On the JWKB expansion and Borel summability, with particular attention to modifications of the radial Schrödinger equation

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Abstract

The JWKB method is generally characterized as an asymptotic expansion for the logarithm of the wave function in powers of $\hbar$. In applications to the radial Schrödinger equation, retention of $\hbar$ in the centrifugal potential leads to ambiguity. Kramers [1] himself implicitly took advantage of the ambiguity by modifying the potential, replacing $\hbar^2 l(l+1)/2mr^2$ by $\hbar^2(l+1/2)^2/2mr^2$, to get better results. Following Kramers, many modifications of the centrifugal potential have been proposed, which can best be understood in terms of a two-$\hbar$ analysis, one $\hbar$ for expansion, the other for the centrifugal potential, to be set equal at the end to recover the physical problem [2].

When provable, Borel summability greatly enhances the significance of a JWKB expansion, making it equivalent to an exact solution and providing a method for obtaining accurate numerical results. Proof of Borel summability of the some of the two-$\hbar$ JWKB expansions for the radial Coulomb problem will given.

References

[1] H. A. Kramers, *Wave mechanics and half-integral quantisation*, Z. Phys. **39** (1926), no. 10/11, 828–840.

[2] Tatsuya Koike and Harris J. Silverstone, *Rereading Langer's influential 1937 JWKB paper: the unnecessary Langer transformation; the two $\hbar$'s*, (submitted for publication).