

Convergence of ray sequences of Padé approximants for ${}_2F_1(a, 1; c; z)$, for $c > a > 0^*$

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Abstract

The Padé table of ${}_2F_1(a, 1; c; z)$ is normal for $c > a > 0$ (cf. [2]). For $m \geq n - 1$ and $c \notin \mathbb{Z}^-$, the denominator polynomial $Q_{mn}(z)$ in the $[m/n]$ Padé approximant $P_{mn}(z)/Q_{mn}(z)$ for ${}_2F_1(a, 1; c; z)$ and the remainder term $Q_{mn}(z){}_2F_1(a, 1; c; z) - P_{mn}(z)$ were explicitly evaluated by Padé (cf. [1], [3] or [4]). We show that for $c > a > 0$ and $m \geq n - 1$, the poles of $P_{mn}(z)/Q_{mn}(z)$ lie on the cut $(1, \infty)$. We deduce that the sequence of approximants $P_{mn}(z)/Q_{mn}(z)$ converges to ${}_2F_1(a, 1; c; z)$ as $m \rightarrow \infty$, $n/m \rightarrow \rho$ with $0 < \rho \leq 1$, uniformly on compact subsets of the unit disc $|z| < 1$ for $c > a > 0$.

References

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