Richiami Analisi 1 ½ :

$\varphi : U \overset{\circ}{\subset} \mathbb{R}^n \longrightarrow \mathbb{R}^m$ funzione differenziabile

$d\varphi : U \longrightarrow \mathrm{Hom}_{\mathbb{R}}(\mathbb{R}^n, \mathbb{R}^m)$

$P \longmapsto d\varphi(P) : \mathbb{R}^n \longrightarrow \mathbb{R}^m$

funzione lineare (e continua)

che meglio approssima φ vicino a P:

$d\varphi(P)\, v := \partial_v \varphi(P)$

$= \sum_i v_i \partial_i \varphi(P) \qquad v = \begin{pmatrix} v_1 \\ \vdots \\ v_n \end{pmatrix}$

TEOREMA DELLA F. INVERSA:

$\varphi : U \underset{\subseteq \mathbb{R}^n}{} \longrightarrow \mathbb{R}^n$ diff

$P \in U$: $d\varphi(P)$ rango massimo ($= n$)

$\Rightarrow$

$\exists$ intorni aperti U' di P e V' di $\varphi(P)$

tali che:

$\varphi_{|U'} : U' \longrightarrow V'$ isomorfismo

ovvero $\exists\ \psi : V' \longrightarrow U'$, $\psi = (\varphi_{|U'})^{-1}$

e $(d\psi)(\varphi P) = (d\varphi(P))^{-1}$

$\qquad = (d\varphi^{-1})(\varphi P)$

$(d\psi)(Q) = (d\varphi(\varphi^{-1} Q))^{-1}$

$\qquad = (d\varphi(\psi Q))^{-1}$.

TEOREMA delle F. IMPLICITA (Dini):

$F : U \times V \longrightarrow \mathbb{R}^n$ diff, $\quad U \subseteq \mathbb{R}^m,\ V \subseteq \mathbb{R}^n$

$(P, Q) \longmapsto F(P,Q) = 0$

$d_V F(P,Q)$ sia di rango massimo ($= n$)

$\Rightarrow$

$\exists\ U' \times V'$ intorno aperto di (P,Q)

$\exists\ f : U' \longrightarrow V'$

t.c.: $\Gamma(f) = Z(F) = \{(x,y) \in U' \times V' : F(x,y) = 0\}$

$\Gamma(f) = \{(x, f(x)) : x \in U'\}$

e $(df)(P) = -\left(d_V F(P, fP)\right)^{-1} d_U F(P, fP)$

(nota nei casi 1: $\dfrac{dy}{dx} = -\dfrac{\partial F/\partial x}{\partial F/\partial y}$ ...)

Problema: i due teoremi sono equivalenti:

TF INV. $\Rightarrow$ TF. IMPL. $\Rightarrow$ TF. INV.

$\Rightarrow$ $F : U \times V \longrightarrow \mathbb{R}^n$

usiamo

$U \times V \longrightarrow U \times \mathbb{R}^n$

$(x, y) \longmapsto (x, F(x,y))$

applico t.f. inversa:

$U' \times V' \longleftrightarrow U' \times V_0'$

$(x, \phi(x,z)) \longleftarrow\!\shortmid\ (x, z)$

$\varphi(x) := \Phi(x, 0)$.

$\Leftarrow$

composizione di differenziali:

$\mathbb{R} \longrightarrow \mathbb{R}^2 \longrightarrow \mathbb{R}^n$

$(u,v) \longmapsto \sigma(u,v)$

$t \longmapsto (u(t), v(t)) \longmapsto \sigma(u(t), v(t)) = \gamma(t)$

$$\boxed{\frac{d\gamma}{dt}(t) = \frac{d}{dt}\sigma(u(t), v(t)) = \frac{\partial \sigma}{\partial u}(u(t), v(t))\frac{du}{dt}(t) + \frac{\partial \sigma}{\partial v}(u(t), v(t))\frac{dv}{dt}(t)}$$

$$\boxed{\gamma' = \sigma_u \cdot u' + \sigma_v \cdot v'}$$

$$\begin{tikzcd} \mathbb{R}^2 \arrow[r] \arrow[rr, bend left, "\tilde{\sigma}"] & \mathbb{R}^2 \arrow[r, "\sigma"] & \mathbb{R}^n \end{tikzcd}$$

$(u,v) \longmapsto \sigma(u,v)$

$(s,t) \longmapsto (u(s,t), v(s,t)) \longmapsto \tilde{\sigma}(s,t) = \sigma(u(s,t), v(s,t))$

$$\begin{cases} \dfrac{\partial}{\partial s}\tilde{\sigma}(s,t) = \dots \\[2mm] \dfrac{\partial}{\partial t}\tilde{\sigma}(s,t) = \dots \end{cases}$$

$$\boxed{\begin{array}{l} \begin{cases} \tilde{\sigma}_s = \sigma_u \cdot u_s + \sigma_v \cdot v_s \\ \tilde{\sigma}_t = \sigma_u \cdot u_t + \sigma_v \cdot v_t \end{cases} \\[4mm] \begin{pmatrix} \tilde{\sigma}_s & \tilde{\sigma}_t \end{pmatrix} = \begin{pmatrix} \sigma_u & \sigma_v \end{pmatrix} \begin{pmatrix} u_s & u_t \\ v_s & v_t \end{pmatrix} \end{array}}$$