# CONTENTS

1 User Guide 3
1.1 Installing scikit-learn ................................. 3
1.2 Tutorials: From the bottom up with scikit-learn ................................. 6
1.3 Supervised learning ........................................ 35
1.4 Unsupervised learning .................................... 99
1.5 Model Selection ........................................... 154
1.6 Dataset transformations ................................. 204
1.7 Dataset loading utilities ................................. 218
1.8 Reference ................................................ 226

2 Example Gallery 661
2.1 Examples ................................................ 661

3 Development 981
3.1 Contributing .............................................. 981
3.2 How to optimize for speed .............................. 988
3.3 Utilities for Developers ................................. 993
3.4 Developers’ Tips for Debugging ...................... 997
3.5 About us .................................................. 997
3.6 Support ................................................... 999
3.7 0.12 .................................................... 1000
3.8 0.11 .................................................... 1000
3.9 0.10 .................................................... 1004
3.10 0.9 .................................................... 1007
3.11 0.8 .................................................... 1010
3.12 0.7 .................................................... 1012
3.13 0.6 .................................................... 1013
3.14 0.5 .................................................... 1014
3.15 0.4 .................................................... 1016
3.16 Presentations and Tutorials on Scikit-Learn ......................... 1017

Bibliography 1019
Python Module Index 1023
Python Module Index 1025
Index 1027
scikit-learn is a Python module integrating classic machine learning algorithms in the tightly-knit scientific Python world (numpy, scipy, matplotlib). It aims to provide simple and efficient solutions to learning problems, accessible to everybody and reusable in various contexts: machine-learning as a versatile tool for science and engineering.

License: Open source, commercially usable: BSD license (3 clause)

Documentation for scikit-learn version 0.12-git. For other versions and printable format, see Documentation resources.
1.1 Installing scikit-learn

There are different ways to get scikit-learn installed:

- Install the version of scikit-learn provided by your operating system distribution. This is the quickest option for those who have operating systems that distribute scikit-learn.
- Install an official release. This is the best approach for users who want a stable version number and aren’t concerned about running a slightly older version of scikit-learn.
- Install the latest development version. This is best for users who want the latest-and-greatest features and aren’t afraid of running brand-new code.

Note: If you wish to contribute to the project, it’s recommended you install the latest development version.

1.1.1 Installing an official release

Installing from source

Installing from source requires you to have installed python (>= 2.6), numpy (>= 1.3), scipy (>= 0.7), setuptools, python development headers and a working C++ compiler. Under Debian-based systems you can get all this by executing with root privileges:

```bash
sudo apt-get install python-dev python-numpy python-numpy-dev python-setuptools python-numpy-dev python-scipy libatlas-dev g++
```

Note: In order to build the documentation and run the example code contains in this documentation you will need matplotlib:

```bash
sudo apt-get install python-matplotlib
```

Note: On Ubuntu LTS (10.04) the package libatlas-dev is called libatlas-headers

Easy install

This is usually the fastest way to install the latest stable release. If you have pip or easy_install, you can install or update with the command:
pip install -U scikit-learn

or:

easy_install -U scikit-learn

for easy_install. Note that you might need root privileges to run these commands.

From source package

Download the package from http://pypi.python.org/pypi/scikit-learn/, unpack the sources and cd into archive. This packages uses distutils, which is the default way of installing python modules. The install command is:

cd
python setup.py install

Windows installer

You can download a windows installer from downloads in the project’s web page. Note that must also have installed the packages numpy and setuptools.

This package is also expected to work with python(x,y) as of 2.6.5.5.

Installing on Windows 64bit

To install a 64bit version of the scikit, you can download the binaries from http://www.lfd.uci.edu/~gohlke/pythonlibs/#scikit-learn Note that this will require a compatible version of numpy, scipy and matplotlib. The easiest option is to also download them from the same URL.

Building on windows

To build scikit-learn on windows you will need a C/C++ compiler in addition to numpy, scipy and setuptools. At least MinGW (a port of GCC to Windows OS) and the Microsoft Visual C++ 2008 should work out of the box. To force the use of a particular compiler, write a file named setup.cfg in the source directory with the content:

[bld]

[build]

[build_ext]

compiler=my_compiler

where my_compiler should be one of mingw32 or msvc.

When the appropriate compiler has been set, and assuming Python is in your PATH (see Python FAQ for windows for more details), installation is done by executing the command:

python setup.py install

To build a precompiled package like the ones distributed at the downloads section, the command to execute is:

python setup.py bdist_wininst -b doc/logos/scikit-learn-logo.bmp

This will create an installable binary under directory dist/.
1.1.2 Third party distributions of scikit-learn

Some third-party distributions are now providing versions of scikit-learn integrated with their package-management systems.

These can make installation and upgrading much easier for users since the integration includes the ability to automatically install dependencies (numpy, scipy) that scikit-learn requires.

The following is a list of Linux distributions that provide their own version of scikit-learn:

**Debian and derivatives (Ubuntu)**

The Debian package is named python-sklearn (formerly python-scikits-learn) and can be installed using the following commands with root privileges:

```
apt-get install python-sklearn
```

Additionally, backport builds of the most recent release of scikit-learn for existing releases of Debian and Ubuntu are available from NeuroDebian repository.

**Python(x, y)**

The Python(x, y) distributes scikit-learn as an additional plugin, which can be found in the Additional plugins page.

**Enthought Python distribution**

The Enthought Python Distribution already ships a recent version.

**Macports**

The macport's package is named `py26-sklearn` or `py27-sklearn` depending on the version of Python. It can be installed by typing the following command:

```
sudo port install py26-scikits-learn
```

or:

```
sudo port install py27-scikits-learn
```

depending on the version of Python you want to use.

**NetBSD**

scikit-learn is available via pkgsrc-wip:

```
http://pkgsrc.se/wip/py-scikit_learn
```

1.1.3 Bleeding Edge

See section Retrieving the latest code on how to get the development version.
1.1.4 Testing

Testing requires having the nose library. After installation, the package can be tested by executing from outside the source directory:

```
nosetests sklearn --exe
```

This should give you a lot of output (and some warnings) but eventually should finish with the a text similar to:

```
Ran 601 tests in 27.920s
OK (SKIP=2)
```

otherwise please consider posting an issue into the bug tracker or to the Mailing List.

---

**Note:** Alternative testing method

If for some reason the recommended method is failing for you, please try the alternate method:

```
python -c "import sklearn; sklearn.test()"
```

This method might display doctest failures because of nosetests issues.

scikit-learn can also be tested without having the package installed. For this you must compile the sources inplace from the source directory:

```
python setup.py build_ext --inplace
```

Test can now be run using nosetests:

```
nosetests sklearn/
```

This is automated in the commands:

```
made in
```

and:

```
made test
```

1.2 Tutorials: From the bottom up with scikit-learn

Quick start

In this section, we introduce the machine learning vocabulary that we use throughout scikit-learn and give a simple learning example.

1.2.1 An Introduction to machine learning with scikit-learn

Section contents

In this section, we introduce the machine learning vocabulary that we use throughout scikit-learn and give a simple learning example.
Machine learning: the problem setting

In general, a learning problem considers a set of n samples of data and try to predict properties of unknown data. If each sample is more than a single number, and for instance a multi-dimensional entry (aka multivariate data), is it said to have several attributes, or features.

We can separate learning problems in a few large categories:

- **supervised learning**, in which the data comes with additional attributes that we want to predict (Click here to go to the Scikit-Learn supervised learning page). This problem can be either:
  - **classification**: samples belong to two or more classes and we want to learn from already labeled data how to predict the class of unlabeled data. An example of classification problem would be the digit recognition example, in which the aim is to assign each input vector to one of a finite number of discrete categories.
  - **regression**: if the desired output consists of one or more continuous variables, then the task is called regression. An example of a regression problem would be the prediction of the length of a salmon as a function of its age and weight.

- **unsupervised learning**, in which the training data consists of a set of input vectors x without any corresponding target values. The goal in such problems may be to discover groups of similar examples within the data, where it is called clustering, or to determine the distribution of data within the input space, known as density estimation, or to project the data from a high-dimensional space down to two or thee dimensions for the purpose of visualization (Click here to go to the Scikit-Learn unsupervised learning page).

### Training set and testing set

Machine learning is about learning some properties of a data set and applying them to new data. This is why a common practice in machine learning to evaluate an algorithm is to split the data at hand in two sets, one that we call a **training set** on which we learn data properties, and one that we call a **testing set**, on which we test these properties.

### Loading an example dataset

scikit-learn comes with a few standard datasets, for instance the iris and digits datasets for classification and the boston house prices dataset for regression.:

```python
>>> from sklearn import datasets
>>> iris = datasets.load_iris()
>>> digits = datasets.load_digits()
```

A dataset is a dictionary-like object that holds all the data and some metadata about the data. This data is stored in the .data member, which is a n_samples, n_features array. In the case of supervised problem, explanatory variables are stored in the .target member. More details on the different datasets can be found in the dedicated section.

For instance, in the case of the digits dataset, digits.data gives access to the features that can be used to classify the digits samples:

```python
>>> print digits.data
[[ 0.  0.  5. ...,  0.  0.  0.]
 [ 0.  0.  0. ..., 10.  0.  0.]
 [ 0.  0.  0. ..., 16.  9.  0.]
 ..., 
 [ 0.  0.  1. ...,  6.  0.  0.]
 [ 0.  0.  2. ..., 12.  0.  0.]
 [ 0.  0. 10. ..., 12.  1.  0.]]
```
and `digits.target` gives the ground truth for the digit dataset, that is the number corresponding to each digit image that we are trying to learn:

```python
>>> digits.target
array([0, 1, 2, ..., 8, 9, 8])
```

### Shape of the data arrays

The data is always a 2D array, `n_samples, n_features`, although the original data may have had a different shape. In the case of the digits, each original sample is an image of shape `8, 8` and can be accessed using:

```python
>>> digits.images[0]
array([[ 0., 0., 5., 13., 9., 1., 0., 0.],
      [ 0., 0., 13., 15., 10., 15., 0., 0.],
      [ 0., 3., 15., 2., 0., 11., 8., 0.],
      [ 0., 4., 12., 0., 0., 8., 8., 0.],
      [ 0., 5., 8., 0., 0., 9., 8., 0.],
      [ 0., 4., 11., 0., 1., 12., 7., 0.],
      [ 0., 2., 14., 5., 10., 12., 0., 0.],
      [ 0., 0., 6., 13., 10., 0., 0., 0.]])
```

The simple example on this dataset illustrates how starting from the original problem one can shape the data for consumption in the scikit-learn.

### Learning and Predicting

In the case of the digits dataset, the task is to predict the value of a hand-written digit from an image. We are given samples of each of the 10 possible classes on which we fit an estimator to be able to predict the labels corresponding to new data.

In scikit-learn, an estimator is just a plain Python class that implements the methods `fit(X, Y)` and `predict(T)`.

An example of estimator is the class `sklearn.svm.SVC` that implements Support Vector Classification. The constructor of an estimator takes as arguments the parameters of the model, but for the time being, we will consider the estimator as a black box:

```python
>>> from sklearn import svm
>>> clf = svm.SVC(gamma=0.001, C=100.)
```

### Choosing the parameters of the model

In this example we set the value of `gamma` manually. It is possible to automatically find good values for the parameters by using tools such as `grid search` and `cross validation`.

We call our estimator instance `clf` as it is a classifier. It now must be fitted to the model, that is, it must learn from the model. This is done by passing our training set to the `fit` method. As a training set, let us use all the images of our dataset apart from the last one:

```python
>>> clf.fit(digits.data[:-1], digits.target[:-1])
SVC(C=100.0, cache_size=200, class_weight=None, coef0=0.0, degree=3,
   gamma=0.001, kernel='rbf', probability=False, shrinking=True, tol=0.001,
   verbose=False)
```

Now you can predict new values, in particular, we can ask to the classifier what is the digit of our last image in the `digits` dataset, which we have not used to train the classifier:

```python
```
The corresponding image is the following:

As you can see, it is a challenging task: the images are of poor resolution. Do you agree with the classifier?

A complete example of this classification problem is available as an example that you can run and study: Recognizing hand-written digits.

**Model persistence**

It is possible to save a model in the scikit by using Python’s built-in persistence model, namely pickle:

```python
>>> from sklearn import svm
>>> from sklearn import datasets

>>> clf = svm.SVC()
>>> iris = datasets.load_iris()
>>> X, y = iris.data, iris.target
>>> clf.fit(X, y)
SVC(C=1.0, cache_size=200, class_weight=None, coef0=0.0, degree=3, gamma=0.25,
    kernel='rbf', probability=False, shrinking=True, tol=0.001,
    verbose=False)
```

```python
>>> import pickle

>>> s = pickle.dumps(clf)
>>> clf2 = pickle.loads(s)
>>> clf2.predict(X[0])
array([0.])
>>> y[0]
0
```

In the specific case of the scikit, it may be more interesting to use joblib’s replacement of pickle (joblib.dump & joblib.load), which is more efficient on big data, but can only pickle to the disk and not to a string:

```python
>>> from sklearn.externals import joblib

>>> joblib.dump(clf, 'filename.pkl')
```

### Statistical-learning Tutorial

This tutorial covers some of the models and tools available to do data-processing with Scikit Learn and how to learn from your data.

### 1.2.2 A tutorial on statistical-learning for scientific data processing
Statistical learning

Machine learning is a technique with a growing importance, as the size of the datasets experimental sciences are facing is rapidly growing. Problems it tackles range from building a prediction function linking different observations, to classifying observations, or learning the structure in an unlabeled dataset.

This tutorial will explore statistical learning, that is the use of machine learning techniques with the goal of statistical inference: drawing conclusions on the data at hand.

**sklearn** is a Python module integrating classic machine learning algorithms in the tightly-knit world of scientific Python packages (*numpy*, *scipy*, *matplotlib*).

**Warning:** In scikit-learn release 0.9, the import path has changed from *scikits.learn* to *sklearn*. To import with cross-version compatibility, use:

```python
try:
    from sklearn import something
except ImportError:
    from scikits.learn import something
```

Statistical learning: the setting and the estimator object in the scikit-learn

Datasets

The *scikit-learn* deals with learning information from one or more datasets that are represented as 2D arrays. They can be understood as a list of multi-dimensional observations. We say that the first axis of these arrays is the samples axis, while the second is the features axis.

A simple example shipped with the scikit: iris dataset

```python
>>> from sklearn import datasets
>>> iris = datasets.load_iris()
>>> data = iris.data
>>> data.shape
(150, 4)
```

It is made of 150 observations of irises, each described by 4 features: their sepal and petal length and width, as detailed in *iris.DESCR*.

When the data is not initially in the (*n_samples*, *n_features*) shape, it needs to be preprocessed to be used by the scikit.
An example of reshaping data: the digits dataset

The digits dataset is made of 1797 8x8 images of hand-written digits

```python
>>> digits = datasets.load_digits()
>>> digits.images.shape
(1797, 8, 8)
>>> import pylab as pl
>>> pl.imshow(digits.images[-1], cmap=pl.cm.gray_r)
<matplotlib.image.AxesImage object at ...>
```

To use this dataset with the scikit, we transform each 8x8 image in a feature vector of length 64

```python
>>> data = digits.images.reshape((digits.images.shape[0], -1))
```

Estimators objects

**Fitting data:** The core object of the scikit-learn is the estimator object. All estimator objects expose a `fit` method, that takes a dataset (2D array):

```python
>>> estimator.fit(data)
```

**Estimator parameters:** All the parameters of an estimator can be set when it is instanciated, or by modifying the corresponding attribute:

```python
>>> estimator = Estimator(param1=1, param2=2)
>>> estimator.param1
1
```

**Estimated parameters:** When data is fitted with an estimator, parameters are estimated from the data at hand. All the estimated parameters are attributes of the estimator object ending by an underscore:

```python
>>> estimator.estimated_param_
```

Supervised learning: predicting an output variable from high-dimensional observations

**The problem solved in supervised learning**

Supervised learning consists in learning the link between two datasets: the observed data $X$, and an external variable $y$ that we are trying to predict, usually called target or labels. Most often, $y$ is a 1D array of length $n_{samples}$.

All supervised estimators in the scikit-learn implement a $fit(X, y)$ method to fit the model, and a $predict(X)$ method that, given unlabeled observations $X$, returns the predicted labels $y$. 
Vocabulary: classification and regression

If the prediction task is to classify the observations in a set of finite labels, in other words to “name” the objects observed, the task is said to be a **classification** task. On the opposite, if the goal is to predict a continuous target variable, it is said to be a **regression** task.

In the *scikit-learn*, for classification tasks, $y$ is a vector of integers.

Note: See the Introduction to machine learning with Scikit-learn Tutorial for a quick run-through on the basic machine learning vocabulary used within Scikit-learn.

Nearest neighbor and the curse of dimensionality

Classifying irises:

The iris dataset is a classification task consisting in identifying 3 different types of irises (Setosa, Versicolour, and Virginica) from their petal and sepal length and width:

```python
>>> import numpy as np
>>> from sklearn import datasets
>>> iris = datasets.load_iris()
>>> iris_X = iris.data
>>> iris_y = iris.target
>>> np.unique(iris_y)
array([0, 1, 2])
```

**k-Nearest neighbors classifier**  
The simplest possible classifier is the nearest neighbor: given a new observation $X_{test}$, find in the training set (i.e. the data used to train the estimator) the observation with the closest feature vector. (Please see the Nearest Neighbors section of the online Scikit-learn documentation for more information about this type of classifier.)

Training set and testing set

When experimenting with learning algorithm, it is important not to test the prediction of an estimator on the data used to fit the estimator, as this would not be evaluating the performance of the estimator on new data. This is why datasets are often split into *train* and *test* data.
KNN (k nearest neighbors) classification example:

```python
>>> # Split iris data in train and test data
>>> # A random permutation, to split the data randomly
>>> np.random.seed(0)
>>> indices = np.random.permutation(len(iris_X))
>>> iris_X_train = iris_X[indices[:-10]]
>>> iris_y_train = iris_y[indices[:-10]]
>>> iris_X_test = iris_X[indices[-10:]]
>>> iris_y_test = iris_y[indices[-10:]]
>>> from sklearn.neighbors import KNeighborsClassifier
>>> knn = KNeighborsClassifier()
>>> knn.fit(iris_X_train, iris_y_train)
KNeighborsClassifier(algorithm='auto', leaf_size=30, n_neighbors=5, p=2,
                    warn_on_equidistant=True, weights='uniform')
>>> knn.predict(iris_X_test)
array([1, 2, 1, 0, 0, 0, 2, 1, 2, 0])
>>> iris_y_test
array([1, 1, 1, 0, 0, 0, 2, 1, 2, 0])
```

The curse of dimensionality

For an estimator to be effective, you need the distance between neighboring points to be less than some value $d$, which depends on the problem. In one dimension, this requires on average $n \sim 1/d$ points. In the context of the above KNN example, if the data is only described by one feature, with values ranging from 0 to 1 and with $n$ training observations, new data will thus be no further away than $1/n$. Therefore, the nearest neighbor decision rule will be efficient as soon as $1/n$ is small compared to the scale of between-class feature variations.

If the number of features is $p$, you now require $n \sim 1/d^p$ points. Let’s say that we require 10 points in one dimension: Now $10^p$ points are required in $p$ dimensions to pave the $[0, 1]$ space. As $p$ becomes large, the number of training points required for a good estimator grows exponentially.

For example, if each point is just a single number (8 bytes), then an effective KNN estimator in a paltry $p\sim20$ dimensions would require more training data than the current estimated size of the entire internet! ($\pm1000$ Exabytes or so).

This is called the curse of dimensionality and is a core problem that machine learning addresses.
Linear model: from regression to sparsity

Diabetes dataset

The diabetes dataset consists of 10 physiological variables (age, sex, weight, blood pressure) measure on 442 patients, and an indication of disease progression after one year:

```python
>>> diabetes = datasets.load_diabetes()
>>> diabetes_X_train = diabetes.data[:-20]
>>> diabetes_X_test = diabetes.data[-20:]
>>> diabetes_y_train = diabetes.target[:-20]
>>> diabetes_y_test = diabetes.target[-20:]
```

The task at hand is to predict disease progression from physiological variables.

Linear regression  LinearRegression, in it's simplest form, fits a linear model to the data set by adjusting a set of parameters, in order to make the sum of the squared residuals of the model as small as possible.

```python
>>> from sklearn import linear_model
>>> regr = linear_model.LinearRegression()
>>> regr.fit(diabetes_X_train, diabetes_y_train)
LinearRegression(copy_X=True, fit_intercept=True, normalize=False)
>>> print regr.coef_
[ 0.30349955 -237.63931533 510.53060544 327.73698041 -814.13170937
  492.81458798 102.84845219 184.60648906 743.51961675 76.09517222]
```

- $X$: data
- $y$: target variable
- $\beta$: Coefficients
- $\epsilon$: Observation noise

```python
>>> from sklearn import linear_model
>>> regr = linear_model.LinearRegression()
>>> regr.fit(diabetes_X_train, diabetes_y_train)
LinearRegression(copy_X=True, fit_intercept=True, normalize=False)
```

- # The mean square error
- # Explained variance score: 1 is perfect prediction
- # and 0 means that there is no linear relationship
- # between $X$ and $Y$.

```python
>>> np.mean((regr.predict(diabetes_X_test)-diabetes_y_test)**2)
2004.56760268...
```

```python
>>> regr.score(diabetes_X_test, diabetes_y_test)
0.5850753022690...
```
Shrinkage  If there are few data points per dimension, noise in the observations induces high variance:

```python
>>> X = np.c_[.5, 1].T
>>> y = [.5, 1]
>>> test = np.c_[0, 2].T
>>> regr = linear_model.LinearRegression()

>>> import pylab as pl
>>> pl.figure()

>>> np.random.seed(0)
>>> for _ in range(6):
...    this_X = .1*np.random.normal(size=(2, 1)) + X
...    regr.fit(this_X, y)
...    pl.plot(test, regr.predict(test))
...    pl.scatter(this_X, y, s=3)
```

A solution, in high-dimensional statistical learning, is to shrink the regression coefficients to zero: any two randomly chosen set of observations are likely to be uncorrelated. This is called Ridge regression:

```python
>>> regr = linear_model.Ridge(alpha=.1)

>>> pl.figure()

>>> np.random.seed(0)
>>> for _ in range(6):
...    this_X = .1*np.random.normal(size=(2, 1)) + X
...    regr.fit(this_X, y)
...    pl.plot(test, regr.predict(test))
...    pl.scatter(this_X, y, s=3)
```
This is an example of **bias/variance tradeoff**: the larger the ridge alpha parameter, the higher the bias and the lower the variance.

We can choose *alpha* to minimize left out error, this time using the diabetes dataset, rather than our synthetic data:

```python
>>> alphas = np.logspace(-4, -1, 6)
>>> print [regr.set_params(alpha=alpha ... ).fit(diabetes_X_train, diabetes_y_train, ...
... ).score(diabetes_X_test, diabetes_y_test) for alpha in alphas]
[0.585110683883..., 0.5852073015444..., 0.5854677540698..., 0.5855512036503..., 0.5830717085554..., 0.57058999437...]
```

**Note:** Capturing in the fitted parameters noise that prevents the model to generalize to new data is called **overfitting**. The bias introduced by the ridge regression is called a **regularization**.

---

**Sparsity**  
**Fitting only features 1 and 2**

---

**Note:** A representation of the full diabetes dataset would involve 11 dimensions (10 feature dimensions, and one of the target variable). It is hard to develop an intuition on such representation, but it may be useful to keep in mind that it would be a fairly **empty** space.

We can see that although feature 2 has a strong coefficient on the full model, it conveys little information on *y* when considered with feature 1.

To improve the conditioning of the problem (mitigate the **The curse of dimensionality**), it would be interesting to select only the informative features and set non-informative ones, like feature 2 to 0. Ridge regression will decrease their contribution, but not set them to zero. Another penalization approach, called **Lasso** (least absolute shrinkage and selection operator), can set some coefficients to zero. Such methods are called **sparse method**, and sparsity can be seen as an application of Occam’s razor: **prefer simpler models**.
Different algorithms for a same problem

Different algorithms can be used to solve the same mathematical problem. For instance the Lasso object in the scikit-learn solves the lasso regression using a coordinate decent method, that is efficient on large datasets. However, the scikit-learn also provides the LassoLars object, using the LARS which is very efficient for problems in which the weight vector estimated is very sparse, that is problems with very few observations.

Classification

For classification, as in the labeling iris task, linear regression is not the right approach, as it will give too much weight to data far from the decision frontier. A linear approach is to fit a sigmoid function, or logistic function:

\[
y = \text{sigmoid}(X\beta - \text{offset}) + \epsilon = \frac{1}{1 + \exp(-X\beta + \text{offset})} + \epsilon
\]

>> logit = linear_model.LogisticRegression(C=1e5)
>> logit.fit(iris_X_train, iris_y_train)
LogisticRegression(C=100000.0, class_weight=None, dual=False, 
fit_intercept=True, intercept_scaling=1, penalty='l2', 
tol=0.0001)
This is known as LogisticRegression.

### Multiclass classification

If you have several classes to predict, an option often used is to fit one-versus-all classifiers, and use a voting heuristic for the final decision.

### Shrinkage and sparsity with logistic regression

The $C$ parameter controls the amount of regularization in the LogisticRegression object: a large value for $C$ results in less regularization. `penalty="l2"` gives Shrinkage (i.e. non-sparse coefficients), while `penalty="l1"` gives Sparsity.

### Exercise

Try classifying the digits dataset with nearest neighbors and a linear model. Leave out the last 10% and test prediction performance on these observations.

```python
from sklearn import datasets, neighbors, linear_model
digits = datasets.load_digits()
X_digits = digits.data
y_digits = digits.target

# Solution:
```

### Support vector machines (SVMs)

**Linear SVMs** Support Vector Machines belong to the discriminant model family: they try to find a combination of samples to build a plane maximizing the margin between the two classes. Regularization is set by the $C$ parameter: a small value for $C$ means the margin is calculated using many or all of the observations around the separating line (more regularization); a large value for $C$ means the margin is calculated on observations close to the separating line (less regularization).
SVMs can be used in regression — SVR (Support Vector Regression) — or in classification — SVC (Support Vector Classification).

```python
>>> from sklearn import svm
>>> svc = svm.SVC(kernel='linear')
>>> svc.fit(iris_X_train, iris_y_train)
```

```
SVC(C=1.0, cache_size=200, class_weight=None, coef0=0.0, degree=3, gamma=0.0,
    kernel='linear', probability=False, shrinking=True, tol=0.001,
    verbose=False)
```

**Warning:** Normalizing data
For many estimators, including the SVMs, having datasets with unit standard deviation for each feature is important to get good prediction.

**Using kernels** Classes are not always linearly separable in feature space. The solution is to build a decision function that is not linear but that may be for instance polynomial. This is done using the *kernel trick* that can be seen as creating an decision energy by positioning kernels on observations:
Linear kernel

Polynomial kernel

```python
>>> svc = svm.SVC(kernel='linear')
```

```python
>>> svc = svm.SVC(kernel='poly',
...    degree=3)
>>> # degree: polynomial degree
```

RBF kernel (Radial Basis Function)

```python
>>> svc = svm.SVC(kernel='rbf')
>>> # gamma: inverse of size of
>>> # radial kernel
```

Interactive example

See the SVM GUI to download `svm_gui.py`; add data points of both classes with right and left button, fit the model and change parameters and data.
Exercise

Try classifying classes 1 and 2 from the iris dataset with SVMs, with the 2 first features. Leave out 10% of each class and test prediction performance on these observations.

**Warning:** the classes are ordered, do not leave out the last 10%, you would be testing on only one class.

**Hint:** You can use the `decision_function` method on a grid to get intuitions.

```python
gaussian = datasets.load_iris()
X = iris.data
y = iris.target

X = X[y != 0, :2]
y = y[y != 0]

Solution: ../../auto_examples/exercises/plot_iris_exercise.py
```

Model selection: choosing estimators and their parameters

Score, and cross-validated scores

As we have seen, every estimator exposes a `score` method that can judge the quality of the fit (or the prediction) on new data. **Bigger is better.**

```python
>>> from sklearn import datasets, svm
>>> digits = datasets.load_digits()
>>> X_digits = digits.data
>>> y_digits = digits.target
>>> svc = svm.SVC(C=1, kernel='linear')
>>> svc.fit(X_digits[:-100], y_digits[:-100]).score(X_digits[-100:], y_digits[-100:])
0.9799999999999998
```

To get a better measure of prediction accuracy (which we can use as a proxy for goodness of fit of the model), we can successively split the data in **folds** that we use for training and testing:

```python
>>> import numpy as np
>>> X_folds = np.array_split(X_digits, 3)
>>> y_folds = np.array_split(y_digits, 3)
>>> scores = list()
>>> for k in range(3):
...     # We use 'list' to copy, in order to 'pop' later on
...     X_train = list(X_folds)
...     X_test = X_train.pop(k)
...     X_train = np.concatenate(X_train)
...     y_train = list(y_folds)
...     y_test = y_train.pop(k)
...     y_train = np.concatenate(y_train)
...     scores.append(svc.fit(X_train, y_train).score(X_test, y_test))
>>> print scores
[0.93489148580968284, 0.95659432387312182, 0.93989983305509184]
```

This is called a **KFold** cross validation.

Cross-validation generators

The code above to split data in train and test sets is tedious to write. The *sklearn* exposes cross-validation generators to generate list of indices for this purpose:
The cross-validation can then be implemented easily:

```python
>>> kfold = cross_validation.KFold(len(X_digits), k=3)
>>> [svc.fit(X_digits[train], y_digits[train]).score(X_digits[test], y_digits[test])
... for train, test in kfold]
[0.93489148580968284, 0.95659432387312182, 0.93989983305509184]
```

To compute the `score` method of an estimator, the sklearn exposes a helper function:

```python
>>> cross_validation.cross_val_score(svc, X_digits, y_digits, cv=kfold, n_jobs=-1)
array([ 0.93489149, 0.95659432, 0.93989983])
```

`n_jobs=-1` means that the computation will be dispatched on all the CPUs of the computer.

**Cross-validation generators**

<table>
<thead>
<tr>
<th>KFold (n, k)</th>
<th>StratifiedKFold (y, k)</th>
<th>LeaveOneOut (n)</th>
<th>LeaveOneLabelOut (labels)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Split it K folds, train on K-1, test on left-out</td>
<td>Make sure that all classes are even accross the folds</td>
<td>Leave one observation out</td>
<td>Takes a label array to group observations</td>
</tr>
</tbody>
</table>
Exercise

On the digits dataset, plot the cross-validation score of a SVC estimator with an RBF kernel as a function of parameter C (use a logarithmic grid of points, from 1 to 10).

```python
from sklearn import cross_validation, datasets, svm
digits = datasets.load_digits()
X = digits.data
y = digits.target

svc = svm.SVC()
C_s = np.logspace(1, 10, 10)
scores = list()
scores_std = list()
```

Solution: `../../auto_examples/exercises/plot_cv_digits.py`

Grid-search and cross-validated estimators

**Grid-search**  The sklearn provides an object that, given data, computes the score during the fit of an estimator on a parameter grid and chooses the parameters to maximize the cross-validation score. This object takes an estimator during the construction and exposes an estimator API:

```python
>>> from sklearn.grid_search import GridSearchCV
>>> gammas = np.logspace(-6, -1, 10)
>>> clf = GridSearchCV(estimator=svc, param_grid=dict(gamma=gammas),
...                     n_jobs=-1)
>>> clf.fit(X_digits[:1000], y_digits[:1000])
```

```python
GridSearchCV(cv=None,...
>>> clf.best_score_
0.988991985997974
>>> clf.best_estimator_.gamma
9.9999999999999995e-07
```

```python
>>> # Prediction performance on test set is not as good as on train set
>>> clf.score(X_digits[1000:], y_digits[1000:])
0.94228356336260977
```

By default the `GridSearchCV` uses a 3-fold cross-validation. However, if it detects that a classifier is passed, rather than a regressor, it uses a stratified 3-fold.

**Nested cross-validation**

```python
>>> cross_validation.cross_val_score(clf, X_digits, y_digits)
array([ 0.97996661, 0.98163606, 0.98330551])
```

Two cross-validation loops are performed in parallel: one by the `GridSearchCV` estimator to set `gamma`, the other one by `cross_val_score` to measure the prediction performance of the estimator. The resulting scores are unbiased estimates of the prediction score on new data.

**Warning:** You cannot nest objects with parallel computing (n_jobs different than 1).
Cross-validated estimators  Cross-validation to set a parameter can be done more efficiently on an algorithm-by-algorithm basis. This is why, for certain estimators, the sklearn exposes Cross-Validation: evaluating estimator performance estimators, that set their parameter automatically by cross-validation:

```python
>>> from sklearn import linear_model, datasets
>>> diabetes = datasets.load_diabetes()
>>> X_diabetes = diabetes.data
>>> y_diabetes = diabetes.target
>>> lasso = linear_model.LassoCV()
>>> lasso.fit(X_diabetes, y_diabetes)
LassoCV(alphas=array([2.14804, 2.00327, ..., 0.0023, 0.00215]),
       copy_X=True, cv=None, eps=0.001, fit_intercept=True,
       max_iter=1000, n_alphas=100, normalize=False,
       precompute='auto', tol=0.0001, verbose=False)
>>> # The estimator chose automatically its lambda:
>>> lasso.alpha
0.01318...
```

These estimators are called similarly to their counterparts, with ‘CV’ appended to their name.

**Exercise**

On the diabetes dataset, find the optimal regularization parameter alpha.

**Bonus:** How much can you trust the selection of alpha?

```python
import numpy as np
import pylab as pl

from sklearn import cross_validation, datasets, linear_model

diabetes = datasets.load_diabetes()
X = diabetes.data
y = diabetes.target

lasso = linear_model.Lasso()
alphas = np.logspace(-4, -1, 20)

Solution: ../../auto_examples/exercises/plot_cv_diabetes.py
```

Unsupervised learning: seeking representations of the data

Clustering: grouping observations together

**The problem solved in clustering**

Given the iris dataset, if we knew that there were 3 types of iris, but did not have access to a taxonomist to label them: we could try a clustering task: split the observations in well-separated group called clusters.
**K-means clustering**  Note that there exists a lot of different clustering criteria and associated algorithms. The simplest clustering algorithm is the *K-means*.

```python
>>> from sklearn import cluster, datasets
>>> iris = datasets.load_iris()
>>> X_iris = iris.data
>>> y_iris = iris.target

>>> k_means = cluster.KMeans(n_clusters=3)
>>> k_means.fit(X_iris)
KMeans(copy_x=True, init='k-means++', ...)
>>> print k_means.labels_[::10]
[1 1 1 1 0 0 0 0 2 2 2 2 2]
>>> print y_iris[::10]
[0 0 0 0 0 1 1 1 1 1 2 2 2 2 2]

**Warning:** There is absolutely no guarantee of recovering a ground truth. First choosing the right number of clusters is hard. Second, the algorithm is sensitive to initialization, and can fall in local minima, although in the *sklearn* package we play many tricks to mitigate this issue.

<table>
<thead>
<tr>
<th>Bad initialization</th>
<th>8 clusters</th>
<th>Ground truth</th>
</tr>
</thead>
<tbody>
<tr>
<td>Don’t over-interpret clustering results</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Application example: vector quantization

Clustering in general and KMeans in particular, can be seen as a way of choosing a small number of exemplars to compress the information, a problem sometimes known as vector quantization. For instance, this can be used to posterize an image:

```python
>>> import scipy as sp
>>> try:
...     lena = sp.lena()
... except AttributeError:
...     from scipy import misc
...     lena = misc.lena()
>>> X = lena.reshape((-1, 1))  # We need an (n_sample, n_feature) array
>>> k_means = cluster.KMeans(n_clusters=5, n_init=1)
>>> k_means.fit(X)
KMeans(copy_x=True, init='k-means++', ...)
>>> values = k_means.cluster_centers_.squeeze()
>>> labels = k_means.labels_
>>> lena_compressed = np.choose(labels, values)
>>> lena_compressed.shape = lena.shape

Raw image | K-means quantization | Equal bins | Image histogram
```

Hierarchical agglomerative clustering: Ward  

A **Hierarchical clustering** method is a type of cluster analysis that aims to build a hierarchy of clusters. In general, the various approaches of this technique are either:

- **Agglomerative** - bottom-up approaches, or
- **Divisive** - top-down approaches.

For estimating a large number of clusters, top-down approaches are both statistically ill-posed, and slow - due to it starting with all observations as one cluster, which it splits recursively. Agglomerative hierarchical-clustering is a bottom-up approach that successively merges observations together and is particularly useful when the clusters of interest are made of only a few observations. **Ward** clustering minimizes a criterion similar to k-means in a bottom-up approach. When the number of clusters is large, it is much more computationally efficient than k-means.

**Connectivity-constrained clustering**  
With Ward clustering, it is possible to specify which samples can be clustered together by giving a connectivity graph. Graphs in the scikit are represented by their adjacency matrix. Often a sparse matrix is used. This can be useful for instance to retrieve connect regions when clustering an image:
# Generate data
lena = sp.misc.lena()
# Downsample the image by a factor of 4
lena = lena[:-2, :-2] + lena[1:-2, :-2] + lena[:-2, 1:-2] + lena[1:-2, 1:-2]
X = np.reshape(lena, (-1, 1))

# Define the structure A of the data. Pixels connected to their neighbors.
connectivity = grid_to_graph(*lena.shape)

# Compute clustering
print "Compute structured hierarchical clustering..."
st = time.time()
n_clusters = 15  # number of regions
ward = Ward(n_clusters=n_clusters, connectivity=connectivity).fit(X)
label = np.reshape(ward.labels_, lena.shape)
print "Elapsed time: ", time.time() - st
print "Number of pixels: ", label.size
print "Number of clusters: ", np.unique(label).size

Feature agglomeration We have seen that sparsity could be used to mitigate the curse of dimensionality, i.e. the insufficiency of observations compared to the number of features. Another approach is to merge together similar features: **feature agglomeration**. This approach can be implementing by clustering in the feature direction, in other

words clustering the transposed data.

```python
>>> digits = datasets.load_digits()
>>> images = digits.images
>>> X = np.reshape(images, (len(images), -1))
>>> connectivity = grid_to_graph(*images[0].shape)

>>> agglo = cluster.WardAgglomeration(connectivity=connectivity,
... n_clusters=32)

>>> agglo.fit(X)
WardAgglomeration(connectivity=...)

>>> X_reduced = agglo.transform(X)

>>> X_approx = agglo.inverse_transform(X_reduced)
>>> images_approx = np.reshape(X_approx, images.shape)
```
**transform and inverse_transform methods**

Some estimators expose a *transform* method, for instance to reduce the dimensionality of the dataset.

---

**Decompositions: from a signal to components and loadings**

**Components and loadings**

If \( X \) is our multivariate data, the problem that we are trying to solve is to rewrite it on a different observation basis: we want to learn loadings \( L \) and a set of components \( C \) such that \( X = L C \). Different criteria exist to choose the components.

**Principal component analysis: PCA**  
*Principal component analysis (PCA)* selects the successive components that explain the maximum variance in the signal.

The point cloud spanned by the observations above is very flat in one direction: one of the 3 univariate features can almost be exactly computed using the 2 other. PCA finds the directions in which the data is not flat. When used to *transform* data, PCA can reduce the dimensionality of the data by projecting on a principal subspace.

```python
>>> # Create a signal with only 2 useful dimensions
>>> x1 = np.random.normal(size=100)
>>> x2 = np.random.normal(size=100)
>>> x3 = x1 + x2
>>> X = np.c_[x1, x2, x3]

>>> from sklearn import decomposition

>>> pca = decomposition.PCA()
>>> pca.fit(X)
PCA(copy=True, n_components=None, whiten=False)
>>> print pca.explained_variance_
[ 2.18565811e+00 1.19346747e+00 8.43026679e-32]

>>> # As we can see, only the 2 first components are useful
>>> pca.n_components = 2
>>> X_reduced = pca.fit_transform(X)
>>> X_reduced.shape
(100, 2)
```
Independent Component Analysis: ICA  

Independent component analysis (ICA) selects components so that the distribution of their loadings carries a maximum amount of independent information. It is able to recover non-

Gaussian independent signals:

```python
>>> # Generate sample data
>>> time = np.linspace(0, 10, 2000)
>>> s1 = np.sin(2 * time)  # Signal 1 : sinusoidal signal
>>> s2 = np.sign(np.sin(3 * time))  # Signal 2 : square signal
>>> S = np.c_[s1, s2]
>>> S += 0.2 * np.random.normal(size=S.shape)  # Add noise
>>> S /= S.std(axis=0)  # Standardize data
>>> # Mix data
>>> A = np.array([[1, 1], [0.5, 2]])  # Mixing matrix
>>> X = np.dot(S, A.T)  # Generate observations

>>> # Compute ICA
>>> ica = decomposition.FastICA()
>>> S_ = ica.fit(X).transform(X)  # Get the estimated sources
>>> A_ = ica.get_mixing_matrix()  # Get estimated mixing matrix
>>> np.allclose(X, np.dot(S_, A_.T))
True
```
Putting it all together

Pipelining

We have seen that some estimators can transform data, and some estimators can predict variables. We can create combined estimators:

```python
import pylab as pl
from sklearn import linear_model, decomposition, datasets
logistic = linear_model.LogisticRegression()

pca = decomposition.PCA()
from sklearn.pipeline import Pipeline
pipe = Pipeline(steps=[('pca', pca), ('logistic', logistic)])
```

```python
digits = datasets.load_digits()
X_digits = digits.data
y_digits = digits.target
```

```python
# Plot the PCA spectrum
pca.fit(X_digits)
pl.figure(1, figsize=(4, 3))
pl.clf()
pl.axes([.2, .2, .7, .7])
pl.plot(pca.explained_variance_, linewidth=2)
pl.axis('tight')
pl.xlabel('n_components')
pl.ylabel('explained_variance_')

# Prediction
from sklearn.grid_search import GridSearchCV
n_components = [20, 40, 64]
Cs = np.logspace(-4, 4, 3)

# Parameters of pipelines can be set using '__' separated parameter names:
estimator = GridSearchCV(pipe,
dict(pca__n_components=n_components,
    logistic__C=Cs))
```
estimator.fit(X_digits, y_digits)

pl.axvline(estimator.best_estimator_.named_steps['pca'].n_components,
           linestyle=':', label='n_components chosen')
pl.legend(prop=dict(size=12))

Face recognition with eigenfaces

The dataset used in this example is a preprocessed excerpt of the “Labeled Faces in the Wild”, aka LFW:


Expected results for the top 5 most represented people in the dataset:

<table>
<thead>
<tr>
<th>Name</th>
<th>Precision</th>
<th>Recall</th>
<th>F1-score</th>
<th>Support</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gerhard_Schroeder</td>
<td>0.91</td>
<td>0.75</td>
<td>0.82</td>
<td>28</td>
</tr>
<tr>
<td>Donald_Rumsfeld</td>
<td>0.84</td>
<td>0.82</td>
<td>0.83</td>
<td>33</td>
</tr>
<tr>
<td>Tony_Blair</td>
<td>0.65</td>
<td>0.82</td>
<td>0.73</td>
<td>34</td>
</tr>
<tr>
<td>Colin_Powell</td>
<td>0.78</td>
<td>0.88</td>
<td>0.83</td>
<td>58</td>
</tr>
<tr>
<td>George_W_Bush</td>
<td>0.93</td>
<td>0.86</td>
<td>0.90</td>
<td>129</td>
</tr>
</tbody>
</table>
| avg / total       | 0.86      | 0.84   | 0.85     | 282     

Face recognition example using eigenfaces and SVMs

The dataset used in this example is a preprocessed excerpt of the "Labeled Faces in the Wild", aka LFW:


.. _LFW: http://vis-www.cs.umass.edu/lfw/

Expected results for the top 5 most represented people in the dataset:

<table>
<thead>
<tr>
<th>Name</th>
<th>Precision</th>
<th>Recall</th>
<th>F1-score</th>
<th>Support</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gerhard_Schroeder</td>
<td>0.91</td>
<td>0.75</td>
<td>0.82</td>
<td>28</td>
</tr>
<tr>
<td>Donald_Rumsfeld</td>
<td>0.84</td>
<td>0.82</td>
<td>0.83</td>
<td>33</td>
</tr>
<tr>
<td>Tony_Blair</td>
<td>0.65</td>
<td>0.82</td>
<td>0.73</td>
<td>34</td>
</tr>
<tr>
<td>Colin_Powell</td>
<td>0.78</td>
<td>0.88</td>
<td>0.83</td>
<td>58</td>
</tr>
<tr>
<td>George_W_Bush</td>
<td>0.93</td>
<td>0.86</td>
<td>0.90</td>
<td>129</td>
</tr>
</tbody>
</table>
| avg / total       | 0.86      | 0.84   | 0.85     | 282     

from time import time
import logging
import pylab as pl
from sklearn.cross_validation import train_test_split
from sklearn.datasets import fetch_lfw_people
from sklearn.grid_search import GridSearchCV
from sklearn.metrics import classification_report
from sklearn.metrics import confusion_matrix
from sklearn.decomposition import RandomizedPCA
from sklearn.svm import SVC

# Display progress logs on stdout
logging.basicConfig(level=logging.INFO, format='%(asctime)s %(message)s')

1.2. Tutorials: From the bottom up with scikit-learn

31
# Download the data, if not already on disk and load it as numpy arrays

```python
lfw_people = fetch_lfw_people(min_faces_per_person=70, resize=0.4)
```

# introspect the images arrays to find the shapes (for plotting)
```
n_samples, h, w = lfw_people.images.shape
```

# for machine learning we use the 2 data directly (as relative pixel positions info is ignored by this model)
```
X = lfw_people.data
n_features = X.shape[1]
```

# the label to predict is the id of the person
```
y = lfw_people.target
target_names = lfw_people.target_names	n_classes = target_names.shape[0]
```

```
print "Total dataset size:
print "n_samples: %d" % n_samples
print "n_features: %d" % n_features
print "n_classes: %d" % n_classes
```

# Split into a training set and a test set using a stratified k fold
```
# split into a training and testing set
X_train, X_test, y_train, y_test = train_test_split(X, y, test_fraction=0.25)
```

# Compute a PCA (eigenfaces) on the face dataset (treated as unlabeled dataset): unsupervised feature extraction / dimensionality reduction
```
n_components = 150
```

```
print "Extracting the top %d eigenfaces from %d faces" % (n_components, X_train.shape[0])
```

```
t0 = time()
pca = RandomizedPCA(n_components=n_components, whiten=True).fit(X_train)
```

```
print "done in %0.3fs" % (time() - t0)
eigenfaces = pca.components_.reshape((n_components, h, w))
```

```
print "Projecting the input data o the eigenfaces orthonormal basis"
t0 = time()
X_train_pca = pca.transform(X_train)
X_test_pca = pca.transform(X_test)
```

