A simple multi-objective optimization problem
Introduction

Let’s introduce a geometrical optimization problem, named **cones problem**, with the following characteristics:

- **multi-objective** problem (two objective functions): the solution is not a single optimum design, but instead it is represented by the set of designs belonging to the *Pareto frontier*
- **simple** mathematical formulation: easy and quick implementation from scratch of the relevant modeFRONTIER project
- **constrained** problem: objectives space and designs space present *feasible* and *unfeasible* regions
Problem definition

Right circular cone:

\[ r = \text{base radius} \]
\[ h = \text{height} \]
\[ s = \text{slant height} \]

\( V = \text{volume} \)
\( B = \text{base area} \)
\( S = \text{lateral surface area} \)
\( T = \text{total area} \)

\[
s = \sqrt{r^2 + h^2}
\]

\[
V = \frac{\pi}{3} r^2 h
\]

\[
B = \pi r^2
\]

\[
S = \pi r s
\]

\[
T = B + S = \pi r (r + s)
\]
Cones problem

- two input variables: \( r, h \)
  \[
  r \in [0, 10] \text{ cm} , \quad h \in [0, 20] \text{ cm}
  \]
- two objectives:
  \[
  \text{min } S \\
  \text{min } T
  \]
- one constraint:
  \[
  V > 200 \text{ cm}^3
  \]

The cone shape (i.e. the design) is defined univocally when both \( r \) and \( h \) are given.

We want to minimize both the lateral surface area and the total surface area.

A constraint for the cone volume is given, in order to guarantee a minimum volume.
Project building

Let’s build from scratch the pertinent modeFRONTIER project:

1. Work Flow setup: fill the work canvas with the project’s building blocks
2. Script Node setup: use your favourite math tool
   - Jython script
   - Matlab node
   - Excel Workbook node
   - OpenOffice Spreadsheet node
Work Flow setup

**Cones: Two-objective Optimization Problem**

- **Two input variables**: \( r \) and \( h \)
- **Two design variables**: \( r \) and \( h \)
- **DOE Sequence**
- **Three output variables**: volume, minimum_volume, lateral, min_lateral, base, min_total
- **Two objectives**: min_total
- **One constraint**: one constraint

**Logic Flow**

For more information visit: www.esteco.com or send an e-mail to: modeFRONTIER@esteco.com
Work Flow setup

cones: two-objective optimization problem

two design variables

DOE Sequence

DOE
Script node: Jython

Jython (Python) script case:

Write down the formulae

Load math module

Note the syntax of mathematical functions and constants

\[ V = \frac{\pi}{3} r^2 h \]

\[ s = \sqrt{r^2 + h^2} \]

\[ S = \pi r s \]

\[ B = \pi r^2 \]
Script node: Matlab

Matlab case:

Write down the formulae:

\[ V = \frac{\pi}{3} r^2 h \]

\[ s = \sqrt{r^2 + h^2} \]

\[ S = \pi r s \]

\[ B = \pi r^2 \]

Check Matlab version

Load the matlab file
Script node: Excel

Excel Workbook case:

Build the spreadsheet

Load the xls file

Link variables to cells

Insert the formulae

$$s = \sqrt{r^2 + h^2}$$

$$V = \frac{\pi}{3} r^2 h$$

$$S = \pi rs$$

$$B = \pi r^2$$
Script node: OpenOffice

OpenOffice Spreadsheet case:

Build the spreadsheet

Load the sxc file

Link variables to cells

Insert the formulae

\[ s = \sqrt{r^2 + h^2} \]

\[ V = \frac{\pi}{3} r^2 h \]

\[ S = \pi r s \]

\[ B = \pi r^2 \]
Runs examples

Let’s see some examples of runs with different DOEs and/or schedulers:

- **Full Factorial** DOE
- random samplings: **Random Sequence** and **Sobol** DOEs
- genetic algorithms: **MOGA-II**, **NSGA-II**
- **MOSA**
- **NBI-NLPQLP**
Full Factorial DOE
10 levels per variable
100 eval. designs
Random Sequence

Random Sequence DOE
1000 eval. designs
Sobol

Sobol DOE
1000 eval. designs
MOGA-II

20 individuals (Sobol)
50 generations
1000 eval. designs
MOGA-II

20 individuals (Sobol)
50 generations
1000 eval. designs
NSGA-II

20 individuals (Sobol)
50 generations
1000 eval. designs
MOSA

10 points (Sobol)
100 iterations
1000 eval. designs
NBI-NLPQLP

NBI-NLPQLP
(DOE: 10 Sobol)
20 NBI-subproblems
346 eval. designs
Final considerations

Let’s consider the difference between

- single-objective problem solutions: two different minima
- multi-objective problem solutions: the Pareto frontier
Single-objectives minima

Each design represents the optimum solution for its corresponding single-objective problem.

...but what about the in between designs?

...we would like to get a compromise solution. A trade-off of the two objectives...

What we want is the Pareto frontier!

\[ \begin{align*}
\min S \\
r &= 5.131 \text{ cm} \\
h &= 7.256 \text{ cm} \\
V &= 200 \text{ cm}^3 \\
S &= 143.23 \text{ cm}^2 \\
T &= 225.92 \text{ cm}^2
\end{align*} \]

\[ \begin{align*}
\min T \\
r &= 4.072 \text{ cm} \\
h &= 11.518 \text{ cm} \\
V &= 200 \text{ cm}^3 \\
S &= 156.28 \text{ cm}^2 \\
T &= 208.38 \text{ cm}^2
\end{align*} \]
The Pareto frontier