Quantum Appell polynomials and their quadratic decomposition *

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Abstract

By performing the quadratic decomposition of H_q-Appell sequences, where H_q is the lowering (or annihilating) operator defined by $H_q f(x) = \dfrac{f(qx) - f(x)}{(q-1)x}$, another lowering operator $\mathcal{L}_{q;\varepsilon}$ (with $\varepsilon^2 = 1$) arises, since the two polynomial sequences lying in the principal diagonal are $\mathcal{L}_{q;\varepsilon}$-Appell. Triggered by this result, after developing the concept of the $\mathcal{L}_{q;\varepsilon}$-Appell sequences, all the orthogonal $\mathcal{L}_{q;\varepsilon}$-Appell sequences are sought, which outcome was the *Little q-Laguerre* polynomial sequences - they are indeed the unique ones fulfilling both properties. These latter are not only H_q but also $\mathcal{L}_{q;\varepsilon}$-classical sequences in Hahn's sense, which opens up the problem of finding all the orthogonal sequences $\{P_n\}_{n\geqslant 0}$ such that their orthogonality is preserved by the operator $\mathcal{L}_{q;\varepsilon}$.

References

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