Laboratorio di Apprendimento Automatico

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Probabilistic Classifiers

- Compute $P(c_j|d_i)$ by means of the Bayes' theorem
  - $P(c_j|d_i) = P(d_i|c_j)P(c_j)/P(d_i)$
  - Maximum a posteriori Hypothesys (MAP) argmax $P(c_j|d_i)$
- Classes are viewed as generators of documents
- The prior probability $P(c_j)$ is the probability that a document d is in c_j

Naive Bayes Classifiers

Task: Classify a new instance D based on a tuple of attribute values $D = \langle x_1, x_2, \ldots, x_n \rangle$ into one of the classes $c_j \in C$

$$c_{MAP} = \underset{c_j \in C}{\operatorname{argmax}} P(c_j \mid x_1, x_2, \ldots, x_n)$$

$$= \underset{c_j \in C}{\operatorname{argmax}} \frac{P(x_1, x_2, \ldots, x_n \mid c_j) P(c_j)}{P(x_1, x_2, \ldots, x_n)}$$

$$= \underset{c_j \in C}{\operatorname{argmax}} P(x_1, x_2, \ldots, x_n \mid c_j) P(c_j)$$

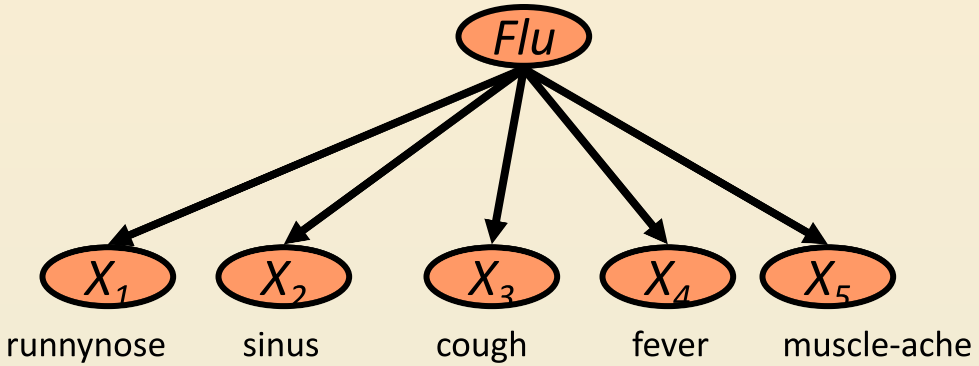
Naïve Bayes Classifier: Assumption

- $P(c_j)$
  - Can be estimated from the frequency of classes in the training examples.
- $P(x_1,x_2,\ldots,x_n|c_j)$
  - $O(|X|^n \cdot |C|)$ parameters
  - Could only be estimated if a very, very large number of training examples was available.

Naïve Bayes Conditional Independence Assumption:

- Assume that the probability of observing the conjunction of attributes is equal to the product of the individual probabilities $P(x_i|c_j)$.

The Naïve Bayes Classifier

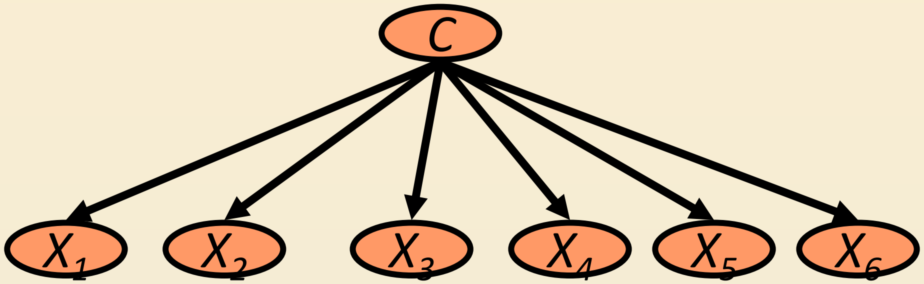


- **Conditional Independence Assumption:** features are independent of each other given the class:

$$P(X_1,\ldots,X_5 \mid C) = P(X_1 \mid C) \bullet P(X_2 \mid C) \bullet \cdots \bullet P(X_5 \mid C)$$

- Only n|C| parameters (+|C|) to estimate

Learning the Model



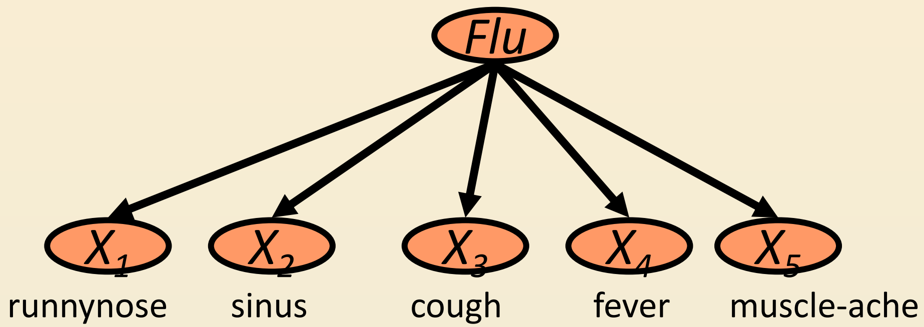
maximum likelihood estimates: most likely value of each parameter given the training data

– i.e. simply use the frequencies in the data

$$\hat{P}(c_j) = \frac{N(C = c_j)}{N}$$

$$\hat{P}(x_i \mid c_j) = \frac{N(X_i = x_i, C = c_j)}{N(C = c_j)}$$

Problem with Max Likelihood



$$P(X_1,\ldots,X_5 \mid C) = P(X_1 \mid C) \bullet P(X_2 \mid C) \bullet \cdots \bullet P(X_5 \mid C)$$

- What if we have seen no training cases where patient had no flu and muscle aches?

$$\hat{P}(X_5 = t \mid C = nf) = \frac{N(X_5 = t, C = nf)}{N(C = nf)} = 0$$

- Zero probabilities cannot be conditioned away, no matter the other evidence!

$$\ell = \arg\max_c \hat{P}(c) \prod_i \hat{P}(x_i \mid c)$$

Smoothing to Avoid Overfitting

$$\hat{P}(x_i \mid c_j) = \frac{N(X_i = x_i, C = c_j) + 1}{N(C = c_j) + k}$$

# of values of X_i