Divergent series in quantum mechanics

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Abstract

This talk is concerned with two themes: how to obtain the large-order behavior of the divergent series (LOBDS) and how to use it for the summation of the series.

As shown explicitly for the fisrt time in [1] the large-order behavior of the perturbation coefficients for wide class of the eigenvalue problems can be through dispersion relations ireduced to the WKB calculation of the tunneling through potential barrier. Several recent improvements in constructing the WKB approximation will be described, namely the following.

First, it is not necessary to deal with inadequacy of the WKB approximation at the classical turning points [2]. Second, the WKB approximation can be formulated directly as an expansion in pertinent coupling constant without any reference to quantum-classical correspondence [2]. Third, for multidimensional problems the WKB approximation can be formulated in such a way that the calculation can be reduced to the quadratures and at the same time the method yields systematic approximation to the probability flux [3]. Applying this method to Zeeman effect in hydrogen we were able for the first time to carry out the multidimensional WKB approximation explicitly beyond the leading order [4].

The use of LOBDS for the summation of the series will be discussed next. We show that first LOBDS naturally leads to the form of the sequence transformation [5]. Second LOBDS yields the singularities of the Borel transform [6], third it improves the summation to the smallest term and Padé summation for Stieltjes series.

References

[1] C. M. Bender and T. T. Wu, Phys. Rev. D **7**, 1620 (1973).

[2] J. Zamastil, J. Čížek and L. Skála, Phys. Rev. Lett. **84**, 5683 (2000). See also J. Zamastil, J. Čížek and L. Skála, Phys. Rev. A **63**, 022107 (2001).

[3] J. Zamastil, Phys. Rev. A **72**, 024101 (2005). See also J. Zamastil, V. Špirko, J. Čížek, L. Skála, and O. Bludský, Phys. Rev. A **64**, 042101 (2001).

[4] J. Zamastil and L. Skála, J. Math. Phys **47**, 022106 (2006).

[5] J. Čížek, J. Zamastil and L. Skála, J. Math. Phys. **44**, 962 (2003).

[6] J. C. Le Guillou and J. Zinn-Justin Eds. *Large-Order Behaviour of Perturbation Theory*, North-Holland, Amsterdam, 1990.