CONVERGENCE OF THE SOLUTIONS OF THE DISCOUNTED EQUATION

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Abstract. We consider the discounted equation

$$\lambda u_\lambda(x) + H(x, d_x u_\lambda) = c \qquad \text{in } M,$$

set on a compact and connected Riemannian manifold M, where λ is a positive parameter, H is a continuous Hamiltonian, coercive in the momentum, and c is the associated critical value. Under these assumptions, the corresponding solutions $u_\lambda : M \to \mathbb{R}$ are equi–bounded and equi–Lipschitz, hence they uniformly converge, along subsequences as the discount factor λ goes to 0, to a viscosity solution of the critical equation

$$H(x, d_x u) = c \qquad \text{in } M.$$

Due to the lack of a uniqueness result for the critical equation, it is not clear at this point that the solutions selected at the limit along different subsequences are the same. When H is additionally assumed convex in the momentum, we prove that the u_λ uniformly converge to a specific solution of the critical equation, characterized in terms of a class of probability measures introduced in the framework of weak KAM Theory. This is a joint work with A. Fathi, R. Iturriaga and M. Zavidovique.