Pre-vote negotiations and the outcome of collective decisions

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joint work with

Umberto Grandi (Padova) & Davide Grossi (Liverpool)
Cicero used to say that it was not in the senate chamber that the real business of the republic was done, but outside, in the open-air lobby known as the senaculum, where the senators were obliged to wait until they constituted a quorum.
The aim

- Capture the structure of collective decisions in political settings (voting, coherence, ideology);
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- Capture the structure of negotiations before a collective decision (lobbying, do ut des);
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- Understand how pre-vote negotiations affect collective decisions (achievable (un)desirable properties);
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- Capture the structure of negotiations before a collective decision (lobbying, do ut des);
- Understand how pre-vote negotiations affect collective decisions (achievable (un)desirable properties);
- Ideally, a framework for political analysis.
Pre-vote negotiations

Three steps

1. *Voting* games;

Possibility of investing resources beforehand to convince others to change their vote; goals as ideological positions (that cannot be changed by monetary offers).
Three steps

- Voting games;
  - vote over a set of (interdependent) issues;
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  - have preferred electoral outcomes, i.e., goals;
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- **Voting** games with resources;

- Possibility of investing resources beforehand to convince the others to change their vote;

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  - get resources (payoff received as result of a vote);
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- **Voting games with resources and pre-vote negotiations**;
  - possibility of investing resources beforehand to convince others to change their vote;
  - goals as ideological positions (that cannot be changed by monetary offers).
societies of voters that express a yes/no opinion on several issues at stake;
societies of voters that express a yes/no opinion on several issues at stake;

issues are logically interdependent, and might be subjected to satisfy a given formula, i.e., a given *integrity constraint*. 
A key reference

Umberto Grandi and Ulle Endriss
Lifting integrity constraints in binary aggregation.
Definition (BA structure)

A binary aggregation structure (BA structure) is a tuple \( S = \langle \mathcal{N}, \mathcal{I}, \text{IC} \rangle \) where:

\[ \mathcal{N} = \{ 1, \ldots, n \} \]
\[ \mathcal{I} = \{ 1, \ldots, m \} \]
\[ \text{IC} \text{ is a propositional formula of } \mathcal{L}_{\text{PS}}, \text{ an algebraic language constructed over the set } \mathcal{P} = \{ p_1, \ldots, p_m \}. \]
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A binary aggregation structure (BA structure) is a tuple \( S = \langle \mathcal{N}, \mathcal{I}, IC \rangle \) where:

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- $IC$ is a propositional formula of $\mathcal{L}_{PS}$, a propositional language constructed over the set $PS = \{p_1, \ldots, p_m\}$.
Atomic weapons

Example
A parliament is to decide whether to build nuclear power plants (N) and develop atomic weapons (W). If importing nuclear technology from abroad is not an option, the development of atomic weapons involves the construction of nuclear power plants, i.e., $IC = (W \rightarrow N)$. 
Definition (Aggregation procedure)

An *aggregation procedure* for BA structure $S$ is a function

$$F : \text{Mod}(\text{IC})^N \rightarrow D$$

mapping every profile of IC-consistent ballots ($\text{Mod}(\text{IC})^N$) to a binary ballot ($D$).
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An **aggregation procedure** for BA structure $S$ is a function

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- Majority, unanimity etc.
Definition (Aggregation procedure)

An \textit{aggregation procedure} for BA structure $S$ is a function

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mapping every profile of IC-consistent ballots ($\text{Mod}(IC)^N$) to a binary ballot ($\mathcal{D}$).