```
print "done in %0.3fs" % (time() - t0)
```

# Train a SVM classification model
```
print "Fitting the classifier to the training set"
t0 = time()
param_grid = {
```
'C': [1e3, 5e3, 1e4, 5e4, 1e5],
'gamma': [0.0001, 0.0005, 0.001, 0.005, 0.01, 0.1],
}
clf = GridSearchCV(SVC(kernel='rbf', class_weight='auto'), param_grid)
clf = clf.fit(X_train_pca, y_train)
print "done in %0.3fs" % (time() - t0)
print "Best estimator found by grid search:"
print clf.best_estimator_

# Quantitative evaluation of the model quality on the test set
print "Predicting the people names on the testing set"
t0 = time()
y_pred = clf.predict(X_test_pca)
print "done in %0.3fs" % (time() - t0)
print classification_report(y_test, y_pred, target_names=target_names)
print confusion_matrix(y_test, y_pred, labels=range(n_classes))

# Qualitative evaluation of the predictions using matplotlib

def plot_gallery(images, titles, h, w, n_row=3, n_col=4):
    """Helper function to plot a gallery of portraits""
    pl.figure(figsize=(1.8 * n_col, 2.4 * n_row))
    pl.subplots_adjust(bottom=0, left=.01, right=.99, top=.90, hspace=.35)
    for i in range(n_row * n_col):
        pl.subplot(n_row, n_col, i + 1)
        pl.imshow(images[i].reshape((h, w)), cmap=pl.cm.gray)
        pl.title(titles[i], size=12)
        pl.xticks(())
        pl.yticks(())

# plot the result of the prediction on a portion of the test set

def title(y_pred, y_test, target_names, i):
    pred_name = target_names[y_pred[i]].rsplit(' ', 1)[-1]
    true_name = target_names[y_test[i]].rsplit(' ', 1)[-1]
    return 'predicted: %s
true: %s' % (pred_name, true_name)
prediction_titles = [title(y_pred, y_test, target_names, i)
    for i in range(y_pred.shape[0])]
plot_gallery(X_test, prediction_titles, h, w)

# plot the gallery of the most significative eigenfaces
eigenface_titles = ['"eigenface %d" % i' for i in range(eigenfaces.shape[0])]
plot_gallery(eigenfaces, eigenface_titles, h, w)
pl.show()
Expected results for the top 5 most represented people in the dataset:

<table>
<thead>
<tr>
<th>Name</th>
<th>Precision</th>
<th>Recall</th>
<th>F1-score</th>
<th>Support</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gerhard_Schroeder</td>
<td>0.91</td>
<td>0.75</td>
<td>0.82</td>
<td>28</td>
</tr>
<tr>
<td>Donald_Rumsfeld</td>
<td>0.84</td>
<td>0.82</td>
<td>0.83</td>
<td>33</td>
</tr>
<tr>
<td>Tony_Blair</td>
<td>0.65</td>
<td>0.82</td>
<td>0.73</td>
<td>34</td>
</tr>
<tr>
<td>Colin_Powell</td>
<td>0.78</td>
<td>0.88</td>
<td>0.83</td>
<td>58</td>
</tr>
<tr>
<td>George_W_Bush</td>
<td>0.93</td>
<td>0.86</td>
<td>0.90</td>
<td>129</td>
</tr>
<tr>
<td><strong>avg / total</strong></td>
<td><strong>0.86</strong></td>
<td><strong>0.84</strong></td>
<td><strong>0.85</strong></td>
<td><strong>282</strong></td>
</tr>
</tbody>
</table>

**Open problem: Stock Market Structure**

Can we predict the variation in stock prices for Google?

*Visualizing the stock market structure*

**Finding help**

**The project mailing list**

If you encounter a bug with scikit-learn or something that needs clarification in the docstring or the online documentation, please feel free to ask on the Mailing List

**Q&A communities with Machine Learning practitioners**

1. **Metaoptimize/QA** A forum for Machine Learning, Natural Language Processing and other Data Analytics discussions (similar to what Stackoverflow is for developers):
   http://metaoptimize.com/qa
   A good starting point is the discussion on good freely available textbooks on machine learning

2. **Quora.com** Quora has a topic for Machine Learning related questions that also features some interesting discussions:
   http://quora.com/Machine-Learning
   Have a look at the best questions section, e.g: What are some good resources for learning about machine learning.

**Note: Videos**

Videos with tutorials can also be found in the Videos section.
Note: Doctest Mode

The code-examples in the above tutorials are written in a python-console format. If you wish to easily execute these examples in iPython, use:

```
%doctest_mode
```

in the iPython-console. You can then simply copy and paste the examples directly into iPython without having to worry about removing the `>>>` manually.

### 1.3 Supervised learning

#### 1.3.1 Generalized Linear Models

The following are a set of methods intended for regression in which the target value is expected to be a linear combination of the input variables. In mathematical notion, if $\hat{y}$ is the predicted value.

$$
\hat{y}(w, x) = w_0 + w_1 x_1 + \ldots + w_p x_p
$$

Across the module, we designate the vector $w = (w_1, \ldots, w_p)$ as `coef_` and $w_0$ as `intercept_`.

To perform classification with generalized linear models, see *Logistic regression*.

#### Ordinary Least Squares

`LinearRegression` fits a linear model with coefficients $w = (w_1, \ldots, w_p)$ to minimize the residual sum of squares between the observed responses in the dataset, and the responses predicted by the linear approximation. Mathematically it solves a problem of the form:

$$
\min_w \|Xw - y\|_2^2
$$
LinearRegression will take in its fit method arrays X, y and will store the coefficients w of the linear model in its coef_ member:

```python
>>> from sklearn import linear_model
>>> clf = linear_model.LinearRegression()
>>> clf.fit ([[0, 0], [1, 1], [2, 2]], [0, 1, 2])
LinearRegression(copy_X=True, fit_intercept=True, normalize=False)
>>> clf.coef_
array([ 0.5, 0.5])
```

However, coefficient estimates for Ordinary Least Squares rely on the independence of the model terms. When terms are correlated and the columns of the design matrix X have an approximate linear dependence, the design matrix becomes close to singular and as a result, the least-squares estimate becomes highly sensitive to random errors in the observed response, producing a large variance. This situation of multicollinearity can arise, for example, when data are collected without an experimental design.

### Examples:
- **Linear Regression Example**

### Ordinary Least Squares Complexity

This method computes the least squares solution using a singular value decomposition of X. If X is a matrix of size (n, p) this method has a cost of $O(np^2)$, assuming that $n \geq p$.

### Ridge Regression

Ridge regression addresses some of the problems of Ordinary Least Squares by imposing a penalty on the size of coefficients. The ridge coefficients minimize a penalized residual sum of squares,

$$
\min_w \|Xw - y\|_2^2 + \alpha\|w\|_2^2
$$

Here, $\alpha \geq 0$ is a complexity parameter that controls the amount of shrinkage: the larger the value of $\alpha$, the greater the amount of shrinkage and thus the coefficients become more robust to collinearity.

As with other linear models, Ridge will take in its fit method arrays X, y and will store the coefficients $w$ of the linear model in its coef_ member:
>>> from sklearn import linear_model
>>> clf = linear_model.Ridge (alpha = .5)
>>> clf.fit ([[0, 0], [0, 0], [1, 1]], [0, .1, 1])
Ridge(alpha=0.5, copy_X=True, fit_intercept=True, normalize=False, tol=0.001)
>>> clf.coef_
array([ 0.34545455, 0.34545455])
>>> clf.intercept_
0.13636...

Examples:
- Plot Ridge coefficients as a function of the regularization
- Classification of text documents using sparse features

Ridge Complexity

This method has the same order of complexity than an Ordinary Least Squares.

Setting the regularization parameter: generalized Cross-Validation

RidgeCV implements ridge regression with built-in cross-validation of the alpha parameter. The object works in the same way as GridSearchCV except that it defaults to Generalized Cross-Validation (GCV), an efficient form of leave-one-out cross-validation:

>>> from sklearn import linear_model
>>> clf = linear_model.RidgeCV(alphas=[0.1, 1.0, 10.0])
>>> clf.fit([[0, 0], [0, 0], [1, 1]], [0, .1, 1])
RidgeCV(alphas=[0.1, 1.0, 10.0], cv=None, fit_intercept=True, loss_func=None, normalize=False, score_func=None)
>>> clf.best_alpha
0.1

References
- “Notes on Regularized Least Squares”, Rifkin & Lippert (technical report, course slides).

Lasso

The Lasso is a linear model that estimates sparse coefficients. It is useful in some contexts due to its tendency to prefer solutions with fewer parameter values, effectively reducing the number of variables upon which the given solution is dependent. For this reason, the Lasso and its variants are fundamental to the field of compressed sensing. Under certain conditions, it can recover the exact set of non-zero weights (see Compressive sensing: tomography reconstruction with L1 prior (Lasso)).

Mathematically, it consists of a linear model trained with \( \ell_1 \) prior as regularizer. The objective function to minimize is:

\[
\min_w \frac{1}{2n_{\text{samples}}} \|Xw - y\|_2^2 + \alpha \|w\|_1
\]
The lasso estimate thus solves the minimization of the least-squares penalty with \( \alpha ||w||_1 \) added, where \( \alpha \) is a constant and \( ||w||_1 \) is the \( \ell_1 \)-norm of the parameter vector.

The implementation in the class Lasso uses coordinate descent as the algorithm to fit the coefficients. See Least Angle Regression for another implementation:

```python
>>> clf = linear_model.Lasso(alpha = 0.1)
>>> clf.fit([[0, 0], [1, 1]], [0, 1])
Lasso(alpha=0.1, copy_X=True, fit_intercept=True, max_iter=1000,
    normalize=False, positive=False, precompute='auto', tol=0.0001,
    warm_start=False)
>>> clf.predict([[1, 1]])
array([0.8])
```

Also useful for lower-level tasks is the function lasso_path that computes the coefficients along the full path of possible values.

### Examples:
- Lasso and Elastic Net for Sparse Signals
- Compressive sensing: tomography reconstruction with L1 prior (Lasso)

**Note:** Feature selection with Lasso

As the Lasso regression yields sparse models, it can thus be used to perform feature selection, as detailed in L1-based feature selection.

### Setting regularization parameter

The alpha parameter control the degree of sparsity of the coefficients estimated.

Using cross-validation scikit-learn exposes objects that set the Lasso alpha parameter by cross-validation: LassoCV and LassoLarsCV. LassoLarsCV is based on the Least Angle Regression algorithm explained below.

For high-dimensional datasets with many collinear regressors, LassoCV is most often preferrable. How, LassoLarsCV has the advantage of exploring more relevant values of alpha parameter, and if the number of samples is very small compared to the number of observations, it is often faster than LassoCV.
Information-criteria based model selection  Alternatively, the estimator `LassoLarsIC` proposes to use the Akaike information criterion (AIC) and the Bayes Information criterion (BIC). It is a computationally cheaper alternative to find the optimal value of alpha as the regularization path is computed only once instead of k+1 times when using k-fold cross-validation. However, such criteria needs a proper estimation of the degrees of freedom of the solution, are derived for large samples (asymptotic results) and assume the model is correct, i.e. that the data are actually generated by this model. They also tend to break when the problem is badly conditioned (more features than samples).

\[
\begin{align*}
\text{min}_{w} & \quad \frac{1}{2n_{\text{samples}}} ||X w - y||_2^2 + \alpha \rho ||w||_1 + \frac{\alpha(1-\rho)}{2} ||w||_2^2 \\
\end{align*}
\]

The class `ElasticNetCV` can be used to set the parameters alpha and rho by cross-validation.

Examples:
- Lasso model selection: Cross-Validation / AIC / BIC
- Lasso and Elastic Net for Sparse Signals
- Lasso and Elastic Net

Least Angle Regression

Least-angle regression (LARS) is a regression algorithm for high-dimensional data, developed by Bradley Efron, Trevor Hastie, Iain Johnstone and Robert Tibshirani.

The advantages of LARS are:
• It is numerically efficient in contexts where \( p \gg n \) (i.e., when the number of dimensions is significantly greater than the number of points)

• It is computationally just as fast as forward selection and has the same order of complexity as an ordinary least squares.

• It produces a full piecewise linear solution path, which is useful in cross-validation or similar attempts to tune the model.

• If two variables are almost equally correlated with the response, then their coefficients should increase at approximately the same rate. The algorithm thus behaves as intuition would expect, and also is more stable.

• It is easily modified to produce solutions for other estimators, like the Lasso.

The disadvantages of the LARS method include:

• Because LARS is based upon an iterative refitting of the residuals, it would appear to be especially sensitive to the effects of noise. This problem is discussed in detail by Weisberg in the discussion section of the Efron et al. (2004) Annals of Statistics article.

The LARS model can be used using estimator `Lars`, or its low-level implementation `lars_path`.

### LARS Lasso

`LassoLars` is a lasso model implemented using the LARS algorithm, and unlike the implementation based on coordinate_descent, this yields the exact solution, which is piecewise linear as a function of the norm of its coefficients.

```python
>>> from sklearn import linear_model
>>> clf = linear_model.LassoLars(alpha=.1)
>>> clf.fit([[0, 0], [1, 1]], [0, 1])
LassoLars(alpha=0.1, copy_X=True, eps=..., fit_intercept=True,
          max_iter=500, normalize=True, precompute='auto', verbose=False)
>>> clf.coef_
array([ 0.717157..., 0. ])
```

**Examples:**

- **Lasso path using LARS**
The Lars algorithm provides the full path of the coefficients along the regularization parameter almost for free, thus a common operation consist of retrieving the path with function \texttt{lars_path}.

**Mathematical formulation**

The algorithm is similar to forward stepwise regression, but instead of including variables at each step, the estimated parameters are increased in a direction equiangular to each one’s correlations with the residual.

Instead of giving a vector result, the LARS solution consists of a curve denoting the solution for each value of the L1 norm of the parameter vector. The full coefficients path is stored in the array \texttt{coef_path}, which has size \((n\text{\_features}, \text{max\_features}+1)\). The first column is always zero.

**References:**

- Original Algorithm is detailed in the paper *Least Angle Regression* by Hastie et al.

**Orthogonal Matching Pursuit (OMP)**

\texttt{OrthogonalMatchingPursuit} and \texttt{orthogonal\_mp} implements the OMP algorithm for approximating the fit of a linear model with constraints imposed on the number of non-zero coefficients (ie. the L0 pseudo-norm).

Being a forward feature selection method like *Least Angle Regression*, orthogonal matching pursuit can approximate the optimum solution vector with a fixed number of non-zero elements:

\[
\text{arg min } ||y - X\gamma||_2^2 \quad \text{subject to } ||\gamma||_0 \leq n_{\text{nonzero\_coefs}}
\]

Alternatively, orthogonal matching pursuit can target a specific error instead of a specific number of non-zero coefficients. This can be expressed as:

\[
\text{arg min } ||\gamma||_0 \quad \text{subject to } ||y - X\gamma||_2^2 \leq \text{tol}
\]

OMP is based on a greedy algorithm that includes at each step the atom most highly correlated with the current residual. It is similar to the simpler matching pursuit (MP) method, but better in that at each iteration, the residual is recomputed using an orthogonal projection on the space of the previously chosen dictionary elements.
Bayesian Regression

Bayesian regression techniques can be used to include regularization parameters in the estimation procedure: the regularization parameter is not set in a hard sense but tuned to the data at hand.

This can be done by introducing uninformative priors over the hyper parameters of the model. The $\ell_2$ regularization used in Ridge Regression is equivalent to finding a maximum a-postiori solution under a Gaussian prior over the parameters $w$ with precision $\lambda^{-1}$. Instead of setting $\lambda$ manually, it is possible to treat it as a random variable to be estimated from the data.

To obtain a fully probabilistic model, the output $y$ is assumed to be Gaussian distributed around $Xw$:

$$p(y|X, w, \alpha) = \mathcal{N}(y|Xw, \alpha)$$

Alpha is again treated as a random variable that is to be estimated from the data.

The advantages of Bayesian Regression are:

- It adapts to the data at hand.
- It can be used to include regularization parameters in the estimation procedure.

The disadvantages of Bayesian regression include:

- Inference of the model can be time consuming.

References

- A good introduction to Bayesian methods is given in C. Bishop: Pattern Recognition and Machine learning
- Original Algorithm is detailed in the book Bayesian learning for neural networks by Radford M. Neal

Bayesian Ridge Regression

$\texttt{BayesianRidge}$ estimates a probabilistic model of the regression problem as described above. The prior for the parameter $w$ is given by a spherical Gaussian:

$$p(w|\lambda) = \mathcal{N}(w|0, \lambda^{-1}I_p)$$

The priors over $\alpha$ and $\lambda$ are choosen to be gamma distributions, the conjugate prior for the precision of the Gaussian.

The resulting model is called Bayesian Ridge Regression, and is similar to the classical Ridge. The parameters $w$, $\alpha$ and $\lambda$ are estimated jointly during the fit of the model. The remaining hyperparameters are the parameters of the
gamma priors over $\alpha$ and $\lambda$. These are usually chosen to be non-informative. The parameters are estimated by maximizing the marginal log likelihood.

By default $\alpha_1 = \alpha_2 = \lambda_1 = \lambda_2 = 1.e^{-6}$.

Bayesian Ridge Regression is used for regression:

```python
>>> from sklearn import linear_model
>>> X = [[0., 0.], [1., 1.], [2., 2.], [3., 3.]]
>>> Y = [0., 1., 2., 3.]
>>> clf = linear_model.BayesianRidge()
>>> clf.fit(X, Y)
BayesianRidge(alpha_1=1e-06, alpha_2=1e-06, compute_score=False, copy_X=True,
fit_intercept=True, lambda_1=1e-06, lambda_2=1e-06, n_iter=300,
normalize=False, tol=0.001, verbose=False)
```

After being fitted, the model can then be used to predict new values:

```python
>>> clf.predict ([[1, 0.]])
array([ 0.50000013])
```

The weights $w$ of the model can be accessed:

```python
>>> clf.coef_
array([ 0.49999993, 0.49999993])
```

Due to the Bayesian framework, the weights found are slightly different to the ones found by Ordinary Least Squares. However, Bayesian Ridge Regression is more robust to ill-posed problem.

Examples:

- Bayesian Ridge Regression

References

- More details can be found in the article Bayesian Interpolation by MacKay, David J. C.
Automatic Relevance Determination - ARD

ARDRegression is very similar to Bayesian Ridge Regression, but can lead to sparser weights $w$. ARDRegression poses a different prior over $w$, by dropping the assumption of the Gaussian being spherical.

Instead, the distribution over $w$ is assumed to be an axis-parallel, elliptical Gaussian distribution. This means each weight $w_i$ is drawn from a Gaussian distribution, centered on zero and with a precision $\lambda_i$:

$$p(w|\lambda) = N(w|0, A^{-1})$$

with $\text{diag}(A) = \lambda = \{\lambda_1, ..., \lambda_p\}$.

In contrast to Bayesian Ridge Regression, each coordinate of $w_i$ has its own standard deviation $\lambda_i$. The prior over all $\lambda_i$ is chosen to be the same gamma distribution given by hyperparameters $\lambda_1$ and $\lambda_2$.

Examples:

- Automatic Relevance Determination Regression (ARD)

References:

Logistic regression

If the task at hand is to choose which class a sample belongs to given a finite (hopefully small) set of choices, the learning problem is a classification, rather than regression. Linear models can be used for such a decision, but it is best to use what is called a logistic regression, that doesn’t try to minimize the sum of square residuals, as in regression, but rather a “hit or miss” cost.

The LogisticRegression class can be used to do L1 or L2 penalized logistic regression. L1 penalization yields sparse predicting weights. For L1 penalization, sklearn.svm.l1_min_c allows to calculate the lower bound for $C$ in order to get a non “null” (all feature weights to zero) model.

$^1$ David Wipf and Srikantan Nagarajan: A new view of automatic relevance determination.
Examples:
- L1 Penalty and Sparsity in Logistic Regression
- Path with L1-Logistic Regression

Note: Feature selection with sparse logistic regression
A logistic regression with L1 penalty yields sparse models, and can thus be used to perform feature selection, as detailed in L1-based feature selection.

Stochastic Gradient Descent - SGD

Stochastic gradient descent is a simple yet very efficient approach to fit linear models. It is particularly useful when the number of samples (and the number of features) is very large.

The classes SGDClassifier and SGDRegressor provide functionality to fit linear models for classification and regression using different (convex) loss functions and different penalties.

References
- Stochastic Gradient Descent

Perceptron

The Perceptron is another simple algorithm suitable for large scale learning. By default:
- It does not require a learning rate.
- It is not regularized (penalized).
- It updates its model only on mistakes.

The last characteristic implies that the Perceptron is slightly faster to train than SGD with the hinge loss and that the resulting models are sparser.

1.3.2 Support Vector Machines

Support vector machines (SVMs) are a set of supervised learning methods used for classification, regression and outliers detection.

The advantages of support vector machines are:
- Effective in high dimensional spaces.
- Still effective in cases where number of dimensions is greater than the number of samples.
- Uses a subset of training points in the decision function (called support vectors), so it is also memory efficient.
- Versatile: different Kernel functions can be specified for the decision function. Common kernels are provided, but it is also possible to specify custom kernels.

The disadvantages of support vector machines include:
- If the number of features is much greater than the number of samples, the method is likely to give poor performances.
SVMs do not directly provide probability estimates, these are calculated using five-fold cross-validation, and thus performance can suffer.

The support vector machines in scikit-learn support both dense (\texttt{numpy.ndarray} and convertible to that by \texttt{numpy.asarray}) and sparse (any \texttt{scipy.sparse}) sample vectors as input. However, to use an SVM to make predictions for sparse data, it must have been fit on such data. For optimal performance, use C-ordered \texttt{numpy.ndarray} (dense) or \texttt{scipy.sparse.csr_matrix} (sparse) with \texttt{dtype=\texttt{float64}}.

In previous versions of scikit-learn, sparse input support existed only in the \texttt{sklearn.svm.sparse} module which duplicated the \texttt{sklearn.svm} interface. This module still exists for backward compatibility, but is deprecated and will be removed in scikit-learn 0.12.

**Classification**

\texttt{SVC}, \texttt{NuSVC} and \texttt{LinearSVC} are classes capable of performing multi-class classification on a dataset.

\texttt{SVC} and \texttt{NuSVC} are similar methods, but accept slightly different sets of parameters and have different mathematical formulations (see section \texttt{Mathematical formulation}). On the other hand, \texttt{LinearSVC} is another implementation of Support Vector Classification for the case of a linear kernel. Note that \texttt{LinearSVC} does not accept keyword `kernel`, as this is assumed to be linear. It also lacks some of the members of \texttt{SVC} and \texttt{NuSVC}, like \texttt{support}_.

As other classifiers, \texttt{SVC}, \texttt{NuSVC} and \texttt{LinearSVC} take as input two arrays: an array \(X\) of size \([\text{n\_samples}, \text{n\_features}]\) holding the training samples, and an array \(Y\) of integer values, size \([\text{n\_samples}]\), holding the class labels for the training samples:
```python
>>> from sklearn import svm
>>> X = [[0, 0], [1, 1]]
>>> Y = [0, 1]
>>> clf = svm.SVC()
>>> clf.fit(X, Y)
SVC(C=1.0, cache_size=200, class_weight=None, coef0=0.0, degree=3,
gamma=0.5, kernel='rbf', probability=False, shrinking=True, tol=0.001,
verbose=False)
After being fitted, the model can then be used to predict new values:
```n
```python
>>> clf.predict([[2., 2.]])
array([ 1.])
```

SVMs decision function depends on some subset of the training data, called the support vectors. Some properties of these support vectors can be found in members `support_vectors_`, `support_` and `n_support`:

```python
>>> # get support vectors
... clf.support_vectors_
array([[ 0., 0.],
       [ 1., 1.]])
>>> # get indices of support vectors
... clf.support_
array([0, 1,...]
```n
```python
>>> # get number of support vectors for each class
... clf.n_support_
array([1, 1,...]
```

**Multi-class classification**

`SVC` and `NuSVC` implement the “one-against-one” approach (Knerr et al., 1990) for multi-class classification. If `n_class` is the number of classes, then `n_class * (n_class - 1)/2` classifiers are constructed and each one trains data from two classes:

```python
>>> X = [[0], [1], [2], [3]]
>>> Y = [0, 1, 2, 3]
>>> clf = svm.SVC()
>>> clf.fit(X, Y)
SVC(C=1.0, cache_size=200, class_weight=None, coef0=0.0, degree=3,
gamma=1.0, kernel='rbf', probability=False, shrinking=True, tol=0.001,
verbose=False)
```n
```python
>>> dec = clf.decision_function([[1]])
>>> dec.shape[1] # 4 classes: 4*3/2 = 6
6
```n
On the other hand, `LinearSVC` implements “one-vs-the-rest” multi-class strategy, thus training `n_class` models. If there are only two classes, only one model is trained:

```python
>>> lin_clf = svm.LinearSVC()
>>> lin_clf.fit(X, Y)
LinearSVC(C=1.0, class_weight=None, dual=True, fit_intercept=True,
intercept_scaling=1, loss='l2', multi_class='ovr', penalty='l2',
tol=0.0001, verbose=0)
>>> dec = lin_clf.decision_function([[1]])
>>> dec.shape[1]
4
```n
See **Mathematical formulation** for a complete description of the decision function.
Note that the LinearSVC also implements an alternative multi-class strategy, the so-called multi-class SVM formulated by Crammer and Singer, by using the option “multi_class=’crammer_singer’”. This method is consistent, which is not true for one-vs-rest classification. In practice, on-vs-rest classification is usually preferred, since the results are mostly similar, but the runtime is significantly less.

For “one-vs-rest” LinearSVC the attributes coef_ and intercept_ have the shape [n_class, n_features] and [n_class] respectively. Each row of the coefficients corresponds to one of the n_class many “one-vs-rest” classifiers and similar for the intercepts, in the order of the “one” class.

In the case of “one-vs-one” SVC, the layout of the attributes is a little more involved. In the case of having a linear kernel, the layout of coef_ and intercept_ is similar to the one described for LinearSVC described above, except that the shape of coef_is [n_class * (n_class - 1) / 2, corresponding to as many binary classifiers. The order for classes 0 to n is “0 vs 1”, “0 vs 2”, ... “0 vs n”, “1 vs 2”, “1 vs 3”, “1 vs n”, ... “n-1 vs n”.

The shape of dual_coef_is [n_class-1, n_SV] with a somewhat hard to grasp layout. The columns correspond to the support vectors involved in any of the n_class * (n_class - 1) / 2 “one-vs-one” classifiers. Each of the support vectors is used in n_class - 1 classifiers. The n_class - 1 entries in each row correspond to the dual coefficients for these classifiers.

This might be made more clear by an example:

Consider a three class problem with with class 0 having 3 support vectors v_0^0, v_0^1, v_0^2 and class 1 and 2 having two support vectors v_1^0, v_1^1 and v_0^0, v_1^1 respectively. For each support vector v_i^j, there are 2 dual coefficients. Let’s call the coefficient of support vector v_i^j in the classifier between classes i and k α_i^j,k.

Then dual_coef_ looks like this:

Unbalanced problems

In problems where it is desired to give more importance to certain classes or certain individual samples keywords class_weight and sample_weight can be used.

SVC (but not NuSVC) implement a keyword class_weight in the fit method. It’s a dictionary of the form {class_label : value}, where value is a floating point number > 0 that sets the parameter C of class class_label to C * value.

SVC, NuSVC, SVR, NuSVR and OneClassSVM implement also weights for individual samples in method fit through keyword sample_weight.

Examples:

- Plot different SVM classifiers in the iris dataset,
- SVM: Maximum margin separating hyperplane,
- SVM: Separating hyperplane for unbalanced classes
- SVM-Anova: SVM with univariate feature selection,
- Non-linear SVM
- SVM: Weighted samples,

Regression

The method of Support Vector Classification can be extended to solve regression problems. This method is called Support Vector Regression.

The model produced by support vector classification (as described above) depends only on a subset of the training data, because the cost function for building the model does not care about training points that lie beyond the margin.
1.3. Supervised learning

![Graph showing comparison between models with and without weights]

- **no weights**
- **with weights**

![Contour plot comparing models with and without weights]
Analogously, the model produced by Support Vector Regression depends only on a subset of the training data, because the cost function for building the model ignores any training data close to the model prediction.

There are two flavors of Support Vector Regression: SVR and NuSVR.

As with classification classes, the fit method will take as argument vectors X, y, only that in this case y is expected to have floating point values instead of integer values:

```python
>>> from sklearn import svm
>>> X = [[0, 0], [2, 2]]
>>> y = [0.5, 2.5]
>>> clf = svm.SVR()
>>> clf.fit(X, y)
SVR(C=1.0, cache_size=200, coef0=0.0, degree=3, epsilon=0.1, gamma=0.5, kernel='rbf', probability=False, shrinking=True, tol=0.001, verbose=False)
>>> clf.predict([[1, 1]])
array([ 1.5])
```

**Examples:**

- Support Vector Regression (SVR) using linear and non-linear kernels

**Density estimation, novelty detection**

One-class SVM is used for novelty detection, that is, given a set of samples, it will detect the soft boundary of that set so as to classify new points as belonging to that set or not. The class that implements this is called `OneClassSVM`.

In this case, as it is a type of unsupervised learning, the fit method will only take as input an array X, as there are no class labels.

See, section *Novelty and Outlier Detection* for more details on this usage.
Examples:

- One-class SVM with non-linear kernel (RBF)
- Species distribution modeling

Complexity

Support Vector Machines are powerful tools, but their compute and storage requirements increase rapidly with the number of training vectors. The core of an SVM is a quadratic programming problem (QP), separating support vectors from the rest of the training data. The QP solver used by this libsvm-based implementation scales between $O(n_{\text{features}} \times n_{\text{samples}}^2)$ and $O(n_{\text{features}} \times n_{\text{samples}}^3)$ depending on how efficiently the libsvm cache is used in practice (dataset dependent). If the data is very sparse $n_{\text{features}}$ should be replaced by the average number of non-zero features in a sample vector.

Also note that for the linear case, the algorithm used in LinearSVC by the liblinear implementation is much more efficient than its libsvm-based SVC counterpart and can scale almost linearly to millions of samples and/or features.

Tips on Practical Use

- Avoiding data copy: For SVC, SVR, NuSVC and NuSVR, if the data passed to certain methods is not C-ordered contiguous, and double precision, it will be copied before calling the underlying C implementation. You can check whether a give numpy array is C-contiguous by inspecting its flags attribute.

  For LinearSVC (and LogisticRegression) any input passed as a numpy array will be copied and converted to the liblinear internal sparse data representation (double precision floats and int32 indices of non-zero components). If you want to fit a large-scale linear classifier without copying a dense numpy C-contiguous double precision array as input we suggest to use the SGDClassifier class instead. The objective function can be configured to be almost the same as the LinearSVC model.

- Kernel cache size: For SVC, SVR, nuSVC and NuSVR, the size of the kernel cache has a strong impact on run times for larger problems. If you have enough RAM available, it is recommended to set cache_size to a higher value than the default of 200(MB), such as 500(MB) or 1000(MB).

- Setting C: C is 1 by default and it’s a reasonable default choice. If you have a lot of noisy observations you should decrease it. It corresponds to regularize more the estimation.

  Support Vector Machine algorithms are not scale invariant, so it is highly recommended to scale your data. For example, scale each attribute on the input vector X to [0,1] or [-1,+1], or standardize it to have mean 0 and variance 1. Note that the same scaling must be applied to the test vector to obtain meaningful results. See section Preprocessing data for more details on scaling and normalization.

  Parameter nu in NuSVC/OneClassSVM/NuSVR approximates the fraction of training errors and support vectors.

  In SVC, if data for classification are unbalanced (e.g. many positive and few negative), set class_weight='auto' and/or try different penalty parameters C.

  The underlying LinearSVC implementation uses a random number generator to select features when fitting the model. It is thus not uncommon, to have slightly different results for the same input data. If that happens, try with a smaller tol parameter.

  Using L1 penalization as provided by LinearSVC(loss='l2', penalty='l1', dual=False) yields a sparse solution, i.e. only a subset of feature weights is different from zero and contribute to the decision function. Increasing C yields a more complex model (more feature are selected). The C value that yields a “null” model (all weights equal to zero) can be calculated using l1_min_c.
Kernel functions

The kernel function can be any of the following:

- **linear**: \( <x_i, x'_j> \).
- **polynomial**: \((\gamma <x, x'> + r)^d\). \(d\) is specified by keyword `degree`, \(r\) by `coef0`.
- **rbf** (\(\exp(-\gamma |x - x'|^2), \gamma > 0\)). \(\gamma\) is specified by keyword `gamma`.
- **sigmoid** (\(\tanh(<x_i, x_j > + r)\)), where \(r\) is specified by `coef0`.

Different kernels are specified by keyword `kernel` at initialization:

```python
>>> linear_svc = svm.SVC(kernel='linear')
>>> linear_svc.kernel
'linear'
>>> rbf_svc = svm.SVC(kernel='rbf')
>>> rbf_svc.kernel
'rbf'
```

Custom Kernels

You can define your own kernels by either giving the kernel as a python function or by precomputing the Gram matrix.

Classifiers with custom kernels behave the same way as any other classifiers, except that:

- Field `support_vectors_` is now empty, only indices of support vectors are stored in `support_`
- A reference (and not a copy) of the first argument in the fit() method is stored for future reference. If that array changes between the use of fit() and predict() you will have unexpected results.

Using python functions as kernels  You can also use your own defined kernels by passing a function to the keyword `kernel` in the constructor.

Your kernel must take as arguments two matrices and return a third matrix.

The following code defines a linear kernel and creates a classifier instance that will use that kernel:

```python
>>> import numpy as np
>>> from sklearn import svm
>>> def my_kernel(x, y):
...     return np.dot(x, y.T)
...
>>> clf = svm.SVC(kernel=my_kernel)
```

Examples:

- **SVM with custom kernel.**

Using the Gram matrix  Set kernel=‘precomputed’ and pass the Gram matrix instead of X in the fit method. At the moment, the kernel values between all training vectors and the test vectors must be provided.

```python
>>> import numpy as np
>>> from sklearn import svm
>>> X = np.array([[0, 0], [1, 1]])
>>> y = [0, 1]
```
Parameters of the RBF Kernel

When training an SVM with the Radial Basis Function (RBF) kernel, two parameters must be considered: $C$ and $\gamma$. The parameter $C$, common to all SVM kernels, trades off misclassification of training examples against simplicity of the decision surface. A low $C$ makes the decision surface smooth, while a high $C$ aims at classifying all training examples correctly. $\gamma$ defines how much influence a single training example has. The larger $\gamma$ is, the closer other examples must be to be affected.

Proper choice of $C$ and $\gamma$ is critical to the SVM’s performance. One is advised to use GridSearchCV with $C$ and $\gamma$ spaced exponentially far apart to choose good values.

Examples:

- RBF SVM parameters

Mathematical formulation

A support vector machine constructs a hyper-plane or set of hyper-planes in a high or infinite dimensional space, which can be used for classification, regression or other tasks. Intuitively, a good separation is achieved by the hyper-plane that has the largest distance to the nearest training data points of any class (so-called functional margin), since in general the larger the margin the lower the generalization error of the classifier.

SVC

Given training vectors $x_i \in \mathbb{R}^p$, $i=1,...,n$, in two classes, and a vector $y \in \mathbb{R}^n$ such that $y_i \in 1, -1$, SVC solves the following primal problem:

$$ \min_{w,b,\zeta} \frac{1}{2} w^T w + C \sum_{i=1}^{n} \zeta_i $$

subject to $y_i (w^T \phi(x_i) + b) \geq 1 - \zeta_i$,

$$ \zeta_i \geq 0, i = 1,...,n $$

Its dual is

$$ \min_{\alpha} \frac{1}{2} \alpha^T Q \alpha - e^T \alpha $$

subject to $y^T \alpha = 0$

$$ 0 \leq \alpha_i \leq C, i = 1,...,l $$

where $e$ is the vector of all ones, $C > 0$ is the upper bound, $Q$ is an $n$ by $n$ positive semidefinite matrix, $Q_{ij} \equiv K(x_i, x_j)$ and $\phi(x_i)^T \phi(x)$ is the kernel. Here training vectors are mapped into a higher (maybe infinite) dimensional space by the function $\phi$. 

1.3. Supervised learning 53
The decision function is:

\[ \text{sgn}(\sum_{i=1}^{n} y_i \alpha_i K(x_i, x) + \rho) \]

**Note:** While SVM models derived from libsvm and liblinear use \( C \) as regularization parameter, most other estimators use \( \text{alpha} \). The relation between both is \( C = \frac{n_{\text{samples}}}{\text{alpha}} \).

This parameters can be accessed through the members \texttt{dual_coef_} which holds the product \( y_i \alpha_i \), \texttt{support_vectors_} which holds the support vectors, and \texttt{intercept_} which holds the independent term \(-\rho\).

**References:**

**NuSVC**

We introduce a new parameter \( \nu \) which controls the number of support vectors and training errors. The parameter \( \nu \in (0, 1] \) is an upper bound on the fraction of training errors and a lower bound of the fraction of support vectors.

It can be shown that the \( nu \)-SVC formulation is a reparametrization of the \( C \)-SVC and therefore mathematically equivalent.
Implementation details

Internally, we use libsvm and liblinear to handle all computations. These libraries are wrapped using C and Cython.

References:
For a description of the implementation and details of the algorithms used, please refer to
- LIBSVM: a library for Support Vector Machines
- LIBLINEAR – A Library for Large Linear Classification

1.3.3 Stochastic Gradient Descent

Stochastic Gradient Descent (SGD) is a simple yet very efficient approach to discriminative learning of linear classifiers under convex loss functions such as (linear) Support Vector Machines and Logistic Regression. Even though SGD has been around in the machine learning community for a long time, it has received a considerable amount of attention just recently in the context of large-scale learning.

SGD has been successfully applied to large-scale and sparse machine learning problems often encountered in text classification and natural language processing. Given that the data is sparse, the classifiers in this module easily scale to problems with more than 10^5 training examples and more than 10^5 features.

The advantages of Stochastic Gradient Descent are:
- Efficiency.
- Ease of implementation (lots of opportunities for code tuning).

The disadvantages of Stochastic Gradient Descent include:
- SGD requires a number of hyperparameters such as the regularization parameter and the number of iterations.
- SGD is sensitive to feature scaling.

Classification

Warning: Make sure you permute (shuffle) your training data before fitting the model or use shuffle=True to shuffle after each iteration.

The class SGDClassifier implements a plain stochastic gradient descent learning routine which supports different loss functions and penalties for classification.

As other classifiers, SGD has to be fitted with two arrays: an array X of size [n_samples, n_features] holding the training samples, and an array Y of size [n_samples] holding the target values (class labels) for the training samples:

```python
>>> from sklearn.linear_model import SGDClassifier
>>> X = [[0., 0.], [1., 1.]]
>>> y = [0, 1]
>>> clf = SGDClassifier(loss="hinge", penalty="l2")
>>> clf.fit(X, y)
```

After being fitted, the model can then be used to predict new values:
SGD fits a linear model to the training data. The member `coef_` holds the model parameters:

```python
>>> clf.coef_
array([[ 9.90090187,  9.90090187]])
```

Member `intercept_` holds the intercept (aka offset or bias):

```python
>>> clf.intercept_
array([-9.990...])
```

Whether or not the model should use an intercept, i.e. a biased hyperplane, is controlled by the parameter `fit_intercept`.

To get the signed distance to the hyperplane use `decision_function`:

```python
>>> clf.decision_function([[2., 2.]])
array([ 29.61357756])
```

The concrete loss function can be set via the `loss` parameter. `SGDClassifier` supports the following loss functions:

- `loss="hinge"`: (soft-margin) linear Support Vector Machine,
- `loss="modified_huber"`: smoothed hinge loss,
- `loss="log"`: Logistic Regression,
- and all regression losses below.

The first two loss functions are lazy, they only update the model parameters if an example violates the margin constraint, which makes training very efficient and may result in sparser models, even when L2 penalty is used.

In the case of binary classification and `loss="log"` or `loss="modified_huber"` you get a probability estimate \( P(y=C|x) \) using `predict_proba`, where \( C \) is the largest class label:
clf = SGDClassifier(loss="log").fit(X, y)
clf.predict_prob�浒([1., 1.])
array([0.99999949])

The concrete penalty can be set via the penalty parameter. SGD supports the following penalties:

- penalty="l2": L2 norm penalty on coef_.
- penalty="l1": L1 norm penalty on coef_.
- penalty="elasticnet": Convex combination of L2 and L1; rho * L2 + (1 - rho) * L1.

The default setting is penalty="l2". The L1 penalty leads to sparse solutions, driving most coefficients to zero. The Elastic Net solves some deficiencies of the L1 penalty in the presence of highly correlated attributes. The parameter rho has to be specified by the user.

SGDClassifier supports multi-class classification by combining multiple binary classifiers in a “one versus all” (OVA) scheme. For each of the K classes, a binary classifier is learned that discriminates between that and all other K-1 classes. At testing time, we compute the confidence score (i.e. the signed distances to the hyperplane) for each classifier and choose the class with the highest confidence. The Figure below illustrates the OVA approach on the iris dataset. The dashed lines represent the three OVA classifiers; the background colors show the decision surface induced by the three classifiers.

In the case of multi-class classification coef_ is a two-dimensionaarray of shape [n_classes, n_features] and intercept_ is a one dimensional array of shape [n_classes]. The i-th row of coef_ holds the weight vector of the OVA classifier for the i-th class; classes are indexed in ascending order (see attribute classes). Note that, in principle, since they allow to create a probability model, loss="log" and loss="modified_huber" are more suitable for one-vs-all classification.

SGDClassifier supports both weighted classes and weighted instances via the fit parameters class_weight and sample_weight. See the examples below and the doc string of SGDClassifier.fit for further information.
Examples:

- \textit{SGD}: Maximum margin separating hyperplane,
- Plot multi-class SGD on the iris dataset
- \textit{SGD}: Separating hyperplane with weighted classes
- \textit{SGD}: Weighted samples

\section*{Regression}

The class \texttt{SGDRegressor} implements a plain stochastic gradient descent learning routine which supports different loss functions and penalties to fit linear regression models. \texttt{SGDRegressor} is well suited for regression problems with a large number of training samples (> 10.000), for other problems we recommend \texttt{Ridge}, \texttt{Lasso}, or \texttt{ElasticNet}.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{regression_plot}
\caption{Graph showing regression model fit.}
\end{figure}

The concrete loss function can be set via the \texttt{loss} parameter. \texttt{SGDRegressor} supports the following loss functions:

- \texttt{loss=“squared\_loss”}: Ordinary least squares,
- \texttt{loss=“huber”}: Huber loss for robust regression,
- \texttt{loss=“epsilon\_insensitive”}: linear Support Vector Regression.

The Huber and epsilon-insensitive loss functions can be used for robust regression. The width of the insensitive region has to be specified via the parameter \texttt{epsilon}. This parameter depends on the scale of the target variables.

Examples:

- Ordinary Least Squares with SGD,
Stochastic Gradient Descent for sparse data

**Note:** The sparse implementation produces slightly different results than the dense implementation due to a shrunk learning rate for the intercept.

There is built-in support for sparse data given in any matrix in a format supported by scipy.sparse. For maximum efficiency, however, use the CSR matrix format as defined in `scipy.sparse.csr_matrix`.

**Examples:**

- *Classification of text documents using sparse features*

**Complexity**

The major advantage of SGD is its efficiency, which is basically linear in the number of training examples. If X is a matrix of size (n, p) training has a cost of $O(\bar{p}k)$, where $k$ is the number of iterations (epochs) and $\bar{p}$ is the average number of non-zero attributes per sample.

Recent theoretical results, however, show that the runtime to get some desired optimization accuracy does not increase as the training set size increases.

**Tips on Practical Use**

- Stochastic Gradient Descent is sensitive to feature scaling, so it is highly recommended to scale your data. For example, scale each attribute on the input vector X to [0,1] or [-1,+1], or standardize it to have mean 0 and variance 1. Note that the same scaling must be applied to the test vector to obtain meaningful results. This can be easily done using `Scaler`:

