802: Decimal expansion of the constant $\Pi(5,1)$;
13) A338462: Decimal expansion of the constant $\Pi(5,4)$;
14) A340127: Decimal expansion of $\prod_{p \equiv 4 \bmod 5} \frac{p^{2}}{p^{2}-1}$;
15) A340628: Decimal expansion of $\prod_{p \equiv 4 \bmod 5} \frac{p^{2}+1}{p^{2}-1}$;
16) A340629: Decimal expansion of $\prod_{p \equiv 1 \bmod 5} \frac{p^{2}+1}{p^{2}-1}$;
17) A340711: Decimal expansion of $\prod_{p \equiv 3 \bmod 5} \frac{p^{2}+1}{p^{2}-1}$;
18) A340839: Decimal expansion of Mertens constant $C(5,1)$;
19) A340866: Decimal expansion of Mertens constant $C(5,4)$;
20) A368644: Decimal expansion of the Mertens constant $M(3,2)$ arising in the formula for the sum of reciprocals of primes $p \equiv 2 \bmod 3$;
21) A368645: Decimal expansion of the Mertens constant $M(4,1)$ arising in the formula for the sum of reciprocals of primes $p \equiv 1 \bmod 4$ (negated);
22) A368646 Decimal expansion of the Mertens constant $M(4,3)$ arising in the formula for the sum of reciprocals of primes $p \equiv 1 \bmod 4$.

## Research

## Coauthors (in alphabetical order):

1) S.S. Al-Haj Baddar (University of Jordan, Amman, Jordan)
2) D. Bazzanella (Politecnico di Torino, Torino, Italy)
3) M. Cantarini (Università di Perugia, Perugia, Italy)
4) A. Ciolan
5) A. Gambini (Università di Roma "La Sapienza", Roma, Italy)
6) J. Kaczorowski (Università di Poznan, Poznan, Poland)
7) N. Kandhil (Università di Hong Kong, Hong Kong)
8) B. Kerr (University of New South Wales, Canberra, Australia)
9) Y. Lamzouri (Institut Élie Cartan de Lorraine, Université de Lorraine, Lorraine, France)
10) J. Lee
11) F. Luca (University of the Witwatersrand, South Africa)
12) F. Menegazzo (Università di Padova, Padova, Italy)
13) P. Moree (Scientific Coordinator, Max Planck Institute fur Mathematig, Bonn, Germany)
14) M. Migliardi (Università di Padova, Padova, Italy)
15) M. Morigi (Università di Bologna, Bologna, Italy)
16) A. Perelli (Università di Genova, Genova, Italy)
17) I. Petrykiewicz
18) J. Pintz (Alfred Renyi Mathematical Institute, Budapest, Hungary)
19) L. Righi (Università di Padova, Padova, Italy)
20) S. Saad Eddin (Johann Radon Institute for Computational and Applied Mathematics, Linz, Austria)
21) A. Sedunova (University of Warwick, England)
22) A.I. Suriajaya (Faculty of Mathematics, Kyushu University, Japan)
23) V. Settimi
24) A. Togbé (Purdue University, Purdue, USA)
25) T. Trudgian (University of New South Wales, Canberra, Australia)
26) A. Zaccagnini (Università di Parma, Parma, Italy)

## Research description:

My complete scientific production is of no. 86 papers; of them no. 68 are published scientific papers, no. 10 are prepublications, no. 4 are published textbooks and no. 4 are web-publications. I also wrote no. 7 teaching notes (listed in the section "Other publications" together with my Master and Ph.D. theses).
My main research field is in Analytic Number Theory. In particular I worked on additive problems with prime numbers and on the distribution of the non-trivial zeroes of the Riemann $\zeta$ and Dirichlet $L$-functions. Sometimes I also worked on some related computational problem. In the following, I will briefly describe such research topics. The complete list of my papers is towards the bottom of this file or at the following link: papers. The abstracts of all the papers I published are collected here: papers-abstracts.

## Goldbach's conjecture:

One of my main research topic is about Goldbach's conjecture. In 1742 Golbach conjectured that "every even integer n, $n>2$, is the sum of two prime numbers". A similar, but weaker, statement is: "every sufficiently large even integer $n$ is the sum of two prime numbers".
Both statements are open problems. From now on I will call Goldbach number an even integer which is the sum of two prime numbers. Several papers of mine are about some partial results on Goldbach's conjecture.
For example the papers [82], [80] and [68] are about the size of the set of integers which are not Goldbach numbers. Let $E=\{n \in \mathbb{N} ; n$ is not a Goldbach number $\}$ and let $X$ be a parameter; we try to evaluate the cardinality of the "exceptional set" $E(X)=E \cap(1, X)$ or of the "exceptional set in short intervals" $E(X, H)=E \cap(X, X+H)$, where $X$ and $H$ are sufficiently large but $H=o(X)$. Meaningful results are the ones in which we get $E(X)=o(X)$ or $E(X, H)=o(H)$ for $X \rightarrow+\infty$. In [68] an estimate on $E(X, H)$ is obtained.
Another kind of results is to study the distribution in short intervals of the Goldbach numbers; in other words the question is how large an interval of the type $(X, X+H)$ has to be to be sure that in such an interval there exists at least one Goldbach number. Some of these results depend on the assumption of some conjectures about the distribution of the zeroes of the Riemann $\zeta$ and Dirichlet $L$-functions. Results of this kind were proved in the papers [79], [77] and [75]. The techniques used to prove such results are connected to the following two quantities: the Selberg integral (a mean square result on the distribution of primes in short intervals) and a truncated $L^{2}$-mean for the trigonometric polynomial $\sum_{n \leq x} \Lambda(n) e(n \alpha)$, where $\alpha \in(0,1), e(\tau)=\exp (2 \pi i \tau)$ and $\Lambda(n)$ is the von Mangoldt functions. Some papers of mine, [72] and [74], are about these quantities.
Moreover, in 2010, working with A. Zaccagnini, assuming the validity of the Riemann hypothesis, we improved the estimate of the error term involved in the average number of representations of an even integer as a sum of two primes, see [51]. Such a paper was cited in the OEIS, see A002375.

