Symmetry in sequent calculus and Matte Blanco's Bi-logic

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***Abstract*—We discuss the problem of symmetry, namely of the orientation of the logical consequence, represented by the sequent sign in sequent calculus. We show that the problem is surprisingly entangled with the problem of "being infinite". We present a model based on quantum states and we show that the requirements of Matte Blanco's symmetric mode are satisfied. We briefly discuss the model for symmetry to include correlations, in order to obtain a possible approach to displacement. In this setting, we find a possible reading of the structural rules of sequent calculus, whose role in computation, on one side, and in the representation of human reasoning, on the other, has been debated for a long time.**

I. Introduction

The present kind of work arises from the convergence of different viewpoints about something that could be considered as a unitary fact, we think. Such viewpoints have been developed under the common aim to investigate the origins of logic, namely the treatment of human judgements. For this reason, the author had contributed to the development of a common platform for several extensional logics, termed *basic logic* [SBF], that is developed as a sequent calculus. In particular, basic logic underlies some quantum logics, and so can be adopted to study them. The author has concentrated on the study of the quantum world hoping that this, in particular, could offer a new perspective for the origins of logic.

Basic logic permits to view logical constants as originated putting equations that allow to import some metalinguistic links between judgements into the object language. Such a feature has been exploited to import judgements concerning quantum physics into logic, and then discuss the features of such judgements [Ba], [Ba2]. In particular, we have considered the problem of negation, in terms of duality, and we have shown that it can be discussed in a spin model, where duality is substituted by *symmetry*, due to the uncertainty [Ba3].

Symmetry is useful to make an analysis of the orientation - from the premises to the conclusions - of theorems. This is an important point, since it is becoming more and more apparent that quantum computational "processes" do not obey the usual, input-output oriented, view of a process [Ca], [DE], [BV]. Then, in order to model quantum "processes", and discuss their embedding into the usual input-output view, logic should discuss how an orientation could arise and, most of all, to see what is possible without it. Our search for a hidden logic has found an enlightening connection with Bi-logic [MB] (see [Ba4]).

In the present paper we better focus on the gap between the metalinguistic and the linguistic level, that permits to see that quantum states correspond to infinite sets, and in particular to conceive the "infinite singletons". We think that the concept of infinite singletons could be very useful to model certain psychological features that, up to now, can be hardly touched by logic. We are developing an approach to correlations by infinite singletons, that is originated by the representation of quantum correlations [Ba3]. Here, we make the hypothesis that the structural rules of sequent calculus could be originated by correlations. The framework of basic logic permits to discuss this point, since basic logic underlies also linear logic [Gi], which is founded on the criticism of structural rules.

Our approach finds something opposite with respect to the usual view of structural rules. For, they have mostly been considered as a negative object for computation, whereas quantum correlations represent the best for computation, since they allow the quantum speed up. The problem, we think, is that logic is based on a compositional approach, whereas the quantum world shows holistic features, due to superposition and correlations, that should be considered by logic [DCGL], [DCGLS]. One can keep that holistic features can be an advantage in information processing. This is clear for example in dealing with languages: on one side, the meaning of a sentence is not given by the sum of the words it contains, on the other, it is possible to grasp the real meaning of a sentence even without knowing the meaning of some, or even a lot, of the words it contains.

The key to obtain our model has been the representation of random variables as first order variables, in order to model quantum states and correlations, exploiting the holistic features of first order quantifiers: as is well known, a universal (existential) sentence is not the same as a conjunction (disjunction) of formulae with closed terms, even if the domain of quantification is finite. On one side, this points to the incompleteness theorems of logic, on the other, as the model shows, to the difference between statistical and quantum mechanics. It is the difference between quantifiers and propositional connectives which allows to consider a quantum state as a unique whole, not an ensenble, namely as an infinite singleton. It is logical incompleteness, namely the gap between meta-level and object level, which allows to conceive infinite singletons.

The work presented here arises from putting together foundations of mathematics and physics together, however we think that the approach to correlations so obtained points to the crucial issue of the possibility to identify contextual information in natural vs artificial and formal systems. We hope that our proposal can help in developing a view "by separation" rather than "by composition" in logic. We recall that Matte Blanco ([MB2]) makes the hypothesis that five different layers of possible identification are present in the human mind, corresponding to five different levels of consciousness, the fifth being the total identification. In particular, one could investigate about possible coupling to ultrametric and quantum models of cognitive processes (see [Kh1], [Kh2], [Lg], [Mu]). In more generality, the paper contributes to the new and increasing field of quantum cognition, we quote, e.g., [Ae], [ABGS], [AGS], [AS], [AS2], [BB], [BPFT], [QI08], [QI09], [QI11], [QI12], [QI13].

