

# Block QD Algorithm

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## Abstract

The LR algorithm applied to a  $n \times n$  tridiagonal matrix  $A$  and the qd algorithm are closely connected (see [1]). The LU decomposition of  $A$  is recursively used after having multiplied  $U$  by  $L$  at the previous step. If  $A$  is a positive definite symmetric matrix, if  $E^{(k)}Q^{(k)}$  denotes the  $k$ th decomposition and if  $q_i^{(k)}$  denotes the diagonal entry in row  $i$  in  $Q^{(k)}$ , then it was proved (see [2]) that

$$\begin{aligned} q_1^{(k)} &< q_1^{(k+1)}, & \forall k, \\ q_n^{(k)} &> q_n^{(k+1)}, & \forall k. \end{aligned}$$

This result was extended to the case of a positive definite symmetric band matrix of half width  $\ell$  (that is to say  $A_{i,j} = 0$  if  $|i - j| > \ell$ ) such that  $A_{i,j} \neq 0 \forall i, j$  such that  $|i - j| = \ell$  (see [3]).

When  $\ell = 2$ , the characteristic determinant of  $A$  can be written as a six term recurrence relation. But, when  $\ell > 2$ , the expansion of the characteristic determinant as a recurrence relation is inextricable. On the other hand any  $n\ell \times n\ell$  symmetric band matrix  $A$  with a half width  $\ell$  can always be considered as a  $n \times n$  tridiagonal block matrix whose the size of the blocks are  $\ell \times \ell$ . Then it corresponds to a Jacobi block matrix for some matrix orthogonal polynomials. The block LR (or block qd) algorithm can be applied to such a matrix if  $A_{i,j} \neq 0$  if  $|i - j| = \ell$ . Then we prove that the eigenvalues  $\lambda_i^{(k)}$ ,  $i = 1, \dots, \ell$ , of the first diagonal block  $Q_1^{(k)}$  and those  $\lambda_i^{(k)}$ ,  $i = (n - 1)\ell + 1, \dots, n\ell$ , of the last diagonal block  $Q_n^{(k)}$  are such that

$$\begin{aligned} \lambda_i^{(k)} &< \lambda_i^{(k+1)}, & i = 1, \dots, \ell, \forall k, \\ \lambda_i^{(k)} &> \lambda_i^{(k+1)}, & i = (n - 1)\ell + 1, \dots, n\ell, \forall k. \end{aligned}$$

## References

- [1] C. Brezinski, *Padé-Type Approximation and General Orthogonal Polynomials*, ISNM vol. 50, Birkhäuser, Basel, 1980.
- [2] A. Draux, *Improvement of the formal and numerical estimation of the constant in some Markov-Bernstein inequalities*, Numerical Algorithms **24** (2000), 31-58.
- [3] A. Draux, B. Moalla and M. Sadik, *Generalized qd algorithm and Markov-Bernstein inequalities for Jacobi weight*, to appear in Numerical Algorithms (2009).