- Majority, unanimity etc.
- APs can be studied axiomatically.
Question

- If individuals provide an IC-consistent ballot, will the resulting ballot be IC-consistent as well?
Discursive Dilemma

Example

If buying nuclear energy from the foreign market is an option, i.e., it is possible to vote on the issue \((W \rightarrow N)\), there is a natural IC in \((W \land (W \rightarrow N)) \rightarrow N\), when ballot \((1, 1, 0)\) is outright inadmissible. In a parliament of 3 equally representative parties a Discursive Dilemma can arise with majority voting.

\[
\text{IC} = (W \land (W \rightarrow N)) \rightarrow N
\]

<table>
<thead>
<tr>
<th></th>
<th>(W)</th>
<th>(W \rightarrow N)</th>
<th>(N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Party A</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Party B</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Party C</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Majority</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

Table: A Discursive Dilemma
Voting Games and Goals

- Voting can be studied as a game, votes as individual strategies;
Voting can be studied as a game, votes as individual strategies;
Players’ goals are on the outcome of the vote.
Key reference

Paul Harrenstein, Wiebe van der Hoek, John-Jules Meyer and Cees Witteveen
Boolean games.
TARK 2001
An aggregation game is a tuple \( \mathcal{A} = \langle \mathcal{N}, \mathcal{I}, \text{IC}, F, \{\gamma_i\}_{i \in \mathcal{N}} \rangle \) such that:

All individuals share the same set of IC-consistent strategies, namely the set of IC-consistent ballots Mod(IC).
Aggregation games

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An aggregation game is a tuple $\mathcal{A} = \langle \mathcal{N}, \mathcal{I}, \text{IC}, F, \{\gamma_i\}_{i \in \mathcal{N}} \rangle$ such that:

1. $\langle \mathcal{N}, \mathcal{I}, \text{IC} \rangle$ is a binary aggregation structure;

All individuals share the same set of IC-consistent strategies, namely the set of IC-consistent ballots $\text{Mod}(\text{IC})$. 
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- $\langle \mathcal{N}, \mathcal{I}, \text{IC} \rangle$ is a binary aggregation structure;
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- $\langle \mathcal{N}, \mathcal{I}, \text{IC} \rangle$ is a binary aggregation structure;
- $F$ is an aggregation procedure for $\langle \mathcal{N}, \mathcal{I}, \text{IC} \rangle$;
- each $\gamma_i$ is a propositional formula in $\mathcal{L}_{PS}$ which is consistent with $\text{IC}$;

All individuals share the same set of IC-consistent strategies, namely the set of IC-consistent ballots $\text{Mod(IC)}$. 

Preferences

Definition (Preferences in aggregation games)

Let $A = \langle \mathcal{N}, I, IC, F, \{\gamma_i\}_{i \in \mathcal{N}} \rangle$ be an aggregation game. For ballots $B, B'$

$$B \preceq_i^A B' \iff B' \models \neg \gamma_i \text{ or } B \models \gamma_i$$
Voting strategies

Definition

A strategy $B \in \text{Mod}(\text{IC})$ is *truthful for agent $i$* if it satisfies $\gamma_i$. A strategy profile $B = (B_1, \ldots, B_n)$ is:

- **truthful** if all $B_i$ are truthful;
- **IC-consistent** if $F(B) = \text{IC};$
- **goal-efficient** if $F(B) \models \gamma_i$ for all $i \in N$.

Paolo Turrini
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A strategy $B \in \text{Mod}(\text{IC})$ is truthful for agent $i$ if it satisfies $\gamma_i$.

A strategy profile $B = (B_1, \ldots, B_n)$ is:

- truthful if all $B_i$ are truthful;
- IC-consistent if $F(B) \models \text{IC}$;
- goal-efficient if $F(B) \models \land_i \gamma_i$;
- goal-inefficient if $F(B) \not\models \gamma_i$ for all $i \in N$. 
An aggregation game is consistent if the conjunction of all individual goals is consistent with IC, i.e., if \((\bigwedge_{i \in N} \gamma_i) \land IC\) is consistent.
Pre-vote negotiations
Voting games with goals

Equilibria

Proposition

*Every consistent aggregation game for the majority rule (maj) has an IC-consistent NE that is truthful and goal-efficient.*
Pre-vote negotiations
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Equilibria

Proposition

Every consistent aggregation game for the majority rule (maj) has an IC-consistent NE that is truthful and goal-efficient.

Idea: at unanimous, truthful, IC-consistent and goal-efficient profile $B^* = (B^*)_N$, $maj(B^*) = B^*$. 
Pre-vote negotiations
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Every consistent aggregation game for the majority rule (maj) has an IC-consistent NE that is truthful and goal-efficient.