II. Symmetry in sequent calculus

Sequent calculus for basic logic [SBF] has the following *symmetry theorem*:

Theorem 2.1: A sequent $\Gamma \vdash \Delta$ is derivable if and only if $\Delta^s \vdash \Gamma^s$ is derivable (where s is defined by induction on formulae, putting $\circ^s \equiv$ the *dual* of $\circ$, for every logical constant $\circ$. On literals, s can be the identity.)

Sketch of the proof: The theorem is proved by induction on derivations, since $A \vdash A$ iff $A^s \vdash A^s$, for every choice of s, and since every left (right) rule of the calculus has a symmetric one, that is the right (left) rule on the *dual* constant.

Usually, literals are considered in dual pairs too. This gives rise to a negation, that is defined on formulae rather than being introduced by a unary connective: the so called Girard negation, introduced in linear logic [Gi].

But the symmetry theorem shows that the duality is required at the inductive step only, that is it is due to connectives: our conception of connectives is in *dual pairs* conjunction and disjunction, universal and existential quantifier. This yields the orientation of the turnstyle $\vdash$, that is the consequence relation, and implies that a logic made of symmetric connectives would render the orientation of the turnstyle irrelevant. In which terms is a *symmetric* logic conceivable, namely a logic based on symmetric connectives? One can notice that a symmetric connective is immediately available, considering a quantifier whose domain is any singleton $\{u\}$, since the equivalence

$$(\forall x \in \{u\})A(x) \equiv (\exists x \in \{u\})A(x)$$

is sound in any model and is proved by the rules on $\forall$ and $\exists$ (such rules, for basic logic, have been given in [MS]).

We are going to extend this idea. Quite surprisingly, we have discovered that it is entangled with "being infinite". This meets Matte Blanco's requirements.

III. Finite and infinite sets

The question: Finite or infinite set? can be answered in two opposite ways, for the same set, at two different levels: the meta-level and at the object-level, with respect to a certain logical system.

For, assume that D is any set. By the logical rules on $\exists$ and $=$, it is provable that

$$z \in D \equiv (\exists x \in D)(x = z)$$

However, even if we recognize that

$$D = \{t_1, \ldots, t_n\}$$

is finite, at the metalevel, the sequent

$$z \in D \vdash z = t_1 \vee \ldots \vee z = t_n$$

is not derivable, as is well known.

We keep that it is possible to count the elements of D in a logic, if the equivalence

$$z \in D \equiv z = t_1 \vee \ldots \vee z = t_n$$

holds: D is finite in that logic. Otherwise D is infinite in that logic.

Moreover, one can see that characterizing finite and infinite sets in the above way, means to distinguish between propositional and predicative. For, one can see that

$$z \in D \equiv z = t_1 \vee \ldots \vee z = t_n$$

is provable if and only if

$$(\forall x \in D)A(x) \equiv A(t_1)\& \ldots \& A(t_n)$$

is provable for every A (see[Ba3].)

This makes it possible to adopt a predicative logical model, developed for quantum computation, where to find out Matte Blanco's symmetry and infinite sets .

IV. Representation of quantum states

We see how a universal proposition can describe the state of a particle $\mathcal{A}$, with respect to a fixed observable.

If Z is the random variable given by the measurement of the particle, we consider the proposition with free variable z:

$A(z) \equiv$ "particle $\mathcal{A}$ is found in state $s(z)$ with probability $p\{Z = s(z)\}$".

and the domain

$$D_Z = \{(s(z), p\{Z = s(z)\})\}$$

that is the set of the eventual outcomes $s(z)$ of Z with their probabilities.

Then, given a set of assumptions Γ concerning the measurement (roughly, Γ corresponds to the preparation of the state, in logical terms), we have that assumptions Γ *yield* $A(z)$ *forall* $z \in D_Z$, and hence we write the sequent ([MS]):

$$\Gamma, z \in D_Z \vdash A(z)$$

that means: "assumptions Γ together with $z \in D_Z$ *yield* $A(z)$".