## Explicit formula for prime numbers:

In the meantime, I was also attracted to other connected problems. For example in studying the distribution of prime numbers, a fundamental tool is the explicit formula for Chebyshev's function $\psi(x)=\sum_{n \leq x} \Lambda(n)$. In [78] a weighted form of such explicit formula was developed and in [76] we studied a variation, Landau's formula, which directly connects the distribution of the non-trivial zeros of the Riemann $\zeta$-function with the values of the von Mangoldt function $\Lambda(n)$.

## The Hardy-Littlewood additive problem:

In 2001-2002 I obtained a result on the exceptional set in short intervals for the Hardy-Littlewood problem, see [67]. In 2008, with A. Zaccagnini, I worked on establishing an asymptotic formula for the sum of the number of representations of an integer as a sum of a prime and a power of an integer, see [60]. Later I also studied the same problem under the assumption of the Generalised Riemann Hypothesis, see [58].

## Average number of representations of Goldbach and Hardy-Littlewood numbers:

In this series of papers I introduced the Cesàro weight to study the asymptotic and explicit formulae for the number of representations of Goldbach, Hardy-Littlewood numbers and of their generalised problems. In 2010 I developed a technique based on Laplace transform starting from some Walfisz' papers published in the fifties and then I asked to Zaccagnini to work together on these problems.
In 2012, again working with A. Zaccagnini, we proved that the Cesàro average of the number of representations of an integer as the sum of two primes has an extremely precise explicit formula (see [43]). To obtain this we used the Laplace transforms instead of the circle method. A similar but more complicated idea, lead us to obtain a similar result on the number of representations of an integer as the sum of a prime and a square. In this case the major complication arose from the fact that Bessel's functions of complex order are involved in such an explicit formula, see [46].
In 2016 A. Zaccagnini and myself have proved that the technique used in [51] can be applied to short intervals Cesàro averaged sums for integers representable as sums of two primes, see [35]. Moreover we also improved and generalised our technique used in in [43] and [46] proving that it also works for integers that can be written as a sum of two prime powers and as a sum of a prime power and a squares, see [29] and [30].

## Montgomery's conjecture:

Montgomery's conjecture assumes that the Riemann Hypothesis holds and it is about the asymptotic order of magnitude of a function built as a sum over pairs of imaginary parts of Riemann's zeta function zeros (usually called as the "pair correlation function"). Starting from 1998, see [77], I studied how this conjecture might give results on the distribution of Goldbach numbers.
In 2000, with A. Perelli, we proved an equivalence between the asymptotic formulation of the Montgomery conjecture and the asymptotic behaviour for some exponential sum over primes (this quantity is mainly used in the circle method), see [74]. In 2013, with Zaccagnini, we proved a form of this results in which the connections between the error terms are explicitly specified, see [44].
In 2010-2011, working with A. Perelli and A. Zaccagnini, we proved a result which explicitly connects the size of the error term for the pair-correlation function of the Riemann $\zeta$-function and for the mean-square of primes in short intervals, see [48]. In 2012-13, in a collaboration with A. Perelli and A. Zaccagnini, we obtained a generalisation of the pair-correlation function for the zeros of the Riemann $\zeta$-function; such a generalisation allows to study short averages of primes in short intervals, see [39]. In another paper we proved that such a generalisation holds if the main parameters run in suitable intervals and we further refined the link with the distribution of primes in short intervals, see [34].

## Diophantine approximation with primes and prime powers:

The main novelty introduced in these papers is the use of $L^{2}$-estimates for exponential sums over primes, or prime powers, into the classical method by Davenport and Heilbronn, as improved by Vaughan. This allowed us to enlarge the size of the major arc for this problem thus improving the quality of the final results. For some of these problems, I also implemented the Pintz-Ruzsa algorithm to study the extremal values of the exponential sum over powers of two. All these novelties were included in the vast scientific literature that followed and are now a standard in using the Davenport-Heilbronn method for such problems.
In 2008, working with A. Zaccagnini, I started to study a diophantine approximation problem based on the form in
primes and powers of two given by $\lambda_{1} p_{1}+\lambda_{2} p_{2}+\mu_{1} 2^{m_{1}}+\ldots+\mu_{s} 2^{m_{s}}$, where $\lambda_{i}, \mu_{j}$ are fixed. We improved the estimate about the number of powers of two required to approximate every real number using the values of such linear form in primes and powers of two (see [55]). This paper is cited in the OEIS, see A187549.
Working with V. Settimi, I also studied the number of solutions of a Diophantine approximation problem which involves $\lambda_{1} p_{1}+\lambda_{2} p_{2}^{2}+\lambda_{3} p_{3}^{2}+\mu_{1} 2^{m_{1}}+\ldots+\mu_{s} 2^{m_{s}}$, where $\lambda_{i}, \mu_{j}$ are fixed real numbers. We greatly improved the number of powers of two needed to obtain that every real number can be well approximate with values of this form in primes and powers of two, see [49]). The paper was inserted in the OEIS, see A187549.
More recently, working with A. Zaccagnini, I improved results on the approximability of real numbers using the values of the form $\lambda_{1} p_{1}+\lambda_{2} p_{2}^{2}+\lambda_{3} p_{3}^{2}+\lambda_{4} p_{4}^{2}$, (see [50]), on other diophantine problems with primes and prime powers (see [40]) and with mixed powers of primes (see [47]).
Moreover, in a paper with A. Gambini and A. Zaccagnini, we improved a result on a diophantine problem with prime numbers [33].

## Asymptotic formulae for additive problems with primes and prime powers:

In 2010 I realised that, by adapting the original Hardy-Littlewood formulation of the circle method, I was able to obtain results about the asymptotic formulae for several additive problems with primes and prime powers. The main problem was that we needed to have suitable mean square estimates for the exponential sums over prime powers; these estimates are in fact also useful to study diophantine approximation problems with prime and primes powers (see the relevant paragraph on this).
In 2014, working with A. Zaccagnini, we proved, under the assumption of the Riemann Hypothesis, that every interval $[N, N+H], H \gg(\log N)^{4}, N \rightarrow \infty$, contains at least an integer which is the sum of a prime and of two squares of primes, see [42].
During the same years, again with A. Zaccagnini, we studied the problem of representing integers as sums of a prime and a square, or a prime square, see [37] and [41]. In such papers we prove that suitable asymptotic formulae in short intervals hold for the average number of representation of an integer written as sums of two summands in which the first is a prime, or a prime square, and the second is a square, or a prime square. We also generalised the approach used in [37] and [41] to handle binary problems with density $\leq 1$, see [32].
In 2017-18, working with A. Zaccagnini, we proved that the asymptotic formulae in short intervals for some classical problems in number theory hold: sums of four prime cubes (see [31]), and sums of a prime power and two squares of primes (see [26]).
Further improvements of such results were obtained by refining the previously used techniques. In this way we were able to obtain results on the average Waring-Goldbach problem with $s$ summands and we also improved the results on binary problems with prime powers, see [27] (joint work with A. Zaccagnini). Moreover, in a collaboration with M. Cantarini, A. Gambini, and A. Zaccagnini, we studied a ternary problem with prime powers, see [28].