  ```python
  from sklearn.preprocessing import Scaler
  scaler = Scaler()
  scaler.fit(X_train)  # Don't cheat - fit only on training data
  X_train = scaler.transform(X_train)
  X_test = scaler.transform(X_test)  # apply same transformation to test data
  ```

  If your attributes have an intrinsic scale (e.g. word frequencies or indicator features) scaling is not needed.

- Finding a reasonable regularization term $\alpha$ is best done using `GridSearchCV`, usually in the range $10.0^{**-np.arange(1,7)}$.

- Empirically, we found that SGD converges after observing approx. $10^6$ training samples. Thus, a reasonable first guess for the number of iterations is $n_{iter} = \text{np.ceil}(10^6/n)$, where $n$ is the size of the training set.

- If you apply SGD to features extracted using PCA we found that it is often wise to scale the feature values by some constant $c$ such that the average L2 norm of the training data equals one.

**References:**

Mathematical formulation

Given a set of training examples \((x_1, y_1), \ldots, (x_n, y_n)\) where \(x_i \in \mathbb{R}^n\) and \(y_i \in \{-1, 1\}\), our goal is to learn a linear scoring function \(f(x) = w^T x + b\) with model parameters \(w \in \mathbb{R}^m\) and intercept \(b \in \mathbb{R}\). In order to make predictions, we simply look at the sign of \(f(x)\). A common choice to find the model parameters is by minimizing the regularized training error given by

\[
E(w, b) = \sum_{i=1}^{n} L(y_i, f(x_i)) + \alpha R(w)
\]

where \(L\) is a loss function that measures model (mis)fit and \(R\) is a regularization term (aka penalty) that penalizes model complexity; \(\alpha > 0\) is a non-negative hyperparameter.

Different choices for \(L\) entail different classifiers such as

- Hinge: (soft-margin) Support Vector Machines.
- Log: Logistic Regression.
- Least-Squares: Ridge Regression.
- Epsilon-Insensitive: (soft-margin) Support Vector Regression.

All of the above loss functions can be regarded as an upper bound on the misclassification error (Zero-one loss) as shown in the Figure below.

![Figure showing contours of different regularization terms](image)

Popular choices for the regularization term \(R\) include:

- L2 norm: \(R(w) := \frac{1}{2} \sum_{i=1}^{n} w_i^2\),
- L1 norm: \(R(w) := \sum_{i=1}^{n} |w_i|\), which leads to sparse solutions.
- Elastic Net: \(R(w) := \rho \frac{1}{2} \sum_{i=1}^{n} w_i^2 + (1 - \rho) \sum_{i=1}^{n} |w_i|\), a convex combination of L2 and L1.

The Figure below shows the contours of the different regularization terms in the parameter space when \(R(w) = 1\).
SGD

Stochastic gradient descent is an optimization method for unconstrained optimization problems. In contrast to (batch) gradient descent, SGD approximates the true gradient of $E(w, b)$ by considering a single training example at a time.

The class `SGDClassifier` implements a first-order SGD learning routine. The algorithm iterates over the training examples and for each example updates the model parameters according to the update rule given by

$$ w \leftarrow w - \eta \left( \alpha \frac{\partial R(w)}{\partial w} + \frac{\partial L(w^T x_i + b, y_i)}{\partial w} \right) $$

where $\eta$ is the learning rate which controls the step-size in the parameter space. The intercept $b$ is updated similarly but without regularization.

The learning rate $\eta$ can be either constant or gradually decaying. For classification, the default learning rate schedule (`learning_rate='optimal'`) is given by

$$ \eta(t) = \frac{1}{\alpha(t_0 + t)} $$

where $t$ is the time step (there are a total of $n\_samples * epochs$ time steps), $t_0$ is determined based on a heuristic proposed by Léon Bottou such that the expected initial updates are comparable with the expected size of the weights (this assuming that the norm of the training samples is approx. 1). The exact definition can be found in `init_t` in BaseSGD.

For regression, the default learning rate schedule, inverse scaling (`learning_rate='invscaling'`), is given by

$$ \eta(t) = \frac{\text{eta0}}{t^{\text{power} - t}} $$

where $\text{eta0}$ and $\text{power} - t$ are hyperparameters chosen by the user via $\text{eta0}$ and $\text{power} - t$, resp.

For a constant learning rate use `learning_rate='constant'` and use $\text{eta0}$ to specify the learning rate.

The model parameters can be accessed through the members `coef_` and `intercept_`: 

---

**1.3. Supervised learning**
• Member `coef_` holds the weights $w$
• Member `intercept_` holds $b$

References:


Implementation details

The implementation of SGD is influenced by the Stochastic Gradient SVM of Léon Bottou. Similar to SvmSGD, the weight vector is represented as the product of a scalar and a vector which allows an efficient weight update in the case of L2 regularization. In the case of sparse feature vectors, the intercept is updated with a smaller learning rate (multiplied by 0.01) to account for the fact that it is updated more frequently. Training examples are picked up sequentially and the learning rate is lowered after each observed example. We adopted the learning rate schedule from Shalev-Shwartz et al. 2007. For multi-class classification, a “one versus all” approach is used. We use the truncated gradient algorithm proposed by Tsuruoka et al. 2009 for L1 regularization (and the Elastic Net). The code is written in Cython.

References:

- “Pegasos: Primal estimated sub-gradient solver for svm” S. Shalev-Shwartz, Y. Singer, N. Srebro - In Proceedings of ICML ‘07.

1.3.4 Nearest Neighbors

`sklearn.neighbors` provides functionality for unsupervised and supervised neighbors-based learning methods. Unsupervised nearest neighbors is the foundation of many other learning methods, notably manifold learning and spectral clustering. Supervised neighbors-based learning comes in two flavors: classification for data with discrete labels, and regression for data with continuous labels.

The principle behind nearest neighbor methods is to find a predefined number of training samples closest in distance to the new point, and predict the label from these. The number of samples can be a user-defined constant (k-nearest neighbor learning), or vary based on the local density of points (radius-based neighbor learning). The distance can, in general, be any metric measure: standard Euclidean distance is the most common choice. Neighbors-based methods are known as non-generalizing machine learning methods, since they simply “remember” all of its training data (possibly transformed into a fast indexing structure such as a Ball Tree or KD Tree.).

Despite its simplicity, nearest neighbors has been successful in a large number of classification and regression problems, including handwritten digits or satellite image scenes. It is often successful in classification situations where the decision boundary is very irregular.

The classes in `sklearn.neighbors` can handle either Numpy arrays or `scipy.sparse` matrices as input. Arbitrary Minkowski metrics are supported for searches.
Unsupervised Nearest Neighbors

NearestNeighbors implements unsupervised nearest neighbors learning. It acts as a uniform interface to three different nearest neighbors algorithms: BallTree, scipy.spatial.cKDTree, and a brute-force algorithm based on routines in sklearn.metrics.pairwise. The choice of neighbors search algorithm is controlled through the keyword ‘algorithm’, which must be one of [‘auto’, ‘ball_tree’, ‘kd_tree’, ‘brute’]. When the default value ‘auto’ is passed, the algorithm attempts to determine the best approach from the training data. For a discussion of the strengths and weaknesses of each option, see Nearest Neighbor Algorithms.

Nearest Neighbors Classification

Neighbors-based classification is a type of instance-based learning or non-generalizing learning: it does not attempt to construct a general internal model, but simply stores instances of the training data. Classification is computed from a simple majority vote of the nearest neighbors of each point: a query point is assigned the data class which has the most representatives within the nearest neighbors of the point.

scikit-learn implements two different nearest neighbors classifiers: KNeighborsClassifier implements learning based on the $k$ nearest neighbors of each query point, where $k$ is an integer value specified by the user. RadiusNeighborsClassifier implements learning based on the number of neighbors within a fixed radius $r$ of each training point, where $r$ is a floating-point value specified by the user.

The $k$-neighbors classification in KNeighborsClassifier is the more commonly used of the two techniques. The optimal choice of the value $k$ is highly data-dependent: in general a larger $k$ suppresses the effects of noise, but makes the classification boundaries less distinct.

In cases where the data is not uniformly sampled, radius-based neighbors classification in RadiusNeighborsClassifier can be a better choice. The user specifies a fixed radius $r$, such that points in sparser neighborhoods use fewer nearest neighbors for the classification. For high-dimensional parameter spaces, this method becomes less effective due to the so-called “curse of dimensionality”.

The basic nearest neighbors classification uses uniform weights: that is, the value assigned to a query point is computed from a simple majority vote of the nearest neighbors. Under some circumstances, it is better to weight the neighbors such that nearer neighbors contribute more to the fit. This can be accomplished through the weights keyword. The default value, weights = ‘uniform’, assigns uniform weights to each neighbor. weights = ‘distance’ assigns weights proportional to the inverse of the distance from the query point. Alternatively, a user-defined function of the distance can be supplied which is used to compute the weights.
Examples:

- Nearest Neighbors Classification: an example of classification using nearest neighbors.

Nearest Neighbors Regression

Nearest neighbors-based regression can be used in cases where the data labels are continuous rather than discrete variables. The label assigned to a query point is computed based on the mean of the labels of its nearest neighbors.

scikit-learn implements two different neighbors regressors: `KNeighborsRegressor` implements learning based on the $k$ nearest neighbors of each query point, where $k$ is an integer value specified by the user. `RadiusNeighborsRegressor` implements learning based on the neighbors within a fixed radius $r$ of the query point, where $r$ is a floating-point value specified by the user.

The basic nearest neighbors regression uses uniform weights: that is, each point in the local neighborhood contributes uniformly to the classification of a query point. Under some circumstances, it can be advantageous to weight points such that nearby points contribute more to the regression than faraway points. This can be accomplished through the `weights` keyword. The default value, `weights = 'uniform'`, assigns equal weights to all points. `weights = 'distance'` assigns weights proportional to the inverse of the distance from the query point. Alternatively, a user-defined function of the distance can be supplied, which will be used to compute the weights.

Examples:

- Nearest Neighbors regression: an example of regression using nearest neighbors.

Nearest Neighbor Algorithms

Brute Force

Fast computation of nearest neighbors is an active area of research in machine learning. The most naive neighbor search implementation involves the brute-force computation of distances between all pairs of points in the dataset: for $N$ samples in $D$ dimensions, this approach scales as $O[DN^2]$. Efficient brute-force neighbors searches can be very competitive for small data samples. However, as the number of samples $N$ grows, the brute-force approach quickly becomes infeasible. In the classes within `sklearn.neighbors`, brute-force neighbors searches
are specified using the keyword `algorithm = 'brute'`, and are computed using the routines available in `sklearn.metrics.pairwise`.

**K-D Tree**

To address the computational inefficiencies of the brute-force approach, a variety of tree-based data structures have been invented. In general, these structures attempt to reduce the required number of distance calculations by efficiently encoding aggregate distance information for the sample. The basic idea is that if point \( A \) is very distant from point \( B \), and point \( B \) is very close to point \( C \), then we know that points \( A \) and \( C \) are very distant, without having to explicitly calculate their distance. In this way, the computational cost of a nearest neighbors search can be reduced to \( O(D \log(N)) \) or better. This is a significant improvement over brute-force for large \( N \).

An early approach to taking advantage of this aggregate information was the **KD tree** data structure (short for **K-dimensional tree**), which generalizes two-dimensional **Quad-trees** and 3-dimensional **Oct-trees** to an arbitrary number of dimensions. The KD tree is a tree structure which recursively partitions the parameter space along the data axes, dividing it into nested orthotopic regions into which data points are filed. The construction of a KD tree is very fast: because partitioning is performed only along the data axes, no \( D \)-dimensional distances need to be computed. Once constructed, the nearest neighbor of a query point can be determined with only \( O(\log(N)) \) distance computations. Though the KD tree approach is very fast for low-dimensional \((D < 20)\) neighbors searches, it becomes inefficient as \( D \) grows very large: this is one manifestation of the so-called “curse of dimensionality”. In scikit-learn, KD tree neighbors searches are specified using the keyword `algorithm = 'kd_tree'`, and are computed using the class `scipy.spatial.cKDTree`.

References:

* “Multidimensional binary search trees used for associative searching”, Bentley, J.L., Communications of the ACM (1975)
Ball Tree

To address the inefficiencies of KD Trees in higher dimensions, the ball tree data structure was developed. Where KD trees partition data along Cartesian axes, ball trees partition data in a series of nesting hyper-spheres. This makes tree construction more costly than that of the KD tree, but results in a data structure which allows for efficient neighbors searches even in very high dimensions.

A ball tree recursively divides the data into nodes defined by a centroid $C$ and radius $r$, such that each point in the node lies within the hyper-sphere defined by $r$ and $C$. The number of candidate points for a neighbor search is reduced through use of the triangle inequality:

$$|x + y| \leq |x| + |y|$$

With this setup, a single distance calculation between a test point and the centroid is sufficient to determine a lower and upper bound on the distance to all points within the node. Because of the spherical geometry of the ball tree nodes, its performance does not degrade at high dimensions. In scikit-learn, ball-tree-based neighbors searches are specified using the keyword algorithm = 'ball_tree', and are computed using the class sklearn.neighbors.BallTree. Alternatively, the user can work with the BallTree class directly.

References:


Choice of Nearest Neighbors Algorithm

The optimal algorithm for a given dataset is a complicated choice, and depends on a number of factors:

- number of samples $N$ (i.e. n_samples) and dimensionality $D$ (i.e. n_features).
  
  - Brute force query time grows as $O|DN|
  
  - Ball tree query time grows as approximately $O[D \log(N)]$
  
  - KD tree query time changes with $D$ in a way that is difficult to precisely characterise. For small $D$ (less than 20 or so) the cost is approximately $O[D \log(N)]$, and the KD tree query can be very efficient. For larger $D$, the cost increases to nearly $O[DN]$, and the overhead due to the tree structure can lead to queries which are slower than brute force.

For small data sets ($N$ less than 30 or so), $\log(N)$ is comparable to $N$, and brute force algorithms can be more efficient than a tree-based approach. Both cKDTree and BallTree address this through providing a leaf size parameter: this controls the number of samples at which a query switches to brute-force. This allows both algorithms to approach the efficiency of a brute-force computation for small $N$.

- data structure: intrinsic dimensionality of the data and/or sparsity of the data. Intrinsic dimensionality refers to the dimension $d \leq D$ of a manifold on which the data lies, which can be linearly or nonlinearly embedded in the parameter space. Sparsity refers to the degree to which the data fills the parameter space (this is to be distinguished from the concept as used in “sparse” matrices. The data matrix may have no zero entries, but the structure can still be “sparse” in this sense).

  - Brute force query time is unchanged by data structure.

  - Ball tree and KD tree query times can be greatly influenced by data structure. In general, sparser data with a smaller intrinsic dimensionality leads to faster query times. Because the KD tree internal representation is aligned with the parameter axes, it will not generally show as much improvement as ball tree for arbitrarily structured data.
Datasets used in machine learning tend to be very structured, and are very well-suited for tree-based queries.

- number of neighbors \( k \) requested for a query point.
  - **Brute force** query time is largely unaffected by the value of \( k \)
  - **Ball tree** and **KD tree** query time will become slower as \( k \) increases. This is due to two effects: first, a larger \( k \) leads to the necessity to search a larger portion of the parameter space. Second, using \( k > 1 \) requires internal queueing of results as the tree is traversed.

As \( k \) becomes large compared to \( N \), the ability to prune branches in a tree-based query is reduced. In this situation, Brute force queries can be more efficient.

- number of query points. Both the ball tree and the KD Tree require a construction phase. The cost of this construction becomes negligible when amortized over many queries. If only a small number of queries will be performed, however, the construction can make up a significant fraction of the total cost. If very few query points will be required, brute force is better than a tree-based method.

Currently, \texttt{algorithm = 'auto'} selects 'ball_tree' if \( k < N/2 \), and 'brute' otherwise. This choice is based on the assumption that the number of query points is at least the same order as the number of training points, and that \texttt{leaf_size} is close to its default value of 30.

### Effect of \texttt{leaf_size}

As noted above, for small sample sizes a brute force search can be more efficient than a tree-based query. This fact is accounted for in the ball tree and KD tree by internally switching to brute force searches within leaf nodes. The level of this switch can be specified with the parameter \texttt{leaf_size}. This parameter choice has many effects:

**construction time** A larger \texttt{leaf_size} leads to a faster tree construction time, because fewer nodes need to be created.

**query time** Both a large or small \texttt{leaf_size} can lead to suboptimal query cost. For \texttt{leaf_size} approaching 1, the overhead involved in traversing nodes can significantly slow query times. For \texttt{leaf_size} approaching the size of the training set, queries become essentially brute force. A good compromise between these is \texttt{leaf_size = 30}, the default value of the parameter.

**memory** As \texttt{leaf_size} increases, the memory required to store a tree structure decreases. This is especially important in the case of ball tree, which stores a \( D \)-dimensional centroid for each node. The required storage space for \texttt{BallTree} is approximately \( 1 / \texttt{leaf_size} \) times the size of the training set.

\texttt{leaf_size} is not referenced for brute force queries.

### Nearest Centroid Classifier

The \texttt{NearestCentroid} classifier is a simple algorithm that represents each class by the centroid of its members. In effect, this makes it similar to the label updating phase of the \texttt{sklearn.KMeans} algorithm. It also has no parameters to choose, making it a good baseline classifier. It does, however, suffer on non-convex classes, as well as when classes have drastically different variances, as equal variance in all dimensions is assumed. See Linear Discriminant Analysis (\texttt{sklearn.lda.LDA}) and Quadratic Discriminant Analysis (\texttt{sklearn.qda.QDA}) for more complex methods that do not make this assumption. Usage of the default \texttt{NearestCentroid} is simple:

```python
>>> from sklearn.neighbors.nearest_centroid import NearestCentroid
>>> import numpy as np
>>> X = np.array([[-1, -1], [-2, -1], [-3, -2], [1, 1], [2, 1], [3, 2]])
>>> y = np.array([1, 1, 1, 2, 2, 2])
>>> clf = NearestCentroid()
>>> clf.fit(X, y)
NearestCentroid(metric='euclidean', shrink_threshold=None)
```
>>> print clf.predict([[-0.8, -1]])
[1]

Nearest Shrunken Centroid

The NearestCentroid classifier has a `shrink_threshold` parameter, which implements the nearest shrunken centroid classifier. In effect, the value of each feature for each centroid is divided by the within-class variance of that feature. The feature values are then reduced by `shrink_threshold`. Most notably, if a particular feature value crosses zero, it is set to zero. In effect, this removes the feature from affecting the classification. This is useful, for example, for removing noisy features.

In the example below, using a small shrink threshold increases the accuracy of the model from 0.81 to 0.82.

Examples:

- **Nearest Centroid Classification**: an example of classification using nearest centroid with different shrink thresholds.
1.3.5 Gaussian Processes

Gaussian Processes for Machine Learning (GPML) is a generic supervised learning method primarily designed to solve regression problems. It has also been extended to probabilistic classification, but in the present implementation, this is only a post-processing of the regression exercise.

The advantages of Gaussian Processes for Machine Learning are:

- The prediction interpolates the observations (at least for regular correlation models).
- The prediction is probabilistic (Gaussian) so that one can compute empirical confidence intervals and exceedence probabilities that might be used to refit (online fitting, adaptive fitting) the prediction in some region of interest.
- Versatile: different linear regression models and correlation models can be specified. Common models are provided, but it is also possible to specify custom models provided they are stationary.

The disadvantages of Gaussian Processes for Machine Learning include:

- It is not sparse. It uses the whole samples/features information to perform the prediction.
- It loses efficiency in high dimensional spaces – namely when the number of features exceeds a few dozens. It might indeed give poor performance and it loses computational efficiency.
- Classification is only a post-processing, meaning that one first need to solve a regression problem by providing the complete scalar float precision output $y$ of the experiment one attempt to model.

Thanks to the Gaussian property of the prediction, it has been given varied applications: e.g. for global optimization, probabilistic classification.

Examples

An introductory regression example

Say we want to surrogate the function $g(x) = x \sin(x)$. To do so, the function is evaluated onto a design of experiments. Then, we define a GaussianProcess model whose regression and correlation models might be specified using additional kwargs, and ask for the model to be fitted to the data. Depending on the number of parameters provided at instantiation, the fitting procedure may recourse to maximum likelihood estimation for the parameters or alternatively it uses the given parameters.

```python
gp = gaussian_process.GaussianProcess(theta0=1e-2, thetaL=1e-4, thetaU=1e-1)
gp.fit(X, y)
```

Examples

An introductory regression example

Say we want to surrogate the function $g(x) = x \sin(x)$. To do so, the function is evaluated onto a design of experiments. Then, we define a GaussianProcess model whose regression and correlation models might be specified using additional kwargs, and ask for the model to be fitted to the data. Depending on the number of parameters provided at instantiation, the fitting procedure may recourse to maximum likelihood estimation for the parameters or alternatively it uses the given parameters.

```python
gp = gaussian_process.GaussianProcess(theta0=1e-2, thetaL=1e-4, thetaU=1e-1)
gp.fit(X, y)
```
Fitting Noisy Data

When the data to be fit includes noise, the Gaussian process model can be used by specifying the variance of the noise for each point. `GaussianProcess` takes a parameter `nugget` which is added to the diagonal of the correlation matrix between training points: in general this is a type of Tikhonov regularization. In the special case of a squared-exponential correlation function, this normalization is equivalent to specifying a fractional variance in the input. That is

\[ \text{nugget}_i = \left( \frac{\sigma_i}{y_i} \right)^2 \]

With `nugget` and `corr` properly set, Gaussian Processes can be used to robustly recover an underlying function from noisy data:

Other examples
- Gaussian Processes classification example: exploiting the probabilistic output
Mathematical formulation

The initial assumption

Suppose one wants to model the output of a computer experiment, say a mathematical function:

\[
g : \mathbb{R}^{n_{\text{features}}} \rightarrow \mathbb{R} \\
X \mapsto y = g(X)
\]

GPML starts with the assumption that this function is a conditional sample path of a Gaussian process \( G \) which is additionally assumed to read as follows:

\[
G(X) = f(X)^T \beta + Z(X)
\]

where \( f(X)^T \beta \) is a linear regression model and \( Z(X) \) is a zero-mean Gaussian process with a fully stationary covariance function:

\[
C(X, X') = \sigma^2 R(\|X - X'\|)
\]

\( \sigma^2 \) being its variance and \( R \) being the correlation function which solely depends on the absolute relative distance between each sample, possibly featurewise (this is the stationarity assumption).

From this basic formulation, note that GPML is nothing but an extension of a basic least squares linear regression problem:

\[
g(X) \approx f(X)^T \beta
\]

Except we additionally assume some spatial coherence (correlation) between the samples dictated by the correlation function. Indeed, ordinary least squares assumes the correlation model \( R(\|X - X'\|) \) is one when \( X = X' \) and zero otherwise: a dirac correlation model – sometimes referred to as a nugget correlation model in the kriging literature.

The best linear unbiased prediction (BLUP)

We now derive the best linear unbiased prediction of the sample path \( g \) conditioned on the observations:

\[
\hat{G}(X) = G(X|y_1 = g(X_1), \ldots, y_{n_{\text{samples}}} = g(X_{n_{\text{samples}}}))
\]

It is derived from its given properties:

- It is linear (a linear combination of the observations)

\[
\hat{G}(X) = a(X)^T y
\]

- It is unbiased

\[
\mathbb{E}[G(X) - \hat{G}(X)] = 0
\]

- It is the best (in the Mean Squared Error sense)

\[
\hat{G}(X)^* = \arg \min_{G(X)} \mathbb{E}[(G(X) - \hat{G}(X))^2]
\]

So that the optimal weight vector \( a(X) \) is solution of the following equality constrained optimization problem:

\[
a(X)^* = \arg \min_{a(X)} \mathbb{E}[(G(X) - a(X)^T y)^2] \\
s.t. \mathbb{E}[G(X) - a(X)^T y] = 0
\]
Rewriting this constrained optimization problem in the form of a Lagrangian and looking further for the first order optimality conditions to be satisfied, one ends up with a closed form expression for the sought predictor – see references for the complete proof.

In the end, the BLUP is shown to be a Gaussian random variate with mean:

$$\mu_{\hat{Y}}(X) = f(X)^T \hat{\beta} + r(X)^T \gamma$$

and variance:

$$\sigma^2_{\hat{Y}}(X) = \sigma^2_Y (1 - r(X)^T R^{-1} r(X) + u(X)^T (F^T R^{-1} F)^{-1} u(X))$$

where we have introduced:

- the correlation matrix whose terms are defined wrt the autocorrelation function and its built-in parameters $\theta$:
  $$R_{ij} = R(|X_i - X_j|, \theta), \ i, j = 1, ..., m$$

- the vector of cross-correlations between the point where the prediction is made and the points in the DOE:
  $$r_i = R(|X - X_i|, \theta), \ i = 1, ..., m$$

- the regression matrix (eg the Vandermonde matrix if $f$ is a polynomial basis):
  $$F_{ij} = f_i(X_j), \ i = 1, ..., p, \ j = 1, ..., m$$

- the generalized least square regression weights:
  $$\hat{\beta} = (F^T R^{-1} F)^{-1} F^T R^{-1} Y$$

- and the vectors:
  $$\gamma = R^{-1} (Y - F \hat{\beta})$$
  $$u(X) = F^T R^{-1} r(X) - f(X)$$

It is important to notice that the probabilistic response of a Gaussian Process predictor is fully analytic and mostly relies on basic linear algebra operations. More precisely the mean prediction is the sum of two simple linear combinations (dot products), and the variance requires two matrix inversions, but the correlation matrix can be decomposed only once using a Cholesky decomposition algorithm.

The empirical best linear unbiased predictor (EBLUP)

Until now, both the autocorrelation and regression models were assumed given. In practice however they are never known in advance so that one has to make (motivated) empirical choices for these models Correlation Models.

Provided these choices are made, one should estimate the remaining unknown parameters involved in the BLUP. To do so, one uses the set of provided observations in conjunction with some inference technique. The present implementation, which is based on the DACE’s Matlab toolbox uses the maximum likelihood estimation technique – see DACE manual in references for the complete equations. This maximum likelihood estimation problem is turned into a global optimization problem onto the autocorrelation parameters. In the present implementation, this global optimization is solved by means of the fmin_cobyla optimization function from scipy.optimize. In the case of anisotropy however, we provide an implementation of Welch’s componentwise optimization algorithm – see references.

For a more comprehensive description of the theoretical aspects of Gaussian Processes for Machine Learning, please refer to the references below:
Correlation Models

Common correlation models match some famous SVM’s kernels because they are mostly built on equivalent assumptions. They must fulfill Mercer’s conditions and should additionally remain stationary. Note however, that the choice of the correlation model should be made in agreement with the known properties of the original experiment from which the observations come. For instance:

- If the original experiment is known to be infinitely differentiable (smooth), then one should use the squared-exponential correlation model.
- If it’s not, then one should rather use the exponential correlation model.
- Note also that there exists a correlation model that takes the degree of derivability as input: this is the Matern correlation model, but it’s not implemented here (TODO).

For a more detailed discussion on the selection of appropriate correlation models, see the book by Rasmussen & Williams in references.

Regression Models

Common linear regression models involve zero- (constant), first- and second-order polynomials. But one may specify its own in the form of a Python function that takes the features X as input and that returns a vector containing the values of the functional set. The only constraint is that the number of functions must not exceed the number of available observations so that the underlying regression problem is not underdetermined.

Implementation details

The present implementation is based on a translation of the DACE Matlab toolbox.

Partial Least Squares

Partial least squares (PLS) models are useful to find linear relations between two multivariate datasets: in PLS the X and Y arguments of the fit method are 2D arrays.

PLS finds the fundamental relations between two matrices (X and Y): it is a latent variable approach to modeling the covariance structures in these two spaces. A PLS model will try to find the multidimensional direction in the X
space that explains the maximum multidimensional variance direction in the Y space. PLS-regression is particularly suited when the matrix of predictors has more variables than observations, and when there is multicollinearity among X values. By contrast, standard regression will fail in these cases.

Classes included in this module are **PLSRegression**, **PLSCanonical**, **CCA** and **PLSSVD**

Reference:
- JA Wegelin *A survey of Partial Least Squares (PLS) methods, with emphasis on the two-block case*

Examples:
- *PLS Partial Least Squares*

### 1.3.7 Naive Bayes

Naive Bayes methods are a set of supervised learning algorithms based on applying Bayes’ theorem with the “naive” assumption of independence between every pair of features. Given a class variable $y$ and a dependent feature vector $x_1$ through $x_n$, Bayes’ theorem states the following relationship:

$$P(y \mid x_1, \ldots, x_n) = \frac{P(y)P(x_1, \ldots, x_n \mid y)}{P(x_1, \ldots, x_n)}$$

Using the naive independence assumption that

$$P(x_i \mid y, x_1, \ldots, x_{i-1}, x_{i+1}, \ldots, x_n) = P(x_i \mid y),$$
for all $i$, this relationship is simplified to

$$P(y | x_1, \ldots, x_n) = \frac{P(y) \prod_{i=1}^{n} P(x_i | y)}{P(x_1, \ldots, x_n)}$$

Since $P(x_1, \ldots, x_n)$ is constant given the input, we can use the following classification rule:

$$P(y | x_1, \ldots, x_n) \propto P(y) \prod_{i=1}^{n} P(x_i | y)$$

$$\downarrow$$

$$\hat{y} = \arg\max_y P(y) \prod_{i=1}^{n} P(x_i | y),$$

and we can use Maximum A Posteriori (MAP) estimation to estimate $P(y)$ and $P(x_i | y)$; the former is then the relative frequency of class $y$ in the training set.

The different Naive Bayes classifiers differ mainly by the assumptions they make regarding the distribution of $P(x_i | y)$.

In spite of their apparently over-simplified assumptions, Naive Bayes classifiers have worked quite well in many real-world situations, famously document classification and spam filtering. They require a small amount of training data to estimate the necessary parameters. (For theoretical reasons why Naive Bayes works well, and on which types of data it does, see the references below.)

Naive Bayes learners and classifiers can be extremely fast compared to more sophisticated methods. The decoupling of the class conditional feature distributions means that each distribution can be independently estimated as a one dimensional distribution. This in turn helps to alleviate problems stemming from the curse of dimensionality.

On the flip side, although Naive Bayes is known as a decent classifier, it is known to be a bad estimator, so the probability outputs from `predict_proba` are not to be taken too seriously.

**References:**


**Gaussian Naive Bayes**

`GaussianNB` implements the Gaussian Naive Bayes algorithm for classification. The likelihood of the features is assumed to be Gaussian:

$$P(x_i | y) = \frac{1}{\sqrt{2\pi \sigma_y^2}} \exp\left(-\frac{(x_i - \mu_y)^2}{2 \sigma_y^2}\right)$$

The parameters $\sigma_y$ and $\mu_y$ are estimated using maximum likelihood.

```python
>>> from sklearn import datasets
>>> iris = datasets.load_iris()
>>> from sklearn.naive_bayes import GaussianNB
>>> gnb = GaussianNB()
>>> y_pred = gnb.fit(iris.data, iris.target).predict(iris.data)
>>> print "Number of mislabeled points : %d" % (iris.target != y_pred).sum()
Number of mislabeled points : 6
```
Multinomial Naive Bayes

MultinomialNB implements the Naive Bayes algorithm for multinomially distributed data, and is one of the two classic Naive Bayes variants used in text classification (where the data are typically represented as word vector counts, although tf-idf vectors are also known to work well in practice). The distribution is parametrized by vectors \( \theta_y = (\theta_{y1}, \ldots, \theta_{yn}) \) for each class \( y \), where \( n \) is the number of features (in text classification, the size of the vocabulary) and \( \theta_{yi} \) is the probability \( P(x_i \mid y) \) of feature \( i \) appearing in a sample belonging to class \( y \).

The parameters \( \theta_y \) is estimated by a smoothed version of maximum likelihood, i.e. relative frequency counting:

\[
\hat{\theta}_{yi} = \frac{N_{yi} + \alpha}{N_y + \alpha n}
\]

where \( N_{yi} = \sum_{x \in T} x_i \) is the number of times feature \( i \) appears in a sample of class \( y \) in the training set \( T \), and \( N_y = \sum_{i=1}^{\#T} N_{yi} \) is the total count of all features for class \( y \).

The smoothing priors \( \alpha \geq 0 \) accounts for features not present in the learning samples and prevents zero probabilities in further computations. Setting \( \alpha = 1 \) is called Laplace smoothing, while \( \alpha < 1 \) is called Lidstone smoothing.

Bernoulli Naive Bayes

BernoulliNB implements the Naive Bayes training and classification algorithms for data that is distributed according to multivariate Bernoulli distributions; i.e., there may be multiple features but each one is assumed to be a binary-valued (Bernoulli, boolean) variable. Therefore, this class requires samples to be represented as binary-valued feature vectors; if handed any other kind of data, a BernoulliNB instance may binarizes its input (depending on the binarize parameter).

The decision rule for Bernoulli Naive Bayes is based on

\[
P(x_i \mid y) = P(i \mid y) x_i \times (1 - P(i \mid y))(1 - x_i)
\]

which differs from multinomial NB’s rule in that it explicitly penalizes the non-occurrence of a feature \( i \) that is an indicator for class \( y \), where the multinomial variant would simply ignore a non-occurring feature.

In the case of text classification, word occurrence vectors (rather than word count vectors) may be used to train and use this classifier. BernoulliNB might perform better on some datasets, especially those with shorter documents. It is advisable to evaluate both models, if time permits.

References:


1.3.8 Decision Trees

Decision Trees (DTs) are a non-parametric supervised learning method used for classification and regression. The goal is to create a model that predicts the value of a target variable by learning simple decision rules inferred from the data features.

For instance, in the example below, decision trees learn from data to approximate a sine curve with a set of if-then-else decision rules. The deeper the tree, the more complex the decision rules and the fitter the model.
Some advantages of decision trees are:

- Simple to understand and to interpret. Trees can be visualised.
- Requires little data preparation. Other techniques often require data normalisation, dummy variables need to be created and blank values to be removed. Note however that this module does not support missing values.
- The cost of using the tree (i.e., predicting data) is logarithmic in the number of data points used to train the tree.
- Able to handle both numerical and categorical data. Other techniques are usually specialised in analysing datasets that have only one type of variable. See algorithms for more information.
- Uses a white box model. If a given situation is observable in a model, the explanation for the condition is easily explained by boolean logic. By contrast, in a black box model (e.g., in an artificial neural network), results may be more difficult to interpret.
- Possible to validate a model using statistical tests. That makes it possible to account for the reliability of the model.
- Performs well even if its assumptions are somewhat violated by the true model from which the data were generated.

The disadvantages of decision trees include:

- Decision-tree learners can create over-complex trees that do not generalise the data well. This is called overfitting. Mechanisms such as pruning (not currently supported), setting the minimum number of samples required at a leaf node or setting the maximum depth of the tree are necessary to avoid this problem.
- Decision trees can be unstable because small variations in the data might result in a completely different tree being generated. This problem is mitigated by using decision trees within an ensemble.
- The problem of learning an optimal decision tree is known to be NP-complete under several aspects of optimality and even for simple concepts. Consequently, practical decision-tree learning algorithms are based on heuristic algorithms such as the greedy algorithm where locally optimal decisions are made at each node. Such algorithms cannot guarantee to return the globally optimal decision tree. This can be mitigated by training multiple trees in an ensemble learner, where the features and samples are randomly sampled with replacement.
• There are concepts that are hard to learn because decision trees do not express them easily, such as XOR, parity or multiplexer problems.

• Decision tree learners create biased trees if some classes dominate. It is therefore recommended to balance the dataset prior to fitting with the decision tree.

Classification

DecisionTreeClassifier is a class capable of performing multi-class classification on a dataset.

As other classifiers, DecisionTreeClassifier take as input two arrays: an array X of size [n_samples, n_features] holding the training samples, and an array Y of integer values, size [n_samples], holding the class labels for the training samples:

```python
>>> from sklearn import tree
>>> X = [[0, 0], [1, 1]]
>>> Y = [0, 1]
>>> clf = tree.DecisionTreeClassifier()
>>> clf = clf.fit(X, Y)
```

After being fitted, the model can then be used to predict new values:

```python
>>> clf.predict([[2., 2.]])
array([1])
```

DecisionTreeClassifier is capable of both binary (where the labels are [-1, 1]) classification and multiclass (where the labels are [0, ..., K-1]) classification.

Using the Iris dataset, we can construct a tree as follows:

```python
>>> from sklearn.datasets import load_iris
>>> from sklearn import tree
>>> iris = load_iris()
>>> clf = tree.DecisionTreeClassifier()
>>> clf = clf.fit(iris.data, iris.target)
```

Once trained, we can export the tree in Graphviz format using the export_graphviz exporter. Below is an example export of a tree trained on the entire iris dataset:

```python
>>> from StringIO import StringIO
>>> out = StringIO()
>>> out = tree.export_graphviz(clf, out_file=out)
```

After being fitted, the model can then be used to predict new values:

```python
>>> clf.predict(iris.data[0, :])
array([0])
```

Examples:

• Plot the decision surface of a decision tree on the iris dataset

Regression

Decision trees can also be applied to regression problems, using the DecisionTreeRegressor class.
petal length (cm) <= 2.45000004768
error = 0.666666686535
samples = 150
value = [ 50.  50.  50.]

error = 0.6
samples = 15
value = [ 6.  8.  6.]

error = 0.0
samples = 1
value = [ 0.  1.  0.]

petal width (cm) <= 1.75
error = 0.5
samples = 100
value = [  0.  50.  50.]

petal length (cm) <= 4.94999980927
error = 0.168038412929
samples = 54
value = [  0.  49.   5.]

petal length (cm) <= 4.85000038147
error = 0.0425330810249
samples = 46
value = [  0.   1.  45.]

petal width (cm) <= 1.65000009537
error = 0.040798611939
samples = 48
value = [  0.  47.   1.]

petal width (cm) <= 1.54999995232
error = 0.444444447756
samples = 6
value = [ 0.  2.  4.]

sepal length (cm) <= 5.94999980927
error = 0.444444447756
samples = 3
value = [ 0.  1.  2.]

error = 0.0
samples = 43
value = [  0.   0.  43.]

error = 0.0
samples = 47
value = [  0.  47.   0.]

error = 0.0
samples = 1
value = [ 0.  0.  1.]

error = 0.0
samples = 3
value = [ 0.  0.  3.]

error = 0.0
samples = 2
value = [ 0.  0.  2.]

error = 0.0
samples = 1
value = [ 0.  1.  0.]

Decision surface of a decision tree using paired features
As in the classification setting, the fit method will take as argument arrays X and y, only that in this case y is expected to have floating point values instead of integer values:

```python
>>> from sklearn import tree

>>> X = [[0, 0], [2, 2]]
>>> y = [0.5, 2.5]
>>> clf = tree.DecisionTreeRegressor()
>>> clf = clf.fit(X, y)
>>> clf.predict([[1, 1]])
array([ 0.5])
```

Examples:

- **Decision Tree Regression**

Complexity

In general, the run time cost to construct a balanced binary tree is $O(n_{\text{samples}} n_{\text{features}} \log(n_{\text{samples}}))$ and query time $O(\log(n_{\text{samples}}))$. Although the tree construction algorithm attempts to generate balanced trees, they will not always be balanced. Assuming that the subtrees remain approximately balanced, the cost at each node consists of searching through $O(n_{\text{features}})$ to find the feature that offers the largest reduction in entropy. This has a cost of $O(n_{\text{features}} n_{\text{samples}} \log(n_{\text{samples}}))$ at each node, leading to a total cost over the entire trees (by summing the cost at each node) of $O(n_{\text{features}} n_{\text{samples}}^2 \log(n_{\text{samples}}))$.

Scikit-learn offers a more efficient implementation for the construction of decision trees. A naive implementation (as above) would recompute the class label histograms (for classification) or the means (for regression) at each new split point along a given feature. By presorting the feature over all relevant samples, and retaining a running label count, we reduce the complexity at each node to $O(n_{\text{features}} \log(n_{\text{samples}}))$, which results in a total cost of $O(n_{\text{features}} n_{\text{samples}} \log(n_{\text{samples}}))$.  

1.3. Supervised learning
This implementation also offers a parameter \texttt{min\_density} to control an optimization heuristic. A sample mask is used to mask data points that are inactive at a given node, which avoids the copying of data (important for large datasets or training trees within an ensemble). Density is defined as the ratio of ‘active’ data samples to total samples at a given node. The minimum density parameter specifies the level below which fancy indexing (and therefore data copied) and the sample mask reset. If \texttt{min\_density} is 1, then fancy indexing is always used for data partitioning during the tree building phase. In this case, the size of memory (as a proportion of the input data \( a \)) required at a node of depth \( n \) can be approximated using a geometric series: \[
\text{size} = a \frac{1-r^n}{1-r},
\] where \( r \) is the ratio of samples used at each node. A best case analysis shows that the lowest memory requirement (for an infinitely deep tree) is \( 2 \times a \), where each partition divides the data in half. A worst case analysis shows that the memory requirement can increase to \( n \times a \). In practice it usually requires 3 to 4 times \( a \). Setting \texttt{min\_density} to 0 will always use the sample mask to select the subset of samples at each node. This results in little to no additional memory being allocated, making it appropriate for massive datasets or within ensemble learners. The default value for \texttt{min\_density} is 0.1 which empirically leads to fast training for many problems. Typically high values of \texttt{min\_density} will lead to excessive reallocation, slowing down the algorithm significantly.

**Tips on practical use**

- Decision trees tend to overfit on data with a large number of features. Getting the right ratio of samples to number of features is important, since a tree with few samples in high dimensional space is very likely to overfit.

- Consider performing dimensionality reduction (\texttt{PCA, ICA, or Feature selection}) beforehand to give your tree a better chance of finding features that are discriminative.

- Visualise your tree as you are training by using the \texttt{export} function. Use \texttt{max\_depth=3} as an initial tree depth to get a feel for how the tree is fitting to your data, and then increase the depth.

- Remember that the number of samples required to populate the tree doubles for each additional level the tree grows to. Use \texttt{max\_depth} to control the size of the tree to prevent overfitting.

- Use \texttt{min\_samples\_split} or \texttt{min\_samples\_leaf} to control the number of samples at a leaf node. A very small number will usually mean the tree will overfit, whereas a large number will prevent the tree from learning the data. Try \texttt{min\_samples\_leaf=5} as an initial value. The main difference between the two is that \texttt{min\_samples\_leaf} guarantees a minimum number of samples in a leaf, while \texttt{min\_samples\_split} can create arbitrary small leaves, though \texttt{min\_samples\_split} is more common in the literature.

- Balance your dataset before training to prevent the tree from creating a tree biased toward the classes that are dominant.

- All decision trees use Fortran ordered \texttt{np.float32} arrays internally. If training data is not in this format, a copy of the dataset will be made.

**Tree algorithms: ID3, C4.5, C5.0 and CART**

What are all the various decision tree algorithms and how do they differ from each other? Which one is implemented in scikit-learn?

\texttt{ID3} (Iterative Dichotomiser 3) was developed in 1986 by Ross Quinlan. The algorithm creates a multiway tree, finding for each node (i.e. in a greedy manner) the categorical feature that will yield the largest information gain for categorical targets. Trees are grown to their maximum size and then a pruning step is usually applied to improve the ability of the tree to generalise to unseen data.

\texttt{C4.5} is the successor to \texttt{ID3} and removed the restriction that features must be categorical by dynamically defining a discrete attribute (based on numerical variables) that partitions the continuous attribute value into a discrete set of intervals. \texttt{C4.5} converts the trained trees (i.e. the output of the \texttt{ID3} algorithm) into sets of if-then rules. These accuracy of each rule is then evaluated to determine the order in which they should be applied. Pruning is done by removing a rule’s precondition if the accuracy of the rule improves without it.
C5.0 is Quinlan’s latest version release under a proprietary license. It uses less memory and builds smaller rulesets than C4.5 while being more accurate.

CART (Classification and Regression Trees) is very similar to C4.5, but it differs in that it supports numerical target variables (regression) and does not compute rule sets. CART constructs binary trees using the feature and threshold that yield the largest information gain at each node.

scikit-learn uses an optimised version of the CART algorithm.

**Mathematical formulation**

Given training vectors \( x_i \in R^n, i = 1, \ldots, l \) and a label vector \( y \in R^l \), a decision tree recursively partitions the space such that the samples with the same labels are grouped together.

Let the data at node \( m \) be represented by \( Q \). For each candidate split \( \theta = (j, t_m) \) consisting of a feature \( j \) and threshold \( t_m \), partition the data into \( Q_{left}(\theta) \) and \( Q_{right}(\theta) \) subsets

\[
Q_{left}(\theta) = (x, y) | x_j \leq t_m \\
Q_{right}(\theta) = Q \setminus Q_{left}(\theta)
\]

The impurity at \( m \) is computed using an impurity function \( H() \), the choice of which depends on the task being solved (classification or regression)

\[
G(Q, \theta) = \frac{n_{left}}{N_m} H(Q_{left}(\theta)) + \frac{n_{right}}{N_m} H(Q_{right}(\theta))
\]

Select the parameters that minimise the impurity

\[
\theta^* = \text{argmin}_\theta G(Q, \theta)
\]

Recurse for subsets \( Q_{left}(\theta^*) \) and \( Q_{right}(\theta^*) \) until the maximum allowable depth is reached, \( N_m < \text{min}_\text{samples} \) or \( N_m = 1 \).

**Classification criteria**

If a target is a classification outcome taking on values 0,1,...,K-1, for node \( m \), representing a region \( R_m \) with \( N_m \) observations, let

\[
p_{mk} = 1/N_m \sum_{x_i \in R_m} I(y_i = k)
\]

be the proportion of class \( k \) observations in node \( m \)

Common measures of impurity are Gini

\[
H(X_m) = \sum_k p_{mk}(1 - p_{mk})
\]

Cross-Entropy

\[
H(X_m) = \sum_k p_{mk}\log(p_{mk})
\]

and Misclassification

\[
H(X_m) = 1 - \max(p_{mk})
\]
Regression criteria

If the target is a continuous value, then for node \( m \), representing a region \( R_m \) with \( N_m \) observations, a common criterion to minimise is the Mean Squared Error

\[
c_m = \frac{1}{N_m} \sum_{i \in N_m} y_i
\]

\[
H(X_m) = \frac{1}{N_m} \sum_{i \in N_m} (y_i - c_m)^2
\]

References:

- http://en.wikipedia.org/wiki/Predictive_analytics

1.3.9 Ensemble methods

The goal of ensemble methods is to combine the predictions of several models built with a given learning algorithm in order to improve generalizability / robustness over a single model.

Two families of ensemble methods are usually distinguished:

- In averaging methods, the driving principle is to build several models independently and then to average their predictions. On average, the combined model is usually better than any of the single model because its variance is reduced.
  
  Examples: Bagging methods, Forests of randomized trees...

- By contrast, in boosting methods, models are built sequentially and one tries to reduce the bias of the combined model. The motivation is to combine several weak models to produce a powerful ensemble.
  
  Examples: AdaBoost, Least Squares Boosting, Gradient Tree Boosting, ...

Forests of randomized trees

The sklearn.ensemble module includes two averaging algorithms based on randomized decision trees: the RandomForest algorithm and the Extra-Trees method. Both algorithms are perturb-and-combine techniques [B1998] specifically designed for trees. This means a diverse set of classifiers is created by introducing randomness in the classifier construction. The prediction of the ensemble is given as the averaged prediction of the individual classifiers.

As other classifiers, forest classifiers have to be fitted with two arrays: an array \( X \) of size \([n\_samples, n\_features]\) holding the training samples, and an array \( Y \) of size \([n\_samples]\) holding the target values (class labels) for the training samples:

```python
>>> from sklearn.ensemble import RandomForestClassifier
>>> X = [[0, 0], [1, 1]]
>>> Y = [0, 1]
>>> clf = RandomForestClassifier(n_estimators=10)
>>> clf = clf.fit(X, Y)
```
Random Forests

In random forests (see RandomForestClassifier and RandomForestRegressor classes), each tree in the ensemble is built from a sample drawn with replacement (i.e., a bootstrap sample) from the training set. In addition, when splitting a node during the construction of the tree, the split that is chosen is no longer the best split among all features. Instead, the split that is picked is the best split among a random subset of the features. As a result of this randomness, the bias of the forest usually slightly increases (with respect to the bias of a single non-random tree) but, due to averaging, its variance also decreases, usually more than compensating for the increase in bias, hence yielding an overall better model.

In contrast to the original publication [B2001], the scikit-learn implementation combines classifiers by averaging their probabilistic prediction, instead of letting each classifier vote for a single class.

Extremely Randomized Trees

In extremely randomized trees (see ExtraTreesClassifier and ExtraTreesRegressor classes), randomness goes one step further in the way splits are computed. As in random forests, a random subset of candidate features is used, but instead of looking for the most discriminative thresholds, thresholds are drawn at random for each candidate feature and the best of these randomly-generated thresholds is picked as the splitting rule. This usually allows to reduce the variance of the model a bit more, at the expense of a slightly greater increase in bias:

```python
>>> from sklearn.cross_validation import cross_val_score
>>> from sklearn.datasets import make_blobs
>>> from sklearn.ensemble import RandomForestClassifier
>>> from sklearn.tree import DecisionTreeClassifier

>>> X, y = make_blobs(n_samples=10000, n_features=10, centers=100,
...                   random_state=0)

>>> clf = DecisionTreeClassifier(max_depth=None, min_samples_split=1,
...                               random_state=0)
>>> scores = cross_val_score(clf, X, y)
>>> scores.mean()
0.978...

>>> clf = RandomForestClassifier(n_estimators=10, max_depth=None,
...                               min_samples_split=1, random_state=0)
>>> scores = cross_val_score(clf, X, y)
>>> scores.mean()
0.999...

