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# Expressive power of definite clauses for verifying authenticity

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# Outline

- 1 Introduction
- 2 The process calculus
- 3 Translation into clauses
- 4 Results
- 5 Future work

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# Aims

Our work aims at clarifying the expressive power of the approach to protocol verification by means of definite clauses. This power relies on

- information embedded into clauses
- verification method

And nodal points are

- conditions for soundness wrt secrecy
- conditions for soundness wrt authenticity
- way of tuning precision
- possibility of achieving completeness (modulo non-termination)

Pole star: Bruno's framework for verification in the formal model

- Translation from protocols into clauses changed in time...
- ...and accordingly the power of the analysis increased!
- But whether a link exists is not explicitly proved

## In nuce

We investigate the relationship between traces and proofs constructed with definite clauses

The main outcomes

- a translation from protocols into definite clauses
  - a verification method
- on which the analysis is both **sound and complete** (modulo non-termination)

These results help us in understanding Bruno's choices

- why clauses changed in time
- how unbounded session handling is achieved
- why Bruno's approach is not complete

# Restrictions of our approach

## Assumptions on protocols

- we assume that protocols communicate only through a public channel

## Limitations of the approach

- we don't have an automatic method for constructing proofs (yet?)
- but we assume that proofs are given

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$c$	channel
$M, N ::=$	terms
$x, y, z \mid a, b \mid f(M_1, \dots, M_n)$	var, name, constr
$P, Q ::=$	processes
$\bar{c}\langle N \rangle^k.P \mid c(x)^k.P \mid 0 \mid$	out, in, nil
$(P \mid Q) \mid !^k P \mid$	par, replication
$(\nu a)^k P \mid \text{begin}(M)^k.P \mid$	restriction, begin
$\text{end}(M)^k.P \mid$	end event
$\text{let}^k x = g(M_1, \dots, M_n) \text{ in } P \text{ else } Q \mid$	destr
$\text{if}^k M = N \text{ then } P \text{ else } Q$	conditional

## Observations

- Each action is labelled by a unique integer  $k$ ,
- Events are used to check authenticity as a correspondence



# A simple example

(non-injective) Authenticity by means of an outgoing test (CP handshake)

Alice  $\longrightarrow$  Bob :  $\{a\}_{bobKey}$   
Bob  $\longrightarrow$  Alice :  $a$

$$Auth2 \equiv (Alice) \mid (Bob)$$

where

$$Alice \equiv !^1 c(partner)^3. (\nu a)^4. let^5 k = getKey(partner) \text{ in } \bar{c}\langle encrypt(a, k) \rangle^6. \\ c(y)^7. if^8 y = a \text{ then } end(partner, a)^9$$

$$Bob \equiv !^2 (\nu bobKey)^{10}. \bar{c}\langle host(bobKey) \rangle^{11}. c(Xa)^{12}. let^{13} b \\ = decrypt(Xa, bobkey) \text{ in } begin(host(bobKey), b)^{14}. \bar{c}\langle b \rangle^{15}$$

$Alice \equiv !^1 c(partner)^3. (\nu a)^4. let^5 k = getKey(partner) \text{ in } \bar{c}\langle encrypt(a, k) \rangle^6. \\ c(y)^7. if^8 y = a \text{ then } end(partner, a)^9$

$(Alice, []) \rightarrow$

## Observation

Input and output are non-blocking since we assume protocols running in presence of a Dolev-Yao adversary

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 $((\nu a)^4 \dots, [sid_1^1]), (Alice, []) \rightarrow$   
 $(\{a \mapsto a[sid_1^1]\}let^5 \dots, [sid_1^1]), (Alice, [])$

## Observation

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## Moral

An action  $A^k$  is uniquely identified by its label  $k$  and the sequence  $L$  of *sid*'s coming from replications preceding  $A^k$ . We call

- $(k, L)$  an **action instance**,
- **session leading to**  $(k, L)$ , the sequence of action instances on the path to  $k$

In the previous example

- $(1, []), (3, [sid_1^1]), (4, [sid_1^1])$  is the  $SL(5, [sid_1^1])$

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# From $\pi$ to definite clauses

A protocol  $P$  is represented by a set  $\llbracket P \rrbracket_{lab}$  of definite clauses of the form

$$att(p_1)[k_1, L_1] \wedge \dots att(p_n)[k_n, L_n] \Rightarrow att(p)[k, L]$$

where

- an output in the head and preceding inputs in the body
- there are clauses also for the adversary

Main differences with Bruno's translations

- 1 each atom is equipped with a pair  $[k, L]$
- 2 bound name representation:  $!^1 \dots !^n \dots (\nu a)^k$  leads to  $a[sid_1, \dots, sid_n]$