Notice that Γ cannot depend on the variable z, in our setting. Then we consider the definition of the universal quantifier $\forall$, that is given by means of the following equation ([MS]):

$$\Gamma \vdash (\forall x \in D_Z)A(x) \quad \equiv \quad \Gamma, z \in D_Z \vdash A(z) \qquad (1)$$

In the above definition, the quantifier $\forall$ imports into the object language the pre-existing metalinguistic link *forall* among the assertions $A(z)$ (this is the usual way to consider the connectives in the framework of basic logic). So the variable z acts as a glue to describe the pure state of the particle $\mathcal{A}$, and the proposition

$$(\forall x \in D_Z)A(x)$$

describes its state [Ba].

Now let us assume that the measurement gives outcomes t_i, $i = 1, \ldots, n$, and hence we recognize the set of outcomes $D_Z = \{t_1, \ldots t_n\}$. Due to measurement, we have $\Gamma \vdash A(t_i)$ for every i. This means that

$$\Gamma \vdash A(t_1)\& \ldots \& A(t_n)$$

where & is a conjunction, due to the definition of & . We recall that, in terms of linear logic, & is the additive conjunction, for the antecedent Γ is in common for all the consequents $A(t_i)$.

As above, we exploit the definition to attribute a state to particle $\mathcal{A}$, and say that the proposition

$$A(t_1)\& \ldots \& A(t_n)$$

describes the mixed state after measurement.

Moreover, we consider the provable sequent

$$(\forall x \in D_Z)A(x) \vdash A(t_1)\& \ldots \& A(t_n)$$

and we say that it describes the collapse from the pure to the mixed state due to measurement. The sequent is is proved by substituting z/t_i ([Ba2]). Substitution is like "the collapse of the variable" and represents measurement.

The converse sequent

$$A(t_1)\& \ldots \& A(t_n) \vdash (\forall x \in D_Z)A(x)$$

is derivable if for every A if and only if D_Z is finite [Ba3]. This means after measurement, namely the state is mixed. The sequent is not true prior to measurement: it is impossible to reconstruct the original pure state from the mixed state obtained by measurement. Then D_Z is infinite.

Our conclusion is that a pure state is an infinite set.

V. Infinite singletons

What about the status of sharp states, namely, certain states? Their domain D_Z is a singleton: a unique state with probability one. Since sharp states are pure states, we can develop an infinite way to conceive a singleton.

Singletons are finite by extensionality: they are sets V for which there is an element u such that, if z is any element of V, then z coincides with u. Then we write $V = \{u\}$. Inside a logic, extensionality is translated into the following natural assumption:

$$z \in V \vdash z = u$$

(where u is a closed term of the logical language denoting the same element). This renders singletons finite in that logic.

However, singletons are not splitted by a disjunction, as it happens for finite sets of cardinality greater than one: they are similar to infinite sets in this. They naturally have a borderline behaviour in logic.

One needs simply to characterize singletons avoiding extensionality: for us a singleton is a set V such that

$$(\forall x \in V)A(x) = (\exists x \in V)A(x)$$

for every A.

An analysis of the definitions of $\exists$ and $\forall$ as given in basic logic shows that, equivalently, one takes a duality d such that

$$z \in V, A(y) \vdash A(z), (y \in V)^d$$

for every A (see [Ba3]). If, given V and d, we assume such such sequents as axioms, for every A, the set V behaves like a singleton.

Such axioms are provable sequents when we consider usual finite, extensional singletons, namely when we have $V = \{u\}$ for some u:

$$z = \{u\}, A(y) \vdash A(z), y \neq \{u\}$$

and equivalently the equality $(\forall x \in \{u\})A(x) = (\exists x \in \{u\})A(x)$ is derivable in such a case.

Sequents can prove that any two elements of any singleton are equal, since $z \in V$ is equivalent to $(\exists x \in V)x = z$, that is by definition $(\forall x \in V)x = z$. The point is that one could have no way to capture that unique element by a closed term of the language, in absence of a substitution rule. Then the element(s) of the corresponding set could not be counted, since no way to distinguish/identify something with certainty with a fixed element, denoted by a closed term, is provided. The set has a floating element, capable to assume different identities. We can think it is inhabited by a random variable, which is not identified with the set of its outcomes, even if it is characterized by them.

