## Logica intuizionista proposizionale $\mathbf{LI}_p$

$$\begin{array}{cc} \text{ax-id} & \text{ax-}\bot \\ A \vdash A & \bot \vdash \end{array}$$

$$\frac{\Gamma \vdash \Sigma}{\Gamma, \Gamma' \vdash \Sigma}\ \mathrm{in}_{\mathrm{sx}} \qquad \frac{\Gamma \vdash \Sigma}{\Gamma \vdash \Sigma, \Sigma'}\ \mathrm{in}_{\mathrm{dx}}$$

$$\frac{\Sigma, \Gamma, \Theta, \Gamma', \Delta \vdash \Sigma}{\Sigma, \Gamma', \Theta, \Gamma, \Delta \vdash \Sigma}\ \mathrm{sc}_{\mathrm{sx}} \qquad \frac{\Gamma \vdash \Sigma, \Delta, \Theta, \Delta', \nabla}{\Gamma \vdash \Sigma, \Delta', \Theta, \Delta, \nabla}\ \mathrm{sc}_{\mathrm{dx}}$$

$$\frac{\Sigma, \Gamma, \Gamma, \Delta \vdash A}{\Sigma, \Gamma, \Delta \vdash A}\ \mathrm{cn}_{\mathrm{sx}} \qquad \frac{\Gamma \vdash \Sigma, \Delta, \Delta, \nabla}{\Gamma \vdash \Sigma, \Delta, \nabla}\ \mathrm{cn}_{\mathrm{dx}}$$

$$\frac{\Gamma \vdash A \quad \Gamma \vdash B}{\Gamma \vdash A\&B}\ \&-\mathrm{F} \qquad \frac{\Gamma, A \vdash C}{\Gamma, A\&B \vdash C}\ \&\mathrm{re}_1 \qquad \frac{\Gamma, B \vdash C}{\Gamma, A\&B \vdash C}\ \&\mathrm{re}_2$$

$$\frac{\Gamma, A \vdash C \quad \Gamma, B \vdash C}{\Gamma, A \vee B \vdash C}\ \vee-\mathrm{F} \qquad \frac{\Gamma \vdash A}{\Gamma \vdash A \vee B}\ \vee\mathrm{re}_1 \qquad \frac{\Gamma \vdash B}{\Gamma \vdash A \vee B}\ \vee\mathrm{re}_2$$

$$\frac{\Gamma, A \vdash B}{\Gamma \vdash A \to B} \to -\mathrm{F} \qquad \frac{\Gamma' \vdash A \quad \Gamma, B \vdash C}{\Gamma, A \to B, \Gamma' \vdash C} \to \mathrm{re}$$

## Regole derivate per $\mathbf{LI}_p$

$$\begin{array}{cc} \text{ax-id}^* & \text{ax-}\bot^* \\ \Gamma, A, \Gamma' \vdash A & \Gamma, \bot, \Gamma' \vdash \Sigma \end{array}$$

$$\frac{\Gamma, A \vdash \bot}{\Gamma \vdash \neg A}\ \neg-\mathrm{F} \qquad \frac{\Gamma' \vdash A}{\Gamma, \neg A, \Gamma' \vdash C}\ \neg\mathrm{re}$$

$$\begin{array}{cc} \neg\text{-ax}_{sx1} & \neg\text{-ax}_{sx2} \\ \Gamma, A, \Gamma', \neg A, \Gamma'' \vdash C & \Gamma, \neg A, \Gamma', A, \Gamma'' \vdash C \end{array}$$

$$\frac{\Gamma, A, B \vdash C}{\Gamma, A\&B \vdash C}\ \&-\mathrm{D}$$

$$\frac{\Gamma, A \to B \vdash A \quad \Gamma, B \vdash C}{\Gamma, A \to B \vdash C} \to \mathrm{re}^*$$

**Logica classica proposizionale** $LC_p$

$$\begin{array}{cc} \text{ax-id} & \text{ax-}\bot \\ A \vdash A & \bot\vdash \end{array}$$

$$\frac{\Gamma \vdash \Sigma}{\Gamma, \Gamma' \vdash \Sigma}\ \text{in}_{\text{sx}} \qquad \frac{\Gamma \vdash \Sigma}{\Gamma \vdash \Sigma, \Sigma'}\ \text{in}_{\text{dx}}$$

$$\frac{\Sigma, \Gamma, \Theta, \Gamma', \Delta \vdash \Sigma}{\Sigma, \Gamma', \Theta, \Gamma, \Delta \vdash \Sigma}\ \text{sc}_{\text{sx}} \qquad \frac{\Gamma \vdash \Sigma, \Delta, \Theta, \Delta', \nabla}{\Gamma \vdash \Sigma, \Delta', \Theta, \Delta, \nabla}\ \text{sc}_{\text{dx}}$$

