Degrees of Polynomial Appoximation in holomorphic Carleman classes

Moulay Taïb BELGHITI Boutayeb EL AMMARI Laurent P. GENDRE

Ibn Tofaïl University, Morocco and Paul Sabatier University, France

Abstract

In this talk, we extend results of M.S. Baouendi and C. Goulaouic (Ann. Inst. Fourrier, 1971 ; Trans. Amer. Math. Soc, 1974), obtained for compacts of $\mathbb{R}^N$ with analytic boundary. If K is a compact of $\mathbb{C}^N \backsimeq \mathbb{R}^{2N}$, $(N \geq 1)$, $\mathcal{H}_M(K)$ is the space of $\overline{\partial}$-Whitney jets on K which are of class $\{M\}$, where $M(t) = t^t e^{t\mu(t)}$, $t >> 0$ and μ belongs to a Hardy field.
We prove that a jet $F := (F^\alpha)_{\alpha \in \mathbb{N}^{2N}} \in \mathcal{H}_M(K)$ if and only if there exist a constant $C > 0$, such that

$$\lim_{n\to\infty} d_n(F^\alpha, K)\exp(C\overline{\omega}_{K,M}(n)) = 0, \quad \text{for all} \quad \alpha \in \mathbb{N}^{2N}, \tag{1}$$

where $d_n(\cdot, K)$ is the distance, for the uniform norm on K to the complex vectorial space of polynomials of degree at most n, and where $\overline{\omega}_{K,M}$ is a weight depending on the class $\{M\}$ and K.

If K is Whitney-regular

$$\mathcal{H}_M(K) \backsimeq \left\{ f \in \mathcal{E}^\infty(K) \cap \mathcal{O}(\dot{K}) : \exists C > 0,\ \exists \rho > 0,\ \|D^\alpha f\|_K \leqslant C\rho^{|\alpha|} M(|\alpha|),\ (\forall \alpha \in \mathbb{N}^N) \right\}.$$

In this situation, $f \in \mathcal{H}_M(K)$ if and only if $\lim\limits_{n\to\infty} d_n(f, K)e^{C\overline{\omega}(n)} = 0$, where $C > 0$ and $\overline{\omega}$ is a weight depending on {M}. Finally, we annonce similar results in the situation where K is a compact of some Stein manifold. A crucial role is played by a new geometric criteria : the Łojasiewicz-Siciak condition for the Green function of K.