In the quantum model substituting means measuring, so we can find infinite singletons in the model, when measurement is inhibited, due to uncertainty. As we see below, a quantum state (namely: the first order domain given by an observable incompatible with the preparation of the state) is the best example of an infinite singleton we can provide. We quote [Be] for the best insights on this.

VI. Infinite sets in the spin model

We consider the observable that is proper only of quantum mechanics, namely the spin. We measure the spin of a particle w.r.t. a fixed axis, say the z axis: two outcomes $\uparrow$ and $\downarrow$ are possible. The sets associated with the sharp states are two singletons. The formulae quantified on them are then equivalent to propositional formulae, say $A_\uparrow$ and $A_\downarrow$.

We put a duality $\perp$ switching $\uparrow$ and $\downarrow$. It translates the Pauli matrix σ_X (namely the NOT gate of computation) into logic. We extend $\perp$ to all formulae and obtain a negation (Girard negation).

The duality $\perp$ satisfies the usual equivalence proper of negation:

$$A \vdash B \quad \text{if and only if} \quad B^\perp \vdash A^\perp$$

In turn, σ_X is a self-adjoint operator representing the observable "spin with respect to the x axis", as is well known. The eigenvectors of σ_X are the so called "dual states" + and −: the superposition of ↑ and ↓ both with probability 1/2. The sets associated with the measurement of particles in states + and − contain the two opposite pieces of information ↑ and ↓. In our model, we represent particles in states + and − by predicative formulae, say A_+ and A_-. By our definition, A_+ and A_- are fixed points for the negation $\perp$.

In turn, + and − are switched by the Pauli matrix σ_Z (our observable "spin with respect to the z axis."). Translating σ_Z into logic, we have a new duality $\top$, that can switch A_+ and A_-.

Changing the measurement context and measuring the spin with respect to the x axis would produce an objective property for + and −, that would be represented by singletons. However, different spin observables are incompatible and so the sets for + and − are infinite singletons, considering the duality $\top$.

So the formulae describing + and − satisfy the equivalence

$$A \vdash B \quad \text{if and only if} \quad B \vdash A \tag{2}$$

Then logic hides a symmetric mode, where negation is meaningless, and sets are infinite, like in Matte Blanco. One could ask: what about the original requirement of Matte Blanco concerning symmetry, that is: only symmetric relations? It is satisfied too, since those sets for which every relation is symmetric are singletons.

Moreover, (2) has a further, immediate, reading: the orientation of logical consequence, represented by the sequent sign $\vdash$, disappears as well, consistently with the logical features of the unconscious thinking outlined by Matte Blanco. We now see how this could be linked to the presence of different correlations between judgements, that make the definition of usual logical implication impossible.

VII. Correlations and structural rules

Assuming axioms of the form $z \in V, A(y) \vdash A(z), (y \in V)^d$, one could prove the equality

$$(\forall x \in V)A_1(x) * A_2(x) = (\forall x \in V)A_1(x) * (\forall x \in V)A_2(x) \tag{3}$$

(where $*$ is a disjunction, the multiplicative disjunction in linear logic) for every pair A_1, A_2. The equality, with disjunction, is sound if and only if only if V is a singleton. In order to extend the equality to infinite singletons, we introduce correlations. To this aim, we widen the action of infinite singletons to the second order, considering "infinite singletons of indices of formulae". The correlation takes place since the same variable is displaced elsewhere, considering another index.

Namely, we consider a family of formulae $A_i(z)$, $i \in I$, where z is a common free first order variable and I is an infinite singleton of indices. We write $i \sim j$ to mean that two indices i and j are equal since they are in the same set of indices I. We write

$$\Gamma, z \in V \vdash A_i(z),_\sim A_j(z)$$

to mean that *forall* $z \in V$, the hypothesis Γ *yields* the correlated results $A_i(z)$ and $A_j(z)$. We represent the correlation in the object language, translating it into a connective $\bowtie$, that extends the multiplicative disjunction $*$, and then, following its definition, given in (1), we apply the quantifier $\forall$. Then the assertion $\Gamma, z \in V \vdash A_i(z),_\sim A_j(z)$ is converted into the sequent $\Gamma \vdash (\forall x \in V)A_i(x) \bowtie A_j(x)$. In turn, switching the applications of $\bowtie$ and of the first order quantifier, as in (3), the same assertion leads to $\Gamma \vdash (\forall x \in V)A_i(x) \bowtie (\forall x \in V)A_j(x)$, namely (see[Ba3]):

$$\Gamma \vdash (\forall x \in V)A_i(x) \bowtie A_j(x) = \Gamma \vdash (\forall x \in V)A_i(x) \bowtie (\forall x \in V)A_j(x)$$