>>> clf = ExtraTreesClassifier(n_estimators=10, max_depth=None,
...                            min_samples_split=1, random_state=0)
>>> scores = cross_val_score(clf, X, y)
>>> scores.mean() > 0.999
True
```

Parameters

The main parameters to adjust when using these methods is n_estimators and max_features. The former is the number of trees in the forest. The larger the better, but also the longer it will take to compute. In addition, note that results will stop getting significantly better beyond a critical number of trees. The latter is the size of the random subsets of features to consider when splitting a node. The lower the greater the reduction of variance, but also the greater the increase in bias. Empiricial good default values are max_features=n_features for regression
problems, and max_features=sqrt(n_features) for classification tasks (where n_features is the number of features in the data). The best results are also usually reached when setting max_depth=None in combination with min_samples_split=1 (i.e., when fully developing the trees). Bear in mind though that these values are usually not optimal. The best parameter values should always be cross-validated. In addition, note that bootstrap samples are used by default in random forests (bootstrap=True) while the default strategy is to use the original dataset for building extra-trees (bootstrap=False).

When training on large datasets, where runtime and memory requirements are important, it might also be beneficial to adjust the min_density parameter, that controls a heuristic for speeding up computations in each tree. See Complexity of trees for details.

Parallelization

Finally, this module also features the parallel construction of the trees and the parallel computation of the predictions through the n_jobs parameter. If n_jobs=k then computations are partitioned into k jobs, and run on k cores of the machine. If n_jobs=-1 then all cores available on the machine are used. Note that because of inter-process communication overhead, the speedup might not be linear (i.e., using k jobs will unfortunately not be k times as fast). Significant speedup can still be achieved though when building a large number of trees, or when building a single tree requires a fair amount of time (e.g., on large datasets).

Examples:

- Plot the decision surfaces of ensembles of trees on the iris dataset
- Pixel importances with a parallel forest of trees

References
Gradient Tree Boosting

Gradient Tree Boosting or Gradient Boosted Regression Trees (GBRT) is a generalization of boosting to arbitrary differentiable loss functions. GBRT is an accurate and effective off-the-shelf procedure that can be used for both regression and classification problems. Gradient Tree Boosting models are used in a variety of areas including Web search ranking and ecology.

The advantages of GBRT are:

• Natural handling of data of mixed type (= heterogeneous features)
• Predictive power
• Robustness to outliers in input space (via robust loss functions)

The disadvantages of GBRT are:

• Scalability, due to the sequential nature of boosting it can hardly be parallelized.

The module `sklearn.ensemble` provides methods for both classification and regression via gradient boosted regression trees.

Classification

`GradientBoostingClassifier` supports both binary and multi-class classification via the deviance loss function (`loss='deviance'`). The following example shows how to fit a gradient boosting classifier with 100 decision stumps as weak learners:

```python
>>> from sklearn.datasets import make_hastie_10_2
>>> from sklearn.ensemble import GradientBoostingClassifier

>>> X, y = make_hastie_10_2(random_state=0)
>>> X_train, X_test = X[:2000], X[2000:]
>>> y_train, y_test = y[:2000], y[2000:]

>>> clf = GradientBoostingClassifier(n_estimators=100, learn_rate=1.0, ...
... max_depth=1, random_state=0).fit(X_train, y_train)

>>> clf.score(X_test, y_test)
0.913...
```

The number of weak learners (i.e. regression trees) is controlled by the parameter `n_estimators`; The maximum depth of each tree is controlled via `max_depth`. The `learn_rate` is a hyper-parameter in the range (0.0, 1.0] that controls overfitting via `shrinkage`.

**Note:** Classification with more than 2 classes requires the induction of `n_classes` regression trees at each at each iteration, thus, the total number of induced trees equals `n_classes * n_estimators`. For datasets with a large number of classes we strongly recommend to use `RandomForestClassifier` as an alternative to GBRT.

Regression

`GradientBoostingRegressor` supports a number of different loss functions for regression which can be specified via the argument `loss`. Currently, supported are least squares (`loss='ls'`) and least absolute deviation (`loss='lad'`), which is more robust w.r.t. outliers. See [F2001] for detailed information.
```python
>>> import numpy as np
>>> from sklearn.metrics import mean_squared_error
>>> from sklearn.datasets import make_friedman1
>>> from sklearn.ensemble import GradientBoostingRegressor

>>> X, y = make_friedman1(n_samples=1200, random_state=0, noise=1.0)
>>> X_train, X_test = X[:200], X[200:]
>>> y_train, y_test = y[:200], y[200:]
>>> clf = GradientBoostingRegressor(n_estimators=100, learn_rate=1.0,
... max_depth=1, random_state=0, loss='ls').fit(X_train, y_train)
>>> mean_squared_error(y_test, clf.predict(X_test))
6.90...
```

The figure below shows the results of applying `GradientBoostingRegressor` with least squares loss and 500 base learners to the Boston house-price dataset (see `sklearn.datasets.load_boston`). The plot on the left shows the train and test error at each iteration. Plots like these are often used for early stopping. The plot on the right shows the feature importances which can be obtained via the `feature_importance` property.

### Mathematical formulation

GBRT considers additive models of the following form:

$$F(x) = \sum_{m=1}^{M} \gamma_m h_m(x)$$

where $h_m(x)$ are the basis functions which are usually called weak learners in the context of boosting. Gradient Tree Boosting uses decision trees of fixed size as weak learners. Decision trees have a number of abilities that make them valuable for boosting, namely the ability to handle data of mixed type and the ability to model complex functions.

Similar to other boosting algorithms GBRT builds the additive model in a forward stagewise fashion:

$$F_m(x) = F_{m-1}(x) + \gamma_m h_m(x)$$

At each stage the decision tree $h_m(x)$ is chosen that minimizes the loss function $L$ given the current model $F_{m-1}$ and its fit $F_{m-1}(x_i)$.
\[ F_m(x) = F_{m-1}(x) + \arg \min_h \sum_{i=1}^{n} L(y_i, F_{m-1}(x_i) - h(x)) \]

The initial model \( F_0 \) is problem specific, for least-squares regression one usually chooses the mean of the target values.

**Note:** The initial model can also be specified via the `init` argument. The passed object has to implement `fit` and `predict`.

Gradient Boosting attempts to solve this minimization problem numerically via steepest descent: The steepest descent direction is the negative gradient of the loss function evaluated at the current model \( F_{m-1} \) which can be calculated for any differentiable loss function:

\[ F_m(x) = F_{m-1}(x) + \gamma_m \sum_{i=1}^{n} \nabla F L(y_i, F_{m-1}(x_i)) \]

Where the step length \( \gamma_m \) is chosen using line search:

\[ \gamma_m = \arg \min_\gamma \sum_{i=1}^{n} L(y_i, F_{m-1}(x_i)) - \gamma \frac{\partial L(y_i, F_{m-1}(x_i))}{\partial F_{m-1}(x_i)} \]

The algorithms for regression and classification only differ in the concrete loss function used.

**Loss Functions** The following loss functions are supported and can be specified using the parameter `loss`:

- **Regression**
  - Least squares (`'ls'`): The natural choice for regression due to its superior computational properties. The initial model is given by the mean of the target values.
  - Least absolute deviation (`'lad'`): A robust loss function for regression. The initial model is given by the median of the target values.

- **Classification**
  - Binomial deviance (`'deviance'`): The negative binomial log-likelihood loss function for binary classification (provides probability estimates). The initial model is given by the log odds-ratio.
  - Multinomial deviance (`'deviance'`): The negative multinomial log-likelihood loss function for multi-class classification with \( n \_classes \) mutually exclusive classes. It provides probability estimates. The initial model is given by the prior probability of each class. At each iteration \( n \_classes \) regression trees have to be constructed which makes GBRT rather inefficient for data sets with a large number of classes.

**Regularization**

**Shrinkage** [F2001] proposed a simple regularization strategy that scales the contribution of each weak learner by a factor \( \nu \):

\[ F_m(x) = F_{m-1}(x) + \nu \gamma_m h_m(x) \]
The parameter $\nu$ is also called the **learning rate** because it scales the step length of the gradient descent procedure; it can be set via the learn_rate parameter.

The parameter learn_rate strongly interacts with the parameter n_estimators, the number of weak learners to fit. Smaller values of learn_rate require larger numbers of weak learners to maintain a constant training error. Empirical evidence suggests that small values of learn_rate favor better test error. [HTF2009] recommend to set the learning rate to a small constant (e.g. learn_rate <= 0.1) and choose n_estimators by early stopping. For a more detailed discussion of the interaction between learn_rate and n_estimators see [R2007].

**Subsampling** [F1999] proposed stochastic gradient boosting, which combines gradient boosting with bootstrap averaging (bagging). At each iteration the base classifier is trained on a fraction subsample of the available training data. The subsample is drawn without replacement. A typical value of subsample is 0.5.

The figure below illustrates the effect of shrinkage and subsampling on the goodness-of-fit of the model. We can clearly see that shrinkage outperforms no-shrinkage. Subsampling with shrinkage can further increase the accuracy of the model. Subsampling without shrinkage, on the other hand, does poorly.

![Graph showing the effect of shrinkage and subsampling on deviance](image)

**Examples:**
- Gradient Boosting regression
- Gradient Boosting regularization

**References**

### 1.3.10 Multiclass and multilabel algorithms

This module implements multiclass and multilabel learning algorithms:
Multiclass classification means classification with more than two classes. Multilabel classification is a different task, where a classifier is used to predict a set of target labels for each instance; i.e., the set of target classes is not assumed to be disjoint as in ordinary (binary or multiclass) classification. This is also called any-of classification.

The estimators provided in this module are meta-estimators: they require a base estimator to be provided in their constructor. For example, it is possible to use these estimators to turn a binary classifier or a regressor into a multiclass classifier. It is also possible to use these estimators with multiclass estimators in the hope that their accuracy or runtime performance improves.

Note: You don’t need to use these estimators unless you want to experiment with different multiclass strategies: all classifiers in scikit-learn support multiclass classification out-of-the-box. Below is a summary of the classifiers supported in scikit-learn grouped by the strategy used.

- Inherently multiclass: Naive Bayes, sklearn.lda.LDA, Decision Trees, Random Forests
- One-Vs-One: sklearn.svm.SVC.

Note: At the moment there are no evaluation metrics implemented for multilabel learnings.

One-Vs-The-Rest

This strategy, also known as one-vs-all, is implemented in OneVsRestClassifier. The strategy consists in fitting one classifier per class. For each classifier, the class is fitted against all the other classes. In addition to its computational efficiency (only \( n \) classes classifiers are needed), one advantage of this approach is its interpretability. Since each class is represented by one and one classifier only, it is possible to gain knowledge about the class by inspecting its corresponding classifier. This is the most commonly used strategy and is a fair default choice. Below is an example:

```python
>>> from sklearn import datasets
>>> from sklearn.multiclass import OneVsRestClassifier
>>> from sklearn.svm import LinearSVC

>>> iris = datasets.load_iris()

>>> X, y = iris.data, iris.target

>>> OneVsRestClassifier(LinearSVC()).fit(X, y).predict(X)
```

Multilabel learning with OvR

OneVsRestClassifier also supports multilabel classification. To use this feature, feed the classifier a list of tuples containing target labels, like in the example below.
Examples:

- **Multilabel classification**

### One-Vs-One

OneVsOneClassifier constructs one classifier per pair of classes. At prediction time, the class which received the most votes is selected. Since it requires to fit $n_{classes} \times (n_{classes} - 1) / 2$ classifiers, this method is usually slower than one-vs-the-rest, due to its $O(n_{classes}^2)$ complexity. However, this method may be advantageous for algorithms such as kernel algorithms which don’t scale well with $n_{samples}$. This is because each individual learning problem only involves a small subset of the data whereas, with one-vs-the-rest, the complete dataset is used $n_{classes}$ times. Below is an example:

```python
>>> from sklearn import datasets
>>> from sklearn.multiclass import OneVsOneClassifier
>>> from sklearn.svm import LinearSVC

>>> iris = datasets.load_iris()
>>> X, y = iris.data, iris.target
>>> OneVsOneClassifier(LinearSVC()).fit(X, y).predict(X)
array([0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0])
```
Error-Correcting Output-Codes

Output-code based strategies are fairly different from one-vs-the-rest and one-vs-one. With these strategies, each class is represented in a euclidean space, where each dimension can only be 0 or 1. Another way to put it is that each class is represented by a binary code (an array of 0 and 1). The matrix which keeps track of the location/code of each class is called the code book. The code size is the dimensionality of the aforementioned space. Intuitively, each class should be represented by a code as unique as possible and a good code book should be designed to optimize classification accuracy. In this implementation, we simply use a randomly-generated code book as advocated in \(^2\) although more elaborate methods may be added in the future.

At fitting time, one binary classifier per bit in the code book is fitted. At prediction time, the classifiers are used to project new points in the class space and the class closest to the points is chosen.

In `OutputCodeClassifier`, the `code_size` attribute allows the user to control the number of classifiers which will be used. It is a percentage of the total number of classes.

A number between 0 and 1 will require fewer classifiers than one-vs-the-rest. In theory, \(\frac{\log_2(n_{classes})}{n_{classes}}\) is sufficient to represent each class unambiguously. However, in practice, it may not lead to good accuracy since \(\log_2(n_{classes})\) is much smaller than \(n_{classes}\).

A number greater than than 1 will require more classifiers than one-vs-the-rest. In this case, some classifiers will in theory correct for the mistakes made by other classifiers, hence the name “error-correcting”. In practice, however, this may not happen as classifier mistakes will typically be correlated. The error-correcting output codes have a similar effect to bagging.

Example:

```python
>>> from sklearn import datasets
>>> from sklearn.multiclass import OutputCodeClassifier
>>> from sklearn.svm import LinearSVC

>>> iris = datasets.load_iris()
>>> X, y = iris.data, iris.target

>>> OutputCodeClassifier(LinearSVC(), code_size=2, random_state=0).fit(X, y).predict(X)
array([0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
       0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
       0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
       0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
       0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
       0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0])
```

References:

1.3.11 Feature selection

The classes in the `sklearn.feature_selection` module can be used for feature selection/dimensionality reduction on sample sets, either to improve estimators’ accuracy scores or to boost their performance on very high-dimensional datasets.

Univariate feature selection

Univariate feature selection works by selecting the best features based on univariate statistical tests. It can seen as a preprocessing step to an estimator. Scikit-Learn exposes feature selection routines as objects that implement the

**transform** method:

- selecting the k-best features `SelectKBest`
- setting a percentile of features to keep `SelectPercentile`
- using common univariate statistical tests for each feature: false positive rate `SelectFpr`, false discovery rate `SelectFdr`, or family wise error `SelectFwe`.

These objects take as input a scoring function that returns univariate p-values:

- For regression: `f_regression`
- For classification: `chi2` or `f_classif`

**Feature selection with sparse data**

If you use sparse data (i.e. data represented as sparse matrices), only `chi2` will deal with the data without making it dense.

**Warning:** Beware not to use a regression scoring function with a classification problem, you will get useless results.

**Examples:**

*Univariate Feature Selection*

**Recursive feature elimination**

Given an external estimator that assigns weights to features (e.g., the coefficients of a linear model), recursive feature elimination (RFE) is to select features by recursively considering smaller and smaller sets of features. First, the estimator is trained on the initial set of features and weights are assigned to each one of them. Then, features whose absolute weights are the smallest are pruned from the current set features. That procedure is recursively repeated on the pruned set until the desired number of features to select is eventually reached.

**Examples:**

- *Recursive feature elimination*: A recursive feature elimination example showing the relevance of pixels in a digit classification task.
- *Recursive feature elimination with cross-validation*: A recursive feature elimination example with automatic tuning of the number of features selected with cross-validation.

**L1-based feature selection**

**Selecting non-zero coefficients**

*Linear models* penalized with the L1 norm have sparse solutions: many of their estimated coefficients are zero. When the goal is to reduce the dimensionality of the data to use with another classifier, they expose a *transform* method to select the non-zero coefficient. In particular, sparse estimators useful for this purpose are the `linear_model.Lasso` for regression, and of `linear_model.LogisticRegression` and `svm.LinearSVC` for classification:
>>> from sklearn.svm import LinearSVC
>>> from sklearn.datasets import load_iris
>>> iris = load_iris()
>>> X, y = iris.data, iris.target
>>> X.shape
(150, 4)
>>> X_new = LinearSVC(C=0.01, penalty="l1", dual=False).fit_transform(X, y)
>>> X_new.shape
(150, 3)

With SVMs and logistic-regression, the parameter $C$ controls the sparsity: the smaller $C$ the fewer features selected. With Lasso, the higher the alpha parameter, the fewer features selected.

Examples:

- Classification of text documents using sparse features: Comparison of different algorithms for document classification including L1-based feature selection.

L1-recovery and compressive sensing

For a good choice of alpha, the Lasso can fully recover the exact set of non-zero variables using only few observations, provided certain specific conditions are met. In particular, the number of samples should be “sufficiently large”, or L1 models will perform at random, where “sufficiently large” depends on the number of non-zero coefficients, the logarithm of the number of features, the amount of noise, the smallest absolute value of non-zero coefficients, and the structure of the design matrix $X$. In addition, the design matrix must display certain specific properties, such as not being too correlated.

There is no general rule to select an alpha parameter for recovery of non-zero coefficients. It can be set by cross-validation (LassoCV or LassoLarsCV), though this may lead to under-penalized models: including a small number of non-relevant variables is not detrimental to prediction score. BIC (LassoLarsIC) tends, on the opposite, to set high values of alpha.


Randomized sparse models

The limitation of L1-based sparse models is that faced with a group of very correlated features, they will select only one. To mitigate this problem, it is possible to use randomization techniques, reestimating the sparse model many times perturbing the design matrix or sub-sampling data and counting how many times a given regressor is selected.

RandomizedLasso implements this strategy for regression settings, using the Lasso, while RandomizedLogisticRegression uses the logistic regression and is suitable for classification tasks. To get a full path of stability scores you can use lasso_stability_path.

Note that for randomized sparse models to be more powerful than standard F statistics at detecting non-zero features, the ground truth model should be sparse, in other words, there should be only a small fraction of features non zero.

Examples:

- Sparse recovery: feature selection for sparse linear models: An example comparing different feature selection approaches and discussing in which situation each approach is to be favored.
Tree-based feature selection

Tree-based estimators (see the sklearn.tree module and forest of trees in the sklearn.ensemble module) can be used to compute feature importances, which in turn can be used to discard irrelevant features:

```python
>>> from sklearn.ensemble import ExtraTreesClassifier
>>> from sklearn.datasets import load_iris
>>> iris = load_iris()
>>> X, y = iris.data, iris.target
>>> X.shape
(150, 4)
>>> clf = ExtraTreesClassifier(compute_importances=True, random_state=0)
>>> X_new = clf.fit(X, y).transform(X)
>>> X_new.shape
(150, 2)
```

Examples:

- **Feature importances with forests of trees**: example on synthetic data showing the recovery of the actually meaningful features.
- **Pixel importances with a parallel forest of trees**: example on face recognition data.
1.3.12 Semi-Supervised

Semi-supervised learning is a situation in which in your training data some of the samples are not labeled. The semi-supervised estimators, in sklearn.semi_supervised are able to make use of this addition unlabeled data to capture better the shape of the underlying data distribution and generalize better to new samples. These algorithms can perform well when we have a very small amount of labeled points and a large amount of unlabeled points.

Unlabeled entries in y

It is important to assign an identifier to unlabeled points along with the labeled data when training the model with the fit method. The identifier that this implementation uses the integer value \(-1\).

Label Propagation

Label propagation denote a few variations of semi-supervised graph inference algorithms.

A few features available in this model:

- Can be used for classification and regression tasks
- Kernel methods to project data into alternate dimensional spaces

sklearn provides two label propagation models: LabelPropagation and LabelSpreading. Both work by constructing a similarity graph over all items in the input dataset.

LabelPropagation and LabelSpreading differ in modifications to the similarity matrix that graph and the clamping effect on the label distributions. Clamping allows the algorithm to change the weight of the true ground labeled data to some degree. The LabelPropagation algorithm performs hard clamping of input labels, which means \(\alpha = 1\). This clamping factor can be relaxed, to say \(\alpha = 0.8\), which means that we will always retain 80 percent of our original label distribution, but the algorithm gets to change it’s confidence of the distribution within 20 percent.

LabelPropagation uses the raw similarity matrix constructed from the data with no modifications. In contrast, LabelSpreading minimizes a loss function that has regularization properties, as such it is often more robust to noise. The algorithm iterates on a modified version of the original graph and normalizes the edge weights by computing the normalized graph Laplacian matrix. This procedure is also used in Spectral clustering.

Label propagation models have two built-in kernel methods. Choice of kernel effects both scalability and performance of the algorithms. The following are available:

- rbf \((\exp(-\gamma|x - y|^2), \gamma > 0)\). \(\gamma\) is specified by keyword gamma.
• \text{knn}(1[x' \in kNN(x)]). \( k \) is specified by keyword \text{n_neighbors}.

RBF kernel will produce a fully connected graph which is represented in memory by a dense matrix. This matrix may be very large and combined with the cost of performing a full matrix multiplication calculation for each iteration of the algorithm can lead to prohibitively long running times. On the other hand, the KNN kernel will produce a much more memory friendly sparse matrix which can drastically reduce running times.

\begin{itemize}
    \item Decision boundary of label propagation versus SVM on the Iris dataset
    \item Label Propagation learning a complex structure
    \item Decision boundary of label propagation versus SVM on the Iris dataset
    \item Label Propagation digits active learning
\end{itemize}

\section*{References}


\subsection*{1.3.13 Linear and Quadratic Discriminant Analysis}

Linear Discriminant Analysis (\texttt{lda.LDA}) and Quadratic Discriminant Analysis (\texttt{qda.QDA}) are two classic classifiers, with, as their names suggest, a linear and a quadratic decision surface, respectively.

These classifiers are attractive because they have closed form solutions that can be easily computed, are inherently multi-class, and have proven to work well in practice. Also there are no parameters to tune for these algorithms.
The plot shows decision boundaries for LDA and QDA. The bottom row demonstrates that LDA can only learn linear boundaries, while QDA can learn quadratic boundaries and is therefore more flexible.

**Examples:**

*Linear and Quadratic Discriminant Analysis with confidence ellipsoid:* Comparison of LDA and QDA on synthetic data.

**References:**

**Dimensionality Reduction using LDA**

`lda.LDA` can be used to perform supervised dimensionality reduction by projecting the input data to a subspace consisting of the most discriminant directions. This is implemented in `lda.LDA.transform`. The desired dimensionality can be set using the `n_components` constructor parameter. This parameter has no influence on `lda.LDA.fit` or `lda.LDA.predict`.

**Mathematical Idea**

Both methods work by modeling the class conditional distribution of the data $P(X|y = k)$ for each class $k$. Predictions can be obtained by using Bayes’ rule:

$$P(y|X) = P(X|y) \cdot P(y)/P(X) = P(X|y) \cdot P(Y)/\sum_{y'} P(X|y') \cdot p(y')$$

In linear and quadratic discriminant analysis, $P(X|y)$ is modeled as a Gaussian distribution. In the case of LDA, the Gaussians for each class are assumed to share the same covariance matrix. This leads to a linear decision surface, as can be seen by comparing the the log-probability ratios $\log[P(y=k|X)/P(y=l|X)]$.

In the case of QDA, there are no assumptions on the covariance matrices of the Gaussians, leading to a quadratic decision surface.

**1.4 Unsupervised learning**

**1.4.1 Gaussian mixture models**

`sklearn.mixture` is a package which enables one to learn Gaussian Mixture Models (diagonal, spherical, tied and full covariance matrices supported), sample them, and estimate them from data. Facilities to help determine the appropriate number of components are also provided.

A Gaussian mixture model is a probabilistic model that assumes all the data points are generated from a mixture of a finite number of Gaussian distributions with unknown parameters. One can think of mixture models as generalizing k-means clustering to incorporate information about the covariance structure of the data as well as the centers of the latent Gaussians.

The scikit-learn implements different classes to estimate Gaussian mixture models, that correspond to different estimation strategies, detailed below.
Figure 1.2: **Two-component Gaussian mixture model:** *data points, and equi-probability surfaces of the model.*

**GMM classifier**

The `GMM` object implements the *expectation-maximization* (EM) algorithm for fitting mixture-of-Gaussian models. It can also draw confidence ellipsoids for multivariate models, and compute the Bayesian Information Criterion to assess the number of clusters in the data. A `GMM.fit` method is provided that learns a Gaussian Mixture Model from train data. Given test data, it can assign to each sample the class of the Gaussian it mostly probably belong to using the `GMM.predict` method.

The `GMM` comes with different options to constrain the covariance of the difference classes estimated: spherical, diagonal, tied or full covariance.

**Examples:**

- See *GMM classification* for an example of using a GMM as a classifier on the iris dataset.
- See *Density Estimation for a mixture of Gaussians* for an example on plotting the density estimation.

**Pros and cons of class `GMM`: expectation-maximization inference**

**Pros**

- **Speed** it is the fastest algorithm for learning mixture models

- **Agnostic** as this algorithm maximizes only the likelihood, it will not bias the means towards zero, or bias the cluster sizes to have specific structures that might or might not apply.

**Cons**

- **Singularities** when one has insufficiently many points per mixture, estimating the covariance matrices becomes difficult, and the algorithm is known to diverge and find solutions with infinite likelihood unless one regularizes the covariances artificially.

- **Number of components** this algorithm will always use all the components it has access to, needing held-out data or information theoretical criteria to decide how many components to use in the absence of external cues.
Selecting the number of components in a classical GMM

The BIC criterion can be used to select the number of components in a GMM in an efficient way. In theory, it recovers the true number of components only in the asymptotic regime (i.e. if much data is available). Note that using a DPGMM avoids the specification of the number of components for a Gaussian mixture model.

Examples:

- See Gaussian Mixture Model Selection for an example of model selection performed with classical GMM.

Estimation algorithm Expectation-maximization

The main difficulty in learning Gaussian mixture models from unlabeled data is that it is one usually doesn’t know which points came from which latent component (if one has access to this information it gets very easy to fit a separate Gaussian distribution to each set of points). Expectation-maximization is a well-fundamented statistical algorithm to get around this problem by an iterative process. First one assumes random components (randomly centered on data points, learned from k-means, or even just normally distributed around the origin) and computes for each point a probability of being generated by each component of the model. Then, one tweaks the parameters to maximize the likelihood of the data given those assignments. Repeating this process is guaranteed to always converge to a local optimum.
VBGMM classifier: variational Gaussian mixtures

The VBGMM object implements a variant of the Gaussian mixture model with variational inference algorithms. The API is identical to GMM. It is essentially a middle-ground between GMM and DPGMM, as it has some of the properties of the Dirichlet process.

Pros and cons of class VBGMM: variational inference

Pros

Regularization due to the incorporation of prior information, variational solutions have less pathological special cases than expectation-maximization solutions. One can then use full covariance matrices in high dimensions or in cases where some components might be centered around a single point without risking divergence.

Cons

Bias to regularize a model one has to add biases. The variational algorithm will bias all the means towards the origin (part of the prior information adds a “ghost point” in the origin to every mixture component) and it will bias the covariances to be more spherical. It will also, depending on the concentration parameter, bias the cluster structure either towards uniformity or towards a rich-get-richer scenario.

Hyperparameters this algorithm needs an extra hyperparameter that might need experimental tuning via cross-validation.

Estimation algorithm: variational inference

Variational inference is an extension of expectation-maximization that maximizes a lower bound on model evidence (including priors) instead of data likelihood. The principle behind variational methods is the same as expectation-maximization (that is both are iterative algorithms that alternate between finding the probabilities for each point to be generated by each mixture and fitting the mixtures to these assigned points), but variational methods add regularization by integrating information from prior distributions. This avoids the singularities often found in expectation-maximization solutions but introduces some subtle biases to the model. Inference is often notably slower, but not usually as much so as to render usage unpractical.
Due to its Bayesian nature, the variational algorithm needs more hyper-parameters than expectation-maximization, the most important of these being the concentration parameter $\alpha$. Specifying a high value of $\alpha$ leads more often to uniformly-sized mixture components, while specifying small (between 0 and 1) values will lead to some mixture components getting almost all the points while most mixture components will be centered on just a few of the remaining points.

**DPGMM classifier: Infinite Gaussian mixtures**

The DPGMM object implements a variant of the Gaussian mixture model with a variable (but bounded) number of components using the Dirichlet Process. The API is identical to GMM. This class doesn’t require the user to choose the number of components, and at the expense of extra computational time the user only needs to specify a loose upper bound on this number and a concentration parameter.

The examples above compare Gaussian mixture models with fixed number of components, to DPGMM models. **On the left** the GMM is fitted with 5 components on a dataset composed of 2 clusters. We can see that the DPGMM is able to limit itself to only 2 components whereas the GMM fits the data fit too many components. Note that with very little observations, the DPGMM can take a conservative stand, and fit only one component. **On the right** we are fitting a dataset not well-depicted by a mixture of Gaussian. Adjusting the $\alpha$ parameter of the DPGMM controls the number of components used to fit this data.

**Examples:**
- See *Gaussian Mixture Model Ellipsoids* for an example on plotting the confidence ellipsoids for both GMM and DPGMM.
- *Gaussian Mixture Model Sine Curve* shows using GMM and DPGMM to fit a sine wave

**Pros and cons of class DPGMM: Diriclet process mixture model**

**Pros**

- **Less sensitivity to the number of parameters** unlike finite models, which will almost always use all components as much as they can, and hence will produce wildly different solutions for different numbers of components, the Dirichlet process solution won’t change much with changes to the parameters, leading to more stability and less tuning.

- **No need to specify the number of components** only an upper bound of this number needs to be provided. Note however that the DPMM is not a formal model selection procedure, and thus provides no guarantee on the result.
Cons

**Speed** the extra parametrization necessary for variational inference and for the structure of the Dirichlet process can and will make inference slower, although not by much.

**Bias** as in variational techniques, but only more so, there are many implicit biases in the Dirichlet process and the inference algorithms, and whenever there is a mismatch between these biases and the data it might be possible to fit better models using a finite mixture.

The Dirichlet Process

Here we describe variational inference algorithms on Dirichlet process mixtures. The Dirichlet process is a prior probability distribution on clusterings with an infinite, unbounded, number of partitions. Variational techniques let us incorporate this prior structure on Gaussian mixture models at almost no penalty in inference time, comparing with a finite Gaussian mixture model.

An important question is how can the Dirichlet process use an infinite, unbounded number of clusters and still be consistent. While a full explanation doesn’t fit this manual, one can think of its chinese restaurant process analogy to help understanding it. The chinese restaurant process is a generative story for the Dirichlet process. Imagine a chinese restaurant with an infinite number of tables, at first all empty. When the first customer of the day arrives, he sits at the first table. Every following customer will then either sit on an occupied table with probability proportional to the number of customers in that table or sit in an entirely new table with probability proportional to the concentration parameter \( \alpha \). After a finite number of customers has sat, it is easy to see that only finitely many of the infinite tables will ever be used, and the higher the value of \( \alpha \) the more total tables will be used. So the Dirichlet process does clustering with an unbounded number of mixture components by assuming a very asymmetrical prior structure over the assignments of points to components that is very concentrated (this property is known as rich-get-richer, as the full tables in the Chinese restaurant process only tend to get fuller as the simulation progresses).

Variational inference techniques for the Dirichlet process still work with a finite approximation to this infinite mixture model, but instead of having to specify a priori how many components one wants to use, one just specifies the concentration parameter and an upper bound on the number of mixture components (this upper bound, assuming it is higher than the “true” number of components, affects only algorithmic complexity, not the actual number of components used).

**Derivation:**

- See here the full derivation of this algorithm.

**Variational Gaussian Mixture Models** The API is identical to that of the `GMM` class, the main difference being that it offers access to precision matrices as well as covariance matrices.

The inference algorithm is the one from the following paper:

- **Variational Inference for Dirichlet Process Mixtures** David Blei, Michael Jordan. Bayesian Analysis, 2006

While this paper presents the parts of the inference algorithm that are concerned with the structure of the dirichlet process, it does not go into detail in the mixture modeling part, which can be just as complex, or even more. For this reason we present here a full derivation of the inference algorithm and all the update and lower-bound equations. If you’re not interested in learning how to derive similar algorithms yourself and you’re not interested in changing/debugging the implementation in the scikit this document is not for you.

The complexity of this implementation is linear in the number of mixture components and data points. With regards to the dimensionality, it is linear when using spherical or diag and quadratic/cubic when using tied or full. For spherical or diag it is \( O(n_{\text{states}} \times n_{\text{points}} \times \text{dimension}) \) and for tied or full it is \( O(n_{\text{states}} \times n_{\text{points}} \times \text{dimension}^2) \).
The bound for \( \sigma \) Gaussian Mixture Models is given. The main parameters of the model, defined for any class \( k \) are the class proportion \( \phi_k \), the mean parameters \( \mu_k \), the covariance parameters \( \Sigma_k \), which is characterized by variational Wishart density, \( \text{Wishart}(\alpha_k, B_k) \), where \( \alpha \) is the degrees of freedom, and \( B \) is the scale matrix. Depending on the covariance parameterization, \( B_k \) can be a positive scalar, a positive vector or a Symmetric Positive Definite matrix.

**The spherical model**  The model then is
\[
\phi_k \sim \text{Beta}(1, \alpha_1) \\
\mu_k \sim \text{Normal}(0, I) \\
\sigma_k \sim \text{Gamma}(1,1) \\
z_i \sim \text{SBP}(\phi) \\
X_i \sim \text{Normal}(\mu_{z_i}, \frac{1}{\sigma_{z_i}}I)
\]

The variational distribution we’ll use is
\[
\phi_k \sim \text{Beta}(\gamma_k,1, \gamma_k,2) \\
\mu_k \sim \text{Normal}(\nu_{\mu_k}, I) \\
\sigma_k \sim \text{Gamma}(a_k, b_k) \\
z_i \sim \text{Discrete}(\nu_{z_i})
\]

**The bound**  The variational bound is
\[
\log P(X) \geq \sum_k (E_q[\log P(\phi_k)] - E_q[\log Q(\phi_k)]) + \sum_k (E_q[\log P(\mu_k)] - E_q[\log Q(\mu_k)]) + \sum_k (E_q[\log P(\sigma_k)] - E_q[\log Q(\sigma_k)]) + \sum_i (E_q[\log P(z_i)] - E_q[\log Q(z_i)]) + \sum_i E_q[\log P(X_i)]
\]

The bound for \( \phi_k \)
\[
E_q[\log \text{Beta}(1, \alpha)] - E[\log \text{Beta}(\gamma_k,1, \gamma_k,2)] = \log \Gamma(1 + \alpha) - \log \Gamma(\alpha) + (\alpha - 1)(\Psi(\gamma_k,2) - \Psi(\gamma_k,1 + \gamma_k,2)) - \log \Gamma(\gamma_k,1 + \gamma_k,2) + \log \Gamma(\gamma_k,1) + \log \Gamma(\gamma_k,2) - (\gamma_k,1 - 1)(\Psi(\gamma_k,1) - \Psi(\gamma_k,1 + \gamma_k,2)) - (\gamma_k,2 - 1)(\Psi(\gamma_k,2) - \Psi(\gamma_k,1 + \gamma_k,2))
\]

The bound for \( \mu_k \)
\[
E_q[\log P(\mu_k)] - E_q[\log Q(\mu_k)] = \int d\mu q(\mu) \log P(\mu) - \int d\mu q(\mu) \log Q(\mu) = -\frac{D}{2} \log 2\pi - \frac{1}{2} ||\nu_{\mu_k}||^2 - \frac{D}{2} + \frac{D}{2} \log 2\pi e
\]

The bound for \( \sigma_k \)
Here I’ll use the inverse scale parametrization of the gamma distribution.
\[
E_q[\log P(\sigma_k)] - E_q[\log Q(\sigma_k)] = \log \Gamma(a_k) - (a_k - 1)\Psi(a_k) - \log b_k + a_k - \frac{a_k}{b_k}
\]
The bound for \( z \)

\[
E_q[\log P(z)] - E_q[\log Q(z)] = \sum_k \left( \left( \sum_{j=k+1}^K \nu_{z_i,j} \right) (\Psi(\gamma_k, 1) - \Psi(\gamma_k, 1 + \gamma_k, 2)) + \nu_{z_i,k} (\Psi(\gamma_k, 1) - \Psi(\gamma_k, 1 + \gamma_k, 2)) - \log \nu_{z_i,k} \right)
\]

The bound for \( X \)

Recall that there is no need for a \( Q(X) \) so this bound is just

\[
E_q[\log P(X_i)] = \sum_k \nu_{z_i,k} \left( -\frac{D}{2} \log 2\pi + \frac{D}{2} (\Psi(a_k) - \log(b_k)) - \frac{a_k}{2b_k} (||X_i - \mu_k||^2 + D) - \log 2\pi e \right)
\]

For simplicity I’ll later call the term inside the parenthesis \( E_q[\log P(X_i|z_i = k)] \)

The updates

Updating \( \gamma \)

\[
\gamma_{k,1} = 1 + \sum_i \nu_{z_i,k} \\
\gamma_{k,2} = \alpha + \sum_i \sum_{j > k} \nu_{z_i,j} 
\]

Updating \( \mu \)

The updates for \( \mu \) essentially are just weighted expectations of \( X \) regularized by the prior. We can see this by taking the gradient of the bound w.r.t. \( \nu_{\mu} \) and setting it to zero. The gradient is

\[
\nabla L = -\nu_{\mu} + \sum_i \frac{\nu_{z_i,k} b_k}{a_k} (X_i - \nu_{\mu})
\]

so the update is

\[
\nu_{\mu} = \frac{\sum_i \nu_{z_i,k} b_k X_i}{1 + \sum_i \nu_{z_i,k}}
\]

Updating \( a \) and \( b \)

For some odd reason it doesn’t really work when you derive the updates for \( a \) and \( b \) using the gradients of the lower bound (terms involving the \( \Psi' \) function show up and \( a \) is hard to isolate). However, we can use the other formula,

\[
\log Q(\sigma_k) = E_{\nu \neq \sigma_k} [\log P] + \text{const}
\]

All the terms not involving \( \sigma_k \) get folded over into the constant and we get two terms: the prior and the probability of \( X \). This gives us

\[
\log Q(\sigma_k) = -\sigma_k + \frac{D}{2} \sum_i \nu_{z_i,k} \log \sigma_k - \frac{\sigma_k}{2} \sum_i \nu_{z_i,k} (||X_i - \mu_k||^2 + D)
\]

This is the log of a gamma distribution, with \( a_k = 1 + \frac{D}{2} \sum_i \nu_{z_i,k} \) and

\[
b_k = 1 + \frac{1}{2} \sum_i \nu_{z_i,k} (||X_i - \mu_k||^2 + D).
\]

You can verify this by normalizing the previous term.

Updating \( z \)

\[
\log \nu_{z_i,k} \propto (\Psi(\gamma_{k,1}) - \Psi(\gamma_{k,1} + \gamma_{k,2}) + E_Q[\log P(X_i|z_i = k)] + \sum_{j < k} (\Psi(\gamma_{j,2}) - \Psi(\gamma_{j,1} + \gamma_{j,2})).
\]
The diagonal model

The model then is

- \( \phi_k \sim \text{Beta}(1, \alpha_1) \)
- \( \mu_k \sim \text{Normal}(0, I) \)
- \( \sigma_{k,d} \sim \text{Gamma}(1, 1) \)
- \( z_i \sim \text{SBP}(\phi) \)
- \( X_t \sim \text{Normal}(\mu_z, \sigma_{z_t}^{-1}) \)

The variational distribution we’ll use is

- \( \phi_k \sim \text{Beta}(\gamma_k, 1, \gamma_k, 2) \)
- \( \mu_k \sim \text{Normal}(\nu \mu_k, I) \)
- \( \sigma_{k,d} \sim \text{Gamma}(a_{k,d}, b_{k,d}) \)
- \( z_i \sim \text{Discrete}(\nu z_i) \)

The lower bound

The changes in this lower bound from the previous model are in the distributions of \( \sigma \) (as there are a lot more \( \sigma \) s now) and \( X \).

The bound for \( \sigma_{k,d} \) is the same bound for \( \sigma_k \) and can be safely omitted.

The bound for \( X \):

The main difference here is that the precision matrix \( \sigma_k \) scales the norm, so we have an extra term after computing the expectation of \( \mu_k^T \sigma_k \mu_k \), which is \( \nu \mu_k^T \sigma_k \nu \mu_k + \sum_d \sigma_{k,d} \). We then have

\[
E_q[\log P(X_i)] = \sum_k \nu z_k \left( -\frac{D}{2} \log 2\pi + \frac{1}{2} \sum_d (\Psi(a_{k,d}) - \log(b_{k,d})) - \frac{1}{2} ((X_i - \mu_k)^T a_k b_k (X_i - \mu_k) + \sum_d \sigma_{k,d}) - \log 2\pi e \right)
\]

The updates

The updates only change for \( \mu \) (to weight them with the new \( \sigma \)), \( z \) (but the change is all folded into the \( E_q[P(X_i|z_i = k)] \) term), and the \( a \) and \( b \) variables themselves.

The update for \( \mu \)

\[
\nu_{\mu_k} = \left( I + \sum_i \frac{\nu z_{i,k} b_k}{a_k} \right)^{-1} \left( \sum_i \frac{\nu z_{i,k} b_k}{a_k} X_i \right)
\]

The updates for \( a \) and \( b \)

Here we’ll do something very similar to the spheric model. The main difference is that now each \( \sigma_{k,d} \) controls only one dimension of the bound:

\[
\log Q(\sigma_{k,d}) = -\sigma_{k,d} + \sum_i \nu z_{i,k} \frac{1}{2} \log \sigma_{k,d} - \frac{\sigma_{k,d}}{2} \sum_i \nu z_{i,k} ((X_{i,d} - \mu_{k,d})^2 + 1)
\]

Hence

\[
a_{k,d} = 1 + \frac{1}{2} \sum_i \nu z_{i,k}
\]

\[
b_{k,d} = 1 + \frac{1}{2} \sum_i \nu z_{i,k} ((X_{i,d} - \mu_{k,d})^2 + 1)
\]
The tied model  The model then is
\[\phi_k \sim \text{Beta}(1, \alpha_1)\]
\[\mu_k \sim \text{Normal}(0, I)\]
\[\Sigma \sim \text{Wishart}(D, I)\]
\[z_i \sim \text{SBP}(\phi)\]
\[X_t \sim \text{Normal}(\mu_{z_i}, \Sigma^{-1})\]

The variational distribution we’ll use is
\[\phi_k \sim \text{Beta}(\gamma_k, 1, \gamma_k, 2)\]
\[\mu_k \sim \text{Normal}(\nu_k\mu_k, I)\]
\[\Sigma \sim \text{Wishart}(a, B)\]
\[z_i \sim \text{Discrete}(\nu_kz_i)\]

The lower bound  There are two changes in the lower-bound: for \(\Sigma\) and for \(X\).

The bound for \(\Sigma\)
\[
\frac{D^2}{2} \log 2 + \sum_d \log \Gamma(D + 1 - d) - \frac{aD}{2} \log 2 + \frac{D}{2} \log |B| + \sum_d \log \Gamma(a + 1 - d) + \frac{D}{2} \log 2 + \log |B| + \frac{1}{2} \text{tr}[B - I]
\]

The bound for \(X\)
\[
E_q[\log P(X_i)] = \sum_k \nu_{z_i,k} \left( -\frac{D}{2} \log 2\pi + \frac{1}{2} \sum_d \Psi \left( \frac{a+1-d}{2} \right) + D \log 2 + \log |B| \right)
- \frac{1}{2} \left( (X_i - \nu_{\mu_k})^T \Sigma (X_i - \nu_{\mu_k}) + \text{tr}(B) \right) - D e
\]

The updates  As in the last setting, what changes are the trivial update for \(z\), the update for \(\mu\) and the update for \(a\) and \(B\).

The update for \(\mu\)
\[\nu_{\mu_k} = \left( I + aB \sum_i \nu_{z_i,k} \right)^{-1} \left( aB \sum_i \nu_{z_i,k} X_i \right)\]

The update for \(a\) and \(B\)
As this distribution is far too complicated I’m not even going to try going at it the gradient way.
\[
\log Q(\Sigma) = +\frac{1}{2} \log |\Sigma| - \frac{1}{2} \text{tr}[\Sigma] + \sum_i \sum_k \nu_{z_i,k} \left( +\frac{1}{2} \log |\Sigma| - \frac{1}{2} (X_i - \nu_{\mu_k})^T \Sigma (X_i - \nu_{\mu_k}) + \text{tr}[\Sigma] \right)
\]
which non-trivially (seeing that the quadratic form with \(\Sigma\) in the middle can be expressed as the trace of something) reduces to
\[
\log Q(\Sigma) = +\frac{1}{2} \log |\Sigma| - \frac{1}{2} \text{tr}[\Sigma] + \sum_i \sum_k \nu_{z_i,k} \left( +\frac{1}{2} \log |\Sigma| - \frac{1}{2} \text{tr}[(X_i - \nu_{\mu_k})(X_i - \nu_{\mu_k})^T \Sigma + \text{tr}[\Sigma]] \right)
\]

hence this (with a bit of squinting) looks like a wishart with parameters
\[a = 2 + D + T\]
and
\[B = \left( I + \sum_i \sum_k \nu_{z_i,k} (X_i - \nu_{\mu_k})(X_i - \nu_{\mu_k})^T \right)^{-1}\]

108 Chapter 1. User Guide
The full model

The model then is

\[ \phi_k \sim \text{Beta}(1, \alpha_1) \]
\[ \mu_k \sim \text{Normal}(0, I) \]
\[ \Sigma_k \sim \text{Wishart}(D, I) \]
\[ z_i \sim \text{SBP}(\phi) \]
\[ X_i \sim \text{Normal}(\mu_{z_i}, \Sigma_{z,i}^{-1}) \]

The variational distribution we’ll use is

\[ \phi_k \sim \text{Beta}(\gamma_k, 1) \]
\[ \mu_k \sim \text{Normal}(\nu_{\mu_k}, I) \]
\[ \Sigma_k \sim \text{Wishart}(a_k, B_k) \]
\[ z_i \sim \text{Discrete}(v_{z_i}) \]

The lower bound All that changes in this lower bound in comparison to the previous one is that there are K priors on different \( \Sigma \) precision matrices and there are the correct indices on the bound for \( X \).

The updates All that changes in the updates is that the update for \( \mu \) uses only the proper sigma and the updates for \( A \) and \( B \) don’t have a sum over \( K \), so

\[ \nu_{\mu_k} = \left( I + a_k B_k \sum_i v_{z_i,k} \right)^{-1} \left( a_k B_k \sum_i v_{z_i,k} X_i \right) \]
\[ a_k = 2 + D + \sum_i v_{z_i,k} \]

and

\[ B = \left( \sum_i v_{z_i,k} + 1 \right) I + \sum_i v_{z_i,k} (X_i - \nu_{\mu_k})(X_i - \nu_{\mu_k})^T \right)^{-1} \]

1.4.2 Manifold learning

Look for the bare necessities
The simple bare necessities
Forget about your worries and your strife
I mean the bare necessities
Old Mother Nature’s recipes
That bring the bare necessities of life

– Baloo’s song [The Jungle Book]

Manifold learning is an approach to nonlinear dimensionality reduction. Algorithms for this task are based on the idea that the dimensionality of many data sets is only artificially high.
High-dimensional datasets can be very difficult to visualize. While data in two or three dimensions can be plotted to show the inherent structure of the data, equivalent high-dimensional plots are much less intuitive. To aid visualization of the structure of a dataset, the dimension must be reduced in some way.

The simplest way to accomplish this dimensionality reduction is by taking a random projection of the data. Though this allows some degree of visualization of the data structure, the randomness of the choice leaves much to be desired. In a random projection, it is likely that the more interesting structure within the data will be lost.
To address this concern, a number of supervised and unsupervised linear dimensionality reduction frameworks have been designed, such as Principal Component Analysis (PCA), Independent Component Analysis, Linear Discriminant Analysis, and others. These algorithms define specific rubrics to choose an “interesting” linear projection of the data. These methods can be powerful, but often miss important nonlinear structure in the data.

Manifold Learning can be thought of as an attempt to generalize linear frameworks like PCA to be sensitive to nonlinear structure in data. Though supervised variants exist, the typical manifold learning problem is unsupervised: it learns the high-dimensional structure of the data from the data itself, without the use of predetermined classifications.

Examples:

- See Manifold learning on handwritten digits: Locally Linear Embedding, Isomap... for an example of dimensionality reduction on handwritten digits.
- See Comparison of Manifold Learning methods for an example of dimensionality reduction on a toy “S-curve” dataset.

The manifold learning implementations available in sklearn are summarized below

**Isomap**

One of the earliest approaches to manifold learning is the Isomap algorithm, short for Isometric Mapping. Isomap can be viewed as an extension of Multi-dimensional Scaling (MDS) or Kernel PCA. Isomap seeks a lower-dimensional
embedding which maintains geodesic distances between all points. Isomap can be performed with the object `Isomap`.

Complexity

The Isomap algorithm comprises three stages:

1. **Nearest neighbor search.** Isomap uses `sklearn.neighbors.BallTree` for efficient neighbor search. The cost is approximately $O[D \log(k)N \log(N)]$, for $k$ nearest neighbors of $N$ points in $D$ dimensions.

2. **Shortest-path graph search.** The most efficient known algorithms for this are Dijkstra’s Algorithm, which is approximately $O[N^2(k + \log(N))]$, or the Floyd-Warshall algorithm, which is $O[N^3]$. The algorithm can be selected by the user with the `path_method` keyword of `Isomap`. If unspecified, the code attempts to choose the best algorithm for the input data.

3. **Partial eigenvalue decomposition.** The embedding is encoded in the eigenvectors corresponding to the $d$ largest eigenvalues of the $N \times N$ isomap kernel. For a dense solver, the cost is approximately $O[dN^2]$. This cost can often be improved using the ARPACK solver. The eigensolver can be specified by the user with the `path_method` keyword of `Isomap`. If unspecified, the code attempts to choose the best algorithm for the input data.

The overall complexity of Isomap is $O[D \log(k)N \log(N)] + O[N^2(k + \log(N))] + O[dN^2]$.

- $N$: number of training data points
- $D$: input dimension
- $k$: number of nearest neighbors
- $d$: output dimension

References:

- “A global geometric framework for nonlinear dimensionality reduction” Tenenbaum, J.B.; De Silva, V.; & Langford, J.C. Science 290 (5500)
Locally Linear Embedding

Locally linear embedding (LLE) seeks a lower-dimensional projection of the data which preserves distances within local neighborhoods. It can be thought of as a series of local Principal Component Analyses which are globally compared to find the best nonlinear embedding.

Locally linear embedding can be performed with function `locally_linear_embedding` or its object-oriented counterpart `LocallyLinearEmbedding`.

Complexity

The standard LLE algorithm comprises three stages:

1. **Nearest Neighbors Search.** See discussion under Isomap above.

2. **Weight Matrix Construction.** \(O[DNk^3]\). The construction of the LLE weight matrix involves the solution of a \(k \times k\) linear equation for each of the \(N\) local neighborhoods.

3. **Partial Eigenvalue Decomposition.** See discussion under Isomap above.

The overall complexity of standard LLE is \(O[D \log(k)N \log(N)] + O[DNk^3] + O[dN^2]\).

- \(N\) : number of training data points
- \(D\) : input dimension
- \(k\) : number of nearest neighbors
- \(d\) : output dimension

References:


Modified Locally Linear Embedding

One well-known issue with LLE is the regularization problem. When the number of neighbors is greater than the number of input dimensions, the matrix defining each local neighborhood is rank-deficient. To address this, standard
LLE applies an arbitrary regularization parameter $r$, which is chosen relative to the trace of the local weight matrix. Though it can be shown formally that as $r \to 0$, the solution converges to the desired embedding, there is no guarantee that the optimal solution will be found for $r > 0$. This problem manifests itself in embeddings which distort the underlying geometry of the manifold.

One method to address the regularization problem is to use multiple weight vectors in each neighborhood. This is the essence of modified locally linear embedding (MLLE). MLLE can be performed with function `locally_linear_embedding` or its object-oriented counterpart `LocallyLinearEmbedding`, with the keyword `method = 'modified'`. It requires $n_{neighbors} > n_{components}$.

### Complexity

The MLLE algorithm comprises three stages:

1. **Nearest Neighbors Search.** Same as standard LLE

2. **Weight Matrix Construction.** Approximately $O[DNk^3] + O[N(k-D)k^2]$. The first term is exactly equivalent to that of standard LLE. The second term has to do with constructing the weight matrix from multiple weights. In practice, the added cost of constructing the MLLE weight matrix is relatively small compared to the cost of steps 1 and 3.

3. **Partial Eigenvalue Decomposition.** Same as standard LLE

The overall complexity of MLLE is $O[D \log(k)N \log(N)] + O[DNk^3] + O[N(k-D)k^2] + O[dN^2]$.

- $N$: number of training data points
- $D$: input dimension
- $k$: number of nearest neighbors
- $d$: output dimension

### References:

- “MLLE: Modified Locally Linear Embedding Using Multiple Weights” Zhang, Z. & Wang, J.
Hessian Eigenmapping

Hessian Eigenmapping (also known as Hessian-based LLE: HLLE) is another method of solving the regularization problem of LLE. It revolves around a hessian-based quadratic form at each neighborhood which is used to recover the locally linear structure. Though other implementations note its poor scaling with data size, sklearn implements some algorithmic improvements which make its cost comparable to that of other LLE variants for small output dimension. HLLE can be performed with function `locally_linear_embedding` or its object-oriented counterpart `LocallyLinearEmbedding`, with the keyword `method = 'hessian'`. It requires `n_neighbors > n_components * (n_components + 3) / 2`.

### Complexity

The HLLE algorithm comprises three stages:

1. **Nearest Neighbors Search.** Same as standard LLE
2. **Weight Matrix Construction.** Approximately $O(DNk^3) + O(Nd^6)$. The first term reflects a similar cost to that of standard LLE. The second term comes from a QR decomposition of the local hessian estimator.
3. **Partial Eigenvalue Decomposition.** Same as standard LLE

The overall complexity of standard HLLE is $O[D \log(k)N \log(N)] + O(DNk^3) + O(Nd^6) + O(dN^2)$.

- $N$: number of training data points
- $D$: input dimension
- $k$: number of nearest neighbors
- $d$: output dimension

### References:

Local Tangent Space Alignment

Though not technically a variant of LLE, Local tangent space alignment (LTSA) is algorithmically similar enough to LLE that it can be put in this category. Rather than focusing on preserving neighborhood distances as in LLE, LTSA seeks to characterize the local geometry at each neighborhood via its tangent space, and performs a global optimization to align these local tangent spaces to learn the embedding. LTSA can be performed with function `locally_linear_embedding` or its object-oriented counterpart `LocallyLinearEmbedding`, with the keyword `method = 'ltsa'`.

Complexity

The LTSA algorithm comprises three stages:

1. **Nearest Neighbors Search.** Same as standard LLE

2. **Weight Matrix Construction.** Approximately $O[DNk^3] + O[k^2d]$. The first term reflects a similar cost to that of standard LLE.

3. **Partial Eigenvalue Decomposition.** Same as standard LLE

The overall complexity of standard LTSA is $O[D \log(k)N \log(N)] + O[DNk^3] + O[k^2d] + O[dN^2]$.

- $N$ : number of training data points
- $D$ : input dimension
- $k$ : number of nearest neighbors
- $d$ : output dimension

References:


**Multi-dimensional Scaling (MDS)**

Multidimensional scaling (MDS) seeks a low-dimensional representation of the data in which the distances respect well the distances in the original high-dimensional space.
In general, is a technique used for analyzing similarity or dissimilarity data. MDS attempts to model similarity or dissimilarity data as distances in a geometric space. The data can be ratings of similarity between objects, interaction frequencies of molecules, or trade indices between countries.

There exists two types of MDS algorithm: metric and non metric. In the scikit-learn, the class MDS implements both. In Metric MDS, the input similarity matrix arises from a metric (and thus respects the triangular inequality), the distances between output two points are then set to be as close as possible to the similarity or dissimilarity data. In the non metric vision, the algorithms will try to preserve the order of the distances, and hence seek for a monotonic relationship between the distances in the embedded space and the similarities/dissimilarities.

Let $S$ be the similarity matrix, and $X$ the coordinates of the $n$ input points. Disparities $\hat{d}_{ij}$ are transformation of the similarities chosen in some optimal ways. The objective, called the stress, is then defined by $\sum_{i<j} d_{ij}(X) - \hat{d}_{ij}(X)$

**Metric MDS**

The simplest metric MDS model, called absolute MDS, disparities are defined by $\hat{d}_{ij} = S_{ij}$. With absolute MDS, the value $S_{ij}$ should then correspond exactly to the distance between point $i$ and $j$ in the embedding point.

Most commonly, disparities are set to $\hat{d}_{ij} = b S_{ij}$.

**Nonmetric MDS**

Non metric MDS focuses on the ordination of the data. If $S_{ij} < S_{kl}$, then the embedding should enforce $d_{ij} < d_{jk}$. A simple algorithm to enforce that is to use a monotonic regression of $d_{ij}$ on $S_{ij}$, yielding disparities $\hat{d}_{ij}$ in the same order as $S_{ij}$.

A trivial solution to this problem is to set all the points on the origin. In order to avoid that, the disparities $\hat{d}_{ij}$ are normalized.

**References:**

- “Nonmetric multidimensional scaling: a numerical method” Kruskal, J. Psychometrika, 29 (1964)
- “Multidimensional scaling by optimizing goodness of fit to a nonmetric hypothesis” Kruskal, J. Psychometrika, 29, (1964)
**Tips on practical use**

- Make sure the same scale is used over all features. Because manifold learning methods are based on a nearest-neighbor search, the algorithm may perform poorly otherwise. See `Scaler` for convenient ways of scaling heterogeneous data.

- The reconstruction error computed by each routine can be used to choose the optimal output dimension. For a \( d \)-dimensional manifold embedded in a \( D \)-dimensional parameter space, the reconstruction error will decrease as \( n_{\text{components}} \) is increased until \( n_{\text{components}} = d \).

- Note that noisy data can “short-circuit” the manifold, in essence acting as a bridge between parts of the manifold that would otherwise be well-separated. Manifold learning on noisy and/or incomplete data is an active area of research.

- Certain input configurations can lead to singular weight matrices, for example when more than two points in the dataset are identical, or when the data is split into disjointed groups. In this case, `method='arpack'` will fail to find the null space. The easiest way to address this is to use `method='dense'` which will work on a singular matrix, though it may be very slow depending on the number of input points. Alternatively, one can attempt to understand the source of the singularity: if it is due to disjoint sets, increasing `n_neighbors` may help. If it is due to identical points in the dataset, removing these points may help.

### 1.4.3 Clustering

Clustering of unlabeled data can be performed with the module `sklearn.cluster`.

Each clustering algorithm comes in two variants: a class, that implements the `fit` method to learn the clusters on train data, and a function, that, given train data, returns an array of integer labels corresponding to the different clusters. For the class, the labels over the training data can be found in the `labels_` attribute.
Input data

One important thing to note is that the algorithms implemented in this module take different kinds of matrix as input. On one hand, MeanShift and KMeans take data matrices of shape \([n_{\text{samples}}, n_{\text{features}}]\). These can be obtained from the classes in the sklearn.feature_extraction module. On the other hand, AffinityPropagation and SpectralClustering take similarity matrices of shape \([n_{\text{samples}}, n_{\text{samples}}]\). These can be obtained from the functions in the sklearn.metrics.pairwise module. In other words, MeanShift and KMeans work with points in a vector space, whereas AffinityPropagation and SpectralClustering can work with arbitrary objects, as long as a similarity measure exists for such objects.

Overview of clustering methods

![Figure 1.3: A comparison of the clustering algorithms in scikit-learn](image)

<table>
<thead>
<tr>
<th>Method name</th>
<th>Parameters</th>
<th>Scalability</th>
<th>Usecase</th>
<th>Geometry (metric used)</th>
</tr>
</thead>
<tbody>
<tr>
<td>K-Means</td>
<td>number of clusters</td>
<td>Very large (n_{\text{samples}}), medium (n_{\text{clusters}}) with MiniBatch code</td>
<td>General-purpose, even cluster size, flat geometry, not too many clusters</td>
<td>Distances between points</td>
</tr>
<tr>
<td>Affinity propagation</td>
<td>damping, sample preference</td>
<td>Not scalable with (n_{\text{samples}})</td>
<td>Many clusters, uneven cluster size, non-flat geometry</td>
<td>Graph distance (e.g. nearest-neighbor graph)</td>
</tr>
<tr>
<td>Mean-shift</td>
<td>bandwidth</td>
<td>Not scalable with (n_{\text{samples}})</td>
<td>Many clusters, uneven cluster size, non-flat geometry</td>
<td>Distances between points</td>
</tr>
<tr>
<td>Spectral clustering</td>
<td>number of clusters</td>
<td>Medium (n_{\text{samples}}), small (n_{\text{clusters}})</td>
<td>Few clusters, even cluster size, non-flat geometry</td>
<td>Graph distance (e.g. nearest-neighbor graph)</td>
</tr>
<tr>
<td>Hierarchical clustering</td>
<td>number of clusters</td>
<td>Large (n_{\text{samples}}) and (n_{\text{clusters}})</td>
<td>Many clusters, possibly connectivity constraints</td>
<td>Distances between points</td>
</tr>
<tr>
<td>DBSCAN</td>
<td>neighborhood size</td>
<td>Very large (n_{\text{samples}}), medium (n_{\text{clusters}})</td>
<td>Non-flat geometry, uneven cluster sizes</td>
<td>Distances between nearest points</td>
</tr>
<tr>
<td>Gaussian mixtures</td>
<td>many</td>
<td>Not scalable</td>
<td>Flat geometry, good for density estimation</td>
<td>Mahalanobis distances to centers</td>
</tr>
</tbody>
</table>
Non-flat geometry clustering is useful when the clusters have a specific shape, i.e. a non-flat manifold, and the standard euclidean distance is not the right metric. This case arises in the two top rows of the figure above.

Gaussian mixture models, useful for clustering, are described in another chapter of the documentation dedicated to mixture models. KMeans can be seen as a special case of Gaussian mixture model with equal covariance per component.

K-means

The KMeans algorithm clusters data by trying to separate samples in n groups of equal variance, minimizing a criterion known as the ‘inertia’ of the groups. This algorithm requires the number of cluster to be specified. It scales well to large number of samples, however its results may be dependent on an initialisation. As a result, the computation is often done several times, with different initialisation of the centroids.

K-means is often referred to as Lloyd’s algorithm. After initialization, k-means consists of looping between two major steps. First the Voronoi diagram of the points is calculated using the current centroids. Each segment in the Voronoi diagram becomes a separate cluster. Secondly, the centroids are updated to the mean of each segment. The algorithm then repeats this until a stopping criteria is fulfilled. Usually, as in this implementation, the algorithm stops when the relative increment in the results between iterations is less than the given tolerance value.

A parameter can be given to allow K-means to be run in parallel, called n_jobs. Giving this parameter a positive value uses that many processors (default=1). A value of -1 uses all processors, with -2 using one less, and so on. Parallelization generally speeds up computation at the cost of memory (in this case, multiple copies of centroids need to be stored, one for each job).

K-means can be used for vector quantization. This is achieved using the transform method of a trained model of KMeans.

Examples:
- A demo of K-Means clustering on the handwritten digits data: Clustering handwritten digits

Mini Batch K-Means

The MiniBatchKMeans is a variant of the KMeans algorithm using mini-batches, random subset of the dataset, to compute the centroids.

Althought the MiniBatchKMeans converge faster than the KMeans version, the quality of the results, measured by the inertia, the sum of the distance of each points to the nearest centroid, is not as good as the KMeans algorithm.

Examples:
- A demo of the K Means clustering algorithm: Comparison of KMeans and MiniBatchKMeans
- Clustering text documents using k-means: Document clustering using sparse MiniBatchKMeans

References:
Affinity propagation

AffinityPropagation clusters data by diffusion in the similarity matrix. This algorithm automatically sets its numbers of cluster. It will have difficulties scaling to thousands of samples.

Examples:

- **Demo of affinity propagation clustering algorithm**: Affinity Propagation on a synthetic 2D datasets with 3 classes.
- **Visualizing the stock market structure** Affinity Propagation on Financial time series to find groups of companies

Mean Shift

MeanShift clusters data by estimating blobs in a smooth density of points matrix. This algorithm automatically sets its numbers of cluster. It will have difficulties scaling to thousands of samples. The utility function `estimate_bandwidth` can be used to guess the optimal bandwidth for `MeanShift` from the data.
Examples:

- *A demo of the mean-shift clustering algorithm*: Mean Shift clustering on a synthetic 2D datasets with 3 classes.

**Spectral clustering**

`SpectralClustering` does a low-dimension embedding of the affinity matrix between samples, followed by a KMeans in the low dimensional space. It is especially efficient if the affinity matrix is sparse and the `pyamg` module is installed. `SpectralClustering` requires the number of clusters to be specified. It works well for a small number of clusters but is not advised when using many clusters.

For two clusters, it solves a convex relaxation of the normalised cuts problem on the similarity graph: cutting the graph in two so that the weight of the edges cut is small compared to the weights in of edges inside each cluster. This criteria is especially interesting when working on images: graph vertices are pixels, and edges of the similarity graph are a function of the gradient of the image.
**Warning:** Shapeless isotropic data
When the data is really shapeless (i.e. generated from a random distribution with no clusters), the spectral-clustering problem is ill-conditioned: the different choices are almost equivalent, and the spectral clustering solver chooses an arbitrary one, putting the first sample alone in one bin.

**Warning:** Transforming distance to well-behaved similarities
Note that if the values of your similarity matrix are not well distributed, e.g. with negative values or with a distance matrix rather than a similarity, the spectral problem will be singular and the problem not solvable. In which case it is advised to apply a transformation to the entries of the matrix. For instance, in the case of a signed distance matrix, is common to apply a heat kernel:

\[
\text{similarity} = \exp(-\beta \cdot \text{distance} / \text{distance.std()})
\]

See the examples for such an application.

**Examples:**
- **Spectral clustering for image segmentation:** Segmenting objects from a noisy background using spectral clustering.
- **Segmenting the picture of Lena in regions:** Spectral clustering to split the image of lena in regions.

**References:**
- “Normalized cuts and image segmentation” Jianbo Shi, Jitendra Malik, 2000
- “On Spectral Clustering: Analysis and an algorithm” Andrew Y. Ng, Michael I. Jordan, Yair Weiss, 2001

**Hierarchical clustering**
Hierarchical clustering is a general family of clustering algorithms that build nested clusters by merging them successively. This hierarchy of clusters represented as a tree (or dendrogram). The root of the tree is the unique cluster that gathers all the samples, the leaves being the clusters with only one sample. See the Wikipedia page for more details.

The **Ward** object performs a hierarchical clustering based on the Ward algorithm, that is a variance-minimizing approach. At each step, it minimizes the sum of squared differences within all clusters (inertia criterion).

This algorithm can scale to large number of samples when it is used jointly with an connectivity matrix, but can be computationally expensive when no connectivity constraints are added between samples: it considers at each step all the possible merges.

**Adding connectivity constraints**
An interesting aspect of the **Ward** object is that connectivity constraints can be added to this algorithm (only adjacent clusters can be merged together), through an connectivity matrix that defines for each sample the neighboring samples following a given structure of the data. For instance, in the swiss-roll example below, the connectivity constraints forbid the merging of points that are not adjacent on the swiss roll, and thus avoid forming clusters that extend across overlapping folds of the roll.
The connectivity constraints are imposed via an connectivity matrix: a scipy sparse matrix that has elements only at the intersection of a row and a column with indices of the dataset that should be connected. This matrix can be constructed from a-priori information, for instance if you wish to cluster web pages, but only merging pages with a link pointing from one to another. It can also be learned from the data, for instance using sklearn.neighbors.kneighbors_graph to restrict merging to nearest neighbors as in the swiss roll example, or using sklearn.feature_extraction.image.grid_to_graph to enable only merging of neighboring pixels on an image, as in the Lena example.

Examples:

- A demo of structured Ward hierarchical clustering on Lena image: Ward clustering to split the image of lena in regions.
- Hierarchical clustering: structured vs unstructured ward: Example of Ward algorithm on a swiss-roll, comparison of structured approaches versus unstructured approaches.
- Feature agglomeration vs. univariate selection: Example of dimensionality reduction with feature agglomeration based on Ward hierarchical clustering.

DBSCAN

The DBSCAN algorithm clusters data by finding core points which have many neighbours within a given radius. After a core point is found, the cluster is expanded by adding its neighbours to the current cluster and recursively checking if any are core points. Formally, a point is considered a core point if it has more than min_points points which are of a similarity greater than the given threshold eps. This is shown in the figure below, where the color indicates cluster membership and large circles indicate core points found by the algorithm. Moreover, the algorithm can detect outliers, indicated by black points below. The outliers are defined as points which do not belong to any current cluster and do not have enough close neighbours to start a new cluster.
Clustering performance evaluation

Evaluating the performance of a clustering algorithm is not as trivial as counting the number of errors or the precision and recall of a supervised classification algorithm. In particular any evaluation metric should not take the absolute values of the cluster labels into account but rather if this clustering define separations of the data similar to some ground truth set of classes or satisfying some assumption such that members belong to the same class are more similar that members of different classes according to some similarity metric.

Inertia

Presentation and usage  TODO: factorize inertia computation out of kmeans and then write me!

Advantages

• No need for the ground truth knowledge of the “real” classes.

Drawbacks

• Inertia makes the assumption that clusters are convex and isotropic which is not always the case especially of the clusters are manifolds with weird shapes: for instance inertia is a useless metrics to evaluate clustering algorithm that tries to identify nested circles on a 2D plane.

• Inertia is not a normalized metrics: we just know that lower values are better and bounded by zero. One potential solution would be to adjust inertia for random clustering (assuming the number of ground truth classes is known).
Adjusted Rand index

Presentation and usage Given the knowledge of the ground truth class assignments labels_true and our clustering algorithm assignments of the same samples labels_pred, the adjusted Rand index is a function that measures the similarity of the two assignments, ignoring permutations and with chance normalization:

```python
>>> from sklearn import metrics

>>> labels_true = [0, 0, 0, 1, 1, 1]
>>> labels_pred = [0, 0, 1, 1, 2, 2]

>>> metrics.adjusted_rand_score(labels_true, labels_pred)
0.24...
```

One can permute 0 and 1 in the predicted labels and rename 2 by 3 and get the same score:

```python
>>> labels_pred = [1, 1, 0, 0, 3, 3]

>>> metrics.adjusted_rand_score(labels_true, labels_pred)
0.24...
```

Furthermore, adjusted_rand_score is symmetric: swapping the argument does not change the score. It can thus be used as a consensus measure:

```python
>>> metrics.adjusted_rand_score(labels_pred, labels_true)
0.24...
```

Perfect labeling is scored 1.0:

```python
>>> labels_pred = labels_true[:]

>>> metrics.adjusted_rand_score(labels_true, labels_pred)
1.0
```

Bad (e.g. independent labelings) have negative or close to 0.0 scores:

```python
>>> labels_true = [0, 1, 2, 0, 3, 4, 5, 1]

>>> labels_pred = [1, 1, 0, 0, 2, 2, 2, 2]

>>> metrics.adjusted_rand_score(labels_true, labels_pred)
-0.12...
```

Advantages

- Random (uniform) label assignments have a ARI score close to 0.0 for any value of n_clusters and n_samples (which is not the case for raw Rand index or the V-measure for instance).
- Bounded range [-1, 1]: negative values are bad (independent labelings), similar clusterings have a positive ARI, 1.0 is the perfect match score.
- No assumption is made on the cluster structure: can be used to compare clustering algorithms such as k-means which assumes isotropic blob shapes with results of spectral clustering algorithms which can find cluster with “folded” shapes.

Drawbacks

- Contrary to inertia, ARI requires knowledge of the ground truth classes while is almost never available in practice or requires manual assignment by human annotators (as in the supervised learning setting).

However ARI can also be useful in a purely unsupervised setting as a building block for a Consensus Index that can be used for clustering model selection (TODO).
Examples:

- **Adjustment for chance in clustering performance evaluation:** Analysis of the impact of the dataset size on the value of clustering measures for random assignments.

**Mathematical formulation** If C is a ground truth class assignment and K the clustering, let us define a and b as:

- a, the number of pairs of elements that are in the same set in C and in the same set in K
- b, the number of pairs of elements that are in different sets in C and in different sets in K

The raw (unadjusted) Rand index is then given by:

\[ \text{RI} = \frac{a + b}{C_n^2} \]

Where \( C_n^2 \) is the total number of possible pairs in the dataset (without ordering).

However the RI score does not guarantee that random label assignments will get a value close to zero (esp. if the number of clusters is in the same order of magnitude as the number of samples).

To counter this effect we can discount the expected RI \( E[\text{RI}] \) of random labelings by defining the adjusted Rand index as follows:

\[ \text{ARI} = \frac{\text{RI} - E[\text{RI}]}{\max(\text{RI}) - E[\text{RI}]} \]

**References**

- Comparing Partitions L. Hubert and P. Arabie, Journal of Classification 1985
- Wikipedia entry for the adjusted Rand index

**Mutual Information based scores**

**Presentation and usage** Given the knowledge of the ground truth class assignments `labels_true` and our clustering algorithm assignments of the same samples `labels_pred`, the Mutual Information is a function that measures the agreement of the two assignments, ignoring permutations. Two different normalized versions of this measure are available, *Normalized Mutual Information (NMI)* and *Adjusted Mutual Information (AMI)*. NMI is often used in the literature while AMI was proposed more recently and is normalized against chance:

```python
>>> from sklearn import metrics
>>> labels_true = [0, 0, 0, 1, 1, 1]
>>> labels_pred = [0, 0, 1, 1, 2, 2]

>>> metrics.adjusted_mutual_info_score(labels_true, labels_pred)
0.22504...
```

One can permute 0 and 1 in the predicted labels and rename 2 by 3 and get the same score:

```python
>>> labels_pred = [1, 1, 0, 0, 3, 3]

>>> metrics.adjusted_mutual_info_score(labels_true, labels_pred)
0.22504...
```
All, `mutual_info_score`, `adjusted_mutual_info_score` and `normalized_mutual_info_score` are symmetric: swapping the argument does not change the score. Thus they can be used as a **consensus measure**:

```python
>>> metrics.adjusted_mutual_info_score(labels_pred, labels_true)
0.22504...
```

Perfect labeling is scored 1.0:

```python
>>> labels_pred = labels_true[:]
>>> metrics.adjusted_mutual_info_score(labels_true, labels_pred)
1.0
```

```python
>>> metrics.normalized_mutual_info_score(labels_true, labels_pred)
1.0
```

This is **not true** for `mutual_info_score`, which is therefore harder to judge:

```python
>>> metrics.mutual_info_score(labels_true, labels_pred)
0.69...
```

Bad (e.g. independent labelings) have non-positive scores:

```python
>>> labels_true = [0, 1, 2, 0, 3, 4, 5, 1]
>>> labels_pred = [1, 1, 0, 0, 2, 2, 2, 2]
>>> metrics.adjusted_mutual_info_score(labels_true, labels_pred)
-0.10526...
```

**Advantages**

- **Random (uniform) label assignments** have a AMI score close to 0.0 for any value of `n_clusters` and `n_samples` (which is not the case for raw Mutual Information or the V-measure for instance).
- **Bounded range [0, 1]**: Values close to zero indicate two label assignments that are largely independent, while values close to one indicate significant agreement. Further, values of exactly 0 indicate purely independent label assignments and a AMI of exactly 1 indicates that the two label assignments are equal (with or without permutation).
- **No assumption is made on the cluster structure**: can be used to compare clustering algorithms such as k-means which assumes isotropic blob shapes with results of spectral clustering algorithms which can find cluster with “folded” shapes.

**Drawbacks**

- Contrary to inertia, **MI-based measures require the knowledge of the ground truth classes** while almost never available in practice or requires manual assignment by human annotators (as in the supervised learning setting).

However MI-based measures can also be useful in purely unsupervised setting as a building block for a Consensus Index that can be used for clustering model selection.

- NMI and MI are not adjusted against chance.

**Examples:**

- **Adjustment for chance in clustering performance evaluation**: Analysis of the impact of the dataset size on the value of clustering measures for random assignments. This example also includes the Adjusted Rand Index.
Mathematical formulation  Assume two label assignments (of the same data), $U$ with $R$ classes and $V$ with $C$ classes. The entropy of either is the amount of uncertainty for an array, and can be calculated as:

$$H(U) = \sum_{i=1}^{R} P(i) \log(P(i))$$

Where $P(i)$ is the number of instances in $U$ that are in class $R_i$. Likewise, for $V$:

$$H(V) = \sum_{j=1}^{C} P'(j) \log(P'(j))$$

Where $P'(j)$ is the number of instances in $V$ that are in class $C_j$.

The mutual information between $U$ and $V$ is calculated by:

$$\text{MI}(U, V) = \sum_{i=1}^{R} \sum_{j=1}^{C} P(i, j) \log \left( \frac{P(i, j)}{P(i)P'(j)} \right)$$

Where $P(i, j)$ is the number of instances with label $R_i$ and also with label $C_j$.

The normalized mutual information is defined as

$$\text{NMI}(U, V) = \frac{\text{MI}(U, V)}{\sqrt{H(U)H(V)}}$$

This value of the mutual information and also the normalized variant is not adjusted for chance and will tend to increase as the number of different labels (clusters) increases, regardless of the actual amount of “mutual information” between the label assignments.

The expected value for the mutual information can be calculated using the following equation, from Vinh, Epps, and Bailey, (2009). In this equation, $a_i$ is the number of instances with label $U_i$ and $b_j$ is the number of instances with label $V_j$.

$$E[\text{MI}(U, V)] = \sum_{i=1}^{R} \sum_{j=1}^{C} \sum_{n_{ij}=a_i+b_j-N}^{\min(a_i,b_j)} \frac{n_{ij}}{N} \log \left( \frac{N.n_{ij}}{a_i b_j} \right) \frac{a_i!b_j!(N-a_i)!(N-b_j)!}{N!n_{ij}!(a_i-n_{ij})!(b_j-n_{ij})!(N-a_i-b_j+n_{ij})!}$$

Using the expected value, the adjusted mutual information can then be calculated using a similar form to that of the adjusted Rand index:

$$\text{AMI} = \frac{\text{MI} - E[\text{MI}]}{\max(H(U), H(V)) - E[\text{MI}]}$$
References

- Wikipedia entry for the (normalized) Mutual Information
- Wikipedia entry for the Adjusted Mutual Information

Homogeneity, completeness and V-measure

Presentation and usage  Given the knowledge of the ground truth class assignments of the samples, it is possible to define some intuitive metric using conditional entropy analysis.

In particular Rosenberg and Hirschberg (2007) define the following two desirable objectives for any cluster assignment:

- **homogeneity**: each cluster contains only members of a single class.
- **completeness**: all members of a given class are assigned to the same cluster.

We can turn those concept as scores `homogeneity_score` and `completeness_score`. Both are bounded below by 0.0 and above by 1.0 (higher is better):

```python
>>> from sklearn import metrics
>>> labels_true = [0, 0, 0, 1, 1, 1]
>>> labels_pred = [0, 0, 1, 1, 2, 2]

>>> metrics.homogeneity_score(labels_true, labels_pred)
0.66...

>>> metrics.completeness_score(labels_true, labels_pred)
0.42...
```

Their harmonic mean called **V-measure** is computed by `v_measure_score`:

```python
>>> metrics.v_measure_score(labels_true, labels_pred)
0.51...
```

The V-measure is actually equivalent to the mutual information (NMI) discussed above normalized by the sum of the label entropies [B2011].

Homogeneity, completeness and V-measure can be computed at once using `homogeneity_completeness_v_measure` as follows:

```python
>>> metrics.homogeneity_completeness_v_measure(labels_true, labels_pred)
...
(0.66..., 0.42..., 0.51...)
```

The following clustering assignment is slightly better, since it is homogeneous but not complete:

```python
>>> labels_pred = [0, 0, 0, 1, 2, 2]
>>> metrics.homogeneity_completeness_v_measure(labels_true, labels_pred)
```
Note: `v_measure_score` is **symmetric**: it can be used to evaluate the **agreement** of two independent assignments on the same dataset.

This is not the case for `completeness_score` and `homogeneity_score`: both are bound by the relationship: `homogeneity_score(a, b) == completeness_score(b, a)`.

**Advantages**

- **Bounded scores**: 0.0 is as bad as it can be, 1.0 is a perfect score.
- Intuitive interpretation: clustering with bad V-measure can be **qualitatively analyzed in terms of homogeneity and completeness** to better feel what ‘kind’ of mistakes is done by the assignement.
- **No assumption is made on the cluster structure**: can be used to compare clustering algorithms such as k-means which assumes isotropic blob shapes with results of spectral clustering algorithms which can find cluster with “folded” shapes.

**Drawbacks**

- The previously introduced metrics are not normalized w.r.t. random labeling: this means that depending on the number of samples, clusters and ground truth classes, a completely random labeling will not always yield the same values for homogeneity, completeness and hence v-measure. In particular **random labeling won’t yield zero scores especially when the number of clusters is large**.

  This problem can safely be ignored when the number of samples is more than a thousand and the number of clusters is less than 10. **For smaller sample sizes or larger number of clusters it is safer to use an adjusted index such as the Adjusted Rand Index (ARI).**

- These metrics **require the knowledge of the ground truth classes** while almost never available in practice or requires manual assignment by human annotators (as in the supervised learning setting).

**Examples:**

- **Adjustment for chance in clustering performance evaluation**: Analysis of the impact of the dataset size on the value of clustering measures for random assignments.

**Mathematical formulation**

Homogeneity and completeness scores are formally given by:

\[
h = 1 - \frac{H(C|K)}{H(C)}
\]

\[
e = 1 - \frac{H(K|C)}{H(K)}
\]
Clustering measures for 2 random uniform labelings with equal number of clusters

- adjusted_rand_score
- v_measure_score
- adjusted_mutual_info_score
- mutual_info_score

Score value

Number of clusters (Number of samples is fixed to 100)
where $H(C|K)$ is the **conditional entropy of the classes given the cluster assignments** and is given by:

$$H(C|K) = - \sum_{c=1}^{\left| C \right|} \sum_{k=1}^{\left| K \right|} \frac{n_{c,k}}{n} \cdot \log \left( \frac{n_{c,k}}{n_k} \right)$$

and $H(C)$ is the **entropy of the classes** and is given by:

$$H(C) = - \sum_{c=1}^{\left| C \right|} \frac{n_c}{n} \cdot \log \left( \frac{n_c}{n} \right)$$

with $n$ the total number of samples, $n_c$ and $n_k$ the number of samples respectively belonging to class $c$ and cluster $k$, and finally $n_{c,k}$ the number of samples from class $c$ assigned to cluster $k$.

The **conditional entropy of clusters given class** $H(K|C)$ and the **entropy of clusters** $H(K)$ are defined in a symmetric manner.

Rosenberg and Hirschberg further define **V-measure** as the **harmonic mean of homogeneity and completeness**:

$$v = 2 \cdot \frac{h \cdot c}{h + c}$$

**References**

**Silhouette Coefficient**

**Presentation and usage** If the ground truth labels are not known, evaluation must be performed using the model itself. The Silhouette Coefficient (**sklearn.metrics.silhouette_score**) is an example of such an evaluation, where a higher Silhouette Coefficient score relates to a model with better defined clusters. The Silhouette Coefficient is defined for each sample and is composed of two scores:

- $a$: The mean distance between a sample and all other points in the same class.
- $b$: The mean distance between a sample and all other points in the next nearest cluster.

The Silhouette Coefficient $s$ for a single sample is then given as:

$$s = \frac{b - a}{\max(a, b)}$$

The Silhouette Coefficient for a set of samples is given as the mean of the Silhouette Coefficient for each sample.

```python
>>> from sklearn import metrics
>>> from sklearn.metrics import pairwise_distances
>>> from sklearn import datasets
>>> dataset = datasets.load_iris()
>>> X = dataset.data
>>> y = dataset.target
```

In normal usage, the Silhouette Coefficient is applied to the results of a cluster analysis.
>>> import numpy as np
>>> from sklearn.cluster import KMeans
>>> kmeans_model = KMeans(n_clusters=3, random_state=1).fit(X)
>>> labels = kmeans_model.labels_
>>> metrics.silhouette_score(X, labels, metric='euclidean')
...
0.55...

References


Advantages

- The score is bounded between -1 for incorrect clustering and +1 for highly dense clustering. Scores around zero indicate overlapping clusters.
- The score is higher when clusters are dense and well separated, which relates to a standard concept of a cluster.

Drawbacks

- The Silhouette Coefficient is generally higher for convex clusters than other concepts of clusters, such as density based clusters like those obtained through DBSCAN.

1.4.4 Decomposing signals in components (matrix factorization problems)

Principal component analysis (PCA)

Exact PCA and probabilistic interpretation

PCA is used to decompose a multivariate dataset in a set of successive orthogonal components that explain a maximum amount of the variance. In scikit-learn, PCA is implemented as a transformer object that learns n components in its fit method, and can be used on new data to project it on these components.

The optional parameter whiten=True parameter make it possible to project the data onto the singular space while scaling each component to unit variance. This is often useful if the models down-stream make strong assumptions on the isotropy of the signal: this is for example the case for Support Vector Machines with the RBF kernel and the K-Means clustering algorithm. However in that case the inverse transform is no longer exact since some information is lost while forward transforming.

In addition, the ProbabilisticPCA object provides a probabilistic interpretation of the PCA that can give a likelihood of data based on the amount of variance it explains. As such it implements a score method that can be used in cross-validation.

Below is an example of the iris dataset, which is comprised of 4 features, projected on the 2 dimensions that explain most variance:

Examples:

- Comparison of LDA and PCA 2D projection of Iris dataset
Approximate PCA

Often we are interested in projecting the data onto a lower dimensional space that preserves most of the variance by dropping the singular vector of components associated with lower singular values.

For instance for face recognition, if we work with 64x64 gray level pixel pictures the dimensionality of the data is 4096 and it is slow to train a RBF Support Vector Machine on such wide data. Furthermore we know that intrinsic dimensionality of the data is much lower than 4096 since all faces pictures look alike. The samples lie on a manifold of much lower dimension (say around 200 for instance). The PCA algorithm can be used to linearly transform the data while both reducing the dimensionality and preserve most of the explained variance at the same time.

The class `RandomizedPCA` is very useful in that case: since we are going to drop most of the singular vectors it is much more efficient to limit the computation to an approximated estimate of the singular vectors we will keep to actually perform the transform.

For instance, the following shows 16 sample portraits (centered around 0.0) from the Olivetti dataset. On the right hand side are the first 16 singular vectors reshaped as portraits. Since we only require the top 16 singular vectors of a dataset with size $n_{\text{samples}} = 400$ and $n_{\text{features}} = 64 \times 64 = 4096$, the computation time it less than 1s:
RandomizedPCA can hence be used as a drop in replacement for PCA minor the exception that we need to give it the size of the lower dimensional space \( n_{\text{components}} \) as mandatory input parameter.

If we note \( n_{\text{max}} = \max(n_{\text{samples}}, n_{\text{features}}) \) and \( n_{\text{min}} = \min(n_{\text{samples}}, n_{\text{features}}) \), the time complexity of RandomizedPCA is \( O(n_{\text{max}}^2 \cdot n_{\text{components}}) \) instead of \( O(n_{\text{max}}^2 \cdot n_{\text{min}}) \) for the exact method implemented in PCA.

The memory footprint of RandomizedPCA is also proportional to \( 2 \cdot n_{\text{max}} \cdot n_{\text{components}} \) instead of \( n_{\text{max}} \cdot n_{\text{min}} \) for the exact method.

Furthermore RandomizedPCA is able to work with \textit{scipy.sparse} matrices as input which make it suitable for reducing the dimensionality of features extracted from text documents for instance.

Note: the implementation of \texttt{inverse\_transform} in RandomizedPCA is not the exact inverse transform of \texttt{transform} even when \texttt{whiten=False} (default).

Examples:

- \textit{Faces recognition example using eigenfaces and SVMs}
- \textit{Faces dataset decompositions}
Kernel PCA

**KernelPCA** is an extension of PCA which achieves non-linear dimensionality reduction through the use of kernels. It has many applications including denoising, compression and structured prediction (kernel dependency estimation). **KernelPCA** supports both `transform` and `inverse_transform`.

**Examples:**
- **Kernel PCA**

Sparse Principal Components Analysis (SparsePCA and MiniBatchSparsePCA)

**SparsePCA** is a variant of PCA, with the goal of extracting the set of sparse components that best reconstruct the data.

Mini Batch Sparse PCA (**MiniBatchSparsePCA**) is a variant of **SparsePCA** that is faster but less accurate. The increased speed is reached by iterating over small chunks of the set of features, for a given number of iterations.

Principal component analysis (**PCA**) has the disadvantage that the components extracted by this method have exclusively dense expressions, i.e. they have non-zero coefficients when expressed as linear combinations of the original variables. This can make interpretation difficult. In many cases, the real underlying components can be more naturally imagined as sparse vectors; for example in face recognition, components might naturally map to parts of faces.
Sparse principal components yields a more parsimonious, interpretable representation, clearly emphasizing which of the original features contribute to the differences between samples.

The following example illustrates 16 components extracted using sparse PCA from the Olivetti faces dataset. It can be seen how the regularization term induces many zeros. Furthermore, the natural structure of the data causes the non-zero coefficients to be vertically adjacent. The model does not enforce this mathematically: each component is a vector $h \in \mathbb{R}^{4096}$, and there is no notion of vertical adjacency except during the human-friendly visualization as 64x64 pixel images. The fact that the components shown below appear local is the effect of the inherent structure of the data, which makes such local patterns minimize reconstruction error. There exist sparsity-inducing norms that take into account adjacency and different kinds of structure; see see [Jen09] for a review of such methods. For more details on how to use Sparse PCA, see the Examples section below.

Note that there are many different formulations for the Sparse PCA problem. The one implemented here is based on [Mrl09]. The optimization problem solved is a PCA problem (dictionary learning) with an $\ell_1$ penalty on the components:

$$ (U^*, V^*) = \arg \min_{U, V} \frac{1}{2} \|X - UV\|_2^2 + \alpha \|V\|_1 $$

subject to $\|U_k\|_2 = 1$ for all $0 \leq k < n_{components}$

The sparsity inducing $\ell_1$ norm also prevents learning components from noise when few training samples are available.
The degree of penalization (and thus sparsity) can be adjusted through the hyperparameter \textit{alpha}. Small values lead to a gently regularized factorization, while larger values shrink many coefficients to zero.

\textbf{Note: } While in the spirit of an online algorithm, the class \texttt{MiniBatchSparsePCA} does not implement \textit{partial_fit} because the algorithm is online along the features direction, not the samples direction.

\textbf{Examples:}

- \textit{Faces dataset decompositions}

\textbf{References:}

\textbf{Dictionary Learning}

\textbf{Sparse coding with a precomputed dictionary}

The \texttt{SparseCoder} object is an estimator that can be used to transform signals into sparse linear combination of atoms from a fixed, precomputed dictionary such as a discrete wavelet basis. This object therefore does not implement a \textit{fit} method. The transformation amounts to a sparse coding problem: finding a representation of the data as a linear combination of as few dictionary atoms as possible. All variations of dictionary learning implement the following transform methods, controllable via the \textit{transform_method} initialization parameter:

- Orthogonal matching pursuit (\textit{Orthogonal Matching Pursuit (OMP)})
- Least-angle regression (\textit{Least Angle Regression})
- Lasso computed by least-angle regression
- Lasso using coordinate descent (\textit{Lasso})
- Thresholding

Thresholding is very fast but it does not yield accurate reconstructions. They have been shown useful in literature for classification tasks. For image reconstruction tasks, orthogonal matching pursuit yields the most accurate, unbiased reconstruction.

The dictionary learning objects offer, via the \textit{split_code} parameter, the possibility to separate the positive and negative values in the results of sparse coding. This is useful when dictionary learning is used for extracting features that will be used for supervised learning, because it allows the learning algorithm to assign different weights to negative loadings of a particular atom, from to the corresponding positive loading.

The split code for a single sample has length $2 \times n\_atoms$ and is constructed using the following rule: First, the regular code of length $n\_atoms$ is computed. Then, the first $n\_atoms$ entries of the split_code are filled with the positive part of the regular code vector. The second half of the split code is filled with the negative part of the code vector, only with a positive sign. Therefore, the split_code is non-negative.

\textbf{Examples:}

- \textit{Sparse coding with a precomputed dictionary}
Generic dictionary learning

Dictionary learning (DictionaryLearning) is a matrix factorization problem that amounts to finding a (usually overcomplete) dictionary that will perform good at sparsely encoding the fitted data.

Representing data as sparse combinations of atoms from an overcomplete dictionary is suggested to be the way the mammal primary visual cortex works. Consequently, dictionary learning applied on image patches has been shown to give good results in image processing tasks such as image completion, inpainting and denoising, as well as for supervised recognition tasks.

Dictionary learning is an optimization problem solved by alternatively updating the sparse code, as a solution to multiple Lasso problems, considering the dictionary fixed, and then updating the dictionary to best fit the sparse code.

\[
(U^*, V^*) = \arg \min_{U, V} \frac{1}{2} \|X - UV\|_2^2 + \alpha \|U\|_1 \\
\text{subject to } \|V_k\|_2 = 1 \text{ for all } 0 \leq k < n_{\text{atoms}}
\]

After using such a procedure to fit the dictionary, the transform is simply a sparse coding step that shares the same implementation with all dictionary learning objects (see Sparse coding with a precomputed dictionary).
The following image shows how a dictionary learned from 4x4 pixel image patches extracted from part of the image of Lena looks like.

Examples:

- Image denoising using dictionary learning

References:


Mini-batch dictionary learning

MiniBatchDictionaryLearning implements a faster, but less accurate version of the dictionary learning algorithm that is better suited for large datasets.

By default, MiniBatchDictionaryLearning divides the data into mini-batches and optimizes in an online manner by cycling over the mini-batches for the specified number of iterations. However, at the moment it does not implement a stopping condition.

The estimator also implements partial_fit, which updates the dictionary by iterating only once over a mini-batch. This can be used for online learning when the data is not readily available from the start, or for when the data does not fit into the memory.

Independent component analysis (ICA)

Independent component analysis separates a multivariate signal into additive subcomponents that are maximally independent. It is implemented in scikit-learn using the Fast ICA algorithm.

It is classically used to separate mixed signals (a problem known as blind source separation), as in the example below:

ICA can also be used as yet another non linear decomposition that finds components with some sparsity:
Examples:

- Blind source separation using FastICA
- FastICA on 2D point clouds
- Faces dataset decompositions

Non-negative matrix factorization (NMF or NNMF)

**NMF** is an alternative approach to decomposition that assumes that the data and the components are non-negative. **NMF** can be plugged in instead of **PCA** or its variants, in the cases where the data matrix does not contain negative values.

Unlike **PCA**, the representation of a vector is obtained in an additive fashion, by superimposing the components, without subtracting. Such additive models are efficient for representing images and text.

It has been observed in [Hoyer, 04] that, when carefully constrained, **NMF** can produce a parts-based representation of the dataset, resulting in interpretable models. The following example displays 16 sparse components found by **NMF** from the images in the Olivetti faces dataset, in comparison with the PCA eigenfaces.

![Eigenfaces - RandomizedPCA - Train time 0.5s](image)

![Non-negative components - NMF - Train time 2.5s](image)

The **init** attribute determines the initialization method applied, which has a great impact on the performance of the method. **NMF** implements the method Nonnegative Double Singular Value Decomposition. NNDSVD is based on two SVD processes, one approximating the data matrix, the other approximating positive sections of the resulting partial...
SVD factors utilizing an algebraic property of unit rank matrices. The basic NNDSVD algorithm is better fit for sparse factorization. Its variants NNDSVDa (in which all zeros are set equal to the mean of all elements of the data), and NNDSVDar (in which the zeros are set to random perturbations less than the mean of the data divided by 100) are recommended in the dense case.

NMF can also be initialized with random non-negative matrices, by passing an integer seed or a RandomState to init. In NMF, sparseness can be enforced by setting the attribute sparseness to data or components. Sparse components lead to localized features, and sparse data leads to a more efficient representation of the data.

Examples:
- Faces dataset decompositions
- Topics extraction with Non-Negative Matrix Factorization

References:
- “Learning the parts of objects by non-negative matrix factorization” D. Lee, S. Seung, 1999
- “Non-negative Matrix Factorization with Sparseness Constraints” P. Hoyer, 2004
- “SVD based initialization: A head start for nonnegative matrix factorization” C. Boutsidis, E. Gallopoulos, 2008

1.4.5 Covariance estimation

Many statistical problems require at some point the estimation of a population’s covariance matrix, which can be seen as an estimation of data set scatter plot shape. Most of the time, such an estimation has to be done on a sample whose properties (size, structure, homogeneity) has a large influence on the estimation’s quality. The sklearn.covariance package aims at providing tools affording an accurate estimation of a population’s covariance matrix under various settings.

We assume that the observations are independent and identically distributed (i.i.d.).

Empirical covariance

The covariance matrix of a data set is known to be well approximated with the classical Maximum Likelihood Estimator (or empirical covariance), provided the number of observations is large enough compared to the number of features (the variables describing the observations). More precisely, the Maximum Likelihood Estimator of a sample is an unbiased estimator of the corresponding population covariance matrix.

The empirical covariance matrix of a sample can be computed using the empirical_covariance function of the package, or by fitting an EmpiricalCovariance object to the data sample with the EmpiricalCovariance.fit method. Be careful that depending whether the data are centered or not, the result will be different, so one may want to use the assume_centered parameter accurately.

Examples:
- See Ledoit-Wolf vs Covariance simple estimation for an example on how to fit an EmpiricalCovariance object to data.
Shrunk Covariance

Basic shrinkage

Despite being an unbiased estimator of the covariance matrix, the Maximum Likelihood Estimator is not a good estimator of the eigenvalues of the covariance matrix, so the precision matrix obtained from its inversion is not accurate. Sometimes, it even occurs that the empirical covariance matrix cannot be inverted for numerical reasons. To avoid such an inversion problem, a transformation of the empirical covariance matrix has been introduced: the \textit{shrinkage}. It consists in reducing the ratio between the smallest and the largest eigenvalue of the empirical covariance matrix. This can be done by simply shifting every eigenvalue according to a given offset, which is equivalent of finding the $l_2$-penalized Maximum Likelihood Estimator of the covariance matrix, or by reducing the highest eigenvalue while increasing the smallest with the help of a convex transformation: $\Sigma_{\text{shrunk}} = (1 - \alpha)\hat{\Sigma} + \alpha \frac{\text{Tr}\hat{\Sigma}}{p} \text{I}_d$. The latter approach has been implemented in scikit-learn.

A convex transformation (with a user-defined shrinkage coefficient) can be directly applied to a pre-computed covariance with the \texttt{shrunk\_covariance} method. Also, a shrunk estimator of the covariance can be fitted to data with a \texttt{ShrunkCovariance} object and its \texttt{ShrunkCovariance.fit} method. Again, depending whether the data are centered or not, the result will be different, so one may want to use the \texttt{assume\_centered} parameter accurately.

Examples:
- See \textit{Ledoit-Wolf vs Covariance simple estimation} for an example on how to fit a \texttt{ShrunkCovariance} object to data.

Ledoit-Wolf shrinkage

In their 2004 paper \cite{1}, O. Ledoit and M. Wolf propose a formula so as to compute the optimal shrinkage coefficient $\alpha$ that minimizes the Mean Squared Error between the estimated and the real covariance matrix in terms of Frobenius norm.

The Ledoit-Wolf estimator of the covariance matrix can be computed on a sample with the \texttt{ledoit\_wolf} function of the \texttt{sklearn.covariance} package, or it can be otherwise obtained by fitting a \texttt{LedoitWolf} object to the same sample.


Examples:
- See \textit{Ledoit-Wolf vs Covariance simple estimation} for an example on how to fit a \texttt{LedoitWolf} object to data and for visualizing the performances of the Ledoit-Wolf estimator in terms of likelihood.

Oracle Approximating Shrinkage

Under the assumption that the data are Gaussian distributed, Chen et al. \cite{2} derived a formula aimed at choosing a shrinkage coefficient that yields a smaller Mean Squared Error than the one given by Ledoit and Wolf’s formula. The resulting estimator is known as the Oracle Shrinkage Approximating estimator of the covariance.

The OAS estimator of the covariance matrix can be computed on a sample with the \texttt{oas} function of the \texttt{sklearn.covariance} package, or it can be otherwise obtained by fitting an \texttt{OAS} object to the same sample. The formula we used to implement the OAS does not correspond to the one given in the article. It has been taken from the MATLAB program available from the author’s webpage (https://tbayes.eecs.umich.edu/yilun/covestimation).

1.4. Unsupervised learning
Sparse inverse covariance

The matrix inverse of the covariance matrix, often called the precision matrix, is proportional to the partial correlation matrix. It gives the partial independence relationship. In other words, if two features are independent conditionally on the others, the corresponding coefficient in the precision matrix will be zero. This is why it makes sense to estimate a sparse precision matrix: by learning independence relations from the data, the estimation of the covariance matrix is better conditioned. This is known as covariance selection.

In the small-samples situation, in which \( n_{\text{samples}} \) is on the order of magnitude of \( n_{\text{features}} \) or smaller, sparse inverse covariance estimators tend to work better than shrunk covariance estimators. However, in the opposite situation, or for very correlated data, they can be numerically unstable. In addition, unlike shrinkage estimators, sparse estimators are able to recover off-diagonal structure.

The GraphLasso estimator uses an \( l_1 \) penalty to enforce sparsity on the precision matrix: the higher its \( \alpha \) parameter, the more sparse the precision matrix. The corresponding GraphLassoCV object uses cross-validation to automatically set the \( \alpha \) parameter.

Note: Structure recovery

Recovering a graphical structure from correlations in the data is a challenging thing. If you are interested in such recovery keep in mind that:

- Recovery is easier from a correlation matrix than a covariance matrix: standardize your observations before running GraphLasso.
Figure 1.4: A comparison of maximum likelihood, shrinkage and sparse estimates of the covariance and precision matrix in the very small samples settings.
• If the underlying graph has nodes with much more connections than the average node, the algorithm will miss some of these connections.

• If your number of observations is not large compared to the number of edges in your underlying graph, you will not recover it.

• Even if you are in favorable recovery conditions, the alpha parameter chosen by cross-validation (e.g. using the GraphLassoCV object) will lead to selecting too many edges. However, the relevant edges will have heavier weights than the irrelevant ones.

The mathematical formulation is the following:

\[
\hat{K} = \arg\min_K (\text{tr} SK - \log\det K + \alpha \|K\|_1)
\]

Where \(K\) is the precision matrix to be estimated, and \(S\) is the sample covariance matrix. \(\|K\|_1\) is the sum of the absolute values of off-diagonal coefficients of \(K\). The algorithm employed to solve this problem is the GLasso algorithm, from the Friedman 2008 Biostatistics paper. It is the same algorithm as in the R glasso package.

Examples:
• **Sparse inverse covariance estimation**: example on synthetic data showing some recovery of a structure, and comparing to other covariance estimators.
• **Visualizing the stock market structure**: example on real stock market data, finding which symbols are most linked.

References:
• Friedman et al, “Sparse inverse covariance estimation with the graphical lasso”, Biostatistics 9, pp 432, 2008

Robust Covariance Estimation

Real data set are often subjects to measurement or recording errors. Regular but uncommon observations may also appear for a variety of reason. Every observation which is very uncommon is called an outlier. The empirical covariance estimator and the shrunk covariance estimators presented above are very sensitive to the presence of outlying observations in the data. Therefore, one should use robust covariance estimators to estimate the covariance of its real data sets. Alternatively, robust covariance estimators can be used to perform outlier detection and discard/downweight some observations according to further processing of the data.

The sklearn.covariance package implements a robust estimator of covariance, the Minimum Covariance Determinant [3].

Minimum Covariance Determinant

The Minimum Covariance Determinant estimator is a robust estimator of a data set’s covariance introduced by P.J.Rousseeuw in [3]. The idea is to find a given proportion (h) of “good” observations which are not outliers and compute their empirical covariance matrix. This empirical covariance matrix is then rescaled to compensate the performed selection of observations (“consistency step”). Having computed the Minimum Covariance Determinant estimator, one can give weights to observations according to their Mahalanobis distance, leading the a reweighted estimate of the covariance matrix of the data set (“reweighting step”).
Rousseuw and Van Driessen [4] developed the FastMCD algorithm in order to compute the Minimum Covariance Determinant. This algorithm is used in scikit-learn when fitting an MCD object to data. The FastMCD algorithm also computes a robust estimate of the data set location at the same time.

Raw estimates can be accessed as `raw_location_` and `raw_covariance_` attributes of a `MinCovDet` robust covariance estimator object.


Examples:

- See Robust vs Empirical covariance estimate for an example on how to fit a `MinCovDet` object to data and see how the estimate remains accurate despite the presence of outliers.
- See Robust covariance estimation and Mahalanobis distances relevance to visualize the difference between `EmpiricalCovariance` and `MinCovDet` covariance estimators in terms of Mahalanobis distance (so we get a better estimate of the precision matrix too).

### Influence of outliers on location and covariance estimates

![Influence of outliers on location and covariance estimates](image1.png)

### Separating inliers from outliers using a Mahalanobis distance

![Separating inliers from outliers using a Mahalanobis distance](image2.png)

#### 1.4.6 Novelty and Outlier Detection

Many applications require being able to decide whether a new observation belongs to the same distribution as exiting observations (it is an inlier), or should be considered as different (it is an outlier). Often, this ability is used to clean real data sets. Two important distinction must be made:

- **novelty detection** The training data is not polluted by outliers, and we are interested in detecting anomalies in new observations.
- **outlier detection** The training data contains outliers, and we need to fit the central mode of the training data, ignoring the deviant observations.

The scikit-learn project provides a set of machine learning tools that can be used both for novelty or outliers detection. This strategy is implemented with objects learning in an unsupervised way from the data:

```python
estimator.fit(X_train)
```

new observations can then be sorted as inliers or outliers with a `predict` method:

```python
estimator.predict(X_test)
```

Inliers are labeled 0, while outliers are labeled 1.
Novelty Detection

Consider a data set of \( n \) observations from the same distribution described by \( p \) features. Consider now that we add one more observation to that data set. Is the new observation so different from the others that we can doubt it is regular? (i.e. does it come from the same distribution?) Or on the contrary, is it so similar to the other that we cannot distinguish it from the original observations? This is the question adressed by the novelty detection tools and methods.

In general, it is about to learn a rough, close frontier delimiting the contour of the initial observations distribution, plotted in embedding \( p \)-dimensional space. Then, if further observations lay within the frontier-delimited subspace, they are considered as coming from the same population than the initial observations. Otherwise, if they lay outside the frontier, we can say that they are abnormal with a given confidence in our assessment.

The One-Class SVM has been introduced in [1] for that purpose and implemented in the Support Vector Machines module in the `svm.OneClassSVM` object. It requires the choice of a kernel and a scalar parameter to define a frontier. The RBF kernel is usually chosen although there exist no exact formula or algorithm to set its bandwith parameter. This is the default in the scikit-learn implementation. The \( \nu \) parameter, also known as the margin of the One-Class SVM, corresponds to the probability of finding a new, but regular, observation outside the frontier.

Examples:

- See One-class SVM with non-linear kernel (RBF) for visualizing the frontier learned around some data by a `svm.OneClassSVM` object.

![Novelty Detection](image.png)

error train: 21/200 ; errors novel regular: 6/20 ; errors novel abnormal: 0/20

Outlier Detection

Outlier detection is similar to novelty detection in the sense that the goal is to separate a core of regular observations from some polluting ones, called “outliers”. Yet, in the case of outlier detection, we don’t have a clean data set representing the population of regular observations that can be used to train any tool.
Fitting an elliptic envelop

One common way of performing outlier detection is to assume that the regular data come from a known distribution (e.g. data are Gaussian distributed). From this assumption, we generally try to define the “shape” of the data, and can define outlying observations as observations which stand far enough from the fit shape.

The scikit-learn provides an object `covariance.EllipticEnvelope` that fits a robust covariance estimate to the data, and thus fits an ellipse to the central data points, ignoring points outside the central mode.

For instance, assuming that the inlier data are Gaussian distributed, it will estimate the inlier location and covariance in a robust way (i.e. without being influenced by outliers). The Mahalanobis distances obtained from this estimate is used to derive a measure of outlyingness. This strategy is illustrated below.

Examples:

- See `Robust covariance estimation and Mahalanobis distances relevance` for an illustration of the difference between using a standard (`covariance.EmpiricalCovariance`) or a robust estimate (`covariance.MinCovDet`) of location and covariance to assess the degree of outlyingness of an observation.

References:

One-class SVM versus elliptic envelop

Strictly-speaking, the One-class SVM is not an outlier-detection method, but a novelty-detection method: it’s training set should not be contaminated by outliers as it may fit them. That said, outlier detection in high-dimension, or without...
any assumptions on the distribution of the inlying data is very challenging, and a One-class SVM gives useful results in these situations.

The examples below illustrate how the performance of the `covariance.EllipticEnvelope` degrades as the data is less and less unimodal. `svm.OneClassSVM` works better on data with multiple modes.

<table>
<thead>
<tr>
<th>Table 1.1: Comparing One-class SVM approach, and elliptic envelopp</th>
</tr>
</thead>
<tbody>
<tr>
<td>For a inlier mode well-centered and elliptic, the <code>svm.OneClassSVM</code> is not able to benefit from the rotational symmetry of the inlier population. In addition, it fits a bit the outliers present in the training set. On the opposite, the decision rule based on fitting an <code>covariance.EllipticEnvelope</code> learns an ellipse, which fits well the inlier distribution.</td>
</tr>
<tr>
<td>As the inlier distribution becomes bimodal, the <code>covariance.EllipticEnvelope</code> does not fit well the inliers. However, we can see that the <code>svm.OneClassSVM</code> tends to overfit: because it has not model of inliers, it interprets a region where, by chance some outliers are clustered, as inliers.</td>
</tr>
<tr>
<td>If the inlier distribution is strongly non Gaussian, the <code>svm.OneClassSVM</code> is able to recover a reasonable approximation, whereas the <code>covariance.EllipticEnvelope</code> completely fails.</td>
</tr>
</tbody>
</table>

Examples:

- See [Outlier detection with several methods](#) for a comparison of the `svm.OneClassSVM` (tuned to perform like an outlier detection method) and a covariance-based outlier detection with `covariance.MinCovDet`.

### 1.4.7 Hidden Markov Models

`sklearn.hmm` implements the algorithms of Hidden Markov Model (HMM). HMM is a generative probabilistic model, in which a sequence of observable $X$ variable is generated by a sequence of internal hidden state $Z$. The hidden states can not be observed directly. The transition of hidden states is assumed to be the first order Markov Chain. It can be specified by the start probability vector $\Pi$ and the transition probability matrix $A$. The emission probability of observable can be any distribution with the parameters $\Theta_i$ conditioned on the current hidden state index. (e.g. Multinomial, Gaussian). Thus the HMM can be completely determined by $\Pi$, $A$ and $\Theta_i$.

There are three fundamental problems of HMM:

- Given the model parameters and observed data, estimate the optimal sequence of hidden states.
- Given the model parameters and observed data, calculate the likelihood of the data.
- Given just the observed data, estimate the model parameters.

The first and the second problem can be solved by the dynamic programing algorithms known as the Viterbi algorithm and the Forward-Backward algorithm respectively. The last one can be solved by an Expectation-Maximization (EM) iterative algorithm, known as Baum-Welch algorithm.

See the ref listed below for further detailed information.
Using HMM

Classes in this module include `MultinomialHMM`, `GaussianHMM`, and `GMMHMM`. They implement HMM with emission probability of Multinomial distribution, Gaussian distribution and the mixture of Gaussian distributions.

Building HMM and generating samples

You can build an HMM instance by passing the parameters described above to the constructor. Then, you can generate samples from the HMM by calling `sample`:

```python
>>> import numpy as np
>>> from sklearn import hmm

>>> startprob = np.array([0.6, 0.3, 0.1])
>>> transmat = np.array([[0.7, 0.2, 0.1], [0.3, 0.5, 0.2], [0.3, 0.3, 0.4]])
>>> means = np.array([[0.0, 0.0], [3.0, -3.0], [5.0, 10.0]])
>>> covars = np.tile(np.identity(2), (3, 1, 1))
>>> model = hmm.GaussianHMM(3, "full", startprob, transmat)
>>> model.means_ = means
>>> model.covars_ = covars
>>> X, Z = model.sample(100)
```
Examples:
  • Demonstration of sampling from HMM

Training HMM parameters and infering the hidden states

You can train the HMM by calling `fit` method. The input is “the list” of the sequence of observed value. Note, since EM-algorithm is a gradient based optimization method, it will generally be stuck at local optimal. You should try to run `fit` with various initialization and select the highest scored model. The score of the model can be calculated by the `score` method. The infered optimal hidden states can be obtained by calling `predict` method. The `predict` method can be specified with decoder algorithm. Currently Viterbi algorithm `viterbi`, and maximum a posteriori estimation `map` is supported. This time, the input is a single sequence of observed values.

```python
>>> model2 = hmm.GaussianHMM(3, "full")
>>> model2.fit([X])
GaussianHMM(algorithm='viterbi',...)
>>> Z2 = model.predict(X)
```

Examples:
  • Gaussian HMM of stock data

Implementing HMMs with other emission probabilities

If you want to implement other emission probability (e.g. Poisson), you have to make you own HMM class by inheriting the `_BaseHMM` and override necessary methods. They should be `__init__`, `_compute_log_likelihood`, `_set` and `_get` for additional parameters, `_initialize_sufficient_statistics`, `_accumulate_sufficient_statistics` and `_do_mstep`.

1.5 Model Selection

1.5.1 Cross-Validation: evaluating estimator performance

Learning the parameters of a prediction function and testing it on the same data is a methodological mistake: a model that would just repeat the labels of the samples that it has just seen would have a perfect score but would fail to predict anything useful on yet-unseen data.

To avoid over-fitting, we have to define two different sets: a training set `X_train`, `y_train` which is used for learning the parameters of a predictive model, and a testing set `X_test`, `y_test` which is used for evaluating the fitted predictive model.

In scikit-learn such a random split can be quickly computed with the `train_test_split` helper function. Let load the iris data set to fit a linear Support Vector Machine model on it:

```python
>>> import numpy as np
>>> from sklearn import cross_validation
>>> from sklearn import datasets
>>> from sklearn import svm

>>> iris = datasets.load_iris()
```
We can now quickly sample a training set while holding out 40% of the data for testing (evaluating) our classifier:

```python
>>> X_train, X_test, y_train, y_test = cross_validation.train_test_split(iris.data, iris.target, test_size=0.4, random_state=0)
```

```python
>>> X_train.shape, y_train.shape
((90, 4), (90,))
>>> X_test.shape, y_test.shape
((60, 4), (60,))
```

```python
>>> clf = svm.SVC(kernel='linear', C=1).fit(X_train, y_train)
>>> clf.score(X_test, y_test)
0.96...
```

However, by defining these two sets, we drastically reduce the number of samples which can be used for learning the model, and the results can depend on a particular random choice for the pair of (train, test) sets.

A solution is to **split the whole data several consecutive times in different train set and test set**, and to return the averaged value of the prediction scores obtained with the different sets. Such a procedure is called **cross-validation**. This approach can be **computationally expensive, but does not waste too much data** (as it is the case when fixing an arbitrary test set), which is a major advantage in problem such as inverse inference where the number of samples is very small.

### Computing cross-validated metrics

The simplest way to use perform cross-validation in to call the `cross_val_score` helper function on the estimator and the dataset.

The following example demonstrates how to estimate the accuracy of a linear kernel Support Vector Machine on the Iris dataset by splitting the data and fitting a model and computing the score 5 consecutive times (with different splits each time):

```python
>>> clf = svm.SVC(kernel='linear', C=1)
>>> scores = cross_validation.cross_val_score(clf, iris.data, iris.target, cv=5)
```

```python
>>> scores
array([ 1. ..., 0.96..., 0.89..., 0.96..., 1. ])
```

The mean score and the standard deviation of the score estimate are hence given by:

```python
>>> print "Accuracy: %0.2f (+/- %0.2f)" % (scores.mean(), scores.std() / 2)
Accuracy: 0.97 (+/- 0.02)
```

By default, the score computed at each CV iteration is the `score` method of the estimator. It is possible to change this by passing a custom scoring function, e.g. from the metrics module:

```python
>>> from sklearn import metrics
>>> cross_validation.cross_val_score(clf, iris.data, iris.target, cv=5, score_func=metrics.f1_score)
```

```python
array([ 1. ..., 0.96..., 0.89..., 0.96..., 1. ])
```

In the case of the Iris dataset, the samples are balanced across target classes hence the accuracy and the F1-score are almost equal.
When the `cv` argument is an integer, `cross_val_score` uses the `KFold` or `StratifiedKFold` strategies by default (depending on the absence or presence of the target array).

It is also possible to use other cross validation strategies by passing a cross validation iterator instead, for instance:

```python
>>> n_samples = iris.data.shape[0]
>>> cv = cross_validation.ShuffleSplit(n_samples, n_iterations=3,
...                                           test_size=0.3, random_state=0)

>>> cross_validation.cross_val_score(clf, iris.data, iris.target, cv=cv)
...
array([ 0.97..., 0.97..., 1. ])
```

The available cross validation iterators are introduced in the following.

### Examples

- Receiver operating characteristic (ROC) with cross validation,
- Recursive feature elimination with cross-validation,
- Parameter estimation using grid search with a nested cross-validation,
- Sample pipeline for text feature extraction and evaluation,

### Cross validation iterators

The following sections list utilities to generate boolean masks or indices that can be used to generate dataset splits according to different cross validation strategies.

#### Boolean mask vs integer indices

Most cross validators support generating both boolean masks or integer indices to select the samples from a given fold.

When the data matrix is sparse, only the integer indices will work as expected. Integer indexing is hence the default behavior (since version 0.10).

You can explicitly pass `indices=False` to the constructor of the CV object (when supported) to use the boolean mask method instead.

#### K-fold

`KFold` divides all the samples in math:`K` groups of samples, called folds (if `K = n`, this is equivalent to the Leave One Out strategy), of equal sizes (if possible). The prediction function is learned using `K - 1` folds, and the fold left out is used for test.

Example of 2-fold:

```python
>>> import numpy as np
>>> from sklearn.cross_validation import KFold
>>> X = np.array([[0., 0.], [1., 1.], [-1., -1.], [2., 2.]])
>>> Y = np.array([0, 1, 0, 1])

>>> kf = KFold(len(Y), 2, indices=False)
>>> print kf
sklearn.cross_validation.KFold(n=4, k=2)
```
>>> for train, test in kf:
...     print train, test
[False False True True] [ True True False False]
[ True True False False] [False False True True]

Each fold is constituted by two arrays: the first one is related to the training set, and the second one to the test set. Thus, one can create the training/test sets using:

>>> X_train, X_test, y_train, y_test = X[train], X[test], Y[train], Y[test]

If X or Y are scipy.sparse matrices, train and test need to be integer indices. It can be obtained by setting the parameter indices to True when creating the cross-validation procedure:

>>> X = np.array([[0., 0.], [1., 1.], [-1., -1.], [2., 2.], [3., 3.], [4., 4.], [0., 1.], ...
>>> Y = np.array([0, 0, 0, 1, 1, 1, 0], [0, 1, 1, 0])

>>> kf = KFold(len(Y), 2, indices=True)

Stratified K-Fold

StratifiedKFold is a variation of K-fold, which returns stratified folds, i.e. which creates folds by preserving the same percentage for each target class as in the complete set.

Example of stratified 2-fold:

>>> from sklearn.cross_validation import StratifiedKFold

>>> X = [[0., 0.], ...
>>>      [1., 1.], ...
>>>      [-1., -1.], ...
>>>      [2., 2.], ...
>>>      [3., 3.], ...
>>>      [4., 4.], ...
>>>      [0., 1.], ...
>>> Y = [0, 0, 0, 1, 1, 1, 0]

>>> skf = StratifiedKFold(Y, 2)

>>> for train, test in skf:
...     print train, test
[1 4 6] [0 2 3 5]
[0 2 3 5] [1 4 6]

Leave-One-Out - LOO

LeaveOneOut (or LOO) is a simple cross-validation. Each learning set is created by taking all the samples except one, the test set being the sample left out. Thus, for n samples, we have n different learning sets and n different tests set. This cross-validation procedure does not waste much data as only one sample is removed from the learning set:

>>> from sklearn.cross_validation import LeaveOneOut

>>> X = np.array([[0., 0.], [1., 1.], [-1., -1.], [2., 2.]])
Y = np.array([0, 1, 0, 1])

loo = LeaveOneOut(len(Y))
print loo
sklearn.cross_validation.LeaveOneOut(n=4)

for train, test in loo:
    print train, test
[1 2 3] [0]
[0 2 3] [1]
[0 1 3] [2]
[0 1 2] [3]

Leave-P-Out - LPO

LeavePOut is very similar to Leave-One-Out, as it creates all the possible training/test sets by removing $P$ samples from the complete set.

Example of Leave-2-Out:

from sklearn.cross_validation import LeavePOut
X = [[0., 0.], [1., 1.], [-1., -1.], [2., 2.]]
Y = [0, 1, 0, 1]
lpo = LeavePOut(len(Y), 2)
print lpo
sklearn.cross_validation.LeavePOut(n=4, p=2)

for train, test in lpo:
    print train, test
[2 3] [0 1]
[1 3] [0 2]
[1 2] [0 3]
[0 3] [1 2]
[0 2] [1 3]
[0 1] [2 3]

Leave-One-Label-Out - LOLO

LeaveOneLabelOut (LOLO) is a cross-validation scheme which holds out the samples according to a third-party provided label. This label information can be used to encode arbitrary domain specific stratifications of the samples as integers.

Each training set is thus constituted by all the samples except the ones related to a specific label.

For example, in the cases of multiple experiments, LOLO can be used to create a cross-validation based on the different experiments: we create a training set using the samples of all the experiments except one:

from sklearn.cross_validation import LeaveOneLabelOut
X = [[0., 0.], [1., 1.], [-1., -1.], [2., 2.]]
Y = [0, 1, 0, 1]
labels = [1, 1, 2, 2]
lolo = LeaveOneLabelOut(labels)
print loolo
sklearn.cross_validation.LeaveOneLabelOut(labels=[1, 1, 2, 2])
Another common application is to use time information: for instance the labels could be the year of collection of the samples and thus allow for cross-validation against time-based splits.

**Leave-P-Label-Out**

*LeavePLabelOut* is similar as *Leave-One-Label-Out*, but removes samples related to $P$ labels for each training/test set.

Example of Leave-2-Label Out:

```python
>>> from sklearn.cross_validation import LeavePLabelOut
>>> X = [[0., 0.], [1., 1.], [-1., -1.], [2., 2.], [3., 3.], [4., 4.]]
>>> Y = [0, 1, 0, 1, 0, 1]
>>> labels = [1, 1, 2, 2, 3, 3]
>>> lplo = LeavePLabelOut(labels, 2)
... for train, test in lplo:
...     print train, test
[4 5] [0 1 2 3]
[2 3] [0 1 4 5]
[0 1] [2 3 4 5]
```

**Random permutations cross-validation a.k.a. Shuffle & Split**

*ShuffleSplit* iterator will generate a user defined number of independent train / test dataset splits. Samples are first shuffled and then splitted into a pair of train and test sets.

It is possible to control the randomness for reproducibility of the results by explicitly seeding the *random_state* pseudo random number generator.

Here is a usage example:

```python
>>> ss = cross_validation.ShuffleSplit(5, n_iterations=3, test_size=0.25,
...                                     random_state=0)
>>> len(ss)
3
>>> for train_index, test_index in ss:
...     print train_index, test_index
...     [1 3 4] [2 0]
[1 4 3] [0 2]
[4 0 2] [1 3]
```
ShuffleSplit is thus a good alternative to KFold cross validation that allows a finer control on the number of iterations and the proportion of samples in on each side of the train / test split.

See also

StratifiedShuffleSplit is a variation of ShuffleSplit, which returns stratified splits, i.e which creates splits by preserving the same percentage for each target class as in the complete set.

Bootstrapping cross-validation

Bootstrapping is a general statistics technique that iterates the computation of an estimator on a resampled dataset.

The Bootstrap iterator will generate a user defined number of independent train / test dataset splits. Samples are then drawn (with replacement) on each side of the split. It furthermore possible to control the size of the train and test subset to make their union smaller than the total dataset if it is very large.

Note: Contrary to other cross-validation strategies, bootstrapping will allow some samples to occur several times in each splits.

```python
>>> bs = cross_validation.Bootstrap(9, random_state=0)
>>> len(bs)
3
>>> print bs
Bootstrap(9, n_bootstraps=3, train_size=5, test_size=4, random_state=0)

>>> for train_index, test_index in bs:
...    print train_index, test_index
...[1 8 7 7 8] [0 3 0 5]
[5 4 2 4 2] [6 7 1 0]
[4 7 0 1 1] [5 3 6 5]
```

Cross validation and model selection

Cross validation iterators can also be used to directly perform model selection using Grid Search for the optimal hyperparameters of the model. This is the topic if the next section: Grid Search: setting estimator parameters.

1.5.2 Grid Search: setting estimator parameters

Grid Search is used to optimize the parameters of a model (e.g. C, kernel and gamma for Support Vector Classifier, alpha for Lasso, etc.) using an internal Cross-Validation: evaluating estimator performance scheme.

GridSearchCV

The main class for implementing hyperparameters grid search in scikit-learn is grid_search.GridSearchCV. This class is passed a base model instance (for example sklearn.svm.SVC()) along with a grid of potential hyper-parameter values such as:
The `grid_search.GridSearchCV` instance implements the usual estimator API: when “fitting” it on a dataset all the possible combinations of hyperparameter values are evaluated and the best combinations is retained.

**Model selection: development and evaluation**

Model selection with `GridSearchCV` can be seen as a way to use the labeled data to “train” the hyperparameters of the grid. When evaluating the resulting model it is important to do it on held-out samples that were not seen during the grid search process: it is recommended to split the data into a development set (to be fed to the `GridSearchCV` instance) and an evaluation set to compute performance metrics. This can be done by using the `cross_validation.train_test_split` utility function.

**Examples**

- See [Parameter estimation using grid search with a nested cross-validation](#) for an example of Grid Search computation on the digits dataset.
- See [Sample pipeline for text feature extraction and evaluation](#) for an example of Grid Search coupling parameters from a text documents feature extractor (n-gram count vectorizer and TF-IDF transformer) with a classifier (here a linear SVM trained with SGD with either elastic net or L2 penalty) using a `pipeline.Pipeline` instance.

**Note:** Computations can be run in parallel if your OS supports it, by using the keyword `n_jobs=-1`, see function signature for more details.

**Alternatives to brute force grid search**

**Model specific cross-validation**

Some models can fit data for a range of value of some parameter almost as efficiently as fitting the estimator for a single value of the parameter. This feature can be leveraged to perform a more efficient cross-validation used for model selection of this parameter.

The most common parameter amenable to this strategy is the parameter encoding the strength of the regularizer. In this case we say that we compute the **regularization path** of the estimator.

Here is the list of such models:

- `linear_model.LarsCV([fit_intercept, ...])`: Cross-validated Least Angle Regression model.
- `linear_model.LassoLarsCV([fit_intercept, ...])`: Cross-validated Lasso, using the LARS algorithm.
- `linear_model.LassoCV([eps, n_alphas, ...])`: Lasso linear model with iterative fitting along a regularization path.

`sklearn.linear_model.RidgeCV`
Ridge regression with built-in cross-validation.

By default, it performs Generalized Cross-Validation, which is a form of efficient Leave-One-Out cross-validation.

**Parameters**

**alphas** : numpy array of shape [n_alpha]:

Array of alpha values to try. Small positive values of alpha improve the conditioning of the problem and reduce the variance of the estimates. Alpha corresponds to \((2+C)^{-1}\) in other linear models such as LogisticRegression or LinearSVC.

**fit_intercept** : boolean

Whether to calculate the intercept for this model. If set to false, no intercept will be used in calculations (e.g. data is expected to be already centered).

**normalize** : boolean, optional

If True, the regressors X are normalized.

**score_func** : callable, optional:

function that takes 2 arguments and compares them in order to evaluate the performance of prediction (big is good) if None is passed, the score of the estimator is maximized.

**loss_func** : callable, optional:

function that takes 2 arguments and compares them in order to evaluate the performance of prediction (small is good) if None is passed, the score of the estimator is maximized.

**cv** : cross-validation generator, optional

If None, Generalized Cross-Validation (efficient Leave-One-Out) will be used.

**See Also:**

RidgeRidge regression

RidgeClassifierRidge classifier

RidgeCVRidge regression with built-in cross validation

**Attributes**

**coef_**

array, shape = [n_features] or [n_classes, n_features]

Weight vector(s).

gcv_mode

[None, ‘auto’, ‘svd’, ‘eigen’], optional

Flag indicating which strategy to use when performing Generalized Cross-Validation. Options are:

‘auto’ : use svd if n_samples > n_
’svd’ : force computation via sing
’eigen’ : force computation via ei

The ‘auto’ mode is the default and is intended to pick the cheaper option of the two depending upon the shape of the training data.
Methods

<table>
<thead>
<tr>
<th>Method</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>decision_function(X)</code></td>
<td>Decision function of the linear model</td>
</tr>
<tr>
<td><code>fit(X, y[, sample_weight])</code></td>
<td>Fit Ridge regression model</td>
</tr>
<tr>
<td><code>get_params([deep])</code></td>
<td>Get parameters for the estimator</td>
</tr>
<tr>
<td><code>predict(X)</code></td>
<td>Predict using the linear model</td>
</tr>
<tr>
<td><code>score(X, y)</code></td>
<td>Returns the coefficient of determination $R^2$ of the prediction.</td>
</tr>
<tr>
<td><code>set_params(**params)</code></td>
<td>Set the parameters of the estimator.</td>
</tr>
</tbody>
</table>

__init__ (alphas=array([ 0.1, 1., 10. ]), fit_intercept=True, normalize=False, score_func=None, loss_func=None, cv=None, gcv_mode=None)

decision_function (X)
Decision function of the linear model

Parameters
X : numpy array of shape [n_samples, n_features]

Returns
C : array, shape = [n_samples]

Returns predicted values.

fit (X, y, sample_weight=1.0)
Fit Ridge regression model

Parameters
X : array-like, shape = [n_samples, n_features]

Training data
y : array-like, shape = [n_samples] or [n_samples, n_responses]

Target values
sample_weight : float or array-like of shape [n_samples]

Sample weight

Returns
self : Returns self.

get_params (deep=True)
Get parameters for the estimator

Parameters
deep : boolean, optional :

If True, will return the parameters for this estimator and contained subobjects that are estimators.

predict (X)
Predict using the linear model

Parameters
X : numpy array of shape [n_samples, n_features]

Returns
C : array, shape = [n_samples]

Returns predicted values.

score (X, y)
Returns the coefficient of determination $R^2$ of the prediction.

The coefficient $R^2$ is defined as $(1 - u/v)$, where $u$ is the regression sum of squares $((y - y\_pred) \times 2)$, and $v$ is the residual sum of squares $((y\_true - y\_true.\_mean()) \times 2)$, sum(). Best possible score is 1.0, lower values are worse.

Parameters
X : array-like, shape = [n_samples, n_features]
Training set.

\[ y : \text{array-like, shape} = [n\_samples] \]

Returns \( z \) : float

`set_params(**params)`
Set the parameters of the estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The former have parameters of the form `component__parameter` so that it's possible to update each component of a nested object.

Returns `self`:

```python
sklearn.linear_model.RidgeClassifierCV
```

`class sklearn.linear_model.RidgeClassifierCV(alphas=array([ 0.1, 1., 10.]), fit_intercept=True, normalize=False, score_func=None, loss_func=None, cv=None, class_weight=None)`

Ridge classifier with built-in cross-validation.

By default, it performs Generalized Cross-Validation, which is a form of efficient Leave-One-Out cross-validation. Currently, only the \( n\_features > n\_samples \) case is handled efficiently.

Parameters `alphas`: `numpy array of shape [n_alpha]`:

Array of alpha values to try. Small positive values of alpha improve the conditioning of the problem and reduce the variance of the estimates. Alpha corresponds to \((2*C)^{-1}\) in other linear models such as LogisticRegression or LinearSVC.

`fit_intercept`: boolean

Whether to calculate the intercept for this model. If set to false, no intercept will be used in calculations (e.g. data is expected to be already centered).

`normalize`: boolean, optional

If True, the regressors \( X \) are normalized.

`score_func`: callable, optional

Function that takes 2 arguments and compares them in order to evaluate the performance of prediction (big is good) if None is passed, the score of the estimator is maximized.

`loss_func`: callable, optional

Function that takes 2 arguments and compares them in order to evaluate the performance of prediction (small is good) if None is passed, the score of the estimator is maximized.

`cv`: cross-validation generator, optional

If None, Generalized Cross-Validation (efficient Leave-One-Out) will be used.

`class_weight`: dict, optional

Weights associated with classes in the form `{class_label : weight}`. If not given, all classes are supposed to have weight one.

See Also:

- `Ridge`: Ridge regression
- `RidgeClassifier`: Ridge classifier
RidgeCV

Ridge regression with built-in cross validation

Notes

For multi-class classification, n_class classifiers are trained in a one-versus-all approach. Concretely, this is implemented by taking advantage of the multi-variate response support in Ridge.

Methods

decision_function(X)

fit(X, y[, sample_weight, class_weight]) Fit the ridge classifier.

get_params([deep]) Get parameters for the estimator

predict(X) Predict target values according to the fitted model.

score(X, y) Returns the coefficient of determination R^2 of the prediction.

set_params(**params) Set the parameters of the estimator.

_init_ (alphas=array([ 0.1, 1., 10. ]), fit_intercept=True, normalize=False, score_func=None, loss_func=None, cv=None, class_weight=None)

fit (X, y, sample_weight=1.0, class_weight=None) Fit the ridge classifier.

Parameters

X : array-like, shape = [n_samples, n_features]
  Training vectors, where n_samples is the number of samples and n_features is the number of features.

y : array-like, shape = [n_samples]
  Target values.

sample_weight : float or numpy array of shape [n_samples]
  Sample weight

class_weight : dict, optional
  Weights associated with classes in the form {class_label : weight}. If not given, all classes are supposed to have weight one.

Returns

self : object
  Returns self.

get_params (deep=True)
  Get parameters for the estimator

Parameters

deep: boolean, optional :
  If True, will return the parameters for this estimator and contained subobjects that are estimators.

predict (X)
  Predict target values according to the fitted model.

Parameters

X : array-like, shape = [n_samples, n_features]

Returns

y : array, shape = [n_samples]
score \((X, y)\)
Returns the coefficient of determination \(R^2\) of the prediction.

The coefficient \(R^2\) is defined as \((1 - u/v)\), where \(u\) is the regression sum of squares \(((y - y_{pred})^2)\).sum() and \(v\) is the residual sum of squares \(((y_{true} - y_{true}.mean())^2)\).sum(). Best possible score is 1.0, lower values are worse.

**Parameters**

\(X\) : array-like, shape \([n\_samples, n\_features]\)
Training set.

\(y\) : array-like, shape \([n\_samples]\)

**Returns**

\(z\) : float

set_params (**params**)
Set the parameters of the estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The former have parameters of the form 
\(<\text{component}>\_<\text{parameter}>\) so that it’s possible to update each component of a nested object.

**Returns**

self : 

```python
sklearn.linear_model.LarsCV

class sklearn.linear_model.LarsCV

Parameters

fit_intercept : boolean
whether to calculate the intercept for this model. If set to false, no intercept will be used in calculations (e.g. data is expected to be already centered).

verbose : boolean or integer, optional
Sets the verbosity amount

normalize : boolean, optional
If True, the regressors \(X\) are normalized

copy_X : boolean, optional, default True
If True, \(X\) will be copied; else, it may be overwritten.

precompute : True | False | ‘auto’ | array-like
Whether to use a precomputed Gram matrix to speed up calculations. If set to ‘auto’ let us decide. The Gram matrix can also be passed as argument.

max_iter : integer, optional
Maximum number of iterations to perform.

cv : crossvalidation generator, optional
see sklearn.cross_validation module. If None is passed, default to a 5-fold strategy

max_n_alphas : integer, optional
The maximum number of points on the path used to compute the residuals in the cross-validation

n_jobs : integer, optional
```
Number of CPUs to use during the cross validation. If ‘-1’, use all the CPUs

**eps:** float, optional

The machine-precision regularization in the computation of the Cholesky diagonal factors. Increase this for very ill-conditioned systems.

**See Also:**

*lars_path*, *LassoLARS*, *LassoLarsCV*

**Attributes**

<table>
<thead>
<tr>
<th>coef_</th>
<th>array, shape = [n_features]</th>
<th>parameter vector (w in the formulation formula)</th>
</tr>
</thead>
<tbody>
<tr>
<td>intercept_</td>
<td>float</td>
<td>independent term in decision function.</td>
</tr>
<tr>
<td>coef_path: array, shape = [n_features, n_alpha]</td>
<td></td>
<td>the varying values of the coefficients along the path</td>
</tr>
</tbody>
</table>

**Methods**

<table>
<thead>
<tr>
<th>Method</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>decision_function(X)</td>
<td>Decision function of the linear model</td>
</tr>
<tr>
<td>fit(X, y)</td>
<td>Fit the model using X, y as training data.</td>
</tr>
<tr>
<td>get_params([deep])</td>
<td>Get parameters for the estimator</td>
</tr>
<tr>
<td>predict(X)</td>
<td>Predict using the linear model</td>
</tr>
<tr>
<td>score(X, y)</td>
<td>Returns the coefficient of determination R^2 of the prediction.</td>
</tr>
<tr>
<td>set_params(**params)</td>
<td>Set the parameters of the estimator.</td>
</tr>
</tbody>
</table>

**__init__** (fit_intercept=True, verbose=False, max_iter=500, normalize=True, precompute='auto', cv=None, max_n_alphas=1000, n_jobs=1, eps=2.220446049250313e-16, copy_X=True)

**decision_function**(X)

Decision function of the linear model

**Parameters** X : numpy array of shape [n_samples, n_features]

**Returns** C : array, shape = [n_samples]

Returns predicted values.

**fit**(X, y)

Fit the model using X, y as training data.

**Parameters** X : array-like, shape = [n_samples, n_features]

Training data.

ty : array-like, shape = [n_samples]

Target values.

**Returns** self : object

returns an instance of self.

**get_params**(deep=True)

Get parameters for the estimator

**Parameters** deep: boolean, optional:
If True, will return the parameters for this estimator and contained subobjects that are estimators.

**predict** *(X)*

Predict using the linear model

**Parameters** X : numpy array of shape [n_samples, n_features]

**Returns** C : array, shape = [n_samples]

Returns predicted values.

**score** *(X, y)*

Returns the coefficient of determination $R^2$ of the prediction.

The coefficient $R^2$ is defined as $(1 - u/v)$, where $u$ is the regression sum of squares $((y - y_{\text{pred}})^2).\text{sum()}$ and $v$ is the residual sum of squares $((y_{\text{true}} - y_{\text{true.mean()}})^2).\text{sum()}$. Best possible score is 1.0, lower values are worse.

**Parameters** X : array-like, shape = [n_samples, n_features]

Training set.

y : array-like, shape = [n_samples]

**Returns** z : float

**set_params** (**params**)

Set the parameters of the estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The former have parameters of the form `<component>__<parameter>` so that it’s possible to update each component of a nested object.

**Returns** self :

```
sklearn.linear_model.LassoLarsCV
class sklearn.linear_model.LassoLarsCV (fit_intercept=True, verbose=False, max_iter=500, normalize=True, precompute='auto', cv=None, max_n_alphas=1000, n_jobs=1, eps=2.2204460492503131e-16, copy_X=True)
```

Cross-validated Lasso, using the LARS algorithm

The optimization objective for Lasso is:

$$(1 / (2 * n_{samples})) * ||y - Xw||_2^2 + \alpha * ||w||_1$$

**Parameters** fit_intercept : boolean

whether to calculate the intercept for this model. If set to false, no intercept will be used in calculations (e.g. data is expected to be already centered).

verbose : boolean or integer, optional

Sets the verbosity amount

normalize : boolean, optional

If True, the regressors $X$ are normalized

precompute : True | False | ‘auto’ | array-like

Whether to use a precomputed Gram matrix to speed up calculations. If set to ‘auto’ let us decide. The Gram matrix can also be passed as argument.
**max_iter**: integer, optional

Maximum number of iterations to perform.

**cv**: cross-validation generator, optional

see sklearn.cross_validation module. If None is passed, default to a 5-fold strategy

**max_n_alphas**: integer, optional

The maximum number of points on the path used to compute the residuals in the cross-validation

**n_jobs**: integer, optional

Number of CPUs to use during the cross validation. If `-1`, use all the CPUs

**eps**: float, optional

The machine-precision regularization in the computation of the Cholesky diagonal factors. Increase this for very ill-conditioned systems.

**copy_X**: boolean, optional, default True

If True, X will be copied; else, it may be overwritten.

---

**See Also**

lars_path, LassoLars, LarsCV, LassoCV

---

**Notes**

The object solves the same problem as the LassoCV object. However, unlike the LassoCV, it find the relevent alphas values by itself. In general, because of this property, it will be more stable. However, it is more fragile to heavily multicollinear datasets.

It is more efficient than the LassoCV if only a small number of features are selected compared to the total number, for instance if there are very few samples compared to the number of features.

---

**Attributes**

<table>
<thead>
<tr>
<th></th>
<th>array, shape =</th>
<th>parameter vector (w in the formulation formula)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>coef_</strong></td>
<td>[n_features]</td>
<td></td>
</tr>
<tr>
<td><strong>intercept_</strong></td>
<td>float</td>
<td>independent term in decision function.</td>
</tr>
<tr>
<td><strong>coef_path</strong></td>
<td>array, shape =</td>
<td>the varying values of the coefficients along the path</td>
</tr>
<tr>
<td></td>
<td>[n_features, n_alpha]</td>
<td></td>
</tr>
<tr>
<td><strong>alphas_</strong></td>
<td>array, shape =</td>
<td>the different values of alpha along the path</td>
</tr>
<tr>
<td></td>
<td>[n_alpha]</td>
<td></td>
</tr>
<tr>
<td><strong>cv_alphas</strong></td>
<td>array, shape =</td>
<td>all the values of alpha along the path for the different folds</td>
</tr>
<tr>
<td></td>
<td>[n_cv_alphas]</td>
<td></td>
</tr>
<tr>
<td><strong>cv_mse_path_</strong></td>
<td>array, shape =</td>
<td>the mean square error on left-out for each fold along the path (alpha values given by cv_alphas)</td>
</tr>
<tr>
<td></td>
<td>[n_folds, n_cv_alphas]</td>
<td></td>
</tr>
</tbody>
</table>

---

**Methods**

<table>
<thead>
<tr>
<th>Method</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>decision_function(X)</td>
<td>Decision function of the linear model</td>
</tr>
</tbody>
</table>

Continued on next page
### Table 1.6 – continued from previous page

<table>
<thead>
<tr>
<th>Method</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>fit(X, y)</td>
<td>Fit the model using X, y as training data.</td>
</tr>
<tr>
<td>get_params()</td>
<td>Get parameters for the estimator</td>
</tr>
<tr>
<td>predict(X)</td>
<td>Predict using the linear model</td>
</tr>
<tr>
<td>score(X, y)</td>
<td>Returns the coefficient of determination $R^2$ of the prediction.</td>
</tr>
<tr>
<td>set_params(**params)</td>
<td>Set the parameters of the estimator.</td>
</tr>
</tbody>
</table>

#### __init__

```python
init__(fit_intercept=True, verbose=False, max_iter=500, normalize=True, precompute='auto', cv=None, max_n_alphas=1000, n_jobs=1, eps=2.2204460492503131e-16, copy_X=True)
```

#### decision_function(X)

Decision function of the linear model

- **Parameters**
  - `X`: numpy array of shape [n_samples, n_features]

- **Returns**
  - `C`: array, shape = [n_samples]
    - Returns predicted values.

#### fit(X, y)

Fit the model using X, y as training data.

- **Parameters**
  - `X`: array-like, shape = [n_samples, n_features]
    - Training data.
  - `y`: array-like, shape = [n_samples]
    - Target values.

- **Returns**
  - `self`: object
    - returns an instance of self.

#### get_params(deep=True)

Get parameters for the estimator

- **Parameters**
  - `deep`: boolean, optional
    - If True, will return the parameters for this estimator and contained subobjects that are estimators.

#### predict(X)

Predict using the linear model

- **Parameters**
  - `X`: numpy array of shape [n_samples, n_features]

- **Returns**
  - `C`: array, shape = [n_samples]
    - Returns predicted values.

#### score(X, y)

Returns the coefficient of determination $R^2$ of the prediction.

- **Parameters**
  - `X`: array-like, shape = [n_samples, n_features]
    - Training set.
  - `y`: array-like, shape = [n_samples]

- **Returns**
  - `z`: float
    - The coefficient $R^2$ is defined as $(1 - u/v)$, where $u$ is the regression sum of squares $(\sum (y - y_{pred})^2)$ and $v$ is the residual sum of squares $(\sum (y_{true} - y_{true.mean})^2)$. Best possible score is 1.0, lower values are worse.
**set_params**(**params**)  
Set the parameters of the estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The former have parameters of the form `<component>__<parameter>` so that it’s possible to update each component of a nested object.

**Returns self** :

```python
sklearn.linear_model.LassoCV
class sklearn.linear_model.LassoCV(eps=0.001, n_alphas=100, alphas=None, fit_intercept=True, normalize=False, precompute='auto', max_iter=1000, tol=0.0001, copy_X=True, cv=None, verbose=False)
```

Lasso linear model with iterative fitting along a regularization path

The best model is selected by cross-validation.

The optimization objective for Lasso is:

\[
(1 / (2 * n_samples)) * \|y - Xw\|^2_2 + alpha * \|w\|_1
\]

**Parameters**  
eps: float, optional  
Length of the path. eps=1e-3 means that alpha_min / alpha_max = 1e-3.

n_alphas: int, optional  
Number of alphas along the regularization path

alphas: numpy array, optional  
List of alphas where to compute the models. If None alphas are set automatically

precompute: True | False | 'auto' | array-like  
Whether to use a precomputed Gram matrix to speed up calculations. If set to ‘auto’ let us decide. The Gram matrix can also be passed as argument.

max_iter: int, optional  
The maximum number of iterations

tol: float, optional  
The tolerance for the optimization: if the updates are smaller than ‘tol’, the optimization code checks the dual gap for optimality and continues until it is smaller than tol.

cv: integer or crossvalidation generator, optional  
If an integer is passed, it is the number of fold (default 3). Specific crossvalidation objects can be passed, see sklearn.cross_validation module for the list of possible objects

verbose: bool or integer  
amount of verbosity

See Also:

lars_path, lasso_path, LassoLars, Lasso, LassoLarsCV

1.5. Model Selection
Notes

See examples/linear_model/lasso_path_with_crossvalidation.py for an example.

To avoid unnecessary memory duplication the X argument of the fit method should be directly passed as a fortran contiguous numpy array.

Attributes

<table>
<thead>
<tr>
<th>alpha_ : float</th>
<th>The amount of penalization chosen by cross validation</th>
</tr>
</thead>
<tbody>
<tr>
<td>coef_ : array, shape = [n_features]</td>
<td>parameter vector (w in the formulation formula)</td>
</tr>
<tr>
<td>intercept_ : float</td>
<td>independent term in decision function.</td>
</tr>
<tr>
<td>mse_path_ : array, shape = [n_alphas, n_folds]</td>
<td>mean square error for the test set on each fold, varying alpha</td>
</tr>
</tbody>
</table>

Methods

| decision_function(X) | Decision function of the linear model |
| fit(X, y) | Fit linear model with coordinate descent along decreasing alphas |
| get_params((deep)) | Get parameters for the estimator |
| path(X, y[, eps, n_alphas, alphas, ...]) | Compute Lasso path with coordinate descent |
| predict(X) | Predict using the linear model |
| score(X, y) | Returns the coefficient of determination $R^2$ of the prediction. |
| set_params(**params) | Set the parameters of the estimator. |

__init__ (eps=0.001, n_alphas=100, alphas= None, fit_intercept=True, normalize=False, precompute='auto', max_iter=1000, tol=0.0001, copy_X=True, cv=None, verbose=False)

decision_function (X)

Decision function of the linear model

Parameters X : numpy array of shape [n_samples, n_features]

Returns C : array, shape = [n_samples]

Returns predicted values.

fit (X, y)

Fit linear model with coordinate descent along decreasing alphas using cross-validation

Parameters X : numpy array of shape [n_samples, n_features]

Training data. Pass directly as fortran contiguous data to avoid unnecessary memory duplication

y : numpy array of shape [n_samples]

Target values

get_params (deep=True)

Get parameters for the estimator

Parameters deep: boolean, optional :
If True, will return the parameters for this estimator and contained subobjects that are estimators.

