Title: **Non-Lipschitz points and the SBV regularity of the minimum time function**

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Abstract: We study the Hausdorff dimension of the set of non-Lipschitz points $\mathcal{S}$ of the minimum time function T under controllability conditions which imply the continuity of T.
We consider first the case of *normal linear* control systems with constant coefficients in $\mathbb{R}^N$. We characterize $\mathcal{S}$ as the set of points which can be reached from the origin by an optimal trajectory (of the reversed dynamics) with vanishing minimized Hamiltonian. Linearity permits an explicit representation of $\mathcal{S}$. Furthermore, we show that $\mathcal{S}$ is $\mathcal{H}^{N-1}$-rectifiable and has positive $\mathcal{H}^{N-1}$-measure.
Second, we consider a class of control-affine *planar* nonlinear systems satisfying a second order controllability condition: we characterize the set $\mathcal{S}$ in a neighborhood of the origin in a similar way and prove its $\mathcal{H}^1$-rectifiability and that $\mathcal{H}^1(\mathcal{S}) > 0$.
In both (linear and nonlinear) cases, T is known to have epigraph with positive reach, hence to be a locally BV function. Since the Cantor part of DT must be concentrated in $\mathcal{S}$, our analysis yields that T is SBV, i.e., the Cantor part of DT vanishes.

This talk is based on a joint work with Giovanni Colombo and Khai T. Nguyen.