- Idea: at unanimous, truthful, IC-consistent and goal-efficient profile $B^* = (B^*)_N$, $maj(B^*) = B^*$.
- Generalizable! (as many results next)
Equilibria

**Proposition**

There exist a consistent aggregation game for maj with a truthful NE that is goal-inefficient and IC-inconsistent.

<table>
<thead>
<tr>
<th></th>
<th>$p_1$</th>
<th>$p_2$</th>
<th>$p_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Voter 1</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Voter 2</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Voter 3</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
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</table>

**Table:** An equilibrium with IC = $p_1 \lor p_2 \lor p_3$ and $\gamma_i = p_i$
Voting games, goals and payoff

- Payoff associated to each possible vote;
Payoff associated to each possible vote;
Goals and payoffs play a role in ballot selection.
Key reference

Aggregation games with payoff

Definition ($A^{\pi}$ games)

An aggregation game with payoff is a tuple

$$\langle A, \{\pi_i\}_{i \in \mathcal{N}} \rangle$$

where:
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An aggregation game with payoff is a tuple

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Pre-vote negotiations
Voting games, goals and payoff

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Definition ($A^\pi$ games)

An aggregation game with payoff is a tuple

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where:

- $A$ is an aggregation game;
- $\pi_i : \text{Mod}(\text{IC})^N \rightarrow \mathbb{R}$ is a payoff function.
Goals and payoffs

- Goal states represent positions upon which players are not willing to negotiate;
Goals and payoffs

- Goal states represent positions upon which players are not willing to negotiate;
- If goals are not an issue, payoffs play a role.
Goals and payoffs

- Goal states represent positions upon which players are not willing to negotiate;
- If goals are not an issue, payoffs play a role.
- A lexicographic (quasi-dichotomous) preference relation.
Goals, payoffs and induced preferences

Definition (Preferences in $A^\pi$ games)

For ballot profiles $B, B'$,

$$B \succeq_i^\pi B'$$

$$\iff$$
Pre-vote negotiations
Voting games, goals and payoff

Goals, payoffs and induced preferences

Definition (Preferences in $A^\pi$ games)

For ballot profiles $B, B'$,

$$B \preceq_i^\pi B'$$

$$\iff$$

- $(F(B') \models \neg \gamma_i$ and $F(B) \models \gamma_i)$ or
Goals, payoffs and induced preferences

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For ballot profiles $B, B'$,

$$B \succeq_i^\pi B'$$

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- $(F(B') \models \gamma_i \iff F(B) \models \gamma_i)$ and $\pi_i(B) \geq \pi_i(B')$.

First we look at the goal, then at the payoff.
Goals, payoffs and induced preferences

Definition (Preferences in $A^\pi$ games)

For ballot profiles $B, B'$,

$$B \succeq^\pi_i B'$$

$$\Leftrightarrow$$

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- First we look at the goal, then at the payoff.
Uniform games

Definition

$A^\pi$-games are *uniform* if, for all $i \in \mathcal{N}$, $\pi_i(B) = \pi_i(B')$ whenever $F(B) = F(B')$. 
Uniform games

Definition

$A^\pi$-games are *uniform* if, for all $i \in N$, $\pi_i(B) = \pi_i(B')$ whenever $F(B) = F(B')$.

- Payoff received only depends on the outcome of the vote.
Uniform payoff: properties

Proposition

Every consistent uniform $A^\pi$-game for maj has an IC-consistent NE that is truthful and goal-efficient.
Uniform payoff: properties

Proposition

Every consistent uniform $A^\pi$-game for maj has an IC-consistent NE that is truthful and goal-efficient.

