$\mathcal{PT}$ symmetry*

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Abstract

The average quantum physicist on the street believes that a quantum-mechanical Hamiltonian must be Dirac Hermitian (symmetric under combined matrix transposition and complex conjugation) in order that the energy eigenvalues are real and that time evolution is unitary. However, the Hamiltonian $H = p^2 + ix^3$, for example, which is clearly not Dirac Hermitian, has a real positive discrete spectrum and generates unitary time evolution, and thus it defines a fully consistent quantum mechanics. Evidently, the axiom of Dirac Hermiticity is too restrictive. The Hamiltonian $H = p^2 + ix^3$ is not Dirac Hermitian, but it is $\mathcal{PT}$ symmetric; that is, it is symmetric under combined space reflection $\mathcal{P}$ and time reversal $\mathcal{T}$. In general, if a Hamiltonian H is not Dirac Hermitian but has an unbroken $\mathcal{PT}$ symmetry, there is a procedure for determining the adjoint operation under which H is Hermitian. (One should not assume that the adjoint operation that interchanges bra and ket vectors in the Hilbert space of states is the Dirac adjoint. This would be like postulating a priori what the metric $g^{\mu\nu}$ in curved space is before solving Einstein's equations.)

In the past year, new table-top experiments have been performed that allow one to observe the transition between theories having a broken and an unbroken $\mathcal{PT}$ symmetry.

In the past a number of interesting quantum theories, such as the Lee model and the Pais-Uhlenbeck model, were abandoned because they were thought to have an incurable disease. The symptom of the disease was the appearance of ghost states (states of negative norm). The cause of the disease was that the Hamiltonians for these models were inappropriately treated as if they were Dirac Hermitian. The disease can be cured because the Hamiltonians for these models are $\mathcal{PT}$ symmetric, and one can calculate exactly and in closed form the appropriate adjoint operation under which each Hamiltonian is Hermitian. When this is done, one can see immediately that there are no ghost states and that these models are perfectly acceptable quantum theories. Thus, generalizing the requirement of Dirac Hermiticity to $\mathcal{PT}$ symmetry allows for the possibility of new kinds of quantum theories.

$\mathcal{PT}$-symmetric quantum theories may be viewed as extensions of ordinary quantum theories into the complex domain. PT quantum theories can be better understood when the associated conventional classical-mechanical theories are extended into the complex domain as well. We will show that by extending classical mechanics into the complex domain, the classical-mechanical theories that one obtains share many of the features of ordinary quantum mechanics.

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