On the numerical solution of integral equations of Mellin type in weighted L^p spaces

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We are interested in the numerical solution of second kind integral equations with fixed singularities of Mellin convolution type given by

$$f(y) + \int_0^1 k(x,y)f(x)dx + \int_0^1 h(x,y)f(x)dx = g(y), \quad y \in (0,1], \tag{1}$$

where f is the unknown, h and g are smooth functions and

$$k(x,y) = \pm\frac{1}{x}\bar{k}\left(\frac{y}{x}\right) \tag{2}$$

is a Mellin kernel, defined by means of a function $\bar{k} : [0,+\infty) \to [0,+\infty)$ satisfying suitable assumptions.
Since the kernel $k(x,y)$ has a fixed-point singularity at $x = y = 0$, the corresponding integral operator

$$(Kf)(y) = \int_0^1 k(x,y)f(x)dx$$

is non-compact. Consequently, the standard stability proofs for numerical methods do not apply and a modification of the classical methods in a neighbourhood of the endpoint $y = 0$ is needed.
Generalizing the results in [1], we propose to approximate the solutions of (1) in weighted L^p spaces by applying a "modified" Nyström method which uses a Gauss-Jacobi quadrature formula. The modification of the classical method essentially consists in a new suitable approximation of the integral transform $(Kf)(y)$ in points very close to 0, where the convergence of the Gaussian rule is not assured.
The stability and the convergence are proved in weighted L^p spaces and error estimates are also given. Moreover, some numerical results show the effectiveness of the method.

References

[1] De Bonis, M. C. and Laurita, C., *A modified Nyström method for integral equations with Mellin type kernels*, J. Comp. Appl. Math. 296 (2016) 512–527.