```python
static path(X, y, eps=0.001, n_alphas=100, alphas=None, precompute='auto', Xy=None,
          fit_intercept=True, normalize=False, copy_X=True, verbose=False, **params)
```

Compute Lasso path with coordinate descent

The optimization objective for Lasso is:

$$(1 / (2 * n_{samples})) * ||y - Xw||^2_2 + alpha * ||w||_1$$

**Parameters**

- **X**: numpy array of shape [n_samples, n_features]
  - Training data. Pass directly as fortran contiguous data to avoid unnecessary memory duplication
- **y**: numpy array of shape [n_samples]
  - Target values
- **eps**: float, optional
  - Length of the path. eps=1e-3 means that alpha_min / alpha_max = 1e-3
- **n_alphas**: int, optional
  - Number of alphas along the regularization path
- **alphas**: numpy array, optional
  - List of alphas where to compute the models. If None alphas are set automatically
- **precompute**: True | False | ‘auto’ | array-like
  - Whether to use a precomputed Gram matrix to speed up calculations. If set to ‘auto’ let us decide. The Gram matrix can also be passed as argument.
- **Xy**: array-like, optional
  - Xy = np.dot(X.T, y) that can be precomputed. It is useful only when the Gram matrix is precomputed.
- **fit_intercept**: bool
  - Fit or not an intercept
- **normalize**: boolean, optional
  - If True, the regressors X are normalized
- **copy_X**: boolean, optional, default True
  - If True, X will be copied; else, it may be overwritten.
- **verbose**: bool or integer
  - Amount of verbosity
- **params**: kwargs
  - keyword arguments passed to the Lasso objects

**Returns models**: a list of models along the regularization path

**See Also**:

- lars_path
- Lasso
- LassoLars
- LassoCV
- LassoLarsCV
- sklearn.decomposition.sparse_encode
Notes

See examples/linear_model/plot_lasso_coordinate_descent_path.py for an example.
To avoid unnecessary memory duplication the X argument of the fit method should be directly passed as a fortran contiguous numpy array.

**predict (X)**

Predict using the linear model

**Parameters**

-X : numpy array of shape [n_samples, n_features]

**Returns**

-C : array, shape = [n_samples]

-Returns predicted values.

**score (X, y)**

Returns the coefficient of determination $R^2$ of the prediction.

The coefficient $R^2$ is defined as $(1 - u/v)$, where $u$ is the regression sum of squares ($(y - y_{pred})^2$).sum() and $v$ is the residual sum of squares ($(y_{true} - y_{true.mean})^2$).sum(). Best possible score is 1.0, lower values are worse.

**Parameters**

-X : array-like, shape = [n_samples, n_features]

-Training set.

-y : array-like, shape = [n_samples]

-Returns z : float

**set_params (**params**)

Set the parameters of the estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The former have parameters of the form `<component>__<parameter>` so that it's possible to update each component of a nested object.

-Returns self:

**sklearn.linear_model.ElasticNetCV**

class sklearn.linear_model.ElasticNetCV (rho=0.5, eps=0.001, n_alphas=100, alphas=None, fit_intercept=True, normalize=False, precompute='auto', max_iter=1000, tol=0.0001, cv=None, copy_X=True, verbose=0, n_jobs=1)

Elastic Net model with iterative fitting along a regularization path

The best model is selected by cross-validation.

**Parameters**

-rho : float, optional

-float between 0 and 1 passed to ElasticNet (scaling between l1 and l2 penalties). For rho = 0 the penalty is an L1 penalty. For rho = 1 it is an L2 penalty. For 0 < rho < 1, the penalty is a combination of L1 and L2 This parameter can be a list, in which case the different values are tested by cross-validation and the one giving the best prediction score is used. Note that a good choice of list of values for rho is often to put more values close to 1 (i.e. Lasso) and less close to 0 (i.e. Ridge), as in [.1, .5, .7, .9, .95, .99, 1]

-eps : float, optional

-Length of the path. eps=1e-3 means that alpha_min / alpha_max = 1e-3.

-n_alphas : int, optional


Number of alphas along the regularization path

**alphas** : numpy array, optional

List of alphas where to compute the models. If None alphas are set automatically

**precompute** : True | False | 'auto' | array-like

Whether to use a precomputed Gram matrix to speed up calculations. If set to ‘auto’ let us decide. The Gram matrix can also be passed as argument.

**max_iter** : int, optional

The maximum number of iterations

**tol** : float, optional

The tolerance for the optimization: if the updates are smaller than ‘tol’, the optimization code checks the dual gap for optimality and continues until it is smaller than tol.

**cv** : integer or crossvalidation generator, optional

If an integer is passed, it is the number of fold (default 3). Specific crossvalidation objects can be passed, see sklearn.cross_validation module for the list of possible objects

**verbose** : bool or integer

amount of verbosity

**n_jobs** : integer, optional

Number of CPUs to use during the cross validation. If ‘-1’, use all the CPUs. Note that this is used only if multiple values for rho are given.

See Also:
enet_path, ElasticNet

Notes

See examples/linear_model/lasso_path_with_crossvalidation.py for an example.

To avoid unnecessary memory duplication the X argument of the fit method should be directly passed as a fortran contiguous numpy array.

The parameter rho corresponds to alpha in the glmnet R package while alpha corresponds to the lambda parameter in glmnet. More specifically, the optimization objective is:

1 / (2 * n_samples) * ||y - Xw||^2_2 +
+ alpha * rho * ||w||_1 + 0.5 * alpha * (1 - rho) * ||w||^2_2

If you are interested in controlling the L1 and L2 penalty separately, keep in mind that this is equivalent to:

\[ a \times L1 + b \times L2 \]

for:

\[ alpha = a + b \text{ and } rho = a / (a + b) \]
Attributes

<table>
<thead>
<tr>
<th>Attribute</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>alpha_</td>
<td>The amount of penalization chosen by cross validation</td>
</tr>
<tr>
<td>rho_</td>
<td>The compromise between l1 and l2 penalization chosen by cross validation</td>
</tr>
<tr>
<td>coef_</td>
<td>Parameter vector (w in the formulation formula)</td>
</tr>
<tr>
<td>intercept_</td>
<td>Independent term in decision function.</td>
</tr>
<tr>
<td>mse_path_</td>
<td>Mean square error for the test set on each fold, varying rho and alpha</td>
</tr>
</tbody>
</table>

Methods

<table>
<thead>
<tr>
<th>Method</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>decision_function(X)</td>
<td>Decision function of the linear model</td>
</tr>
<tr>
<td>fit(X, y)</td>
<td>Fit linear model with coordinate descent along decreasing alphas</td>
</tr>
<tr>
<td>get_params(deep=True)</td>
<td>Get parameters for the estimator</td>
</tr>
<tr>
<td>path(X, y[, rho, eps, n_alphas, alphas, ...])</td>
<td>Compute Elastic-Net path with coordinate descent</td>
</tr>
<tr>
<td>predict(X)</td>
<td>Predict using the linear model</td>
</tr>
<tr>
<td>score(X, y)</td>
<td>Returns the coefficient of determination $R^2$ of the prediction</td>
</tr>
<tr>
<td>set_params(**params)</td>
<td>Set the parameters of the estimator.</td>
</tr>
</tbody>
</table>

__init__(rho=0.5, eps=0.001, n_alphas=100, alphas=None, fit_intercept=True, normalize=False, precompute='auto', max_iter=1000, tol=0.0001, cv=None, copy_X=True, verbose=0, n_jobs=1)

decision_function(X)
Decision function of the linear model

Parameters X: numpy array of shape [n_samples, n_features]

Returns C: array, shape = [n_samples]
Returns predicted values.

fit(X, y)
Fit linear model with coordinate descent along decreasing alphas using cross-validation

Parameters X: numpy array of shape [n_samples, n_features]

    Training data. Pass directly as fortran contiguous data to avoid unnecessary memory duplication

y: numpy array of shape [n_samples]

    Target values

get_params(deep=True)
Get parameters for the estimator

Parameters deep: boolean, optional:
    If True, will return the parameters for this estimator and contained subobjects that are estimators.

static path(X, y, rho=0.5, eps=0.001, n_alphas=100, alphas=None, precompute='auto', Xy=None, fit_intercept=True, normalize=False, copy_X=True, verbose=False, **params)
Compute Elastic-Net path with coordinate descent
The Elastic Net optimization function is:

\[
\frac{1}{2 \times n_{\text{samples}}} \times \|y - Xw\|_2^2 + \\
\alpha \times \rho \times \|w\|_1 + 0.5 \times \alpha \times (1 - \rho) \times \|w\|_2^2
\]

**Parameters**

- **X**: numpy array of shape [n_samples, n_features]
  - Training data. Pass directly as fortran contiguous data to avoid unnecessary memory duplication
- **y**: numpy array of shape [n_samples]
  - Target values
- **rho**: float, optional
  - float between 0 and 1 passed to ElasticNet (scaling between l1 and l2 penalties). rho=1 corresponds to the Lasso
- **eps**: float
  - Length of the path. eps=1e-3 means that alpha_min / alpha_max = 1e-3
- **n_alphas**: int, optional
  - Number of alphas along the regularization path
- **alphas**: numpy array, optional
  - List of alphas where to compute the models. If None alphas are set automatically
- **precompute**: True | False | ‘auto’ | array-like
  - Whether to use a precomputed Gram matrix to speed up calculations. If set to ‘auto’ let us decide. The Gram matrix can also be passed as argument.
- **Xy**: array-like, optional
  - Xy = np.dot(X.T, y) that can be precomputed. It is useful only when the Gram matrix is precomputed.
- **fit_intercept**: bool
  - Fit or not an intercept
- **normalize**: boolean, optional
  - If True, the regressors X are normalized
- **copy_X**: boolean, optional, default True
  - If True, X will be copied; else, it may be overwritten.
- **verbose**: bool or integer
  - Amount of verbosity
- **params**: kwargs
  - keyword arguments passed to the Lasso objects

**Returns**

- **models**: a list of models along the regularization path

**See Also:**

ElasticNet, ElasticNetCV
Notes

See examples/plot_lasso_coordinate_descent_path.py for an example.

**predict** (*X*)

Predict using the linear model

Parameters *X*: numpy array of shape [n_samples, n_features]

Returns *C*: array, shape = [n_samples]

Returns predicted values.

**score** (*X*, *y*)

Returns the coefficient of determination $R^2$ of the prediction.

The coefficient $R^2$ is defined as $(1 - u/v)$, where $u$ is the regression sum of squares $((y - y\_pred)^2).\text{sum()}$ and $v$ is the residual sum of squares $((y\_true - y\_true\_mean)^2).\text{sum()}$. Best possible score is 1.0, lower values are worse.

Parameters *X*: array-like, shape = [n_samples, n_features]

Training set.

*y*: array-like, shape = [n_samples]

Returns *z*: float

**set_params** (**params**)

Set the parameters of the estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The former have parameters of the form `<component>__<parameter>` so that it’s possible to update each component of a nested object.

Returns self:

Information Criterion

Some models can offer an information-theoretic closed-form formula of the optimal estimate of the regularization parameter by computing a single regularization path (instead of several when using cross-validation).

Here is the list of models benefiting from the Akaike Information Criterion (AIC) or the Bayesian Information Criterion (BIC) for automated model selection:

```
linear_model.LassoLarsIC((criterion, ...))  Lasso model fit with Lars using BIC or AIC for model selection
```

```
sklearn.linear_model.LassoLarsIC
class sklearn.linear_model.LassoLarsIC (criterion='aic', fit_intercept=True, verbose=False, normalize=True, precompute='auto', max_iter=500, eps=2.220446049250313e-16, copy_X=True)

Lasso model fit with Lars using BIC or AIC for model selection
```

The optimization objective for Lasso is:

$$(1 / (2 * n\_samples)) * ||y - Xw||^2_2 + alpha * ||w||_1$$

AIC is the Akaike information criterion and BIC is the Bayes Information criterion. Such criteria are useful to select the value of the regularization parameter by making a trade-off between the goodness of fit and the complexity of the model. A good model should explain well the data while being simple.
Parameters criterion: ‘bic’ | ‘aic’:
The type of criterion to use.

fit_intercept: boolean
whether to calculate the intercept for this model. If set to false, no intercept will be used in calculations (e.g. data is expected to be already centered).

verbose: boolean or integer, optional
Sets the verbosity amount

normalize: boolean, optional
If True, the regressors X are normalized

copy_X: boolean, optional, default True
If True, X will be copied; else, it may be overwritten.

precompute: True | False | ‘auto’ | array-like
Whether to use a precomputed Gram matrix to speed up calculations. If set to ‘auto’ let us decide. The Gram matrix can also be passed as argument.

max_iter: integer, optional:
Maximum number of iterations to perform. Can be used for early stopping.

eps: float, optional:
The machine-precision regularization in the computation of the Cholesky diagonal factors. Increase this for very ill-conditioned systems. Unlike the ‘tol’ parameter in some iterative optimization-based algorithms, this parameter does not control the tolerance of the optimization.

See Also:
lars_path, LassoLars, LassoLarsCV

Notes
The estimation of the number of degrees of freedom is given by:

Examples

>>> from sklearn import linear_model
>>> clf = linear_model.LassoLarsIC(criterion='bic')
>>> clf.fit([[-1, 1], [0, 0], [1, 1]], [-1.1111, 0, -1.1111])
...
LassoLarsIC(copy_X=True, criterion='bic', eps=..., fit_intercept=True,
    max_iter=500, normalize=True, precompute='auto',
    verbose=False)
>>> print(clf.coef_)
[ 0. -1.11...]

1.5. Model Selection 179
Attributes

| coef_   | array, shape = [n_features] | parameter vector (w in the formulation formula) |
| intercept_ | float                     | independent term in decision function. |
| alpha_  | float                     | the alpha parameter chosen by the information criterion |

Methods

- `decision_function(X)` Decision function of the linear model
  - Parameters X : numpy array of shape [n_samples, n_features]
  - Returns C : array, shape = [n_samples]
    - Returns predicted values.
- `fit(X, y[, copy_X])` Fit the model using X, y as training data.
  - Parameters x : array-like, shape = [n_samples, n_features]
    - training data.
  - y : array-like, shape = [n_samples]
    - target values.
  - Returns self : object
    - returns an instance of self.
- `get_params([deep])` Get parameters for the estimator
  - Parameters deep: boolean, optional : If True, will return the parameters for this estimator and contained subobjects that are estimators.
- `predict(X)` Predict using the linear model
  - Parameters X : numpy array of shape [n_samples, n_features]
  - Returns C : array, shape = [n_samples]
    - Returns predicted values.
**score** *(X, y)*  
Returns the coefficient of determination R^2 of the prediction.

The coefficient R^2 is defined as (1 - u/v), where u is the regression sum of squares ((y - y_pred) ** 2).sum() and v is the residual sum of squares ((y_true - y_true.mean()) ** 2).sum(). Best possible score is 1.0, lower values are worse.

**Parameters**  

- **X** : array-like, shape = [n_samples, n_features]  
  Training set.

- **y** : array-like, shape = [n_samples]  

**Returns**  

- **z** : float

**set_params** (**params**)  
Set the parameters of the estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The former have parameters of the form `<component>__<parameter>` so that it’s possible to update each component of a nested object.

**Returns**  

- **self** :

### Out of Bag Estimates

When using ensemble methods base upon bagging, i.e. generating new training sets using sampling with replacement, part of the training set remains unused. For each classifier in the ensemble, a different part of the training set is left out.

This left out portion can be used to estimate the generalization error without having to rely on a separate validation set. This estimate comes “for free” as no addititional data is needed and can be used for model selection.

This is currently implemented in the following classes:

<table>
<thead>
<tr>
<th>Class Name</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>ensemble.RandomForestClassifier(...)</code></td>
<td>A random forest classifier.</td>
</tr>
<tr>
<td><code>ensemble.RandomForestRegressor(...)</code></td>
<td>A random forest regressor.</td>
</tr>
<tr>
<td><code>ensemble.ExtraTreesClassifier(...)</code></td>
<td>An extra-trees classifier.</td>
</tr>
<tr>
<td><code>ensemble.GradientBoostingClassifier([loss,...])</code></td>
<td>Gradient Boosting for classification.</td>
</tr>
<tr>
<td><code>ensemble.GradientBoostingRegressor([loss,...])</code></td>
<td>Gradient Boosting for regression.</td>
</tr>
</tbody>
</table>

**sklearn.ensemble.RandomForestClassifier**  

**class sklearn.ensemble.RandomForestClassifier**(\(n_{estimators}=10,\)  
\(\text{criterion=}'\text{gini}',\)  
\(\text{max\_depth=}'\text{None}',\)  
\(\text{min\_samples\_split}=1,\)  
\(\text{min\_samples\_leaf}=1,\)  
\(\text{min\_density}=0.1,\)  
\(\text{max\_features=}'\text{auto}',\)  
\(\text{bootstrap=}'\text{True}',\)  
\(\text{compute\_importances=}'\text{False}',\)  
\(\text{oob\_score=}'\text{False}',\)  
\(\text{n\_jobs}=1,\)  
\(\text{random\_state=}'\text{None}',\)  
\(\text{verbose=}'\text{0}'\))

A random forest classifier.

A random forest is a meta estimator that fits a number of classifical decision trees on various sub-samples of the dataset and use averaging to improve the predictive accuracy and control over-fitting.

**Parameters**  

- **n_estimators** : integer, optional (default=10)  
  The number of trees in the forest.

- **criterion** : string, optional (default=\"gini\")
The function to measure the quality of a split. Supported criteria are “gini” for the Gini impurity and “entropy” for the information gain. Note: this parameter is tree-specific.

**max_depth**: integer or None, optional (default=None)

The maximum depth of the tree. If None, then nodes are expanded until all leaves are pure or until all leaves contain less than min_samples_split samples. Note: this parameter is tree-specific.

**min_samples_split**: integer, optional (default=1)

The minimum number of samples required to split an internal node. Note: this parameter is tree-specific.

**min_samples_leaf**: integer, optional (default=1)

The minimum number of samples in newly created leaves. A split is discarded if after the split, one of the leaves would contain less then min_samples_leaf samples. Note: this parameter is tree-specific.

**min_density**: float, optional (default=0.1)

This parameter controls a trade-off in an optimization heuristic. It controls the minimum density of the sample_mask (i.e. the fraction of samples in the mask). If the density falls below this threshold the mask is recomputed and the input data is packed which results in data copying. If min_density equals to one, the partitions are always represented as copies of the original data. Otherwise, partitions are represented as bit masks (aka sample masks). Note: this parameter is tree-specific.

**max_features**: int, string or None, optional (default=”auto”)

The number of features to consider when looking for the best split:

- If “auto”, then max_features=sqrt(n_features) on classification tasks and max_features=n_features on regression problems.
- If “sqrt”, then max_features=sqrt(n_features).
- If “log2”, then max_features=log2(n_features).
- If None, then max_features=n_features.

Note: this parameter is tree-specific.

**bootstrap**: boolean, optional (default=True)

Whether bootstrap samples are used when building trees.

**compute_importances**: boolean, optional (default=True)

Whether feature importances are computed and stored into the feature_importances_ attribute when calling fit.

**oob_score**: bool

Whether to use out-of-bag samples to estimate the generalization error.

**n_jobs**: integer, optional (default=1)

The number of jobs to run in parallel. If -1, then the number of jobs is set to the number of cores.

**random_state**: int, RandomState instance or None, optional (default=None)
If int, random_state is the seed used by the random number generator; If RandomState instance, random_state is the random number generator; If None, the random number generator is the RandomState instance used by np.random.

`verbose` : int, optional (default=0)

Controls the verbosity of the tree building process.

**See Also:**

DecisionTreeClassifier, ExtraTreesClassifier

**References**

[R59]

**Attributes**

<table>
<thead>
<tr>
<th>feature_importances_</th>
<th>array, shape = [n_features]</th>
<th>The feature importances (the higher, the more important the feature).</th>
</tr>
</thead>
<tbody>
<tr>
<td>oob_score_</td>
<td>float</td>
<td>Score of the training dataset obtained using an out-of-bag estimate.</td>
</tr>
<tr>
<td>oob_decision_function_</td>
<td>array, shape = [n_samples, n_classes]</td>
<td>Decision function computed with out-of-bag estimate on the training set.</td>
</tr>
</tbody>
</table>

**Methods**

<table>
<thead>
<tr>
<th>fit(X, y)</th>
<th>Build a forest of trees from the training set (X, y).</th>
</tr>
</thead>
<tbody>
<tr>
<td>fit_transform(X[, y])</td>
<td>Fit to data, then transform it</td>
</tr>
<tr>
<td>get_params([deep])</td>
<td>Get parameters for the estimator</td>
</tr>
<tr>
<td>predict(X)</td>
<td>Predict class for X.</td>
</tr>
<tr>
<td>predict_log_proba(X)</td>
<td>Predict class log-probabilities for X.</td>
</tr>
<tr>
<td>predict_proba(X)</td>
<td>Predict class probabilities for X.</td>
</tr>
<tr>
<td>score(X, y)</td>
<td>Returns the mean accuracy on the given test data and labels.</td>
</tr>
<tr>
<td>set_params(**params)</td>
<td>Set the parameters of the estimator.</td>
</tr>
<tr>
<td>transform(X[, threshold])</td>
<td>Reduce X to its most important features.</td>
</tr>
</tbody>
</table>

__init__(n_estimators=10, criterion='gini', max_depth=None, min_samples_split=1, min_samples_leaf=1, min_density=0.1, max_features='auto', bootstrap=True, compute_importances=False, oob_score=False, n_jobs=1, random_state=None, verbose=0)

**fit** (X, y)

Build a forest of trees from the training set (X, y).

**Parameters**

X : array-like of shape = [n_samples, n_features]

The training input samples.

y : array-like, shape = [n_samples]

The target values (integers that correspond to classes in classification, real numbers in regression).

**Returns**

self : object
Returns self.

**fit_transform** *(X, y=None, **fit_params)*

Fit to data, then transform it

Fits transformer to X and y with optional parameters fit_params and returns a transformed version of X.

**Parameters**

**X** : numpy array of shape [n_samples, n_features]
    Training set.

**y** : numpy array of shape [n_samples]
    Target values.

**Returns**

**X_new** : numpy array of shape [n_samples, n_features_new]
    Transformed array.

**Notes**

This method just calls fit and transform consecutively, i.e., it is not an optimized implementation of fit_transform, unlike other transformers such as PCA.

**get_params** *(deep=True)*

Get parameters for the estimator

**Parameters**

**deep** : boolean, optional
    If True, will return the parameters for this estimator and contained subobjects that are estimators.

**predict** *(X)*

Predict class for X.

The predicted class of an input sample is computed as the majority prediction of the trees in the forest.

**Parameters**

**X** : array-like of shape = [n_samples, n_features]
    The input samples.

**Returns**

**y** : array of shape = [n_samples]
    The predicted classes.

**predict_log_proba** *(X)*

Predict class log-probabilities for X.

The predicted class log-probabilities of an input sample is computed as the mean predicted class logprobabilities of the trees in the forest.

**Parameters**

**X** : array-like of shape = [n_samples, n_features]
    The input samples.

**Returns**

**p** : array of shape = [n_samples]
    The class log-probabilities of the input samples. Classes are ordered by arithmetical order.

**predict_proba** *(X)*

Predict class probabilities for X.

The predicted class probabilities of an input sample is computed as the mean predicted class probabilities of the trees in the forest.
**Parameters** X:
array-like of shape = [n_samples, n_features]

The input samples.

**Returns** p:
array of shape = [n_samples]

The class probabilities of the input samples. Classes are ordered by arithmetical order.

**score** *(X, y)*

Returns the mean accuracy on the given test data and labels.

**Parameters** X:
array-like, shape = [n_samples, n_features]

Training set.

y:
array-like, shape = [n_samples]

Labels for X.

**Returns** z:
float

**set_params** (**params**)

Set the parameters of the estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The former have parameters of the form `<component>__<parameter>` so that it's possible to update each component of a nested object.

**Returns** self:

**transform** *(X, threshold=None)*

Reduce X to its most important features.

**Parameters** X:
array or scipy sparse matrix of shape [n_samples, n_features]

The input samples.

threshold:
string, float or None, optional (default=None)

The threshold value to use for feature selection. Features whose importance is greater or equal are kept while the others are discarded. If “median” (resp. “mean”), then the threshold value is the median (resp. the mean) of the feature importances. A scaling factor (e.g., “1.25*mean”) may also be used. If None and if available, the object attribute `threshold` is used. Otherwise, “mean” is used by default.

**Returns** X_r:
array of shape [n_samples, n_selected_features]

The input samples with only the selected features.

**sklearn.ensemble.RandomForestRegressor**

class sklearn.ensemble.RandomForestRegressor *(n_estimators=10, criterion='mse', max_depth=None, min_samples_split=1, min_samples_leaf=1, min_density=0.1, max_features='auto', bootstrap=True, compute_importances=False, oob_score=False, n_jobs=1, random_state=None, verbose=0)*

A random forest regressor.

A random forest is a meta estimator that fits a number of classifical decision trees on various sub-samples of the dataset and use averaging to improve the predictive accuracy and control over-fitting.

**Parameters** n_estimators:
integer, optional (default=10)

The number of trees in the forest.

1.5. Model Selection
criterion : string, optional (default="mse")

The function to measure the quality of a split. The only supported criterion is “mse” for
the mean squared error. Note: this parameter is tree-specific.

max_depth : integer or None, optional (default=None)

The maximum depth of the tree. If None, then nodes are expanded until all leaves
are pure or until all leaves contain less than min_samples_split samples. Note: this
parameter is tree-specific.

min_samples_split : integer, optional (default=1)

The minimum number of samples required to split an internal node. Note: this parame-
ter is tree-specific.

min_samples_leaf : integer, optional (default=1)

The minimum number of samples in newly created leaves. A split is discarded if after
the split, one of the leaves would contain less then min_samples_leaf samples.
Note: this parameter is tree-specific.

min_density : float, optional (default=0.1)

This parameter controls a trade-off in an optimization heuristic. It controls the minimum
density of the sample_mask (i.e. the fraction of samples in the mask). If the density falls
below this threshold the mask is recomputed and the input data is packed which results
in data copying. If min_density equals to one, the partitions are always represented
as copies of the original data. Otherwise, partitions are represented as bit masks (aka
sample masks). Note: this parameter is tree-specific.

max_features : int, string or None, optional (default=“auto”)

The number of features to consider when looking for the best split:

• If “auto”, then max_features=sqrt(n_features) on classification tasks and
  max_features=n_features on regression problems.

• If “sqrt”, then max_features=sqrt(n_features).

• If “log2”, then max_features=log2(n_features).

• If None, then max_features=n_features.

Note: this parameter is tree-specific.

bootstrap : boolean, optional (default=True)

Whether bootstrap samples are used when building trees.

compute_importances : boolean, optional (default=True)

Whether feature importances are computed and stored into the
feature_importances_ attribute when calling fit.

oob_score : bool

whether to use out-of-bag samples to estimate the generalization error.

n_jobs : integer, optional (default=1)

The number of jobs to run in parallel. If -1, then the number of jobs is set to the number
of cores.

random_state : int, RandomState instance or None, optional (default=None)
If int, random_state is the seed used by the random number generator; If RandomState instance, random_state is the random number generator; If None, the random number generator is the RandomState instance used by np.random.

verbose : int, optional (default=0)

Controls the verbosity of the tree building process.

See Also:
DecisionTreeRegressor, ExtraTreesRegressor

References

[R60]

Attributes

<table>
<thead>
<tr>
<th>feature_importances_</th>
<th>array of shape = [n_features]</th>
<th>The feature importances (the higher, the more important the feature).</th>
</tr>
</thead>
<tbody>
<tr>
<td>oob_score_</td>
<td>float</td>
<td>Score of the training dataset obtained using an out-of-bag estimate.</td>
</tr>
<tr>
<td>oob_prediction_</td>
<td>array, shape = [n_samples]</td>
<td>Prediction computed with out-of-bag estimate on the training set.</td>
</tr>
</tbody>
</table>

Methods

**fit**(X, y)
Build a forest of trees from the training set (X, y).

**fit_transform**(X[, y])
Fit to data, then transform it

**get_params**(deep)
Get parameters for the estimator

**predict**(X)
Predict regression target for X.

**score**(X, y)
Returns the coefficient of determination R^2 of the prediction.

**set_params**(**params)
Set the parameters of the estimator.

**transform**(X[, threshold])
Reduce X to its most important features.

**init**(n_estimators=10, criterion='mse', max_depth=None, min_samples_split=1, min_samples_leaf=1, min_density=0.1, max_features='auto', bootstrap=True, compute_importances=False, oob_score=False, n_jobs=1, random_state=None, verbose=0)

**fit**(X, y)
Build a forest of trees from the training set (X, y).

**Parameters**

<table>
<thead>
<tr>
<th>X : array-like of shape = [n_samples, n_features]</th>
</tr>
</thead>
<tbody>
<tr>
<td>The training input samples.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>y : array-like, shape = [n_samples]</th>
</tr>
</thead>
<tbody>
<tr>
<td>The target values (integers that correspond to classes in classification, real numbers in regression).</td>
</tr>
</tbody>
</table>

**Returns**

self : object

Returns self.
**fit_transform** *(X, y=None, **fit_params)*

Fit to data, then transform it

Fits transformer to X and y with optional parameters fit_params and returns a transformed version of X.

**Parameters**
- **X**: numpy array of shape [n_samples, n_features]
  - Training set.
- **y**: numpy array of shape [n_samples]
  - Target values.

**Returns**
- **X_new**: numpy array of shape [n_samples, n_features_new]
  - Transformed array.

**Notes**

This method just calls fit and transform consecutively, i.e., it is not an optimized implementation of fit_transform, unlike other transformers such as PCA.

**get_params** *(deep=True)*

Get parameters for the estimator

**Parameters**
- **deep**: boolean, optional
  - If True, will return the parameters for this estimator and contained subobjects that are estimators.

**predict** *(X)*

Predict regression target for X.

The predicted regression target of an input sample is computed as the mean predicted regression targets of the trees in the forest.

**Parameters**
- **X**: array-like of shape = [n_samples, n_features]
  - The input samples.

**Returns**
- **y**: array of shape = [n_samples]
  - The predicted values.

**score** *(X, y)*

Returns the coefficient of determination \( R^2 \) of the prediction.

The coefficient \( R^2 \) is defined as \( 1 - \frac{u}{v} \), where \( u \) is the regression sum of squares \( \sum((y - y_{pred})^2) \) and \( v \) is the residual sum of squares \( \sum((y_{true} - y_{true.mean})^2) \). Best possible score is 1.0, lower values are worse.

**Parameters**
- **X**: array-like, shape = [n_samples, n_features]
  - Training set.
- **y**: array-like, shape = [n_samples]

**Returns**
- **z**: float

**set_params** (**params**)

Set the parameters of the estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The former have parameters of the form \(<component>__<parameter>\) so that it’s possible to update each component of a nested object.
Returns self:

**transform** *(X, threshold=None)*
Reduce X to its most important features.

**Parameters**

- **X**: array or scipy sparse matrix of shape [n_samples, n_features]
  The input samples.
- **threshold**: string, float or None, optional (default=None)
  The threshold value to use for feature selection. Features whose importance is greater or equal are kept while the others are discarded. If “median” (resp. “mean”), then the threshold value is the median (resp. the mean) of the feature importances. A scaling factor (e.g., “1.25*mean”) may also be used. If None and if available, the object attribute threshold is used. Otherwise, “mean” is used by default.

**Returns**

- **X_r**: array of shape [n_samples, n_selected_features]
  The input samples with only the selected features.

**sklearn.ensemble.ExtraTreesClassifier**

```python
class sklearn.ensemble.ExtraTreesClassifier(n_estimators=10, criterion='gini', max_depth=None, min_samples_split=1, min_samples_leaf=1, min_density=0.1, max_features='auto', bootstrap=False, compute_importances=False, oob_score=False, n_jobs=1, random_state=None, verbose=0)
```

An extra-trees classifier.

This class implements a meta estimator that fits a number of randomized decision trees (a.k.a. extra-trees) on various sub-samples of the dataset and use averaging to improve the predictive accuracy and control over-fitting.

**Parameters**

- **n_estimators**: integer, optional (default=10)
  The number of trees in the forest.
- **criterion**: string, optional (default="gini")
  The function to measure the quality of a split. Supported criteria are “gini” for the Gini impurity and “entropy” for the information gain. Note: this parameter is tree-specific.
- **max_depth**: integer or None, optional (default=None)
  The maximum depth of the tree. If None, then nodes are expanded until all leaves are pure or until all leaves contain less than min_samples_split samples. Note: this parameter is tree-specific.
- **min_samples_split**: integer, optional (default=1)
  The minimum number of samples required to split an internal node. Note: this parameter is tree-specific.
- **min_samples_leaf**: integer, optional (default=1)
  The minimum number of samples in newly created leaves. A split is discarded if after the split, one of the leaves would contain less then min_samples_leaf samples. Note: this parameter is tree-specific.
- **min_density**: float, optional (default=0.1)
  This parameter controls a trade-off in an optimization heuristic. It controls the minimum density of the sample_mask (i.e. the fraction of samples in the mask). If the density falls
below this threshold the mask is recomputed and the input data is packed which results in data copying. If \( \text{min\_density} \) equals to one, the partitions are always represented as copies of the original data. Otherwise, partitions are represented as bit masks (aka sample masks). Note: this parameter is tree-specific.

**max_features**: int, string or None, optional (default="auto")

The number of features to consider when looking for the best split.

- If “auto”, then \( \text{max\_features}=\sqrt{n\_features} \) on classification tasks and \( \text{max\_features}=n\_features \) on regression problems.
- If “sqrt”, then \( \text{max\_features}=\sqrt{n\_features} \).
- If “log2”, then \( \text{max\_features}=\log2(n\_features) \).
- If None, then \( \text{max\_features}=n\_features \).

Note: this parameter is tree-specific.

**bootstrap**: boolean, optional (default=False)

Whether bootstrap samples are used when building trees.

**compute_importances**: boolean, optional (default=True)

Whether feature importances are computed and stored into the \text{feature\_importances\_} attribute when calling fit.

**oob_score**: bool

Whether to use out-of-bag samples to estimate the generalization error.

**n_jobs**: integer, optional (default=1)

The number of jobs to run in parallel. If -1, then the number of jobs is set to the number of cores.

**random_state**: int, RandomState instance or None, optional (default=None)

If int, random_state is the seed used by the random number generator; If RandomState instance, random_state is the random number generator; If None, the random number generator is the RandomState instance used by \text{np.random}.

**verbose**: int, optional (default=0)

Controls the verbosity of the tree building process.

See Also:

- \text{sklearn\_tree\_ExtraTreeClassifier} Base classifier for this ensemble.
- \text{RandomForestClassifier} Ensemble Classifier based on trees with optimal splits.

References

[R57]
Attributes

<table>
<thead>
<tr>
<th>feature_importances_</th>
<th>array of shape = [n_features]</th>
<th>The feature importances (the higher, the more important the feature).</th>
</tr>
</thead>
<tbody>
<tr>
<td>oob_score_</td>
<td>float</td>
<td>Score of the training dataset obtained using an out-of-bag estimate.</td>
</tr>
<tr>
<td>oob_decision_function_</td>
<td>array, shape = [n_samples, n_classes]</td>
<td>Decision function computed with out-of-bag estimate on the training set.</td>
</tr>
</tbody>
</table>

Methods

<table>
<thead>
<tr>
<th>fit(X, y)</th>
<th>Build a forest of trees from the training set (X, y).</th>
</tr>
</thead>
<tbody>
<tr>
<td>fit_transform(X[, y])</td>
<td>Fit to data, then transform it</td>
</tr>
<tr>
<td>get_params([deep])</td>
<td>Get parameters for the estimator</td>
</tr>
<tr>
<td>predict(X)</td>
<td>Predict class for X.</td>
</tr>
<tr>
<td>predict_log_proba(X)</td>
<td>Predict class log-probabilities for X.</td>
</tr>
<tr>
<td>predict_proba(X)</td>
<td>Predict class probabilities for X.</td>
</tr>
<tr>
<td>score(X, y)</td>
<td>Returns the mean accuracy on the given test data and labels.</td>
</tr>
<tr>
<td>set_params(**params)</td>
<td>Set the parameters of the estimator.</td>
</tr>
<tr>
<td>transform(X[, threshold])</td>
<td>Reduce X to its most important features.</td>
</tr>
</tbody>
</table>

__init__ (n_estimators=10, criterion='gini', max_depth=None, min_samples_split=1, min_samples_leaf=1, min_density=0.1, max_features='auto', compute_importances=False, oob_score=False, n_jobs=1, random_state=None, verbose=0)

fit (X, y)
Build a forest of trees from the training set (X, y).

Parameters X : array-like of shape = [n_samples, n_features]
The training input samples.

y : array-like, shape = [n_samples]
The target values (integers that correspond to classes in classification, real numbers in regression).

Returns self : object
Returns self.

fit_transform (X, y=None, **fit_params)
Fit to data, then transform it
Fits transformer to X and y with optional parameters fit_params and returns a transformed version of X.

Parameters X : numpy array of shape [n_samples, n_features]
Training set.

y : numpy array of shape [n_samples]
Target values.

Returns X_new : numpy array of shape [n_samples, n_features_new]
Transformed array.
Notes

This method just calls fit and transform consecutively, i.e., it is not an optimized implementation of fit_transform, unlike other transformers such as PCA.

**get_params** *(deep=True)*
Get parameters for the estimator

**Parameters**
- **deep**: boolean, optional
  If True, will return the parameters for this estimator and contained subobjects that are estimators.

**predict** *(X)*
Predict class for X.

The predicted class of an input sample is computed as the majority prediction of the trees in the forest.

**Parameters**
- **X**: array-like of shape = [n_samples, n_features]
  The input samples.

**Returns**
- **y**: array of shape = [n_samples]
  The predicted classes.

**predict_log_proba** *(X)*
Predict class log-probabilities for X.

The predicted class log-probabilities of an input sample is computed as the mean predicted class log-probabilities of the trees in the forest.

**Parameters**
- **X**: array-like of shape = [n_samples, n_features]
  The input samples.

**Returns**
- **p**: array of shape = [n_samples]
  The class log-probabilities of the input samples. Classes are ordered by arithmetical order.

**predict_proba** *(X)*
Predict class probabilities for X.

The predicted class probabilities of an input sample is computed as the mean predicted class probabilities of the trees in the forest.

**Parameters**
- **X**: array-like of shape = [n_samples, n_features]
  The input samples.

**Returns**
- **p**: array of shape = [n_samples]
  The class probabilities of the input samples. Classes are ordered by arithmetical order.

**score** *(X, y)*
Returns the mean accuracy on the given test data and labels.

**Parameters**
- **X**: array-like, shape = [n_samples, n_features]
  Training set.
- **y**: array-like, shape = [n_samples]
  Labels for X.

**Returns**
- **z**: float
**set_params** (**params**)  
Set the parameters of the estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The former have parameters of the form `<component>__<parameter>` so that it’s possible to update each component of a nested object.

**Returns** self:

**transform** (X, threshold=None)  
Reduce X to its most important features.

**Parameters** X : array or scipy sparse matrix of shape [n_samples, n_features]  
The input samples.

threshold : string, float or None, optional (default=None)  
The threshold value to use for feature selection. Features whose importance is greater or equal are kept while the others are discarded. If “median” (resp. “mean”), then the threshold value is the median (resp. the mean) of the feature importances. A scaling factor (e.g., “1.25*mean”) may also be used. If None and if available, the object attribute threshold is used. Otherwise, “mean” is used by default.

**Returns** X_r : array of shape [n_samples, n_selected_features]  
The input samples with only the selected features.