## Other additive problems:

In 2005-2006, in a joint paper with J. Pintz (Budapest, Hungary) and A. Zaccagnini, we obtained a result on the representation of an integer as the sum of two primes and some powers of two, see [62]. The result is a very strong one: it is enough to perturbate the Goldbach problem using only one power of two to obtain an estimate for the cardinality of the exceptional set of this problem that, at the moment, cannot be reached for the original Goldbach problem and it is essentially optimal (depending on the known minor arcs estimates for exponential sums over primes).
In a joint paper with A. Zaccagnini, we improved, under the assumption of the Generalised Riemann Hypothesis, the size of the error term for the explicit formula for the number of representations of an integer as sums of $k \geq 5$ primes, see [52].

## Mertens' product for primes in arithmetic progressions:

In 2005-2006, in a joint paper with A. Zaccagnini, we proved an uniform version of the asymptotic formula for the Mertens product over arithmetic progressions, see [63]. In particular, we showed that the asymptotic formula for $\prod_{p \leq x ; p \equiv a \bmod q}(1-1 / p)$ as $x \rightarrow+\infty$ holds uniformly for $q$ up to a certain bound that depends on the known zero-free region estimates for the Dirichlet $L$-functions (in some sense, this result can be seen as an analogue of the classical Siegel-Walfisz theorem for the Chebyshev $\psi$-functions).

Moreover, we found a new explicit and computable form for the constant $C(q, a)$ we have in the first term of such an asymptotic formula. The central role of the Mertens' product in Number Theory gave to this paper great visibility. The same happened for the computational papers devoted to compute $C(q, a)$, see below. This paper is cited in the OEIS, see A335576-A336798-A340711-A340839-A340866.
In 2007 I continued, with A. Zaccagnini, the study of the asymptotic behaviour of the Mertens product in arithmetic progressions getting estimates of the average order of the involved error term, see [61]. We were also able to find some alternative representations of $C(q, a)$, see [56]. The paper is cited in the OEIS, see A336802-A338462-A340127-A340628-A340629-A340711-A340839-A340866. In another paragraph I will describe the computational aspects related to this problem.

## Miscellanea:

In 2001, for a joint paper with Menegazzo and Morigi, see [69], I proved a result on the distribution of the values of the Euler $\varphi(n)$-function and of the $\Omega(n)$-function.
In 2004-2005, I obtained a result on the representation of an integer of a sum of a prime and of a $k$-free integer, see [64]. In 2009-2010, in a joint paper with D. Bazzanella and A. Zaccagnini, we studied the problem of getting for which $\lambda>1$ there exists a positive proportions of intervals like $(p, p+\lambda \log X], p$ prime, and like $(m, m+\lambda \log X], m$ integral and $X$ sufficiently large, such that it contains at least a prime number or such that it contains no primes. In the paper [57], we develop a technique to estimate moments of prime numbers over short intervals $(p, p+h], p$ prime and $h \leq X$, which allowed us to improve previously known estimates.
In 2009, see [53], I worked with A. Perelli and A. Zaccagnini to prove that the Montgomery-Hooley's formula for the mean square of primes in arithmetic progressions holds in short intervals too.

## Surveys:

The paper [81] is a survey on the Goldbach conjecture. The paper [66] is a survey about two results of mine on Goldbach's and Hardy-Littlewood's conjectures. The paper [38] is a survey about using infinite series instead of finite exponential sums in the circle method. The paper [21] shows the main ideas used to improve the efficiency in computing $L^{\prime} / L(1, \chi)$ by using the reflection formulae of some special functions into the Fast Fourier Transform procedure.

## Papers having a computational part

## Algorithms on Mertens' product in arithmetic progressions:

As I already described, I studied, with A. Zaccagnini, the problem of finding the constant $C(q, a)$ in the leading term of the asymptotic formula for the Mertens product in arithmetic progressions.
Concerning the related computational aspects, in 2008-2009 we computed the numerical values of $C(q, a)$ for every arithmetic progressions of modulo $q \leq 100$, with a precision of at least 100 decimal digits, see [59]. The paper is mentioned in the OEIS, see A340711; some numerical results are also mentioned, see A340127-A340839-A340866. In 2009, in a joint paper with A. Zaccagnini, we continued the study of the computability of the Mertens constants in arithmetic progressions; in [54], we studied the constants involved in the asymptotic formulae by Mertens and Meissel-Mertens

$$
\sum_{p \equiv a \bmod q}\left(\log \left(1-\frac{1}{p}\right)+\frac{1}{p}\right) \quad \text { and } \quad \sum_{\substack{p \leq x \\ p \equiv a \bmod q}} \frac{1}{p} \frac{\log \log x}{\varphi(q)}+M(q, a)+\mathcal{O}_{q}\left(\frac{1}{\log x}\right), \quad \text { as } x \rightarrow \infty
$$

