DIPARTIMENTO DI MATEMATICA TULLIO LEVI-CIVITA
VIA TRIESTE, 63 - 35121 PADOVA (PD) - ITALY

Prof. Roberto Monti
email: monti@math.unipd.it

UNIVERSITÀ DEGLI STUDI DI PADOVA

Optimal transport and elements of higher analysis

Roberto Monti – Unipd – monti@math.unipd.it

The course is an introduction to Optimal Transportation Theory, regarded as an occasion to present ideas, methods and theorems at the core of modern and contemporary analysis.

The course is open to all the students of the SGSS (all years, mathematics, physics and applied sciences) but it is mainly intended for third year students. Even though the topics are advanced, the lectures will be understandable by any student with solid background in Analysis 1 and 2, and Measure Theory. The course can be attended also by Master students and by anyone loving demanding challenges.

Lectures and exam: 30 hours at the blackboard. Final oral exam.

Final schedule: 2+2 = 4 hours weekly starting Monday 7 November 2022:

Monday 18:30-20.30 Auletta A, residenza SGSS

Friday 15:30- 17.30 Auletta A, residenza SGSS

Program:

1. Basics in measure theory. Change of variable formula.
2. Weak topologies on Borel measures. Compactness for measures.
3. Optimal transportation according to Monge's formulation. Examples and applications. Dimension n = 1.
4. Kantorovich dual formulation. From transport plans (measures) to transport maps (functions).
5. Existence of solutions with the direct method.
6. c-duality theory and c-cyclical monotonicity.
7. Kantorovic-Rubistein duality and Convex Analysis.
8. Necessary and sufficient optimality conditions.
9. Brenier, Knott-Smith theorem.
10. Isoperimetric inequality, Brunn-Minkowski and Sobolev-type inequalities.
11. Monge-Ampère equation: introduction to Caffarelli regularity theory.
12. Short introduction to Riemmanian geometry. Optimal transport and Ricci curvature.

Bibliography:

1. Ambrosio-Bruè-Semola, Lectures on Optimal Transport: Springer 2021
2. Santambrogio, Optimal Transport for Applied Mathematicians, Birkhäuser 2015
3. Villani, Topics in Optimal Transportation, AMS 2003
4. Villani, Optimal Transportation. Old and new, Springer 2009
5. Monti, hand-written lectures notes.