$\llbracket Alice \rrbracket_{lab}$

$!^1 c(partner)^3.(\nu a)^4.let^5 k = getkey(partner) \text{ in } \bar{c}\langle encrypt(a, k) \rangle^6.$   
 $c(y)^7.if^8 y = a \text{ then } end(partner, a)^9$



# Translation example

$\llbracket Alice \rrbracket_{lab}$

$!^1 c(\text{partner})^3 . (\nu a)^4 . \text{let}^5 k = \text{getKey}(\text{partner}) \text{ in } \bar{c} \langle \text{encrypt}(a, k) \rangle^6$   
 $. c(y)^7 . \text{if}^8 y = a \text{ then end}(\text{partner}, a)^9$

# Translation example

$\llbracket Alice \rrbracket_{lab}$

$!^1 c(partner)^3 . (\nu a)^4 . let^5 k = getkey(partner) \text{ in } \bar{c}\langle encrypt(a, k) \rangle^6$   
 $.c(y)^7 . if^8 y = a \text{ then } end(partner, a)^9$

- clause of the form  $att(partner) \Rightarrow att(encrypt(a, k))$

# Translation example

$\llbracket \text{Alice} \rrbracket_{lab}$

$!^1 c(\text{partner})^3 . (\nu a)^4 . \text{let}^5 k = \text{getkey}(\text{partner}) \text{ in } \bar{c}\langle \text{encrypt}(a, k) \rangle^6$   
 $. c(y)^7 . \text{if}^8 y = a \text{ then end}(\text{partner}, a)^9$

- clause of the form  $\text{att}(\text{partner}) \Rightarrow \text{att}(\text{encrypt}(a, k))$
- $\text{getkey}$  computes  $\text{partner} \mapsto \text{host}(x), k \mapsto x$

# Translation example

$\llbracket Alice \rrbracket_{lab}$

$!^1 c(\text{partner})^3.(\nu a)^4.let^5 k = \text{getkey}(\text{partner}) \text{ in } \bar{c}\langle \text{encrypt}(a, k) \rangle^6$   
 $.c(y)^7.if^8 y = a \text{ then } end(\text{partner}, a)^9$

- clause of the form  $att(\text{partner}) \Rightarrow att(\text{encrypt}(a, k))$
- $\text{getkey}$  computes  $\text{partner} \mapsto \text{host}(x), k \mapsto x$
- then
  1.  $att(\text{host}(x))[3, \text{sid}_1] \Rightarrow att(\text{encrypt}(a[\text{sid}_1], x))[6, \text{sid}_1]$

# Translation example

$[[!^2 Bob]]_{lab}$

$$\begin{aligned} & !^2(\nu bobKey)^{10}.\bar{c}\langle host(bobKey) \rangle^{11}.c(Xa)^{12}.let^{13} b \\ & = decrypt(Xa, bobkey) \text{ in } begin(host(bobKey), b)^{14}.\bar{c}\langle b \rangle^{15} \end{aligned}$$

- no input/begin preceding the current output
- then
  2.  $\Rightarrow host(bobKey[sid_2])[11, sid_2]$
- and so on

# Verification in action

Properties are verified by means of **backward chaining**

- each clause is renamed apart when selected for unification, *sid*'s included
  - $sid_r$  is renamed as  $sid_r^1, sid_r^2, \dots$
- proofs have the form of trees

Assume we want know whether in *Auth2* Alice is sure to talk with Bob.

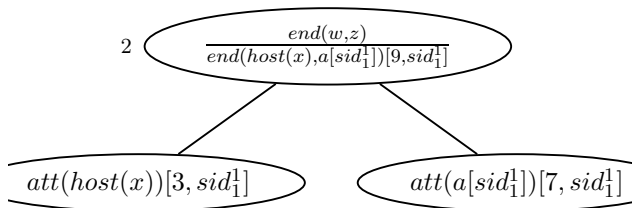
- Property:  $end(w, z) \Rightarrow begin(w, z)$

# Proof tree for *Auth2*

$end(w, z)$

The root node contains (the head of) the query

# Proof tree for *Auth2* [cont.]



Clause 2 is selected for unification:

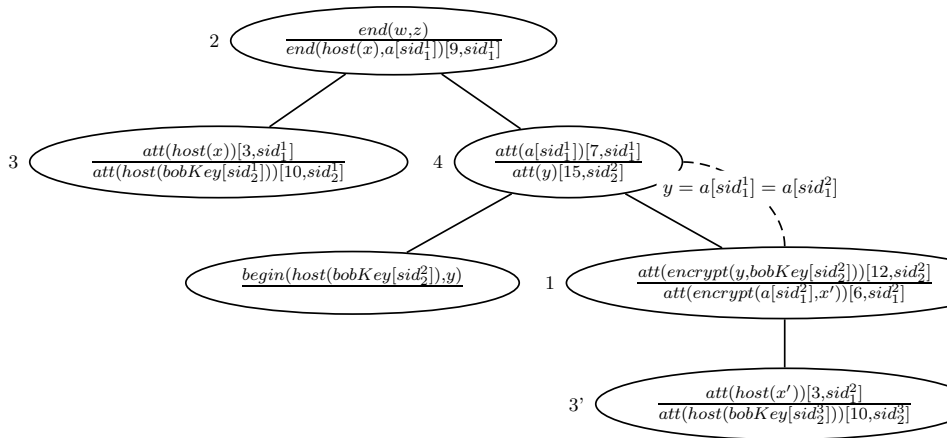
$$att(host(x)) \wedge att(a[sid_1^1]) \Rightarrow end(host(x), a[sid_1^1])$$

we get equations

$$w = host(x), z = a[sid_1^1]$$

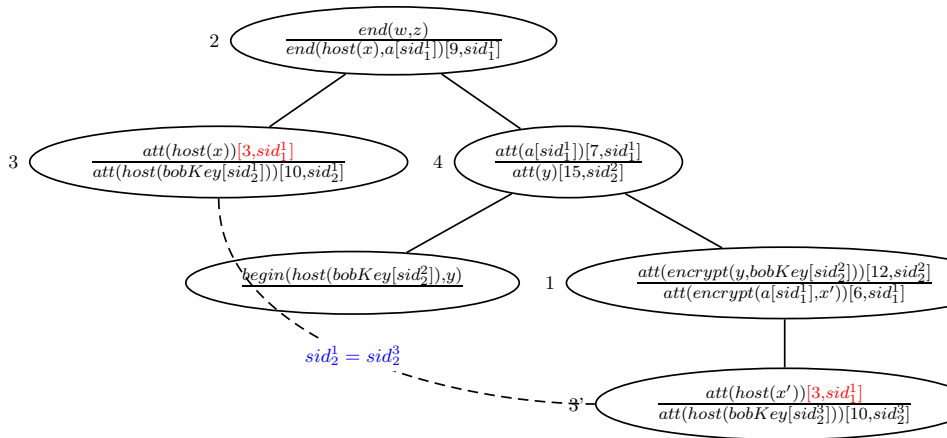


# Proof tree for *Auth2* [cont.]



...and so on.  $Eq(T)$  denotes the set of equations related to nodes of a tree  $T$

# Proof tree for *Auth2* [cont.]



Labels  $[k, L]$  give extra information: same action instances are unified! Iterate until a fix point is reached.

# Solvable proof trees

A proof tree  $T$  is **solvable** if

- the set  $Eq(T)$  is solvable,
- the unifications induced by labels correctly extend a mgu of  $Eq(T)$ ,
- the final mgu is correct wrt destructor applications

When resolution ends correctly and a mgu  $\theta$  is computed,  $\theta$  is applied to  $T$  and the resulting tree is said to be **fully instantiated**.

## Verification method

If the analysis on proof trees is sound wrt trace then

- $P$  supports secrecy of  $M$  if there is no solvable proof trees built with  $\llbracket P \rrbracket_{lab}$  whose root is  $att(M)$
- $P$  supports (non-injective) authenticity if each solvable proof tree built with  $\llbracket P \rrbracket_{lab}$  whose root is  $end(M)$  contains an atom  $begin(M)$

We now give an intuition about the correctness of this approach.

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# Soundness of analysis with proof trees

The correctness of the approach is based on two results

- 1 a strong relationship between traces and clauses
- 2 a strong relationship between terms in traces and in trees

## Lemma

Given a trace leading to an output

$$c(x_1) \dots c(x_2) \dots \bar{c}\langle x_0 \rangle$$

assume that  $t_j$  is the actual value used in the trace for  $x_j$ .

Then there exists a clause

$$C \equiv att(s_1) \wedge att(s_2) \Rightarrow att(s_0)$$

such that  $s_j$  unifies with  $t_j$ .