Several instances of the previous formula were inserted in the OEIS: A161529, A368644, A368645, A368646.
The use of the Fast Fourier Transform in computing $L(1, \chi)$ and $L^{\prime}(1, \chi)$ and the Euler-Kronecker constants: In 2019 I worked on the paper [23] which is about the development of a new algorithm to compute the Euler-Kronecker constants for cyclotomic fields; such a constant is defined as $\gamma_{q}=\sum_{\chi \neq \chi_{0}} L^{\prime} / L(1, \chi)$, where $\chi$ runs over the set of Dirichlet characters mod $q$, and $q$ is an odd prime number.
Such an algorithm combines the Fast Fourier Transform procedure with the reflection formulae for the special functions involved in computing $L(1, \chi)$ and $L^{\prime}(1, \chi)$. These ideas lead us to reduce the overall memory usage and to improve the global computational cost of the algorithm. These facts allowed us to obtain a very large set of informations about
the values of $L(1, \chi)$ and $L^{\prime}(1, \chi)$. Since it is well-known the central role played by these quantities into the Number Theory literature, it is not surprising that, exploiting the knowledge of these data, we were able to obtain several results. In [23], I first confirmed the numerical results obtained by other researchers about the Euler-Kronecker constant for the cyclotomic fields $\mathbb{Q}\left(\zeta_{q}\right)$ and then extended their knowledge to the range $3 \leq q \leq 10^{6}, q$ prime. Moreover, I also found two new counterexamples to Ihara's conjecture on the positivity of such constants. These counterexamples are connected to prime numbers whose sizes are about 10-billions, about ten times larger than the previously known one by Ford, Luca and Moree in 2014. Working with numbers of such a size is quite hard from a computational point of view. This paper is cited in the OEIS, see A135311.
In 2019, in a joint work with P. Moree, S. Saad Eddin and A. Sedunova, I studied the behaviour of $r(q)$, the Kummer ratio between the first factor of the class number of a cyclotomic field $\mathbb{Q}\left(\zeta_{q}\right), q$ prime, $q \geq 3$, and its expected order of magnitude; the computational part of this work is available, in its preliminary form, here: [5]. In such a paper we detected new extremal values for $r(q)$. This paper is cited in the OEIS, see A309520.
In 2020, working with L. Righi (University of Padova), we were able to develop an algorithm to efficiently compute the values of the Ramanujan-Deninger Gamma function, see [25]. This algorithm is much faster that the previous one we used in [23] and hence it allowed us to further enlarge the range of computed values for the Euler-Kronecker constants of cyclotomic fields. In particular, we were able to obtain another counterexample to Ihara's conjecture; the prime number $q$ such that the Euler-Kronecker constant of $\mathbb{Q}\left(\zeta_{q}\right)$ is negative has an order of magnitude of about 50-billions (about fifty times larger than the counterexample known before my contributions). We also obtained the values of every Euler-Kronecker constants for the cyclotomic fields $\mathbb{Q}\left(\zeta_{q}\right)$ in the range $3 \leq q \leq 10^{7}, q$ prime.

Numerical study of Littlewood's bounds for $|L(1, \chi)|$ : More recently (2019-2020), I numerically studied the validity of Littlewood's inequalities about $|L(1, \chi)|$ (see [24]), where $\chi$ is a primitive non-principal Dirichlet character modulo $q, 3 \leq q \leq 10^{7}, q$ prime.

Estimates for $\min _{\chi \neq \chi_{0}}\left|L^{\prime} / L(1, \chi)\right|: \quad$ In 2020-2021, in a collaboration with Y. Lamzouri (University of Nancy, France), we studied the order of magnitude of $\min _{\chi \neq \chi_{0}}\left|L^{\prime} / L(1, \chi)\right|, \chi$ being a primitive non-principal Dirichlet character (see [20]). This result is the first one to give an upper bound for $\min _{\chi \neq \chi_{0}}\left|L^{\prime} / L(1, \chi)\right|$. Moreover, using some computational result, we were also able to obtain suitable lower bounds for the same quantity and for every $q, 3 \leq q \leq 10^{7}, q$ prime. Such computational estimates also proved that $L^{\prime}(1, \chi) \neq 0$ for every non-principal Dirichlet character $\chi$ mod $q$, $3 \leq q \leq 10^{7}, q$ prime, and they lead us to conjecture suitable lower bound estimates for $\left|L^{\prime} / L(1, \chi)\right|$. So far, no theoretical results about such lower bounds are known.

Explicit estimates for $|L(1, \chi)|$ : In 2020, working with T.S. Trudgian (UNSW Canberra, Australia), see [22], we extended the range of validity of the Lamzouri-Li-Soundararajan estimates for $|L(1, \chi)|, \chi \bmod q$, where $\chi \bmod q$ runs ove the non-principal Dirichlet characters and $q \geq 404$ is an integer (their result holds for $q \geq 10^{10}$ only). With a computational part, we then extended this range to $q \geq 3$.

Asymptotic formulae for the generalized divisor function (and related problems): In 2021 I worked with A. Ciolan and P. Moree (Max Planck Institute, Bonn, Germany) on comparing the asymptotic formulae of Landau and Ramanujan about the function that counts the number of integers $n$ such that $q \nmid \sigma_{k}(n)$, where $q$ is a prime number and $\sigma_{k}(n)$ is the generalised divisor function of $n$, [19]. In this work we also treat some classical aspect connected with modular forms, and we also inserted a very accurate computation (up to 47500 decimal digits) for the second order constant in the the asymptotic development of the function that counts the number of integers that are the sum of two squares; see also the OEIS: A227158.

Numerical estimates for the Landau-Siegel zero: Moreover, in 2023 I published a paper that gives a series of numerical estimates about the Landau-Siegel zero, see [15].


#### Abstract

Algorithms for the computation of some special functions: In 2022, I developed a new algorithm to efficiently compute a set of special functions that includes Euler's $\Gamma$-function, the digamma and polygamma functions, the Hurwitz $\zeta$-function $\zeta(s, x)$ and its first partial derivative $(\partial \zeta / \partial s)(s, x)$, the Dirichlet $L$-functions $L(s, \chi)$ and $L^{\prime}(s, \chi)$. In particular, in [17] I showed an efficient algorithm to compute the Hurwitz zeta-function $\zeta(s, x)$ for $s>1$ and $x>0$; and, combining it with the Fast Fourier transform procedure and some suitable reflection formulae, I obtained an efficient algorithm to compute $L(s, \chi), s>1$, for every non-principal Dirichlet character mod $q, q$ being an odd prime number.

Problems about the distribution of integers having two prime factors: In another collaboration with Moree, this time also with F. Luca and A. Togbé, see [11], we obtained explicit values for the constants involved in estimating the differences of two integers of the form $p^{a} \cdot q^{b}$, where $p, q$ are two distinct primes. Letting further $\alpha>1$, we also obtained results, both of theoretical and computational nature, about the smallest $m$ such that for every $n \geq m$ the interval $[n, n \alpha)$ contains an integer of the form $p^{a} \cdot q^{b}$.


