Conjunctive Use of Drinking Water Sources with Multiple Providers
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Contribution
Optimal management of drinking water involves conjunctive use of different sources, one of which is flow and the other stock. This is more complex when multiple providers use the same surface water source (i.e., river) to supply distinct groups of customers. In this paper, we consider the interaction between two profit maximizer providers (upstream and downstream) and specifically determine the optimal conjunctive use at every catchment point.

Model - 2 players - degenerate differential game

Upstream provider (1) with constant demand $d_1$; Downstream provider (2) with constant demand $d_2$

- each one has his own aquifer → 2 AQUIFERS $u_i(t)$ groundwater extraction rate ($m^3$/sec)
- each one abstracts from the same river → 1 RIVER $F$ surface flow ($m^3$/sec)

Upstream provider

$$\max_{u_1 \geq 0} \int_0^T e^{-r t} \left[ (P_{d1}u_1(t) + P_{g1}(d_1 - u_1(t)) - C_{d1}(x_1(t), u_1(t)) \cdot u_1(t)) \right] dt$$

s.t.

$$\dot{x}_1(t) = R_1 - u_1(t), \quad x_1(0) = x_{10}, \quad x_1(t) \geq 0, \quad u_1(t) \in \left[ \max\{0, d_1 - (F - mLF)\}, \min\{d_1, u_{1\text{max}}\} \right]$$

Feasibility condition: $u_{1\text{max}} > d_1 - (F - mLF)$

Downstream provider

$$\max_{u_2 \geq 0} \int_0^T e^{-r t} \left[ (P_{d2}u_2(t) + P_{g2}(d_2 - u_2(t)) - C_{d2}(x_2(t), u_2(t)) \cdot u_2(t)) \right] dt$$

s.t.

$$\dot{x}_2(t) = R_2 - u_2(t), \quad x_2(0) = x_{20}, \quad x_2(t) \geq 0, \quad u_2(t) \in \left[ \max\{0, d_2 - (F - mLF) - u_1^*(t)\}, \min\{d_2, u_{2\text{max}}\} \right]$$

Optimal control problem with mixed constrained

Environmental costs not depending on the extraction rate $C_e(x_i, u_i) = \beta_i u_i / x_i$

$$\dot{u}_i^0(t) = \frac{(P_{g1} - P_{d1}) - \lambda_i(t)}{\beta_i} x_i(t) \text{ feedback solution}$$

$$\lambda_i(t) = \frac{(P_{g1} - P_{d1}) + x_i^2 + x_i t + (P_{g1} - P_{d1})}{x_i^2 - \sqrt{(P_{g1} - P_{d1})}} - \tanh^{-1}\left( \frac{x_i^2 - \sqrt{(P_{g1} - P_{d1})}}{x_i^2 + \sqrt{(P_{g1} - P_{d1})}} \right)$$

$$u_i^*(t) = \max\{u_i, \min\{\hat{u}_i, \dot{u}_i(t)\}\}$$

• Optimal extraction rate depends on the water volume at a given time and on the instant itself
• Being the water volume fixed, the more I approximate to the final time, the more I can abstract. I have no more time left so that I can exploit the aquifer (no condition on final water volume)

$x_i(t) = 0 \Rightarrow u_i^*(t) = 0$ (empty aquifer ⇒ no extraction)

Numerical simulations: Non-cooperative vs Cooperative management strategies

G.A.M.S. Euler method

No First mover advantage:

Non cooperative case $P_{g1} > P_{s1}$, $P_{g2} > P_{s2}$

Cooperative case $P_{g1} < P_{s1}$, $P_{g2} < P_{s2}$

General case $P_{g1} < P_{s1}$, $P_{g2} > P_{s2}$

References

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