Logica intuizionista predicativa LI

$$\begin{array}{c} \text{ax-id} \\ A \vdash A \end{array} \qquad \begin{array}{c} \text{ax-}\bot \\ \bot\vdash \end{array}$$

$$\frac{\Gamma \vdash \Sigma}{\Gamma, \Gamma' \vdash \Sigma}\ \text{in}_{\text{sx}} \qquad \frac{\Gamma \vdash \Sigma}{\Gamma \vdash \Sigma, \Sigma'}\ \text{in}_{\text{dx}}$$

$$\frac{\Sigma, \Gamma, \Theta, \Gamma', \Delta \vdash \Sigma}{\Sigma, \Gamma', \Theta, \Gamma, \Delta \vdash \Sigma}\ \text{sc}_{\text{sx}} \qquad \frac{\Gamma \vdash \Sigma, \Delta, \Theta, \Delta', \nabla}{\Gamma \vdash \Sigma, \Delta', \Theta, \Delta, \nabla}\ \text{sc}_{\text{dx}}$$

$$\frac{\Sigma, \Gamma, \Gamma, \Delta \vdash A}{\Sigma, \Gamma, \Delta \vdash A}\ \text{cn}_{\text{sx}} \qquad \frac{\Gamma \vdash \Sigma, \Delta, \Delta, \nabla}{\Gamma \vdash \Sigma, \Delta, \nabla}\ \text{cn}_{\text{dx}}$$

$$\frac{\Gamma \vdash A \quad \Gamma \vdash B}{\Gamma \vdash A\&B}\ \&-\text{F} \qquad \frac{\Gamma, A \vdash C}{\Gamma, A\&B \vdash C}\ \&-\text{re}_1 \qquad \frac{\Gamma, B \vdash C}{\Gamma, A\&B \vdash C}\ \&-\text{re}_2$$

$$\frac{\Gamma, A \vdash C \quad \Gamma, B \vdash C}{\Gamma, A \vee B \vdash C}\ \vee-\text{F} \qquad \frac{\Gamma \vdash A}{\Gamma \vdash A \vee B}\ \vee-\text{re}_1 \qquad \frac{\Gamma \vdash B}{\Gamma \vdash A \vee B}\ \vee-\text{re}_2$$

$$\frac{\Gamma, A \vdash B}{\Gamma \vdash A \rightarrow B}\ \rightarrow-\text{F} \qquad \frac{\Gamma' \vdash A \quad \Gamma, B \vdash C}{\Gamma, A \rightarrow B, \Gamma' \vdash C}\ \rightarrow-\text{re}$$

$$\frac{\Gamma \vdash A(x)}{\Gamma \vdash \forall x A(x)}\ \forall-\text{F}\ (x \notin VL(\Gamma)) \qquad \frac{\Gamma, A(t) \vdash C}{\Gamma, \forall x\ A(x) \vdash C}\ \forall-\text{re}$$

$$\frac{\Gamma, A(x) \vdash C}{\Gamma, \exists x\ A(x) \vdash C}\ \exists-\text{F}\ (x \notin VL(\Gamma, C)) \qquad \frac{\Gamma \vdash A(t)}{\Gamma \vdash \exists\ x\ A(x)}\ \exists-\text{re}$$

$$\begin{array}{c} =-\text{ax} \\ \vdash t = t \end{array} \qquad \frac{\Gamma(t) \vdash C(t)}{\Gamma(s), t = s \vdash C(s)}\ =-\text{F}$$

Regole derivate per LI

$$\text{ax-id}^*\qquad\qquad \text{ax-}\bot^*$$
$$\Gamma, A, \Gamma' \vdash A \qquad\qquad \Gamma, \bot, \Gamma' \vdash \Sigma$$

$$\frac{\Gamma, A \vdash \bot}{\Gamma \vdash \neg A}\ \neg-\text{F} \qquad\qquad \frac{\Gamma' \vdash A}{\Gamma, \neg A, \Gamma' \vdash C}\ \neg-\text{re}$$

$$\neg\text{-ax}_{sx1}\qquad\qquad \neg\text{-ax}_{sx2}$$
$$\Gamma, A, \Gamma', \neg A, \Gamma'' \vdash C \qquad\qquad \Gamma, \neg A, \Gamma', A, \Gamma'' \vdash C$$