Cryptographic applications (a re-encryption protocol): Recently a paper of mine [16] about a re-encryption protocol for stored data on a digital support, was accepted for publication in the proceedings of the "15th International Conference on Ubiquitous Computing \& Ambient Intelligence", (UCAmI 2023).

## The Dirichlet multinomial log-likelihood function algorithm:

In a collaboration with M. Migliardi (University of Padova), we improved on the known algorithms to compute the Dirichlet multinomial log-likelihood distribution function, see [18]. This distribution involves the computation of $\log \Gamma$-function values at close points, and it is useful to perform statistical investigations in several fields (for example, in genomics, data science, bayesian statistics, machine learning, natural language processing and for image recognition softwares).

## Research in progress (2022-2024)

In 2022-2023, I continued to work with Pieter Moree (Max Planck Institute, Bonn, Germany) and we started a project a about the distribution of the values of the Ramanujan $\tau$-function, the divisor function and of the generalised divisor function in the quadratic and non-quadratic residue classes modulus a prime number $q$; see [6], [7] (in preparation), [10] (submitted). A part of this project includes a collaboration with Bryce Kerr too, [8] (in preparation).
I also started to work again with N. Kandhil, P. Moree, S. Saad Eddin and A. Sedunova, about the study of the Ihara's conjecture on the Euler-Kronecker constants for some cyclotomic subfields, and about the Kummer ratio for the first factor of the class number of the cyclotomic field. The first part of the obtained results is presented in [14].
In 2023-2034, in a collaboration with N. Kandhil and P. Moree [13] we improved a Tatuzawa's result, published in 1953, about the order of magnitude of a suitable normalization of the product of the class number and the regulator of a cyclotomic field.
Concerning applicative topics, working with S.S. Al-Haj Baddar (University of Jordan), and M. Migliardi (University of Padova), see [12], we are further developing the applications of the paper [18]. In particular, the speediness and accuracy of our algorithm to compute the Dirichlet multinomial log-likelihood function are leading us to obtain interesting practical applications in several fields like: bayesian statistics, statistical inference, machine learning, data science, language modeling, facial recognition software, counting data clustering.
I am also working on extending the scope of application of my algorithm in [17] into the $\mathfrak{R}(s)>0$ region. This will have consequences on the known algorithms to compute $L(s, \chi)$ within the critical strip and on the critical line, in particular. Other consequences are on the evaluation of the Dirichlet $L$-functions on the line $\mathfrak{R}(s)=1$. As I did in [17], I will not only show the theoretical algorithm, but I will also provide a running implementation.

## Books in preparation

In 2024, working with J. Lee, P. Moree, I. Petrykiewicz, A.I. Suriajaya e A Sedunova, I started to write a monograph about a computational history of prime numbers and the Riemann zeta function, [9].

In several occasion I also wrote papers on popularisation of mathematics with a particular attention to Number Theory applications. The papers [73], [74], [71], [70] are about these topics. Later, in collaboration with A. Zaccagnini, I wrote a book [3] dedicated to describe the application of Number Theory to Cryptography. A second book on the same topic, that updates and vastly enlarges the previous one, again written in a collaboration with A. Zaccagnini, was published in 2015 [4]. This latest one was cited in the OEIS, see A000466.
Fulfilling a request by the PRISTEM-Bocconi organisation (Milano), working with A. Zaccagnini, we wrote a series of papers ([83], [84], [85] and [86]) on several topics about prime numbers.
In 2014 I wrote chapter 11 of the book of the exposition "Numeri, tutto quello che conta, da zero a infinito" (Numbers: all that counts, from zero to infinity), by C. Bartocci and L. Civalleri, see [45].
In 2017 we wrote a paper on Number Theory, see [36], for the journal "Sapere", the oldest Italian popular science journal.
In 2005 I wrote a column about the prime twins problem on the newspaper "La Voz de Almeria" (see [65]). In 2005-2007 I was the coordinator of the Cryptography group for the "Progetto Nazionale Lauree Scientifiche", Veneto. The book [2], written with A. Zaccagnini, contains the final output of such a work.
In 2016-17, using all the material I collected in the previous twenty years of teaching, I wrote the book [1] about a first course on Mathematical Analysis.

## Programming skills:

Besides being an expert user of $\mathrm{ET}_{\mathrm{E}} \mathrm{X}$, during the last forty years I used several programming languages for work and research. In particular, I am able to write programs using the following languages: EDL (IBM series/1 event driven language), Fortran IV, Fortran 77, MS-DOS Basic, Cobol, Digital PDP-11 Assembler, Pascal, Modula-2, LISP, C. More recently I learned how to use some scripting languages as python and the ones of Pari/Gp and MAXIMA. In several occasions I exploited these abilities in some research projects: they are listed in the following scientific activities section. The actual source codes of these programs are available at the following link: Programs-Languasco.

## List of my research papers and books

## Books (in Italian):

[1] A. Languasco. Analisi Matematica 1. Ulrico Hoepli editore, 2017. Publisher link.
[2] A. Languasco and A. Zaccagnini. Crittografia. CLEUP, Padova, 2006. Progetto Lauree Scientifiche per il Veneto.
[3] A. Languasco and A. Zaccagnini. Introduzione alla Crittografia. Ulrico Hoepli Editore, 2004.
[4] A. Languasco and A. Zaccagnini. Manuale di Crittografia. Ulrico Hoepli Editore, 2015. Publisher link.

## In preparation:

[5] A. Languasco, P. Moree, S. Saad Eddin, and A. Sedunova. Computation of the Kummer ratio of the class number for prime cyclotomic fields. Arxiv, 2019. DOI-link (preliminary version).
[6] A. Languasco and P. Moree. Bias of the Ramanujan tau and the sum of divisors function for even moduli. in preparation, 2022.
[7] A. Languasco and P. Moree. Bias of the Ramanujan tau and the sum of divisors function for odd moduli. in preparation, 2022.
[8] B. Kerr, A. Languasco, and P. Moree. Quadratic residue bias of the divisor function and fake mu's. in preparation, 2022.
[9] A. Languasco, J. Lee, P. Moree, I. Petrykiewicz, A.I. Suriajaya, and A. Sedunova. A computational history of prime numbers and Riemann zeros. 2024. Monograph, in preparation.

