

Unimodularity and Preservation of Volumes in Nonholonomic Mechanics

YURI FEDOROV¹, LUIS C. GARCÍA NARANJO², JUAN CARLOS
MARRERO³

¹ *Department de Matemàtica Aplicada I*
Universitat Politècnica de Catalunya, Barcelona, E-08028 SPAIN.
E-mail address: Yuri.Fedorov@upc.edu

² *Departamento de Matemáticas, ITAM (Instituto Tecnológico Autónomo de México),*
01080–Mexico City, MEXICO.
E-mail address: luis.garcianaranjo@gmail.com

³ *ULL-CSIC Geometría Diferencial y Mecánica Geométrica*
Departamento de Matemática Fundamental, Facultad de Matemáticas, Universidad
de la Laguna, La Laguna, Tenerife, Canary Islands, SPAIN.
E-mail address: jcmarrer@ull.es

The existence of an invariant measure for a system of differential equations is a very important property. From the point of view of dynamical systems, it is a key ingredient for the application of ergodic theory. It is also a crucial hypothesis in Jacobi's theorem of the last multiplier that establishes integrability of the system via quadratures. Moreover, the existence of a smooth invariant measure imposes certain restrictions on the qualitative nature of the fixed points of the system; namely, it prohibits the existence of asymptotic equilibria.

In this talk I will show how the geometric structure that underlies the equations of motion of a nonholonomic mechanical system with symmetry can be exploited to obtain necessary and sufficient conditions for the existence of an invariant volume. As a concrete application, we show that the reduced equations of a rigid body with a planar face that rolls without slipping over a sphere possess an invariant measure if and only if the body is planar, or axially symmetric.