$$\frac{\Gamma, A, B \vdash C}{\Gamma, A\&B \vdash C}\ \&-\text{S}$$

$$\frac{\Gamma, A \to B \vdash A \quad \Gamma, B \vdash C}{\Gamma, A \to B \vdash C}\ \to \text{re}^*$$

$$\frac{\Gamma, \forall x\ A(x), A(t) \vdash C}{\Gamma, \forall x\ A(x) \vdash C}\ \forall-\text{re}^*$$

$$\text{rf}^*\qquad\qquad \text{sm}^*$$
$$\vdash t = t \qquad\qquad \Gamma, t = s \vdash s = t$$

$$\text{tra}^*\qquad\qquad \text{cf}^*$$
$$\Gamma, s = t, t = u \vdash s = u \qquad\qquad \Gamma, t = s \vdash f(t) = f(s)$$

$$\text{cp}^*$$
$$\Gamma, P(t), t = s \vdash P(s)$$

Logica classica predicativa LC

$$\begin{array}{ccc} \text{ax-id} & \qquad & \text{ax-}\bot \\ A \vdash A & & \bot\vdash \end{array}$$

$$\frac{\Gamma \vdash \Sigma}{\Gamma, \Gamma' \vdash \Sigma}\ \text{in}_{\text{sx}} \qquad \frac{\Gamma \vdash \Sigma}{\Gamma \vdash \Sigma, \Sigma'}\ \text{in}_{\text{dx}}$$

$$\frac{\Sigma, \Gamma, \Theta, \Gamma', \Delta \vdash \Sigma}{\Sigma, \Gamma', \Theta, \Gamma, \Delta \vdash \Sigma}\ \text{sc}_{\text{sx}} \qquad \frac{\Gamma \vdash \Sigma, \Delta, \Theta, \Delta', \nabla}{\Gamma \vdash \Sigma, \Delta', \Theta, \Delta, \nabla}\ \text{sc}_{\text{dx}}$$

$$\frac{\Sigma, \Gamma, \Gamma, \Delta \vdash \nabla}{\Sigma, \Gamma, \Delta \vdash \nabla}\ \text{cn}_{\text{sx}} \qquad \frac{\Gamma \vdash \Sigma, \Delta, \Delta, \nabla}{\Gamma \vdash \Sigma, \Delta, \nabla}\ \text{cn}_{\text{dx}}$$

$$\frac{\Gamma \vdash A, \nabla \quad \Gamma \vdash B, \nabla}{\Gamma \vdash A\&B, \nabla}\ \&-\text{D} \qquad \frac{\Gamma, A \vdash \nabla}{\Gamma, A\&B \vdash \nabla}\ \&-\text{re}_1 \qquad \frac{\Gamma, B \vdash \nabla}{\Gamma, A\&B \vdash \nabla}\ \&-\text{re}_2$$

$$\frac{\Gamma, A \vdash \nabla \quad \Gamma, B \vdash \nabla}{\Gamma, A \vee B \vdash \nabla}\ \vee-\text{F} \qquad \frac{\Gamma \vdash A, \nabla}{\Gamma \vdash A \vee B, \nabla}\ \vee-\text{D}_1 \qquad \frac{\Gamma \vdash B, \nabla}{\Gamma \vdash A \vee B, \nabla}\ \vee-\text{D}_2$$

$$\frac{\Gamma, A \vdash B, \nabla}{\Gamma \vdash A \rightarrow B, \nabla} \rightarrow -\text{D} \qquad \frac{\Gamma' \vdash A, \nabla \quad \Gamma, B \vdash \nabla}{\Gamma, A \rightarrow B, \Gamma' \vdash \nabla} \rightarrow -\text{re}_c$$

$$\frac{\Gamma \vdash A(x), \Delta}{\Gamma \vdash \forall x A(x), \Delta}\ \forall-\text{D}\ (x \notin VL(\Gamma, \Delta)) \qquad \frac{\Gamma, A(t) \vdash \Delta}{\Gamma, \forall x\ A(x) \vdash \Delta}\ \forall-\text{S}$$

$$\frac{\Gamma, A(x) \vdash \Delta}{\Gamma, \exists x\ A(x) \vdash \Delta}\ \exists-\text{S}\ (x \notin VL(\Gamma, \Delta)) \qquad \frac{\Gamma \vdash A(t), \Delta}{\Gamma \vdash \exists\ x\ A(x), \Delta}\ \exists-\text{D}$$

$$\begin{array}{c} = -\text{ax} \\ \vdash t = t \end{array} \qquad \frac{\Gamma(t) \vdash \Delta(t)}{\Gamma(s), t = s \vdash \Delta(s)} = -\text{F}$$