But the assertion $\Gamma, z \in V \vdash A_i(z),_\sim A_j(z)$, in our setting, is equivalent to $\Gamma, z \in V, i \sim j \vdash A_i(z)$, since the amount of information contained in $A_i(z)$ is the same amount contained in the correlated results $A_i(z),_\sim A_j(z)$, under the hypothesis $i \sim j$. This permits us to consider a generalized symmetric quantifier, with a first order variable and a variable for indices, both ranging on infinite singletons, which translates the assertion $\Gamma, z \in V, i \sim j \vdash A_i(z)$ and is equivalent to $(\forall x \in V)A_i(x) \bowtie A_j(x)$ or equivalently to $(\forall x \in V)A_i(x) \bowtie (\forall x \in V)A_j(x)$. In the quantum model, we have represented Bell states adopting such a technique.

We have made the hypothesis ([Ba4]) that considering correlations in such a way, namely as a displacement of first order variables on "identical" formulae, could be a way to approach a representation of psychoanalytic displacement, as considered by Matte Blanco. We recall that, following Matte Blanco, displacement takes place by symmetry, since two subclasses are both identified with a larger class (generalization) and then treated as identical. In logic, this is a kind of second-order justification, that we translate into the identification of two indices once they are in the same infinite singleton.

One could wonder if displacement could have a counterpart in our conscious reasoning, namely if it is the symmetric counterpart of some different asymmetric link. A judgement of the form $\Gamma, z \in V \vdash A_i(z),_\sim A_j(z)$, considered above, where the propositions $A_i(z)$ and $A_j(z)$ are correlated, is not suited to be processed in a context-free way. For, A_i and A_j cannot be separated and hence cannot represent a context one with respect to the other, due to the correlation. On the contrary, sequent calculus for classical logic is context free, namely it can treat any formula as a context with respect to the other ones in the sequent. This yields, in particular, the definability of implication ([SBF]). One could consider implication as an asymmetric correlation between two certainties, and hence a sort of natural collapse of correlations, once infinite singletons disappear. It is an intriguing and hard open problem to develop such a point fairly, however we think it is the right way, in logic, to see that quantum processes do not follow the usual input-output orientation, whereas classical processes do. We recall that implication is the standard way to model such an orientation in logic (see the approach to computation "programs as proofs" and the logical semantics of "proofs as programs").

Here, we further develop some observations concerning the symmetry due to infinite singletons, observing that, in our reading, the structural rules of sequent calculus "weakening" and "contraction" could be considered a result of correlations in logic, once correlations have been dropped. Above, due to correlations, we have required the following equivalence:

$$\Gamma, z \in V, i \sim j \vdash A_i(z) \quad \equiv \quad \Gamma, z \in V \vdash A_i(z),_\sim A_j(z)$$

Let us consider the two directions of the equivalence, written as derivation rules. We notice that the direction

$$\frac{\Gamma, z \in V, i \sim j \vdash A_i(z)}{\Gamma, z \in V \vdash A_i(z),_\sim A_j(z)}$$

resembles the structural rules of "weakening" in sequent calculus: namely, in any sequent, one can add conclusions (or premises) preserving its validity:

$$\frac{\Gamma \vdash \Delta}{\Gamma, A \vdash \Delta}\,[lW] \qquad ; \qquad \frac{\Gamma \vdash \Delta}{\Gamma \vdash \Delta, A}\,[rW]$$

The standard interpretation of the comma at the left is conjunction, at the right is disjunction. Left and right weakening are accepted in mathematical reasoning, since the mathematician who has just proved the theorem $\Gamma \vdash \Delta$ simply disregards the new proposition A, considering it as irrelevant. This is due to the perfect context-free attitude shown in this kind of reasoning. On the other side, weakening, expecially left weakening, is not accepted in other kinds of reasoning: for example a medical diagnosis may be dropped after a new symptom is observed, even if the previously observed symptoms continue. Reasoning, in such a case, is context-sensitive, a correlation of the new symptom with the previous ones would enforce the first diagnosis, the lack of correlation would weaken it. Such points have been widely debated in artificial intelligence for a long time, we think it could be useful to approach them from the present point of view.

