NON-LINEAR BOUNDARY VALUE PROBLEMS FOR THE FRACTIONAL LAPLACIAN

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The fractional Laplacian operator is a pseudo-differential operator defined for all measurable function $u : \mathbb{R}^N \to \mathbb{R}$ by

$$(-\Delta)^s u(x) := C(N,s) \lim_{\varepsilon \searrow 0} \int_{\mathbb{R}^N \setminus B_\varepsilon(x)} \frac{u(x) - u(y)}{|x-y|^{N+2s}}\, dy, \qquad x \in \mathbb{R}^N,$$

where $s \in (0,1)$ and $C(N,s) > 0$ is a suitable normalization constant. Nonlocal operators such as $(-\Delta)^s$ naturally arise in many applications, as the outcome of stabilization of Lévy processes (see Caffarelli [3]). The natural functional-analytic framework for the study of such operators is that of fractional Sobolev spaces (see Di Nezza, Palatucci & Valdinoci [4]).

After a brief introduction on some properties of $(-\Delta)^s$, we will focus on the issue of formulating and dealing with the Dirichlet problem on a bounded, smooth domain Ω, which is delicate due to the non-local nature of the operator. We will hint at possible variational interpretation of such problem (mainly that of Servadei & Valdinoci [7]) and some regularity theory (see Ros Oton & Serra [6]), then we will introduce a topological result, proved by Iannizzotto, Mosconi & Squassina [5], on local minimizers for the energy functional of the non-linear problem

$$\begin{cases} (-\Delta)^s\, u = f(x,u) & \text{in } \Omega \\ u = 0 & \text{in } \mathbb{R}^N \setminus \Omega, \end{cases} \tag{1}$$

which extends a classical theorem of Brezis & Nirenberg [2] to the non-local framework (see also Barrios *et al.* [1]) and allows several applications such as existence/multiplicity results for the solutions of (1).

References

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