Calcolo classico predicativo LC^{abbr}

$$\text{ax-id}^* \quad \Gamma, A, \Gamma' \vdash \Delta, A, \Delta' \qquad\qquad \text{ax}^*\text{-}\bot \quad \Gamma, \bot, \Gamma' \vdash \nabla$$

$$\frac{\Sigma, \Gamma, \Theta, \Gamma', \Delta \vdash \Sigma}{\Sigma, \Gamma', \Theta, \Gamma, \Delta \vdash \Sigma}\ \text{sc}_{\text{sx}} \qquad \frac{\Gamma \vdash \Sigma, \Delta, \Theta, \Delta', \nabla}{\Gamma \vdash \Sigma, \Delta', \Theta, \Delta, \nabla}\ \text{sc}_{\text{dx}}$$

$$\frac{\Gamma \vdash A, \nabla \quad \Gamma \vdash B, \nabla}{\Gamma \vdash A\&B, \nabla}\ \&-\text{D} \qquad \frac{\Gamma, A, B \vdash \nabla}{\Gamma, A\&B \vdash \nabla}\ \&-\text{S}$$

$$\frac{\Gamma, A \vdash \nabla \quad \Gamma, B \vdash \nabla}{\Gamma, A \vee B \vdash \nabla}\ \vee-\text{F} \qquad \frac{\Gamma \vdash A, B, \nabla}{\Gamma \vdash A \vee B, \nabla}\ \vee-\text{D}$$

$$\frac{\Gamma, A \vdash B, \nabla}{\Gamma \vdash A \to B, \nabla} \to -\text{D} \qquad \frac{\Gamma \vdash A, \nabla \quad \Gamma, B \vdash \nabla}{\Gamma, A \to B \vdash \nabla} \to -\text{S}$$

$$\frac{\Gamma \vdash A(x), \nabla}{\Gamma \vdash \forall x A(x), \nabla}\ \forall-\text{F}\ (x \notin VL(\Gamma, \nabla)) \qquad \frac{\Gamma, \forall x\ A(x), A(t) \vdash \nabla}{\Gamma, \forall x\ A(x) \vdash \nabla}\ \forall-\text{re}^*$$

$$\frac{\Gamma, A(x) \vdash \nabla}{\Gamma, \exists x\ A(x) \vdash \nabla}\ \exists-\text{F}\ (x \notin VL(\Gamma, \Delta)) \qquad \frac{\Gamma \vdash A(t), \exists\ x\ A(x), \nabla}{\Gamma \vdash \exists x\ A(x), \nabla}\ \exists-\text{re}^*$$

$$= -\text{ax} \quad \vdash t = t, \nabla \qquad\qquad \frac{\Gamma(t) \vdash \Delta(t)}{\Gamma(s), t = s \vdash \Delta(s)} = -\text{F}$$

Regole derivate per LC

$$\frac{\Gamma, A \vdash \nabla}{\Gamma \vdash \neg A, \nabla}\ \neg-\text{F} \qquad \frac{\Gamma \vdash A, \nabla}{\Gamma, \neg A \vdash \nabla}\ \neg-\text{S}$$

$$\neg\text{-ax}_{sx1} \quad \Gamma, A, \Gamma', \neg A, \Gamma'' \vdash \nabla \qquad \neg\text{-ax}_{sx2} \quad \Gamma, \neg A, \Gamma', A, \Gamma'' \vdash \nabla$$

$$\neg\text{-ax}_{dx1} \quad \Gamma \vdash \Sigma, A, \Sigma', \neg A, \Sigma'' \qquad \neg\text{-ax}_{dx2} \quad \Gamma \vdash \Sigma, \neg A, \Sigma', A, \Sigma''$$

$$\text{rf}^* \quad \vdash t = t, \Delta \qquad \text{sm}^* \quad \Gamma, t = s \vdash s = t, \Delta$$

$$\text{tra}^* \quad \Gamma, s = t, t = u \vdash s = u, \Delta \qquad \text{cf}^* \quad \Gamma, t = s \vdash f(t) = f(s), \Delta \qquad \text{cp}^* \quad \Gamma, P(t), t = s \vdash P(s), \Delta$$