The converse direction

$$\frac{\Gamma, z \in V \vdash A_i(z),_\sim A_j(z)}{\Gamma, z \in V, i \sim j \vdash A_i(z)}$$

is exactly the structural rule "contraction". Contraction says that the number of occurrences of the same formula in the conclusions (or premises) of a sequent can be contracted to one (notice that, again, such rules are accepted in an abstract kind of reasoning and are rejected in other situations, when the number of occurrences seems relevant):

$$\frac{\Gamma, A, A \vdash \Delta}{\Gamma, A \vdash \Delta}\,[lC] \qquad ; \qquad \frac{\Gamma \vdash \Delta, A, A}{\Gamma \vdash \Delta, A}\,[rC]$$

It is easy to see that the rule at the right is derived from the above one, interpreting $\sim$ as equality [Ba3]. This means that the set of indices is a singleton. In first order logic, the fact that counting the number of occurrences of a formula implies the ability of recognizing formulae as equal or different is not expressed, but our conception of structural rules as valid rules could be originated from an original attitude of dealing with infinite singletons and displacement of variables, that is preserved, even if logic requires that correlations disappear, in our conscious processing of judgements.

VIII. Some concluding remarks

As is well known, linear logic has rejected structural rules. This has lead to splittig the connective of conjunction into two (additive conjunction and multiplicative conjunction), and the same for disjunction (we have additive and multiplicative disjunction). Linear logic divides logical constants by polarities (negative and positive). Basic logic does the same, adopting the terminology "left" and "right" logical constant, depending on the fact that the equation defining the logical constant itself defines it at the left or at the right of the sequent sign $\vdash$. For example, the quantifier $\forall$ and the additive conjunction &, whose definitions have been exploited above, are defined at the right. Due to their definitions, the additive conjunction and the multiplicative disjunction are right connectives, whereas the multiplicative conjunction and the additive disjunction are left connectives. Adding structural rules identifies multiplicative and additive and hence identifies left and right. In this view, even if a duality between conjunction and disjunction can be defined in classical logic, we can say that it is "more symmetric" than linear logic. This is due to the presence of structural rules. On the other side, classical logic has implication, since it has negation. Then no real symmetry is possible in it.

A different point of view is offered by intuitionistic logic, which has structural rules, and where implication is primitive and negation is defined after it. If our hypothesis that implication could be originated by correlations were correct, intuitionistic logic would be the logic that "can keep as more symmetry as possible", even being asymmetric. It seems that intuitionistic logic can better save the role of variables, even if not as random variables: for, the semantics of intuitionistic implication has variables (see the well known Brouwer-Heyting-Kolmogorov interpretation of intuitionistic logic), or, algebraic semantics considers infinitary structures (the frame given by a sup-lattice of open sets). Matte Blanco, quoting Freud, says that "the Unconscious has no objects". In logical terms, we could say that no closed term is considered, only variables. Unfortunately Matte Blanco, as most of people in this world, was not aware of the existence of intuitionistic logic, since only classical logic is considered and taught, so we cannot exploit his insight in this sense.

An alternative approach to the semantics of implication is the idea of necessity. In formal terms, necessity is expressed by a modality in logic. The well known translation of intuitionistic logic into the modal system S4 shows that the intuitionistic implication link is given adding the modal necessity operator. So it seems that some form of normativity plays a decisive role in defining the meaning of intuitionistic implication, and ultimately, in defining what a logical *judgement* is. In the second part of his life, Sigmund Freud introduced the tripartite theory, where the aspect of normativity is clearly included. Matte Blanco does not love the tripartite theory, he says that it hides Freud's first wonderful intuition of the Unconscious (that is a proper name, not an adjective!), of which the Id is a pale substitute. Trusting again in Matte Blanco's intuition, is this a way to say that the original potentialities of variables are hidden by normativity, creating then what we know as logic? It is really difficult to answer to such a question, however, for what concerns the aims of computation, the ambition to answer even very partially could lead us to a much better way to deal with information, exploiting what the Unconscious can do and we cannot.

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