Beyond B-splines: generalized B-splines, exponential B-splines and pseudo-splines with emphasis on their refinement properties

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Abstract

It is well known that B-splines are a powerful basis for polynomial splines with, beside other nice properties, minimal support with respect to a given degree and smoothness. Any spline function of given degree can be expressed as a linear combination of B-splines of that degree. Starting from a set of control points the latter is in fact the way curve and surface splines are constructed in computer-aided design and computer graphics. B-splines, in particular cardinal B-splines -i.e. B-splines with uniform knots,- find application also in other contexts than design like approximation theory, curve/surface fitting, numerical differentiation and integration, signal and image processing. Due to their refinability property cardinal B-splines are also suitable for multiresolution, multilevel and subdivision approaches which play an important role in numerical analysis. In spite of their celebrity, polynomial B-splines present several drawbacks. They have low approximation order and are not able to exactly reproduce geometries like conic sections which are important in design, biomedical imaging or isogeometric analysis. Also, they do not approximate well causal exponentials that play a fundamental role, for example, in classical system theory. This is why, in the last two decades several generalization of B-splines have been proposed, the most popular being Non Uniform Rational B-splineS (NURBS) that have received an increasing attention in the geometric modeling community, in particular. While NURBS are actually able to exactly reproduce a huge variety of geometries, transcendental curves like helix or cycloid are still excluded and modeling of manifolds with arbitrary topology is conceptually very complicated and extremely expensive. Moreover, NURBS require additional parameters or weights which do not have an evident geometric meaning and whose selection is often unclear. Last but not least, their rational nature is unpleasant with respect to differentiation and integration. To overcome the drawbacks of NURBS, generalized B-splines became, recently, an attractive alternative to the rational model. While classical B-splines are piecewise functions with sections in the space of algebraic polynomials, generalized B-splines are piecewise functions with sections in more general spaces. With a suitable selection of such spaces generalized B-splines allow exact representation of polynomial curves, conic sections, helices and other profiles. They possess all fundamental properties of polynomial B-splines and behave completely similar to B-splines with respect to differentiation and integration. Cardinal exponential B-splines are a crucial instance of such a class of basis suitable for multiresolution, multilevel and subdivision approaches. An other interesting generalization of polynomial B-splines recently emerged is given by pseudo-splines and, more in general by exponential pseudo-splines. Exponential pseudo-splines are a rich family of basis functions meeting various demands for balancing approximation power, regularity, support size, interpolation, reproduction capability and refinability. Their refinability properties combined with high approximation order make them useful, for example, to construct tight wavelet frames to be used in multiresolution analysis approaches in signal and image processing.

The talk will start with a review of polynomial B-splines with special emphasis on their refinement properties and on the corresponding subdivision algorithms for cardinal B-splines. We will then define and discuss exponential B-splines and exponential pseudo-splines in the uniform case yet by the help of a subdivision perspective.