- Same idea: at unanimous, truthful, IC-consistent and goal-efficient profile $B^* = (B^*)^N$, $maj(B^*) = B^*$. It is an equilibrium thanks to $maj$ and uniformity.
Pre-vote negotiations
Voting games, goals and payoff

Non-uniform payoff

**Proposition**

For every uniform $A^\pi$-game $\langle A, \{\pi_i\}_{i \in N} \rangle$ and profile $B^*$ such that $B^*$ is a goal-inefficient NE for $A$, there exists a payoff function $\{\pi'_i\}_{i \in N}$ such that:
Non-uniform payoff

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For every uniform $A^\pi$-game $\langle A, \{\pi_i\}_{i \in N} \rangle$ and profile $B^*$ such that $B^*$ is a goal-inefficient NE for $A$, there exists a payoff function $\{\pi'_i\}_{i \in N}$ such that:

$$\sum_{i \in N} \pi'_i(B) = \sum_{i \in N} \pi_i(B), \text{ for every profile } B;$$
Proposition

For every uniform $A^\pi$-game $\langle A, \{\pi_i\}_{i \in N} \rangle$ and profile $B^*$ such that $B^*$ is a goal-inefficient NE for $A$, there exists a payoff function $\{\pi'_i\}_{i \in N}$ such that:

1. $\sum_{i \in N} \pi'_i(B) = \sum_{i \in N} \pi_i(B)$, for every profile $B$;
2. $B^*$ is not a NE for $\langle A, \{\pi'_i\}_{i \in N} \rangle$. 

Non-uniform payoff
If a NE is goal-inefficient and the game is uniform, then there exists a redistribution of payoffs at each profile that eliminates that equilibrium;
Ideology matters

- If a NE is goal-inefficient and the game is uniform, then there exists a redistribution of payoffs at each profile that eliminates that equilibrium;
- Idea: each voter could pay the others to make their deviation to his goal state profitable for them.
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Ideology matters

- If a NE is goal-inefficient and the game is uniform, then there exists a redistribution of payoffs at each profile that eliminates that equilibrium;
- Idea: each voter could pay the others to make their deviation to his goal state profitable for them. No matter how expensive it is, he is still going to be better off.
A pre-vote phase, where, starting from a uniform $A^\pi$-game, players make simultaneous transfers of payoff at each profile to their fellow players;
Pre-vote negotiations

- A *pre-vote phase*, where, starting from a uniform $A^\pi$-game, players make simultaneous transfers of payoff at each profile to their fellow players;
- A *vote phase*, where players play the original $A^\pi$-game, updated with transfers.
Key references

Paolo Turrini
Endogenous boolean games.
IJCAI 2013.
Key references

Paolo Turrini
Endogenous boolean games.
IJCAI 2013.

Matthew O. Jackson and Simon Wilkie
Endogenous games and mechanisms: Side payments among players.
Endogenous aggregation games

Definition ($\mathcal{A}^T$-games)

An endogenous aggregation game is defined as a tuple

$$\langle \mathcal{A}, \{\pi_i\}_{i \in \mathcal{N}}, \{T_i\}_{i \in \mathcal{N}} \rangle$$

where
Endogenous aggregation games

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- $\langle A, \{\pi_i\}_{i \in \mathcal{N}} \rangle$ is a uniform $A^\pi$ game
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- $\langle A, \{\pi_i\}_{i \in \mathcal{N}} \rangle$ is a uniform $A^\pi$ game
- $\{T_i\}_{i \in \mathcal{N}}$ is the family of sets $T_i$ containing all transfer functions $\tau_i : \text{Mod}(\text{IC})^\mathcal{N} \times \mathcal{N} \rightarrow \mathbb{R}_+$. 
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- $\tau_i(B, j)$ the amount of payoff that a player $i$ gives to player $j$ should a certain profile of votes $B$ be played.
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- $\tau_i(B, j)$ the amount of payoff that a player $i$ gives to player $j$ should a certain profile of votes $B$ be played
- $\tau(A^\pi) = \langle A, \{\pi'_i\}_{i \in N} \rangle$ is the new game where $\pi'_i$ is updated with the payments.
Equilibria in $A^T$ games

**Definition**

Given a $A^T$-game $\langle A, \{\pi_i\}_{i \in N}, \{T_i\}_{i \in N} \rangle$ we call a Nash equilibrium $B$ of $\langle A, \{\pi_i\}_{i \in N} \rangle$ a **surviving Nash equilibrium** if there exist a transfer function $\tau$ and a subgame perfect equilibrium of the two-phase game such that $(\tau, B)$ is played on the equilibrium path.
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- SPEs constructed selecting:
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- SPEs constructed selecting:
  - a pure strategy Nash equilibrium after each transfer, whenever it exists;
Equilibria in $\mathcal{A}^T$ games