## Preprints:

[10] A. Languasco and P. Moree. Quadratic residue bias of the divisor function and Fekete polynomials. submitted, 2022.
[11] A. Languasco, F. Luca, P. Moree, and A. Togbé. Sequences of integers generated by two fixed primes. submitted, 2023. DOI-link:, MR:, ZBL:.
[12] S. Al Haj-Baddar, A. Languasco, and M. Migliardi. Efficient analysis of overdispersed data using an accurate computation of the Dirichlet multinomial distribution. submitted, 2023. TechRxiv. Preprint., CodeOcean capsule.
[13] N. Kandhil, A. Languasco, and P. Moree. The Brauer-Siegel ratio for prime cyclotomic fields. arXiv, submitted, 2024. DOI-link.
[14] N. Kandhil, A. Languasco, P. Moree, S. Saad Eddin, and A. Sedunova. The Kummer ratio of the relative class number for prime cyclotomic fields. to appear in J. Math. Anal. Appl., 2024. DOI-link.

## Published papers (in reverse chronological order):

[15] A. Languasco. Numerical estimates on the Landau-Siegel zero and other related quantities. J. Number Theory, 251:185-209, 2023. DOI-link, MR:4598945, ZBL:7695616.
[16] A. Languasco. A partially decentralised protocol for a distributed encrypted storage system. In José Bravo and Gabriel Urzáiz, editors, Proceedings of the 15th International Conference on Ubiquitous Computing \& Ambient Intelligence (UCAmI 2023), pages 195-204, Cham, 2023. Springer Nature Switzerland. DOI-link.
[17] A. Languasco. A unified strategy to compute some special functions of number-theoretic interest. J. Number Theory, 247:118-161, 2023. DOI-link, MR:4546697, ZBL:7662018.
[18] A. Languasco and M. Migliardi. On the fast computation of the Dirichlet-multinomial log-likelihood function. Comput. Stat., 38:1995-2013, 2023. DOI-link,MR:4672333, ZBL:.
[19] A. Ciolan, A. Languasco, and P. Moree. Landau and Ramanujan approximations for divisor sums and coefficients of cusp forms. J. Math. Anal. Appl., 519:1-48, paper n. 126854, 2023. DOI-link, MR:4511375, ZBL:07624143.
[20] Y. Lamzouri and A. Languasco. Small values of $\left|L^{\prime} / L(1, \chi)\right|$. Exp. Math., 32:362-377, 2023. DOI-link, MR:4592953, ZBL:7708949.
[21] A. Languasco. On computing $L^{\prime} / L(1, \chi)$. Rendiconti Sem. Mat. Univ. Pol. Torino, 80:55-71, 2022. Proceedings of the fifth Number Theory Meeting, Università and Politecnico di Torino, October 26-27th, 2021. DOI-link. MR:4515572, ZBL:7626422.
[22] A. Languasco and T. S. Trudgian. Uniform effective estimates for $|L(1, \chi)| . J$. Number Theory, 236:245-260, 2022. DOI-link, MR:4395349, ZBL:7493024.
[23] A. Languasco. Efficient computation of the Euler-Kronecker constants for prime cyclotomic fields. Res. Number Theory, 7, paper n. 2, 2021. DOI-link, MR:4194178, ZBL:07304549.
 DOI-link, MR:4213696, ZBL:07329220.
[25] A. Languasco and L. Righi. A fast algorithm to compute the Ramanujan-Deninger Gamma function and some number-theoretic applications. Math. Comp., 90:2899-2921, 2021. DOI-link, MR:4305373; ZBL:07390221.
[26] A. Languasco and A. Zaccagnini. Sum of one prime power and two squares of primes in short intervals. Rocky Mountain J. Math., 51:213-224, 2021. DOI-link, MR:4280109, ZBL:07393760.
[27] A. Languasco and A. Zaccagnini. Short intervals asymptotic formulae for binary problems with prime powers, II. J. Aus. Math. Soc., 109:351-370, 2020. DOI-link, MR:4190085, ZBL:07286548.
[28] M. Cantarini, A. Gambini, A. Languasco, and A. Zaccagnini. On a average ternary problem with prime powers. Ramanujan J., 53:155-166, 2020. DOI-link, MR:4148463, ZBL:07176138.
[29] A. Languasco and A. Zaccagnini. A Cesàro average for an additive problem with prime powers. In Proceedings of the conference "Number Theory Week", Poznań, September 4-8, 2017. Banach Center Publications, Institute of Mathematics, Polish Academy of Sciences, Warszawa, volume 118, pages 137-152, 2019. DOI-link, MR:3931260, ZBL:07087893.
[30] A. Languasco and A. Zaccagnini. A Cesàro average for generalised Hardy-Littlewood numbers. Kodai Math. J., 42:358-375, 2019. DOI-link, MR:3981309, ZBL:07108016.
[31] A. Languasco and A. Zaccagnini. Sums of four prime cubes in short intervals. Acta Math. Hungar., 159:150-163, 2019. DOI-link, MR:4003700, ZBL:07119764.
[32] A. Languasco and A. Zaccagnini. Short intervals asymptotic formulae for binary problems with prime powers. J. Théor. Nombres Bordeaux, 30:609-635, 2018. DOI-link, MR:3891329, ZBL:3A07081564.
[33] A. Gambini, A. Languasco, and A. Zaccagnini. A diophantine approximation problem with two primes and one $k$-power of a prime. J. Number Theory, 188:210-228, 2018. DOI-link, MR:3778631, ZBL:06855844.
[34] A. Languasco, A. Perelli, and A. Zaccagnini. An extended pair-correlation conjecture and primes in short intervals. Trans. Amer. Math. Soc., 369(6):4235-4250, 2017. DOI-link, MR:3624407, ZBL:06698813.
[35] A. Languasco and A. Zaccagnini. Cesàro average in short intervals for Goldbach numbers. Proc. Amer. Math. Soc., 145(10):4175-4186, 2017. DOI-link, MR:3690604, ZBL:06767077.
[36] A. Languasco and A. Zaccagnini. Il fascino discreto della teoria dei numeri. Sapere, 1:22-26, 2017. DOI-link.
[37] A. Languasco and A. Zaccagnini. Short intervals asymptotic formulae for binary problems with primes and powers, I: density 3/2. Ramanujan J., 42:371-383, 2017. DOI-link, MR:3596938, ZBL:06692048.
[38] A. Languasco. Applications of some exponential sums on prime powers: a survey. In Proceedings of the "Terzo Incontro Italiano di Teoria dei Numeri", Scuola Normale Superiore, Pisa, 21-24 Settembre 2015. Rivista di Matematica della Università di Parma, volume 7, pages 19-37, 2016. DOI-link, MR:3675401, ZBL:06760984.
[39] A. Languasco, A. Perelli, and A. Zaccagnini. An extension of the pair-correlation conjecture and applications. Math. Res. Lett., 23(1):201-220, 2016. DOI-link, MR:3512883, ZBL:06609432.
[40] A. Languasco and A. Zaccagnini. A Diophantine problem with prime variables. In V. Kumar Murty, D. S. Ramana, and R. Thangadurai, editors, Highly Composite: Papers in Number Theory, Proceedings of the International Meeting on Number Theory, celebrating the 60th Birthday of Professor R. Balasubramanian (Allahabad, 2011), volume 23 of Ramanujan Math. Soc. Lect. Notes Ser., pages 157-168. Ramanujan Math. Soc., Mysore, 2016. DOI-link, MR:3692733, ZBL:1416.11143.
[41] A. Languasco and A. Zaccagnini. Short intervals asymptotic formulae for binary problems with primes and powers, II: density 1. Monatsh. Math., 181:419-435, 2016. DOI-link, MR:3539942, ZBL:1350.11089.
[42] A. Languasco and A. Zaccagnini. Sum of one prime and two squares of primes in short intervals. J. Number Theory, 159:45-58, 2016. DOI-link, MR:3412711, ZBL:06497366.
[43] A. Languasco and A. Zaccagnini. A Cesàro Average of Goldbach numbers. Forum Math., 27:1945-1960, 2015. DOI-link, MR:3365783, ZBL:06458901.
[44] A. Languasco and A. Zaccagnini. Explicit relations between primes in short intervals and exponential sums over primes. Funct. Approx. Comment. Math., 51:379-391, 2014. DOI-link, MR:3282634, ZBL:06380131.
