Accelerating the convergence of continued fraction representations

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Abstract

A lot of well-known constants as well as elementary and special functions in mathematics, physics and engineering enjoy very nice continued fraction representations [9, 7, 10]. In addition, most of these fractions are limit-periodic. There is a lot of literature describing algorithms for the evaluation of these constants or functions making use of their continued fraction representations [8, 6, 4, 5, 3, 2].

The tail or remainder term of a convergent series representation converges to zero. But remarkably, the tail of a convergent continued fraction representation does itself not need to converge at all. A suitable approximation of the usually disregarded continued fraction tail may speed up the convergence of the continued fraction approximants. This idea is further elaborated in this talk [1].

References

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