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- SPEs constructed selecting:
  - a pure strategy Nash equilibrium after each transfer, whenever it exists;
  - a transfer profile, such that no profitable deviation exist for any player by changing his transfer;
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- SPEs constructed selecting:
  - a pure strategy Nash equilibrium after each transfer, whenever it exists;
  - a transfer profile, such that no profitable deviation exist for any player by changing his transfer;

- Assumption: any deviation for a player to a game $\tau(A)$ with no pure strategy Nash equilibrium is never profitable.
Surviving equilibria identify those electoral outcomes that can be rationally sustained by an appropriate pre-vote negotiation.
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We are interested to know whether desirable equilibria, e.g., goal-efficient, can be achieved or maintained in the two-phase game.
Equilibria in $\mathcal{A}^T$ games

**Proposition**

Let $\mathcal{A}^T = \langle \mathcal{A}, \{\pi_i\}_{i \in \mathcal{N}}, \{T_i\}_{i \in \mathcal{N}} \rangle$ be an endogenous aggregation game with more than two players. Every goal-efficient NE of $\langle \mathcal{A}, \{\pi_i\}_{i \in \mathcal{N}} \rangle$ is a surviving equilibrium.
Equilibria in $\mathcal{A}^T$ games

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- Notice: (substantially) independent of aggregation procedure and integrity constraint.
Proposition

Let $A^T = \langle A, \{\pi_i\}_{i \in N}, \{T_i\}_{i \in N} \rangle$ be an endogenous aggregation game for maj such that $\bigwedge_{i \in N} \gamma_i$ is consistent. No goal-inefficient NE of $A$ is a surviving equilibrium.
Avoiding global inconsistency

- What happens if players want to achieve IC?
Avoiding global inconsistency

Definition (Augumented preferences in $A^\pi$ games)

For ballot profiles $B, B'$,

$$B \succeq_i^{(\pi, I_C)} B'$$

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Avoiding global inconsistency

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For ballot profiles $B, B'$,

$B \succeq_i^{(\pi, \neg IC)} B'$

$\iff$

- $(F(B') \models \neg IC$ and $F(B) \models IC)$ or
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For ballot profiles $B, B'$,

$$B \succeq_{i}^{(\pi, IC)} B'$$

$\iff$

- $(F(B') \models \neg IC \text{ and } F(B) \models IC)$ or
- $(F(B') \models \neg IC \iff F(B) \models IC)$ and $B \succeq_{i}^{\pi} B'$
Under the newly defined preference relations

Proposition

Let $A^T = \langle A, \{\pi_i\}_{i \in \mathcal{N}}, \{T_i\}_{i \in \mathcal{N}} \rangle$ be a consistent endogenous aggregation game with more than two players. Every goal-efficient NE of $\langle A, \{\pi_i\}_{i \in \mathcal{N}} \rangle$ is a surviving equilibrium if and only if it satisfies IC.
Avoiding global inconsistency

- Personal interest cannot disrupt safety...
Avoiding global inconsistency

- Personal interest cannot disrupt safety... of the agenda.
Summarizing

- In aggregation games equilibrium outcomes might be goal-inefficient;
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- Redistributing payoff in uniform games can avoid goal-inefficient outcomes;
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- In aggregation games equilibrium outcomes might be goal-inefficient;
- Redistributing payoff in uniform games can avoid goal-inefficient outcomes;
- Pre-vote negotiations avoid goal-inefficient outcomes, whenever goal-efficient ones are possible.
Ideas for the future

- Different sorts of transfers (e.g., not on outcomes but on strategies of other players);
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- Different sorts of transfers (e.g., not on outcomes but on strategies of other players);
- Constraining transfers (somehow, someway);
- Milder incentives to avoid violation of integrity constraints.
- Use different IC for every voter. No logical but political dependence among issues, coherence rather than consistency!