Irregular Tight Wavelet Frames: Matrix Approach

Alberto Viscardi

Department of Mathematics and Applications
University of Milano-Bicocca
Via Cozzi, 55 - 20125 Milano
alberto.viscardi@unimib.it

To construct compactly supported tight wavelet frames in the shift-invariant setting there are powerful tools such as the Unitary Extension Principle, or the Oblique Extension Principles for higher vanishing moments. It is well-known that those principles lead to matrix extension problems. The entries of the corresponding matrices are trigonometric polynomials. In the univariate setting, explicit expressions for such extensions are known for a wide class of trigonometric polynomials arising from refinement equations. Our first goal is to reduce such matrix extension problems to the factorization of positive semi-definite matrices with real entries. In the non shift-invariant setting, Chui, He and Stöckler showed how to construct tight frame elements via the factorization of global positive semi-definite matrices derived from B-splines over irregular knot sequences ([1], [2]). Our second goal is to construct such global matrices for general univariate irregular MRA and to simplify their factorizations. Our simplification leads to the factorization of few positive semi-definite matrices of much smaller size.

[1] C. K. Chui, W. He, J. Stöckler, *"Nonstationary tight wavelet frames, I: Bounded intervals"*, Appl. Comput. Harmon. Anal. 17 (2004), 141–197;

[2] C. K. Chui, W. He, J. Stöckler, *"Nonstationary tight wavelet frames, II: Unbounded intervals"*, Appl. Comput. Harmon. Anal. 18 (2005), 25–66.