$$\frac{\Sigma, \Gamma, \Gamma, \Delta \vdash \nabla}{\Sigma, \Gamma, \Delta \vdash \nabla}\ \text{cn}_{\text{sx}} \qquad \frac{\Gamma \vdash \Sigma, \Delta, \Delta, \nabla}{\Gamma \vdash \Sigma, \Delta, \nabla}\ \text{cn}_{\text{dx}}$$

$$\frac{\Gamma \vdash A, \nabla \quad \Gamma \vdash B, \nabla}{\Gamma \vdash A\&B, \nabla}\ \&-\text{D} \qquad \frac{\Gamma, A \vdash \nabla}{\Gamma, A\&B \vdash \nabla}\ \&\text{re}_1 \qquad \frac{\Gamma, B \vdash \nabla}{\Gamma, A\&B \vdash \nabla}\ \&\text{re}_2$$

$$\frac{\Gamma, A \vdash \nabla \quad \Gamma, B \vdash \nabla}{\Gamma, A \vee B \vdash \nabla}\ \vee-\text{F} \qquad \frac{\Gamma \vdash A, \nabla}{\Gamma \vdash A \vee B, \nabla}\ \vee-\text{D}_1 \qquad \frac{\Gamma \vdash B, \nabla}{\Gamma \vdash A \vee B, \nabla}\ \vee-\text{D}_2$$

$$\frac{\Gamma, A \vdash B, \nabla}{\Gamma \vdash A \to B, \nabla}\ \to -\text{D} \qquad \frac{\Gamma' \vdash A, \nabla \quad \Gamma, B \vdash \nabla}{\Gamma, A \to B, \Gamma' \vdash \nabla}\ \to -\text{re}_c$$

**Calcolo classico proposizionale** $\mathbf{LC}_p^{abbr}$

$$\begin{array}{cc} \text{ax-id}^* & \text{ax}^*\text{-}\bot \\ \Gamma, A, \Gamma' \vdash \Delta, A, \Delta' & \Gamma, \bot, \Gamma' \vdash \nabla \end{array}$$

$$\frac{\Sigma, \Gamma, \Theta, \Gamma', \Delta \vdash \Sigma}{\Sigma, \Gamma', \Theta, \Gamma, \Delta \vdash \Sigma}\ \text{sc}_{\text{sx}} \qquad \frac{\Gamma \vdash \Sigma, \Delta, \Theta, \Delta', \nabla}{\Gamma \vdash \Sigma, \Delta', \Theta, \Delta, \nabla}\ \text{sc}_{\text{dx}}$$

$$\frac{\Gamma \vdash A, \nabla \quad \Gamma \vdash B, \nabla}{\Gamma \vdash A\&B, \nabla}\ \&-\text{D} \qquad \frac{\Gamma, A, B \vdash \nabla}{\Gamma, A\&B \vdash \nabla}\ \&-\text{D}$$

$$\frac{\Gamma, A \vdash \nabla \quad \Gamma, B \vdash \nabla}{\Gamma, A \vee B \vdash \nabla}\ \vee-\text{F} \qquad \frac{\Gamma \vdash A, B, \nabla}{\Gamma \vdash A \vee B, \nabla}\ \vee-\text{D}$$

$$\frac{\Gamma, A \vdash B, \nabla}{\Gamma \vdash A \to B, \nabla}\ \to -\text{D} \qquad \frac{\Gamma \vdash A, \nabla \quad \Gamma, B \vdash \nabla}{\Gamma, A \to B \vdash \nabla}\ \to -\text{S}$$

## Regole derivate per $\mathbf{LC}_p$

$$\frac{\Gamma, A \vdash \nabla}{\Gamma \vdash \neg A, \nabla}\ \neg\!-\!\mathrm{F} \qquad \frac{\Gamma \vdash A, \nabla}{\Gamma, \neg A \vdash \nabla}\ \neg\mathrm{S}$$

$$\overset{\neg\text{-ax}_{sx1}}{\Gamma, A, \Gamma', \neg A, \Gamma'' \vdash \nabla} \qquad \overset{\neg\text{-ax}_{sx2}}{\Gamma, \neg A, \Gamma', A, \Gamma'' \vdash \nabla}$$

$$\overset{\neg\text{-ax}_{dx1}}{\Gamma \vdash \Sigma, A, \Sigma', \neg A, \Sigma''} \qquad \overset{\neg\text{-ax}_{dx2}}{\Gamma \vdash \Sigma, \neg A, \Sigma', A, \Sigma''}$$