Hence,

- if we could simulate with proof trees the output of  $t_1, t_2$
- gluing  $C$  on top we could simulate the whole trace!
- Note: the same let choices of the trace must be taken when translating

# Soundness of analysis with proof trees [cont'ed]

Complication:

- Terms in a tree may differ from terms in a corresponding trace

When different terms in a tree correspond to a unique term in the trace, either

- the resolution process unifies them
- or they remain distinct

but this is sufficient for soundness:

- 1 proof trees contain equal terms only when the corresponding trace uses equal terms
- 2 distinct terms in the trace remain distinct also in the tree
- 3 thus if no trace shows an attack then all the more so trees

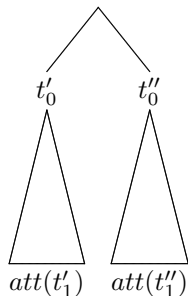
# Completeness of analysis with proof trees

- Given a solvable proof tree we must show that a corresponding trace exists
- The proof does build such a trace

## Intuitively

After iterating on labels, no more unification is needed for the tree to correspond to a trace.

# Trees and traces, soundness and completeness



$$\Leftarrow \dots c(x_1) \dots \bar{c}\langle x_0 \rangle \dots \text{ [Soundness]}$$

$\uparrow \qquad \qquad \uparrow$   
 $t_1 \qquad \qquad t_0$

$$\Rightarrow \dots c(x_1) \dots \bar{c}\langle x_0 \rangle \dots \mid \dots c(x_1) \dots \bar{c}\langle x_0 \rangle \dots \text{ [Completeness]}$$

$\uparrow \qquad \qquad \uparrow \qquad \qquad \uparrow \qquad \qquad \uparrow$   
 $t'_1 \qquad t'_0 \qquad t''_1 \qquad t''_0$



# Glancing at Bruno's formalisations

Various translations have been proposed in time, that differs in bound names representations

- ①  $(\nu a)^k \mapsto a[p_1, \dots, p_m]$  with  $p_i$  previous inputted terms
  - introduced with, and proved sound wrt, secrecy
- ②  $(\nu a)^k \mapsto a[p_1, \dots, p_m, sid_1, \dots, sid_n]$ 
  - introduced with, and proved sound wrt, authenticity

Proof trees help in clarifying that

- *sid*'s are not mandatory to be sound wrt secrecy
- *sid*'s are mandatory to be sound wrt authenticity
- adding input terms and destructor choices help in avoiding some false negative, but not to attain completeness
  - in fact bound names can contain only information about previous actions!

# Authenticity needs session variables

Consider

$$\text{Auth2}' \equiv (\text{Alice}') \mid (\text{Bob})$$

where

$$\begin{aligned} \text{Alice}' \equiv & (\nu a)^3 !^1 c(\text{partner})^4 . \text{let}^5 k = \text{getKey}(\text{partner}) \text{ in } \bar{c} \langle \text{encrypt}(a, k) \rangle^6 . \\ & c(y)^7 . \text{if}^8 y = a \text{ then } \text{end}(\text{partner}, y)^9 \end{aligned}$$

## The protocol is flawed!

- After a run is completed the attacker knows  $\text{bobKey}[]$  and  $a[]$
- No way to observe that *begin* and *end* events do not match, since *partner* and *y* are represented by bound names containing no parameter
- But is simple, in fact, to fool *Alice* by simply replaying them

A flawed protocol is passed as correct  $\Rightarrow$  the method is not sound

# Completeness needs labels

Consider the protocol  $P \equiv !^1 A \mid !^2 B \mid !^3 C$ , where

$$\begin{aligned} A &= (\nu a)^4. \bar{c}\langle a \rangle^5. c(b)^6. \text{if}^7 b = P(a) \text{ then } \bar{c}\langle K1(a) \rangle^8 \text{ else } \bar{c}\langle K2(a) \rangle^9 \\ B &= c(x)^{10}. \bar{c}\langle P(x) \rangle^{11}. c(y1)^{12}. c(y2)^{13}. \text{if}^{14} y1 = K1(x) \\ &\quad \text{then if}^{15} y2 = K2(x) \text{ then } \bar{c}\langle OK \rangle^{16} \\ C &= c(w)^{17}. \bar{c}\langle S(w) \rangle^{18} \end{aligned}$$

Clauses have no control on input 6, since no bound name carry information about it (its session)

- In the proof tree input 6 is used two times, but the read values are not unified
- Thus we find that  $K1(a[])$  and  $K2(a[])$  can be read in the same session
- And so that  $OK$  is outputted...but this is not the case!

A correct protocol is passed as flawed  $\Rightarrow$  the method is not complete

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A line for future work include

- studying for a feasible implementation of our strategy
- considering injective–authenticity in the light of proof trees
- studying termination: what protocols have a finite number of proof trees for a given property
  - = tagged protocols? (on which ProVerif terminates)
- extending the established relationship between definite clauses and types up to consider authenticity in this light