Equazioni Differenziali 2 — Foglio 6

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Exercise 1 Let $(y_n)_{n\in\mathbb{N}}$ be a sequence of functions in $C([0,1];\mathbb{R})$ and let $y \in C([0,1];\mathbb{R})$. Assume that:

i) There exists a dense set $A \subset [0,1]$ such that $y_n(x) \to y(x)$ as $n \to \infty$ for all $x \in A$;

ii) Any subsequence of $(y_n)_{n\in\mathbb{N}}$ has a subsequence which converges uniformly.

Prove that $y_n \to y$ uniformly as $n \to +\infty$.

Exercise 2 Study existence of periodic and global solutions $y \in C^\infty(\mathbb{R})$ to the equation

$$y'' = \alpha^2 - y^2,$$

where α is a real parameter.

Exercise 3 Let $\varphi \in C^1(\mathbb{R})$ be a function. Compute the solution $u \in C^1(U)$, $U \subset \mathbb{R}^2$ open neighborhood of $(0,1) \in \mathbb{R}^2$, to the Cauchy Problem

$$\begin{cases} x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = 2u, & \text{in } U, \\ u(x,1) = \varphi(x), & \text{for } (x,1) \in U. \end{cases}$$

Determine the maximal domain U of the solution.

Exercise 4 For $\alpha > 0$ and $\lambda \in \mathbb{R}$, consider the differential problem

$$\begin{cases} y' = \dfrac{y \sin y}{1 + x^\alpha}, & x > 0, \\ \lim\limits_{x\to+\infty} y(x) = \lambda. \end{cases}$$

For given α and λ, study existence and uniqueness of solutions $y \in C^1(0,+\infty)$. Even partial answers are welcome.