[45] A. Languasco. Numeri primi. In C. Bartocci and L. Civalleri, editors, Numeri. Tutto quello che conta. Da zero a infinito, pages 183-193. Codice Edizioni, Torino, Italy, 2014. (Chapter 11 of the volume).
[46] A. Languasco and A. Zaccagnini. A Cesàro Average of Hardy-Littlewood numbers. J. Math. Anal. Appl., 401:568-577, 2013. DOI-link, MR:3018008, ZBL:06156267.
[47] A. Languasco and A. Zaccagnini. On a ternary Diophantine problem with mixed powers of primes. Acta Arith., 159:345-362, 2013. DOI-link, MR:3080797, ZBL:06184261.
[48] A. Languasco, A. Perelli, and A. Zaccagnini. Explicit relations between pair correlation of zeros and primes in short intervals. J. Math. Anal. Appl., 394:761-771, 2012. DOI-link, MR:2927496, ZBL:06062862.
[49] A. Languasco and V. Settimi. On a Diophantine problem with one prime, two squares of primes and $s$ powers of two. Acta Arith., 154:385-412, 2012. DOI-link, MR:2949876, ZBL:06055436.
[50] A. Languasco and A. Zaccagnini. A Diophantine problem with a prime and three squares of primes. J. Number Theory, 132:3016-3028, 2012. DOI-link, MR:2965205, ZBL:06097276.
[51] A. Languasco and A. Zaccagnini. The number of Goldbach representations of an integer. Proc. Amer. Math. Soc., 140:795-804, 2012. DOI-link, MR:2869064, ZBL:1252.11078.
[52] A. Languasco and A. Zaccagnini. Sums of many primes. J. Number Theory, 132:1265-1283, 2012. DOI-link, MR:2899803, ZBL:06031097.
[53] A. Languasco, A. Perelli, and A. Zaccagnini. On the Montgomery-Hooley theorem in short intervals. Mathematika, 52:231-243, 2010. DOI-link, MR:2678027, ZBL:1238.11087.
[54] A. Languasco and A. Zaccagnini. Computing the Mertens and Meissel-Mertens constants for sums over arithmetic progressions. Exp. Math., 19:279-284, 2010. With an appendix by Karl K. Norton. DOI-link, MR:2743571, ZBL:06074851.
[55] A. Languasco and A. Zaccagnini. On a Diophantine problem with two primes and $s$ powers of two. Acta Arith., 145:193-208, 2010. DOI-link, MR:2733083, ZBL:1222.11049.
[56] A. Languasco and A. Zaccagnini. On the constant in the Mertens product for arithmetic progressions. I. Identities. Funct. Approx. Comment. Math., 42:17-27, 2010. DOI-link, MR:2640766, ZBL:1206.11112.
[57] D. Bazzanella, A. Languasco, and A. Zaccagnini. Prime numbers in logarithmic intervals. Trans. Amer. Math. Soc., 362:2667-2684, 2010. DOI-link, MR:2584615, ZBL:1200.11072.
[58] A. Languasco. A conditional result on the exceptional set for Hardy-Littlewood numbers in short intervals. Int. J. Number Theory, 5:933-951, 2009. DOI-link, MR:2569737, ZBL:1251.11068.
[59] A. Languasco and A. Zaccagnini. On the constant in the Mertens product for arithmetic progressions. II. Numerical values. Math. Comp., 78:315-326, 2009. DOI-link, MR:2448709, ZBL:1214.11108.
[60] A. Languasco and A. Zaccagnini. On the Hardy-Littlewood problem in short intervals. Int. J. Number Theory, 4:715-723, 2008. DOI-link, MR:2458837, ZBL:1251.11069.
[61] A. Languasco and A. Zaccagnini. Some estimates for the average of the error term of the Mertens product for arithmetic progressions. Funct. Approx. Comment. Math., 38:41-47, 2008. DOI-link, MR:2433787, ZBL:1233.11100.
[62] A. Languasco, J. Pintz, and A. Zaccagnini. On the sum of two primes and $k$ powers of two. Bull. Lond. Math. Soc., 39:771-780, 2007. DOI-link, MR:2365226, ZBL:1137.11066.
[63] A. Languasco and A. Zaccagnini. A note on Mertens' formula for arithmetic progressions. J. Number Theory, 127:37-46, 2007. DOI-link, MR:2351662, ZBL:1210.11105.
[64] A. Languasco. On the sum of a prime and a $k$-free number. Funct. Approx. Comment. Math., 34:19-26, 2005. DOI-link, MR:2269661, ZBL:1228.11156.
[65] A. Languasco. Primos Gemelos. La Voz de Almeria, Seccion Matematica, 2005. (pubblicato il 04/09/2005 in lingua spagnola, traduzione in Spagnolo di Juan Cuadra Diaz).
[66] A. Languasco. The exceptional set in short intervals for two additive problems with primes: a survey. Riv. Mat. Univ. Parma (7), 3*:223-231, 2004. DOI-link, MR:2128851, ZBL:1166.11348.
[67] A. Languasco. On the exceptional set for Hardy-Littlewood's numbers in short intervals. Tsukuba J. Math., 28:169-192, 2004. DOI-link, MR:2082228, ZBL:1068.11066. Corrigendum ibid., DOI-link, Tsukuba J. Math., 30 (2006), 237-240, MR:2248294, ZBL:1201.11095.
[68] A. Languasco. On the exceptional set of Goldbach's problem in short intervals. Monatsh. Math., 141:147-169, 2004. DOI-link, MR:2037990, ZBL:1059.11059.
[69] A. Languasco, F. Menegazzo, and M. Morigi. On the composition length of finite primitive linear groups. Arch. Math., 79:408-417, 2002. DOI-link, MR:1966776, ZBL:1015.20034.
[70] A. Languasco and A. Perelli. Crittografia e firma digitale. In M. Emmer and M. Manaresi, editors, Matematica, Arte, Tecnologia, Cinema, pages 99-106, Bologna, 2002. Springer-Verlag, Milano. English translation in Mathematics, Art, Technology, and Cinema, Springer-Verlag, Berlin, Heidelberg, New York, 2003.
[71] A. Languasco. An Introduction to Cryptography. Queen's Papers in Pure and Applied Mathematics, 119(121-140), 2000. in The Curves Seminar at Queen's , vol. 13, ed. da A.V. Geramita.
[72] A. Languasco. Some refinements of error terms estimates for certain additive problems with primes. J. Number Theory, 81:149-161, 2000. DOI-link, MR:1743499, ZBL:1003.11047.
[73] A. Languasco and A. Perelli. Numeri Primi e Crittografia. In M. Emmer, editor, Matematica e Cultura 2000, pages 227-233, Venezia, 2000. Springer-Verlag, Milano. English translation in Mathematics and Culture I, Springer-Verlag, Berlin, Heidelberg, New York, 2003.
[74] A. Languasco and A. Perelli. Pair correlation of zeros, primes in short intervals and exponential sums over primes. J. Number Theory, 84:292-304, 2000. DOI-link, MR:1796516, ZBL:0973.11081.
[75] D. Bazzanella and A. Languasco. On the asymptotic formula for Goldbach numbers in short intervals. Studia Sci. Math. Hungar., 36:185-199, 2000. DOI-link, MR:1768230, ZBL:0973.11089.
[76] J. Kaczorowski, A. Languasco, and A. Perelli. A note on Landau's formula. Funct. Approx. Comment. Math., 28:173-186, 2000. Dedicated to W lodzimierz Staś on the occasion of his 75th birthday. DOI-link, MR:1824002, ZBL:1034.11049.
[77] A. Languasco. A conditional result on Goldbach numbers in short intervals. Acta Arith., 83:93-103, 1998. DOI-link, MR:1490641, ZBL:0940.11045.
[78] A. Languasco. A note on primes and Goldbach numbers in short intervals. Acta Math. Hungar., 79:191-206, 1998. DOI-link, MR:1616038, ZBL:0940.11046.
[79] A. Languasco. A singular series average and Goldbach numbers in short intervals. Acta Arith., 83:171-179, 1998. DOI-link, MR:1490647, ZBL:0894.11037.
[80] A. Languasco and A. Perelli. A pair correlation hypothesis and the exceptional set in Goldbach's problem. Mathematika, 43:349-361, 1996. DOI-link, MR:1433280, ZBL:0884.11042.
[81] A. Languasco. Some results on Goldbach's problem. Rend. Sem. Mat. Univ. Politec. Torino, 53(4):325-337, 1995. DOI-link, MR:1452389, ZBL:0882.11055.
[82] A. Languasco and A. Perelli. On Linnik's theorem on Goldbach numbers in short intervals and related problems. Ann. Inst. Fourier, 44:307-322, 1994. DOI-link, MR:1296733, ZBL:0799.11040.

