Padé approximant in complex points revisited

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Abstract

It is *"well known"* that the interlaced zeros and poles of Padé approximants describe the position of cuts of considered function f. More, the Padé approximants choose automaticaly this position, "in principal" in the direction joining the point of developpement of f and its ramification point. The *"well known"* property was studied by J.S.R.Chisholm and A.C.Genz et M.Pusterla forthy years aqo for *ln(1-z)* function at the complex points, but the *"well known"* results produced by the authors are false. A number of numerical examples show that the positions of zeros and poles deviate from the supposed *"well known"* position. We show also, that only N-point Padé approximants computed with pairs of points, complex and complex conjugate, leads to the traditional position of cuts.

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