

# The convergence of Fourier-Takenaka-Malmquist series\*

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## Abstract

The Takenaka-Malmquist basis is defined as

$$\mathcal{B}_0 = 1, \quad \mathcal{B}_n(z) = \frac{\sqrt{1 - |\alpha_n|^2}}{1 - \bar{\alpha}_n z} \prod_{k=1}^{n-1} \frac{z - \alpha_k}{1 - \bar{\alpha}_k z}, = \overline{\mathcal{B}_{-n}(1/\bar{z})} \quad n = 1, 2, \dots, \quad z \in \mathbb{C}$$

where  $\{\alpha_k\}_1^\infty$  is a sequence of points in the open unit disk  $\mathbb{D} = \{z \in \mathbb{C} : |z| < 1\}$ . These form an orthogonal system with respect to the Lebesgue measure on the unit circle  $\mathbb{T} = \{z \in \mathbb{C} : |z| = 1\}$ . See for example [2].

If  $f(\omega)$  is a  $2\pi$ -periodic function, we discuss the convergence of the truncated Fourier series

$$f_n(\omega) = \sum_{k < |n|} a_k \mathcal{B}_k(e^{i\omega})$$

to  $f(\omega)$  as  $n \rightarrow \infty$ .

Some of this work is published in [1].

## References

- [1] A. Bultheel and P. Carrette. Algebraic and spectral properties of general Toeplitz matrices. *SIAM J. Control Optim.*, 41(5):1413–1439, 2003.
- [2] J.L. Walsh. *Interpolation and approximation*, volume 20 of *Amer. Math. Soc. Colloq. Publ.* Amer. Math. Soc., Providence, Rhode Island, 3rd edition, 1960. First edition 1935.

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