## Web-publications:

[83] A. Languasco and A. Zaccagnini. Alcune proprietà dei numeri primi, I. Sito web Bocconi-Pristem, 2005. link1; link2.
[84] A. Languasco and A. Zaccagnini. Alcune proprietà dei numeri primi, II. Sito web Bocconi-Pristem, 2005. link1; link2.
[85] A. Languasco and A. Zaccagnini. Esistono piccoli intervalli fra primi consecutivi! Sito web Bocconi-Pristem, 2005. link1; link2.
[86] A. Languasco and A. Zaccagnini. Intervalli fra numeri primi consecutivi. Sito web Bocconi-Pristem, 2005. link1; link2.

## Lecture notes and other publications (in Italian):

[87] A. Languasco. Dispense per il Corso di Fondamenti di Analisi Matematica 2. Lecture Notes, Manuscript, (italian), 2020.
[88] A. Languasco. Dispense per il Corso di Metodi Matematici per la Statistica (parte di Analisi Matematica). Lecture Notes, Manuscript, (italian), 2005.
[89] B. Bruno and A. Languasco. Dispense integrative per il Corso di Istituzioni di Analisi Matematica II. Lecture Notes, Manuscript, (italian), 2003.
[90] A. Languasco. Dispense di Algebra Lineare, Geometria e Calcolo Differenziale in più variabili (Matematica B). Lecture Notes, Manuscript, (italian), 2002.
[91] A. Languasco. Dispense di Analisi Matematica 1. Lecture Notes, Manuscript, (italian), 1999.
[92] A. Languasco. La congettura di Goldbach. PhD thesis, Politecnico di Torino, Università di Torino, Università di Genova, 1995. (italian).
[93] A. Languasco. Codici a chiave pubblica ed Algoritmi di Primalità. Master's thesis, Università di Genova, 1989. (italian).

Padova, March 19, 2024


