

# Approximation of hypersingular integral transforms on the real axis

Maria Carmela De Bonis, Donatella Occorsio\*

In this talk we propose some numerical procedures for approximating hypersingular integrals of the type

$$\mathbf{H}_p(fw_{\alpha,\beta}, t) = \oint_{-\infty}^{+\infty} \frac{f(x)}{(x-t)^{p+1}} w_{\alpha,\beta}(x) dx, \quad t \in \mathbb{R}, \quad (1)$$

where  $w_{\alpha,\beta}(x) = |x|^\alpha e^{-|x|^\beta}$  is a generalized Freud weight with  $\alpha \geq 0, \beta > 1$  and  $0 \leq p \in \mathbb{N}$ .

This topic is of interest, for instance, in the numerical solution of hypersingular integral equations, which are often models for physics and engineering problems (see [5, 2, 4]). To our knowledge, most of the papers available in the literature deal with the approximation of Hadamard integrals on bounded intervals (see for instance [4] and the references therein) and the case on the real semiaxis has been considered recently in [1, 3]. We propose here different procedures, which are differently convenient, according that the computation of the integral is required in “many” or “few” values of the parameter  $t$ . The convergence and stability of the proposed methods are proved and error estimates are given. Some numerical tests are shown in order to compare their performances.

## References

- [1] A. Aimi, M. Diligenti, *Numerical integration schemes for hypersingular integrals on the real line*, Communications to SIMAI Congress, ISSN 1827-9015, Vol. 2 (2007).
- [2] Y. Chan, A. C. Fannjiang, G. H. Paulino, B. Feng, *Finite part integrals and hypersingular kernels*, Advances in Dynamical Systems, **14** (2007), 264–269.
- [3] M. C. De Bonis, D. Occorsio, *Approximation of Hilbert and Hadamard transforms on  $(0, +\infty)$* , submitted.
- [4] G. Monegato, *Definitions, properties and applications of finite-part integrals*, J. Comput. Appl. Math., **229** (2009) 425–439.
- [5] I.K. Lifanov, L.N. Poltavskii, G. M. Vainikko, *Hypersingular Integral Equations and their Applications*, Chapman & Hall CRC, 2003.

---

\*Department of Mathematics, Computer Science and Economics, University of Basilicata, Viale dell’Ateneo Lucano 10, Potenza, ITALY  
 mariacarmela.debonis@unibas.it, donatella.occorsio@unibas.it