```python
sklearn.ensemble.ExtraTreesRegressor
```

class sklearn.ensemble.ExtraTreesRegressor(n_estimators=10, criterion='mse', max_depth=None, min_samples_split=1, min_samples_leaf=1, min_density=0.1, max_features='auto', bootstrap=False, compute_importances=False, oob_score=False, n_jobs=1, random_state=None, verbose=0)

An extra-trees regressor.

This class implements a meta estimator that fits a number of randomized decision trees (a.k.a. extra-trees) on various sub-samples of the dataset and use averaging to improve the predictive accuracy and control over-fitting.

**Parameters** n_estimators : integer, optional (default=10)  
The number of trees in the forest.

criterion : string, optional (default="mse")  
The function to measure the quality of a split. The only supported criterion is “mse” for the mean squared error. Note: this parameter is tree-specific.

max_depth : integer or None, optional (default=None)  
The maximum depth of the tree. If None, then nodes are expanded until all leaves are pure or until all leaves contain less than min_samples_split samples. Note: this parameter is tree-specific.

min_samples_split : integer, optional (default=1)  
The minimum number of samples required to split an internal node. Note: this parameter is tree-specific.

min_samples_leaf : integer, optional (default=1)  

The minimum number of samples in newly created leaves. A split is discarded if after
the split, one of the leaves would contain less then min_samples_leaf samples.
Note: this parameter is tree-specific.

**min_density**: float, optional (default=0.1)

This parameter controls a trade-off in an optimization heuristic. It controls the minimum
density of the sample_mask (i.e. the fraction of samples in the mask). If the density falls
below this threshold the mask is recomputed and the input data is packed which results
in data copying. If min_density equals to one, the partitions are always represented
as copies of the original data. Otherwise, partitions are represented as bit masks (aka
sample masks). Note: this parameter is tree-specific.

**max_features**: int, string or None, optional (default=”auto”)

The number of features to consider when looking for the best split:

- If “auto”, then max_features=\sqrt{\text{n_features}} on classification tasks and
  max_features=\text{n_features} on regression problems.
- If “sqrt”, then max_features=\sqrt{\text{n_features}}.
- If “log2”, then max_features=log2(\text{n_features}).
- If None, then max_features=\text{n_features}.

Note: this parameter is tree-specific.

**bootstrap**: boolean, optional (default=False)

Whether bootstrap samples are used when building trees. Note: this parameter is tree-
specific.

**compute_importances**: boolean, optional (default=True)

Whether feature importances are computed and stored into the
feature_importances_ attribute when calling fit.

**oob_score**: bool

Whether to use out-of-bag samples to estimate the generalization error.

**n_jobs**: integer, optional (default=1)

The number of jobs to run in parallel. If -1, then the number of jobs is set to the number
of cores.

**random_state**: int, RandomState instance or None, optional (default=None)

If int, random_state is the seed used by the random number generator; If RandomState
instance, random_state is the random number generator; If None, the random number
generator is the RandomState instance used by np.random.

**verbose**: int, optional (default=0)

Controls the verbosity of the tree building process.

**See Also:**

*sklearn.tree.ExtraTreeRegressor* Base estimator for this ensemble.

*RandomForestRegressor* Ensemble regressor using trees with optimal splits.
References

[R58]

Attributes

<table>
<thead>
<tr>
<th>feature_importances_</th>
<th>array of shape = [n_features]</th>
<th>The feature importances (the higher, the more important the feature).</th>
</tr>
</thead>
<tbody>
<tr>
<td>oob_score_</td>
<td>float</td>
<td>Score of the training dataset obtained using an out-of-bag estimate.</td>
</tr>
<tr>
<td>oob_prediction_</td>
<td>array, shape = [n_samples]</td>
<td>Prediction computed with out-of-bag estimate on the training set.</td>
</tr>
</tbody>
</table>

Methods

<table>
<thead>
<tr>
<th>fit(X, y)</th>
<th>Build a forest of trees from the training set (X, y).</th>
</tr>
</thead>
<tbody>
<tr>
<td>fit_transform(X[, y])</td>
<td>Fit to data, then transform it</td>
</tr>
<tr>
<td>get_params([deep])</td>
<td>Get parameters for the estimator</td>
</tr>
<tr>
<td>predict(X)</td>
<td>Predict regression target for X.</td>
</tr>
<tr>
<td>score(X, y)</td>
<td>Returns the coefficient of determination R^2 of the prediction.</td>
</tr>
<tr>
<td>set_params(**params)</td>
<td>Set the parameters of the estimator.</td>
</tr>
<tr>
<td>transform(X[, threshold])</td>
<td>Reduce X to its most important features.</td>
</tr>
</tbody>
</table>

__init__(n_estimators=10, criterion='mse', max_depth=None, min_samples_split=1, min_samples_leaf=1, min_density=0.1, max_features='auto', bootstrap=False, compute_importances=False, oob_score=False, n_jobs=1, random_state=None, verbose=0)

fit (X, y)
Build a forest of trees from the training set (X, y).

Parameters X : array-like of shape = [n_samples, n_features]
The training input samples.

y : array-like, shape = [n_samples]
The target values (integers that correspond to classes in classification, real numbers in regression).

Returns self : object
Returns self.

fit_transform (X, y=None, **fit_params)
Fit to data, then transform it
Fits transformer to X and y with optional parameters fit_params and returns a transformed version of X.

Parameters X : numpy array of shape [n_samples, n_features]
Training set.

y : numpy array of shape [n_samples]
Target values.

Returns X_new : numpy array of shape [n_samples, n_features_new]
Transformed array.

Notes

This method just calls fit and transform consecutively, i.e., it is not an optimized implementation of fit_transform, unlike other transformers such as PCA.

get_params (deep=True)

Get parameters for the estimator

Parameters deep: boolean, optional:
If True, will return the parameters for this estimator and contained subobjects that are estimators.

predict (X)

Predict regression target for X.
The predicted regression target of an input sample is computed as the mean predicted regression targets of the trees in the forest.

Parameters X : array-like of shape = [n_samples, n_features]
The input samples.

Returns y : array of shape = [n_samples] :
The predicted values.

cscore (X, y)

Returns the coefficient of determination R^2 of the prediction.
The coefficient R^2 is defined as (1 - u/v), where u is the regression sum of squares ((y_true - y_pred)**2).sum() and v is the residual sum of squares ((y_true - y_true.mean())**2).sum(). Best possible score is 1.0, lower values are worse.

Parameters X : array-like, shape = [n_samples, n_features]
Training set.

y : array-like, shape = [n_samples]

Returns z : float

set_params (**params)

Set the parameters of the estimator.
The method works on simple estimators as well as on nested objects (such as pipelines). The former have parameters of the form <component>__<parameter> so that it’s possible to update each component of a nested object.

Returns self :

transform (X, threshold=None)

Reduce X to its most important features.

Parameters X : array or scipy sparse matrix of shape [n_samples, n_features]
The input samples.

threshold : string, float or None, optional (default=None)
The threshold value to use for feature selection. Features whose importance is greater or equal are kept while the others are discarded. If “median” (resp. “mean”), then the threshold value is the median (resp. the mean) of the feature importances. A scaling factor (e.g., “1.25*mean”) may also be used. If None and if available, the object attribute threshold is used. Otherwise, “mean” is used by default.

**Returns X_r**: array of shape [n_samples, n_selected_features]

The input samples with only the selected features.

### sklearn.ensemble.GradientBoostingClassifier

class sklearn.ensemble.GradientBoostingClassifier (loss='deviance', learn_rate=0.1,
n_estimators=100, subsample=1.0, min_samples_split=1,
min_samples_leaf=1, max_depth=3,
init=None, random_state=None)

Gradient Boosting for classification.

GB builds an additive model in a forward stage-wise fashion; it allows for the optimization of arbitrary differentiable loss functions. In each stage \( n_{classes} \) regression trees are fit on the negative gradient of the binomial or multinomial deviance loss function. Binary classification is a special case where only a single regression tree is induced.

**Parameters**

- **loss**: {‘deviance’, ‘ls’}, optional (default=’deviance’)
  - loss function to be optimized. ‘deviance’ refers to deviance (= logistic regression) for classification with probabilistic outputs. ‘ls’ refers to least squares regression.

- **learn_rate**: float, optional (default=0.1)
  - learning rate shrinks the contribution of each tree by \( learn_rate \). There is a trade-off between learn_rate and n_estimators.

- **n_estimators**: int (default=100)
  - The number of boosting stages to perform. Gradient boosting is fairly robust to overfitting so a large number usually results in better performance.

- **max_depth**: integer, optional (default=3)
  - maximum depth of the individual regression estimators. The maximum depth limits the number of nodes in the tree. Tune this parameter for best performance; the best value depends on the interaction of the input variables.

- **min_samples_split**: integer, optional (default=1)
  - The minimum number of samples required to split an internal node.

- **min_samples_leaf**: integer, optional (default=1)
  - The minimum number of samples required to be at a leaf node.

- **subsample**: float, optional (default=1.0)
  - The fraction of samples to be used for fitting the individual base learners. If smaller than 1.0 this results in Stochastic Gradient Boosting. subsample interacts with the parameter n_estimators.

**See Also:**

sklearn.tree.DecisionTreeClassifier, RandomForestClassifier
References


Friedman, Stochastic Gradient Boosting, 1999


Examples

```python
>>> samples = [[0, 0, 2], [1, 0, 0]]
>>> labels = [0, 1]
>>> from sklearn.ensemble import GradientBoostingClassifier
>>> gb = GradientBoostingClassifier().fit(samples, labels)
>>> print gb.predict([[0.5, 0, 0]])
[0]
```

Methods

- **fit**(X, y)
  Fit the gradient boosting model.

- **fit_stage**(i, X, X_argsorted, y, y_pred, ...)
  Fit another stage of n_classes_ trees to the boosting model.

- **get_params**(deep=True)
  Get parameters for the estimator.

- **predict**(X)
  Predict class for X.

- **predict_proba**(X)
  Predict class probabilities for X.

- **score**(X, y)
  Returns the mean accuracy on the given test data and labels.

- **set_params**(**params)
  Set the parameters of the estimator.

- **staged_decision_function**(X)
  Compute decision function for X.

**__init__**(loss='deviance', learn_rate=0.1, n_estimators=100, subsample=1.0, min_samples_split=1, min_samples_leaf=1, max_depth=3, init=None, random_state=None)

**fit**(X, y)
Fit the gradient boosting model.

**Parameters**

- **X**: array-like, shape = [n_samples, n_features]
  Training vectors, where n_samples is the number of samples and n_features is the number of features. Use fortran-style to avoid memory copies.

- **y**: array-like, shape = [n_samples]
  Target values (integers in classification, real numbers in regression) For classification, labels must correspond to classes 0, 1, ..., n_classes_-1

**Returns**

- **self**: object
  Returns self.

**fit_stage**(i, X, X_argsorted, y, y_pred, sample_mask)
Fit another stage of n_classes_ trees to the boosting model.

**get_params**(deep=True)
Get parameters for the estimator

**Parameters**

- **deep**: boolean, optional
If True, will return the parameters for this estimator and contained subobjects that are
estimators.

**predict** (*X*)
Predict class for *X*.

**Parameters**

- **X**: array-like of shape = [n_samples, n_features]
  The input samples.

**Returns**

- **y**: array of shape = [n_samples]
  The predicted classes.

**predict_proba** (*X*)
Predict class probabilities for *X*.

**Parameters**

- **X**: array-like of shape = [n_samples, n_features]
  The input samples.

**Returns**

- **p**: array of shape = [n_samples]
  The class probabilities of the input samples. Classes are ordered by arithmetical order.

**score** (*X*, *y*)
Returns the mean accuracy on the given test data and labels.

**Parameters**

- **X**: array-like, shape = [n_samples, n_features]
  Training set.

- **y**: array-like, shape = [n_samples]
  Labels for *X*.

**Returns**

- **z**: float

**set_params** (**params**)
Set the parameters of the estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The former have
parameters of the form `<component>__<parameter>` so that it’s possible to update each component
of a nested object.

**Returns**

- **self**:

**staged_decision_function** (*X*)
Compute decision function for *X*.

This method allows monitoring (i.e. determine error on testing set) after each stage.

**Parameters**

- **X**: array-like of shape = [n_samples, n_features]
  The input samples.

**Returns**

- **f**: array of shape = [n_samples, n_classes]
  The decision function of the input samples. Classes are ordered by arithmetical order.
  Regression and binary classification are special cases with `n_classes == 1`.

**sklearn.ensemble.GradientBoostingRegressor**
Gradient Boosting for regression.

GB builds an additive model in a forward stage-wise fashion; it allows for the optimization of arbitrary differentiable loss functions. In each stage a regression tree is fit on the negative gradient of the given loss function.

**Parameters**

- **loss**: {'ls', 'lad'}, optional (default='ls')
  - loss function to be optimized. ‘ls’ refers to least squares regression. ‘lad’ (least absolute deviation) is a highly robust loss function solely based on order information of the input variables.

- **learn_rate**: float, optional (default=0.1)
  - learning rate shrinks the contribution of each tree by learn_rate. There is a trade-off between learn_rate and n_estimators.

- **n_estimators**: int (default=100)
  - The number of boosting stages to perform. Gradient boosting is fairly robust to over-fitting so a large number usually results in better performance.

- **max_depth**: integer, optional (default=3)
  - maximum depth of the individual regression estimators. The maximum depth limits the number of nodes in the tree. Tune this parameter for best performance; the best value depends on the interaction of the input variables.

- **min_samples_split**: integer, optional (default=1)
  - The minimum number of samples required to split an internal node.

- **min_samples_leaf**: integer, optional (default=1)
  - The minimum number of samples required to be at a leaf node.

- **subsample**: float, optional (default=1.0)
  - The fraction of samples to be used for fitting the individual base learners. If smaller than 1.0 this results in Stochastic Gradient Boosting. subsample interacts with the parameter n_estimators.

**See Also:**

- sklearn.tree.DecisionTreeRegressor, RandomForestRegressor

**References**


10. Friedman, Stochastic Gradient Boosting, 1999

Examples

```python
>>> samples = [[0, 0, 2], [1, 0, 0]]
>>> labels = [0, 1]
>>> from sklearn.ensemble import GradientBoostingRegressor
>>> gb = GradientBoostingRegressor().fit(samples, labels)
>>> print(gb.predict([[0, 0, 0]]))
[1.32806997e-05]
```

Attributes

<table>
<thead>
<tr>
<th>feature_importances_</th>
<th>array, shape [n_features]</th>
<th>The feature importances (the higher, the more important the feature).</th>
</tr>
</thead>
<tbody>
<tr>
<td>oob_score_</td>
<td>array, shape [n_estimators]</td>
<td>Score of the training dataset obtained using an out-of-bag estimate. The i-th score oob_score_[i] is the deviance (= loss) of the model at iteration i on the out-of-bag sample.</td>
</tr>
<tr>
<td>train_score_</td>
<td>array, shape [n_estimators]</td>
<td>The i-th score train_score_[i] is the deviance (= loss) of the model at iteration i on the in-bag sample. If subsample == 1 this is the deviance on the training data.</td>
</tr>
</tbody>
</table>

Methods

<table>
<thead>
<tr>
<th>Method</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>fit(X, y)</td>
<td>Fit the gradient boosting model.</td>
</tr>
<tr>
<td>fit_stage(i, X, X_argsorted, y, y_pred, ...)</td>
<td>Fit another stage of n_classes_ trees to the boosting model.</td>
</tr>
<tr>
<td>get_params([deep])</td>
<td>Get parameters for the estimator</td>
</tr>
<tr>
<td>predict(X)</td>
<td>Predict regression target for X.</td>
</tr>
<tr>
<td>score(X, y)</td>
<td>Returns the coefficient of determination R^2 of the prediction.</td>
</tr>
<tr>
<td>set_params(**params)</td>
<td>Set the parameters of the estimator.</td>
</tr>
<tr>
<td>staged_decision_function(X)</td>
<td>Compute decision function for X.</td>
</tr>
<tr>
<td>staged_predict(X)</td>
<td>Predict regression target at each stage for X.</td>
</tr>
</tbody>
</table>

```

```
__init__(loss='ls', learn_rate=0.1, n_estimators=100, subsample=1.0, min_samples_split=1, min_samples_leaf=1, max_depth=3, init=None, random_state=None)

fit(X, y)
Fit the gradient boosting model.

Parameters

- **X**: array-like, shape = [n_samples, n_features]
  Training vectors, where n_samples is the number of samples and n_features is the number of features. Use fortran-style to avoid memory copies.

- **y**: array-like, shape = [n_samples]
  Target values (integers in classification, real numbers in regression) For classification, labels must correspond to classes 0, 1, ..., n_classes_-1

Returns

- **self**: object

fit_stage(i, X, X_argsorted, y, y_pred, sample_mask)
Fit another stage of n_classes_ trees to the boosting model.
get_params(deep=True)
Get parameters for the estimator

Parameters deep: boolean, optional:
If True, will return the parameters for this estimator and contained subobjects that are estimators.

predict(X)
Predict regression target for X.

Parameters X: array-like of shape = [n_samples, n_features]
The input samples.

Returns y: array of shape = [n_samples]:
The predicted values.

score(X, y)
Returns the coefficient of determination $R^2$ of the prediction.
The coefficient $R^2$ is defined as $(1 - u/v)$, where $u$ is the regression sum of squares $((y - y_{pred}) ** 2).sum()$ and $v$ is the residual sum of squares $((y_{true} - y_{true.mean}) ** 2).sum()$. Best possible score is 1.0, lower values are worse.

Parameters X: array-like, shape = [n_samples, n_features]
Training set.

y: array-like, shape = [n_samples]

Returns z: float

set_params(**params)
Set the parameters of the estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The former have parameters of the form <component>__<parameter> so that it’s possible to update each component of a nested object.

Returns self:

staged_decision_function(X)
Compute decision function for X.
This method allows monitoring (i.e. determine error on testing set) after each stage.

Parameters X: array-like of shape = [n_samples, n_features]
The input samples.

Returns f: array of shape = [n_samples, n_classes]
The decision function of the input samples. Classes are ordered by arithmetical order. Regression and binary classification are special cases with n_classes == 1.

staged_predict(X)
Predict regression target at each stage for X.
This method allows monitoring (i.e. determine error on testing set) after each stage.

Parameters X: array-like of shape = [n_samples, n_features]
The input samples.

Returns y: array of shape = [n_samples]
The predicted value of the input samples.

1.5.3 Pipeline: chaining estimators

Pipeline can be used to chain multiple estimators into one. This is useful as there is often a fixed sequence of steps in processing the data, for example feature selection, normalization and classification. Pipeline serves two purposes here:

Convenience: You only have to call fit and predict once on your data to fit a whole sequence of estimators.

Joint parameter selection: You can grid search over parameters of all estimators in the pipeline at once.

For estimators to be usable within a pipeline, all except the last one need to have a transform function. Otherwise, the dataset can not be passed through this estimator.

Usage

The Pipeline is build using a list of (key, value) pairs, where the key a string containing the name you want to give this step and value is an estimator object:

```python
>>> from sklearn.pipeline import Pipeline
>>> from sklearn.svm import SVC
>>> from sklearn.decomposition import PCA

>>> estimators = [('reduce_dim', PCA()), ('svm', SVC())]
>>> clf = Pipeline(estimators)
>>> clf
Pipeline(steps=[('reduce_dim', PCA(copy=True, n_components=None, whiten=False)), ('svm', SVC(C=1.0, cache_size=200, class_weight=None, coef0=0.0, degree=3, gamma=0.0, kernel='rbf', probability=False, shrinking=True, tol=0.001, verbose=False))])
```

The estimators of the pipeline are stored as a list in the steps attribute:

```python
>>> clf.steps[0]
('reduce_dim', PCA(copy=True, n_components=None, whiten=False))
```

and as a dict in named_steps:

```python
>>> clf.named_steps['reduce_dim']
PCA(copy=True, n_components=None, whiten=False)
```

Parameters of the estimators in the pipeline can be accessed using the <estimator>__<parameter> syntax:

```python
>>> clf.set_params(svm__C=10) # NORMALIZE_WHITESPACE
Pipeline(steps=[('reduce_dim', PCA(copy=True, n_components=None, whiten=False)), ('svm', SVC(C=10, cache_size=200, class_weight=None, coef0=0.0, degree=3, gamma=0.0, kernel='rbf', probability=False, shrinking=True, tol=0.001, verbose=False))])
```

This is particularly important for doing grid searches:

```python
>>> from sklearn.grid_search import GridSearchCV
>>> params = dict(reduce_dim__n_components=[2, 5, 10],
...    svm__C=[0.1, 10, 100])
>>> grid_search = GridSearchCV(clf, param_grid=params)
```
Examples:

- Pipeline Anova SVM
- Sample pipeline for text feature extraction and evaluation
- Pipelining: chaining a PCA and a logistic regression
- Explicit feature map approximation for RBF kernels
- SVM-Anova: SVM with univariate feature selection

Notes

Calling `fit` on the pipeline is the same as calling `fit` on each estimator in turn, `transform` the input and pass it on to the next step. The pipeline has all the methods that the last estimator in the pipeline has, i.e. if the last estimator is a classifier, the `Pipeline` can be used as a classifier. If the last estimator is a transformer, again, so is the pipeline.

1.6 Dataset transformations

1.6.1 Preprocessing data

The `sklearn.preprocessing` package provides several common utility functions and transformer classes to change raw feature vectors into a representation that is more suitable for the downstream estimators.

Standardization or Mean Removal and Variance Scaling

Standardization of datasets is a common requirement for many machine learning estimators implemented in the scikit: they might behave badly if the individual feature do not more or less look like standard normally distributed data: Gaussian with zero mean and unit variance.

In practice we often ignore the shape of the distribution and just transform the data to center it by removing the mean value of each feature, then scale it by dividing non-constant features by their standard deviation.

For instance, many elements used in the objective function of a learning algorithm (such as the RBF kernel of Support Vector Machines or the l1 and l2 regularizers of linear models) assume that all features are centered around zero and have variance in the same order. If a feature has a variance that is orders of magnitude larger that others, it might dominate the objective function and make the estimator unable to learn from other features correctly as expected.

The function `scale` provides a quick and easy way to perform this operation on a single array-like dataset:

```python
>>> from sklearn import preprocessing
>>> X = [[ 1., -1., 2.],
...     [ 2., 0., 0.],
...     [ 0., 1., -1.]]
>>> X_scaled = preprocessing.scale(X)

>>> X_scaled
array([[ 0. ..., -1.22..., 1.33...],
       [ 1.22..., 0. ..., -0.26...],
       [-1.22..., 1.22..., -1.06...]])
```

Scaled data has zero mean and unit variance:
>>> X_scaled.mean(axis=0)
array([ 0., 0., 0.])

>>> X_scaled.std(axis=0)
array([ 1., 1., 1.])

The preprocessing module further provides a utility class `Scaler` that implements the Transformer API to compute the mean and standard deviation on a training set so as to be able to later reapply the same transformation on the testing set. This class is hence suitable for use in the early steps of a `sklearn.pipeline.Pipeline`:

```python
>>> scaler = preprocessing.Scaler().fit(X)
```

```python
>>> scaler
Scaler(copy=True, with_mean=True, with_std=True)
```

```python
>>> scaler.mean_
array([ 1. ..., 0. ..., 0.33 ...])
```

```python
>>> scaler.std_
array([ 0.81 ..., 0.81 ..., 1.24 ...])
```

```python
>>> scaler.transform(X)
array([[ 0. ..., -1.22 ..., 1.33 ...],
       [ 1.22 ..., 0. ..., -0.26 ...],
       [-1.22 ..., 1.22 ..., -1.06 ...]])
```

The scaler instance can then be used on new data to transform it the same way it did on the training set:

```python
>>> scaler.transform([[-1., 1., 0.]])
array([[-2.44 ..., 1.22 ..., -0.26 ...]])
```

It is possible to disable either centering or scaling by either passing `with_mean=False` or `with_std=False` to the constructor of `Scaler`.

**References:**

Further discussion on the importance of centering and scaling data is available on this FAQ: Should I normalize/standardize/rescale the data?

**Scaling vs Whitening**

It is sometimes not enough to center and scale the features independently, since a downstream model can further make some assumption on the linear independence of the features.

To address this issue you can use `sklearn.decomposition.PCA` or `sklearn.decomposition.RandomizedPCA` with `whiten=True` to further remove the linear correlation across features.
Sparse input

scale and Scaler accept scipy.sparse matrices as input only when with_mean=False is explicitly passed to the constructor. Otherwise a ValueError will be raised as silently centering would break the sparsity and would often crash the execution by allocating excessive amounts of memory unintentionally. If the centered data is expected to be small enough, explicitly convert the input to an array using the toarray method of sparse matrices instead. For sparse input the data is converted to the Compressed Sparse Rows representation (see scipy.sparse.csr_matrix). To avoid unnecessary memory copies, it is recommended to choose the CSR representation upstream.

Scaling target variables in regression

scale and Scaler work out-of-the-box with 1d arrays. This is very useful for scaling the target / response variables used for regression.

Normalization

Normalization is the process of scaling individual samples to have unit norm. This process can be useful if you plan to use a quadratic form such as the dot-product or any other kernel to quantify the similarity of any pair of samples.

This assumption is the base of the Vector Space Model often used in text classification and clustering contexts.

The function normalize provides a quick and easy way to perform this operation on a single array-like dataset, either using the 11 or 12 norms:

```python
>>> X = [[ 1., -1., 2.],
      ... [ 2., 0., 0.],
      ... [ 0., 1., -1.]]
>>> X_normalized = preprocessing.normalize(X, norm='l2')
```

```python
array([[ 0.40..., -0.40..., 0.81...],
       [ 1. ..., 0. ..., 0. ...],
       [ 0. ..., 0.70..., -0.70...]])
```

The preprocessing module further provides a utility class Normalizer that implements the same operation using the Transformer API (even though the fit method is useless in this case: the class is stateless as this operation treats samples independently).

This class is hence suitable for use in the early steps of a sklearn.pipeline.Pipeline:

```python
>>> normalizer = preprocessing.Normalizer().fit(X)  # fit does nothing
>>> normalizer
Normalizer(copy=True, norm='l2')
```

The normalizer instance can then be used on sample vectors as any transformer:

```python
>>> normalizer.transform(X)
array([[ 0.40..., -0.40..., 0.81...],
       [ 1. ..., 0. ..., 0. ...],
       [ 0. ..., 0.70..., -0.70...]])
>>> normalizer.transform([[-1., 1., 0.]])
array([[-0.70..., 0.70..., 0. ...]])
```
Sparse input

normalize and Normalizer accept both dense array-like and sparse matrices from scipy.sparse as input.

For sparse input the data is converted to the Compressed Sparse Rows representation (see scipy.sparse.csr_matrix) before being fed to efficient Cython routines. To avoid unnecessary memory copies, it is recommended to choose the CSR representation upstream.

Binarization

Feature binarization

Feature binarization is the process of thresholding numerical features to get boolean values. This can be useful for downstream probabilistic estimators that make assumption that the input data is distributed according to a multi-variate Bernoulli distribution. For instance, this is the case for the most common class of (Restricted) Boltzmann Machines (not yet implemented in the scikit).

It is also common among the text processing community to use binary feature values (probably to simplify the probabilistic reasoning) even if normalized counts (a.k.a. term frequencies) or TF-IDF valued features often perform slightly better in practice.

As for the Normalizer, the utility class Binarizer is meant to be used in the early stages of sklearn.pipeline.Pipeline. The fit method does nothing as each sample is treated independently of others:

```python
>>> X = [[ 1., -1., 2.],
       [ 2., 0., 0.],
       [ 0., 1., -1.]]
>>> binarizer = preprocessing.Binarizer().fit(X)  # fit does nothing
>>> binarizer
Binarizer(copy=True, threshold=0.0)
>>> binarizer.transform(X)
array([[ 1., 0., 1.],
       [ 1., 0., 0.],
       [ 0., 1., 0.]])
```

It is possible to adjust the threshold of the binarizer:

```python
>>> binarizer = preprocessing.Binarizer(threshold=1.1)
>>> binarizer.transform(X)
array([[ 0., 0., 1.],
       [ 1., 0., 0.],
       [ 0., 0., 0.]])
```

As for the Scaler and Normalizer classes, the preprocessing module provides a companion function binarize to be used when the transformer API is not necessary.
Sparse input

`binarize` and `Binarizer` accept both dense array-like and sparse matrices from `scipy.sparse` as input. For sparse input the data is converted to the Compressed Sparse Rows representation (see `scipy.sparse.csr_matrix`). To avoid unnecessary memory copies, it is recommended to choose the CSR representation upstream.

Label preprocessing

Label binarization

`LabelBinarizer` is a utility class to help create a label indicator matrix from a list of multi-class labels:

```python
>>> lb = preprocessing.LabelBinarizer()
>>> lb.fit([1, 2, 6, 4, 2])
LabelBinarizer(neg_label=0, pos_label=1)
>>> lb.classes_
array([1, 2, 4, 6])
>>> lb.transform([1, 6])
array([[1., 0., 0., 0.],
      [0., 0., 0., 1.]])
```

`LabelBinarizer` also supports multiple labels per instance:

```python
>>> lb.fit_transform([(1, 2), (3,)])
array([[1., 1., 0.],
      [0., 0., 1.]])
>>> lb.classes_
array([1, 2, 3])
```

Label encoding

`LabelEncoder` is a utility class to help normalize labels such that they contain only values between 0 and n_classes-1. This is sometimes useful for writing efficient Cython routines. `LabelEncoder` can be used as follows:

```python
>>> from sklearn import preprocessing
>>> le = preprocessing.LabelEncoder()
>>> le.fit([1, 2, 2, 6])
LabelEncoder()
>>> le.classes_
array([1, 2, 6])
>>> le.transform([1, 1, 2, 6])
array([0, 0, 1, 2])
>>> le.inverse_transform([0, 0, 1, 2])
array([1, 1, 2, 6])
```

It can also be used to transform non-numerical labels (as long as they are hashable and comparable) to numerical labels:

```python
>>> le = preprocessing.LabelEncoder()
>>> le.fit([“paris”, “paris”, “tokyo”, “amsterdam”])
LabelEncoder()
>>> list(le.classes_)
[‘amsterdam’, ‘paris’, ‘tokyo’]
>>> le.transform([“tokyo”, “tokyo”, “paris”])
```
array([2, 2, 1])
>>> list(le.inverse_transform([2, 2, 1]))
['tokyo', 'tokyo', 'paris']

## 1.6.2 Feature extraction

The `sklearn.feature_extraction` module can be used to extract features in a format supported by machine learning algorithms from datasets consisting of formats such as text and image.

**Note:** Feature extraction is very different from Feature selection: the former consists in transforming arbitrary data, such as text or images, into numerical features usable for machine learning. The later is a machine learning technique applied on these features.

### Loading features from dicts

The class `DictVectorizer` can be used to convert feature arrays represented as lists of standard Python `dict` objects to the NumPy/SciPy representation used by scikit-learn estimators.

While not particularly fast to process, Python’s `dict` has the advantages of being convenient to use, being sparse (absent features need not be stored) and storing feature names in addition to values.

`DictVectorizer` implements what is called one-of-K or “one-hot” coding for categorical (aka nominal, discrete) features. Categorical features are “attribute-value” pairs where the value is restricted to a list of discrete of possibilities without ordering (e.g. topic identifiers, types of objects, tags, names...).

In the following, “city” is a categorical attribute while “temperature” is a traditional numerical feature:

```python
>>> measurements = [...
...      {'city': 'Dubai', 'temperature': 33.},
...      {'city': 'London', 'temperature': 12.},
...      {'city': 'San Fransisco', 'temperature': 18.},
...  ]

>>> from sklearn.feature_extraction import DictVectorizer

>>> vec = DictVectorizer()

>>> vec.fit_transform(measurements).toarray()
array([[ 1.,  0.,  0.,  33.],
       [ 0.,  1.,  0.,  12.],
       [ 0.,  0.,  1.,  18.]])

>>> vec.get_feature_names()
['city=Dubai', 'city=London', 'city=San Fransisco', 'temperature']
```

`DictVectorizer` is also a useful representation transformation for training sequence classifiers in Natural Language Processing models that typically work by extracting feature windows around a particular word of interest.

For example, suppose that we have a first algorithm that extracts Part of Speech (PoS) tags that we want to use as complementary tags for training a sequence classifier (e.g. a chunker). The following dict could be such a window of feature extracted around the word ‘sat’ in the sentence ‘The cat sat on the mat.’:

```python
>>> pos_window = [...
...      {...
...        'word-2': 'the',
...        'pos-2': 'DT',
```
This description can be vectorized into a sparse two-dimensional matrix suitable for feeding into a classifier (maybe after being piped into a `text.TfidfTransformer` for normalization):

```python
>>> vec = DictVectorizer()
>>> pos_vectorized = vec.fit_transform(pos_window)
>>> pos_vectorized
<1x6 sparse matrix of type '<type 'numpy.float64'>'
  with 6 stored elements in COOrdinate format>
>>> pos_vectorized.toarray()
array([[ 1., 1., 1., 1., 1., 1.]])
>>> vec.get_feature_names()
['pos+1=PP', 'pos-1=NN', 'pos-2=DT', 'word+1=on', 'word-1=cat', 'word-2=the']
```

As you can imagine, if one extracts such a context around each individual word of a corpus of documents the resulting matrix will be very wide (many one-hot-features) with most of them being valued to zero most of the time. So as to make the resulting data structure able to fit in memory the `DictVectorizer` class uses a `scipy.sparse` matrix by default instead of a `numpy.ndarray`.

**Text feature extraction**

**The Bag of Words representation**

Text Analysis is a major application field for machine learning algorithms. However the raw data, a sequence of symbols cannot be fed directly to the algorithms themselves as most of them expect numerical feature vectors with a fixed size rather than the raw text documents with variable length.

In order to address this, scikit-learn provides utilities for the most common ways to extract numerical features from text content, namely:

- **tokenizing** strings and giving an integer id for each possible token, for instance by using whitespaces and punctuation as token separators.
- **counting** the occurrences of tokens in each document.
- **normalizing** and weighting with diminishing importance tokens that occur in the majority of samples / documents.

In this scheme, features and samples are defined as follows:

- each **individual token occurrence frequency** (normalized or not) is treated as a **feature**.
- the vector of all the token frequencies for a given **document** is considered a multivariate **sample**.

A corpus of documents can thus be represented by a matrix with one row per document and one column per token (e.g. word) occurring in the corpus.

We call **vectorization** the general process of turning a collection of text documents into numerical feature vectors. This specific strategy (tokenization, counting and normalization) is called the **Bag of Words** or “Bag of n-grams” representation. Documents are described by word occurrences while completely ignoring the relative position information of the words in the document.

When combined with **TF-IDF normalization**, the bag of words encoding is also known as the **Vector Space Model**.
Sparsity

As most documents will typically use a very subset of the words used in the corpus, the resulting matrix will have many feature values that are zeros (typically more than 99% of them).

For instance a collection of 10,000 short text documents (such as emails) will use a vocabulary with a size in the order of 100,000 unique words in total while each document will use 100 to 1000 unique words individually.

In order to be able to store such a matrix in memory but also to speed up algebraic operations matrix / vector, implementations will typically use a sparse representation such as the implementations available in the `scipy.sparse` package.

Common Vectorizer usage

`CountVectorizer` implements both tokenization and occurrence counting in a single class:

```python
>>> from sklearn.feature_extraction.text import CountVectorizer
```

This model has many parameters, however the default values are quite reasonable (please see the reference documentation for the details):

```python
>>> vectorizer = CountVectorizer()
>>> vectorizer
CountVectorizer(analyzer='word', binary=False, charset='utf-8',
                charset_error='strict', dtype=<type 'long'>, input='content',
                lowercase=True, max_df=1.0, max_features=None, max_n=1, min_n=1,
                preprocessor=None, stop_words=None, strip_accents=None,
                token_pattern=u'\b\w\w+\b', tokenizer=None, vocabulary=None)
```

Let’s use it to tokenize and count the word occurrences of a minimalistic corpus of text documents:

```python
>>> corpus = [
... 'This is the first document.',
... 'This is the second document.',
... 'And the third one.',
... 'Is this the first document?',
... ]
>>> X = vectorizer.fit_transform(corpus)
>>> X
<4x9 sparse matrix of type '<type 'numpy.int64'>'
with 19 stored elements in COOrdinate format>
```

The default configuration tokenizes the string by extracting words of at least 2 letters. The specific function that does this step can be requested explicitly:

```python
>>> analyze = vectorizer.build_analyzer()
>>> analyze("This is a text document to analyze.")
[u’this’, u’is’, u’text’, u’document’, u’to’, u’analyze’]
```

Each term found by the analyzer during the fit is assigned a unique integer index corresponding to a column in the resulting matrix. This interpretation of the columns can be retrieved as follows:

```python
>>> vectorizer.get_feature_names()
[u’and’, u’document’, u’first’, u’is’, u’one’, u’second’, u’the’, u’nthird’, u’this’]
```

```python
>>> X.toarray()
array([[0, 1, 1, 1, 0, 0, 1, 0, 1],
       [0, 1, 0, 1, 0, 2, 1, 0, 1],
       ...], dtype=int64)
```
The converse mapping from feature name to column index is stored in the `vocabulary_` attribute of the vectorizer:

```python
>>> vectorizer.vocabulary_.get('document')
1
```

Hence words that were not seen in the training corpus will be completely ignored in future calls to the transform method:

```python
>>> vectorizer.transform(['Something completely new.']).toarray()
...
array([[0, 0, 0, 0, 0, 0, 0, 0, 0]])
```

Note that in the previous corpus, the first and the last documents have exactly the same words hence are encoded in equal vectors. In particular we lose the information that the last document is an interrogative form. To preserve some of the local ordering information we can extract 2-grams of words in addition to the 1-grams (the words themselves):

```python
bigram_vectorizer = CountVectorizer(min_n=1, max_n=2, token_pattern=ur'\b\w+\b')
```

```python
analyze = bigram_vectorizer.build_analyzer()
```

```python
analyze('Bi-grams are cool!')
```

[u‘bi’, u‘grams’, u‘are’, u‘cool’, u‘bi grams’, u‘grams are’, u‘are cool’]

The vocabulary extracted by this vectorizer is hence much bigger and can now resolve ambiguities encoded in local positioning patterns:

```python
>>> X_2 = bigram_vectorizer.fit_transform(corpus).toarray()
```

```python
array([[0, 0, 1, 1, 1, 0, 0, 0, 0, 0, 1, 1, 0, 0, 0, 1, 1, 0, 0, 0, 1, 0],
       [0, 0, 1, 1, 0, 0, 0, 1, 1, 0, 0, 0, 1, 0, 0, 0, 0, 1, 0, 0, 0],
       [1, 1, 0, 0, 0, 0, 0, 1, 1, 0, 0, 1, 0, 0, 1, 0, 0, 1, 0, 0, 0],
       [0, 0, 1, 1, 1, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 1, 1, 0, 0, 0, 1]])
```

In particular the interrogative form “Is this” is only present in the last document:

```python
>>> feature_index = bigram_vectorizer.vocabulary_.get(u‘is this’)  
```

```python
>>> X_2[:, feature_index]  
array([0, 0, 1])
```

### TF-IDF normalization

In a large text corpus, some words will be very present (e.g. “the”, “a”, “is” in English) hence carrying very little meaningful information about the actual contents of the document. If we were to feed the direct count data directly to a classifier those very frequent terms would shadow the frequencies of rarer yet more interesting terms.

In order to re-weight the count features into floating point values suitable for usage by a classifier it is very common to use the tf–idf transform.

Tf means term-frequency while tf–idf means term-frequency times inverse document-frequency. This is a originally a term weighting scheme developed for information retrieval (as a ranking function for search engines results), that has also found good use in document classification and clustering.

This normalization is implemented by the `text.TfidfTransformer` class:
Again please see the reference documentation for the details on all the parameters.

Let’s take an example with the following counts. The first term is present 100% of the time hence not very interesting. The two other features only in less than 50% of the time hence probably more representative of the content of the documents:

```python
counts = [[3, 0, 1],
          [2, 0, 0],
          [3, 0, 0],
          [4, 0, 0],
          [3, 2, 0],
          [3, 0, 2]]
```

```python
tfidf = transformer.fit_transform(counts)
```

```python
tfidf
<6x3 sparse matrix of type '<type 'numpy.float64'>'
with 9 stored elements in Compressed Sparse Row format>
```

```python
tfidf.toarray()
array([[ 0.85..., 0. ..., 0.52...],
       [ 1. ..., 0. ..., 0. ...],
       [ 1. ..., 0. ..., 0. ...],
       [ 1. ..., 0. ..., 0. ...],
       [ 0.55..., 0.83..., 0. ...],
       [ 0.63..., 0. ..., 0.77...]])
```

Each row is normalized to have unit euclidean norm. The weights of each feature computed by the fit method call are stored in a model attribute:

```python
transformer.idf_
array([ 1. ..., 2.25..., 1.84...])
```

As tf–idf is a very often used for text features, there is also another class called TfidfVectorizer that combines all the option of CountVectorizer and TfidfTransformer in a single model:

```python
from sklearn.feature_extraction.text import TfidfVectorizer
vectorizer = TfidfVectorizer()
vectorizer.fit_transform(corpus)
...
<4x9 sparse matrix of type '<type 'numpy.float64'>'
with 19 stored elements in Compressed Sparse Row format>
```

While the tf–idf normalization is often very useful, there might be cases where the binary occurrence markers might offer better features. This can be achieved by using the binary parameter of CountVectorizer. In particular, some estimators such as Bernoulli Naive Bayes explicitly model discrete boolean random variables. Also very short text are likely to have noisy tf–idf values while the binary occurrence info is more stable.

As usual the only way how to best adjust the feature extraction parameters is to use a cross-validated grid search, for instance by pipelining the feature extractor with a classifier:

- Sample pipeline for text feature extraction and evaluation
Applications and examples

The bag of words representation is quite simplistic but surprisingly useful in practice.

In particular in a **supervised setting** it can be successfully combined with fast and scalable linear models to train **document classifiers**, for instance:

- Classification of text documents using sparse features

In an **unsupervised setting** it can be used to group similar documents together by applying clustering algorithms such as **K-means**:

- Clustering text documents using k-means

Finally it is possible to discover the main topics of a corpus by relaxing the hard assignment constraint of clustering, for instance by using **Non-negative matrix factorization (NMF or NNMF)**:

- Topics extraction with Non-Negative Matrix Factorization

Limitations of the Bag of Words representation

While some local positioning information can be preserved by extracting n-grams instead of individual words, Bag of Words and Bag of n-grams destroy most of the inner structure of the document and hence most of the meaning carried by that internal structure.

In order to address the wider task of Natural Language Understanding, the local structure of sentences and paragraphs should thus be taken into account. Many such models will thus be casted as “Structured output” problems which are currently outside of the scope of scikit-learn.

Customizing the vectorizer classes

It is possible to customize the behavior by passing some callable as parameters of the vectorizer:

```python
>>> def my_tokenizer(s):
...     return s.split()
...
>>> vectorizer = CountVectorizer(tokenizer=my_tokenizer)
>>> vectorizer.build_analyzer()(u"Some... punctuation!")
[u'\u201csome...', u'\u201cpunctuation!\u2019']
```

In particular we name:

- **preprocessor** a callable that takes a string as input and return another string (removing HTML tags or converting to lower case for instance)
- **tokenizer** a callable that takes a string as input and output a sequence of feature occurrences (a.k.a. the tokens).
- **analyzer** a callable that wraps calls to the preprocessor and tokenizer and further perform some filtering or n-grams extractions on the tokens.

To make the preprocessor, tokenizer and analyzers aware of the model parameters it is possible to derive from the class and override the `build_preprocessor`, `build_tokenizer` and `build_analyzer` factory method instead.

Customizing the vectorizer can be very useful to handle Asian languages that do not use an explicit word separator such as the whitespace for instance.
Image feature extraction

Patch extraction

The `extract_patches_2d` function extracts patches from an image stored as a two-dimensional array, or three-dimensional with color information along the third axis. For rebuilding an image from all its patches, use `reconstruct_from_patches_2d`. For example let use generate a 4x4 pixel picture with 3 color channels (e.g. in RGB format):

```python
>>> import numpy as np
>>> from sklearn.feature_extraction import image

>>> one_image = np.arange(4 * 4 * 3).reshape((4, 4, 3))

>>> one_image[:, :, 0]  # R channel of a fake RGB picture
array([[ 0,  3,  6,  9],
       [12, 15, 18, 21],
       [24, 27, 30, 33],
       [36, 39, 42, 45]])

>>> patches = image.extract_patches_2d(one_image, (2, 2), max_patches=2, ...
...    random_state=0)
>>> patches.shape
(2, 2, 2, 3)

>>> patches[:, :, :, 0]
array([[ 0,  3],
       [12, 15],
       [15, 18],
       [27, 30]])

>>> patches = image.extract_patches_2d(one_image, (2, 2))
>>> patches.shape
(9, 2, 2, 3)

>>> patches[4, :, :, 0]
array([[15, 18],
       [27, 30]])
```

Let us now try to reconstruct the original image from the patches by averaging on overlapping areas:

```python
>>> reconstructed = image.reconstruct_from_patches_2d(patches, (4, 4, 3))
>>> np.testing.assert_array_equal(one_image, reconstructed)
```

The `PatchExtractor` class works in the same way as `extract_patches_2d`, only it supports multiple images as input. It is implemented as an estimator, so it can be used in pipelines. See:

```python
>>> five_images = np.arange(5 * 4 * 4 * 3).reshape(5, 4, 4, 3)
>>> patches = image.PatchExtractor((2, 2)).transform(five_images)
>>> patches.shape
(45, 2, 2, 3)
```

Connectivity graph of an image

Several estimators in the scikit-learn can use connectivity information between features or samples. For instance Ward clustering (Hierarchical clustering) can cluster together only neighboring pixels of an image, thus forming contiguous patches:

For this purpose, the estimators use a ‘connectivity’ matrix, giving which samples are connected.
The function `img_to_graph` returns such a matrix from a 2D or 3D image. Similarly, `grid_to_graph` builds a connectivity matrix for images given the shape of the images.

These matrices can be used to impose connectivity in estimators that use connectivity information, such as Ward clustering (Hierarchical clustering), but also to build precomputed kernels, or similarity matrices.

**Note:** Examples

- A demo of structured Ward hierarchical clustering on Lena image
- Spectral clustering for image segmentation
- Feature agglomeration vs. univariate selection

### 1.6.3 Kernel Approximation

This submodule contains functions that approximate the feature mappings that correspond to certain kernels, as they are used for example in support vector machines (see Support Vector Machines). The following feature functions perform non-linear transformations of the input, which can serve as a basis for linear classification or other algorithms.

The advantage of using approximate explicit feature maps compared to the kernel trick, which makes use of feature maps implicitly, is that explicit mappings can be better suited for online learning and can significantly reduce the cost of learning with very large datasets. Standard kernelized SVMs do not scale well to large datasets, but using an approximate kernel map it is possible to use much more efficient linear SVMs. In particular, the combination of kernel map approximations with `SGDClassifier` can make nonlinear learning on large datasets possible.

Since there has not been much empirical work using approximate embeddings, it is advisable to compare results against exact kernel methods when possible.

**Radial Basis Function Kernel**

The `RBFSampler` constructs an approximate mapping for the radial basis function kernel.

The mapping relies on a Monte Carlo approximation to the kernel values. The `fit` function performs the Monte Carlo sampling, whereas the `transform` method performs the mapping of the data. Because of the inherent randomness of the process, results may vary between different calls to the `fit` function.

The `fit` function takes two arguments: `n_components`, which is the target dimensionality of the feature transform, and `gamma`, the parameter of the RBF-kernel. A higher `n_components` will result in a better approximation of the kernel and will yield results more similar to those produced by a kernel SVM. Note that “fitting” the feature function does not actually depend on the data given to the `fit` function. Only the dimensionality of the data is used. Details on the method can be found in [RR2007].
Additive Chi Squared Kernel

The chi squared kernel is a kernel on histograms, often used in computer vision.

The chi squared kernel is given by

\[ k(x, y) = \sum_i \frac{2x_i y_i}{x_i + y_i} \]

Since the kernel is additive, it is possible to treat all components \( x_i \) separately for embedding. This makes it possible to sample the Fourier transform in regular intervals, instead of approximating using Monte Carlo sampling.

The class `AdditiveChi2Sampler` implements this component wise deterministic sampling. Each component is sampled \( n \) times, yielding \( 2n+1 \) dimensions per input dimension (the multiple of two stems from the real and complex part of the Fourier transform). In the literature, \( n \) is usually choosen to be 1 or 2, transforming the dataset to size \( n_{\text{samples}} \times 5 \times n_{\text{features}} \) (in the case of \( n=2 \)).

The approximate feature map provided by `AdditiveChi2Sampler` can be combined with the approximate feature map provided by `RBFSampler` to yield an approximate feature map for the exponentiated chi squared kernel. See the [VZ2010] for details and [VVZ2010] for combination with the `RBFSampler`.

Skewed Chi Squared Kernel

The skewed chi squared kernel is given by:

\[ k(x, y) = \prod_i \frac{2\sqrt{x_i + c\sqrt{y_i + c}}}{x_i + y_i + 2c} \]

It has properties that are similar to the exponentiated chi squared kernel often used in computer vision, but allows for a simple Monte Carlo approximation of the feature map.

The usage of the `SkewedChi2Sampler` is the same as the usage described above for the `RBFSampler`. The only difference is in the free parameter, that is called \( c \). For a motivation for this mapping and the mathematical details see [LS2010].
Mathematical Details

Kernel methods like support vector machines or kernelized PCA rely on a property of reproducing kernel Hilbert spaces. For any positive definite kernel function $k$ (a so-called Mercer kernel), it is guaranteed that there exists a mapping $\phi$ into a Hilbert space $\mathcal{H}$, such that

$$k(x, y) = \langle \phi(x), \phi(y) \rangle$$

Where $\langle \cdot, \cdot \rangle$ denotes the inner product in the Hilbert space.

If an algorithm, such as a linear support vector machine or PCA, relies only on the scalar product of data points $x_i$, one may use the value of $k(x_i, x_j)$, which corresponds to applying the algorithm to the mapped data points $\phi(x_i)$. The advantage of using $k$ is that the mapping $\phi$ never has to be calculated explicitly, allowing for arbitrary large features (even infinite).

One drawback of kernel methods is that it might be necessary to store many kernel values $k(x_i, x_j)$ during optimization. If a kernelized classifier is applied to new data $y_j$, $k(x_i, y_j)$ needs to be computed to make predictions, possibly for many different $x_i$ in the training set.

The classes in this submodule allow to approximate the embedding $\phi$, thereby working explicitly with the representations $\phi(x_i)$, which obviates the need to apply the kernel or store training examples.

References:

1.7 Dataset loading utilities

The `sklearn.datasets` package embeds some small toy datasets as introduced in the Getting Started section.

To evaluate the impact of the scale of the dataset (n_samples and n_features) while controlling the statistical properties of the data (typically the correlation and informativeness of the features), it is also possible to generate synthetic data.

This package also features helpers to fetch larger datasets commonly used by the machine learning community to benchmark algorithm on data that comes from the ‘real world’.

1.7.1 General dataset API

There are three distinct kinds of dataset interfaces for different types of datasets. The simplest one is the interface for sample images, which is described below in the Sample images section.

The dataset generation functions and the svmlight loader share a simplistic interface, returning a tuple $(X, y)$ consisting of a n_samples x n_features numpy array $X$ and an array of length n_samples containing the targets $y$.

The toy datasets as well as the ‘real world’ datasets and the datasets fetched from mldata.org have more sophisticated structure. These functions return a bunch (which is a dictionary that is accessible with the ’dict.key’ syntax). All datasets have at least two keys, data, containing an array of shape n_samples x n_features (except for 20newsgroups) and target, a numpy array of length n_features, containing the targets.

The datasets also contain a description in DESCRI and some contain feature_names and target_names. See the dataset descriptions below for details.
1.7.2 Toy datasets

scikit-learn comes with a few small standard datasets that do not require to download any file from some external website.

<table>
<thead>
<tr>
<th>Function</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>load_boston()</td>
<td>Load and return the boston house-prices dataset (regression).</td>
</tr>
<tr>
<td>load_iris()</td>
<td>Load and return the iris dataset (classification).</td>
</tr>
<tr>
<td>load_diabetes()</td>
<td>Load and return the diabetes dataset (regression).</td>
</tr>
<tr>
<td>load_digits([n_class])</td>
<td>Load and return the digits dataset (classification).</td>
</tr>
<tr>
<td>load_linnerud()</td>
<td>Load and return the linnerud dataset (multivariate regression).</td>
</tr>
</tbody>
</table>

These datasets are useful to quickly illustrate the behavior of the various algorithms implemented in the scikit. They are however often too small to be representative of real world machine learning tasks.

1.7.3 Sample images

The scikit also embed a couple of sample JPEG images published under Creative Commons license by their authors. Those image can be useful to test algorithms and pipeline on 2D data.

<table>
<thead>
<tr>
<th>Function</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>load_sample_images()</td>
<td>Load sample images for image manipulation.</td>
</tr>
<tr>
<td>load_sample_image(image_name)</td>
<td>Load the numpy array of a single sample image</td>
</tr>
</tbody>
</table>

![Sample Image](image.jpg)

**Warning:** The default coding of images is based on the uint8 dtype to spare memory. Often algorithms work best if the input is converted to a floating point representation first. Also, `pylab.imshow` don’t forget to scale to the range 0 - 1 as done in the following example.

**Examples:**

- *Color Quantization using K-Means*
1.7.4 Sample generators

In addition, scikit-learn includes various random sample generators that can be used to build artificial datasets of controlled size and complexity.

```
make_classification([n_samples, n_features, ...])  # Generate a random n-class classification problem.
make_multilabel_classification([n_samples, ...])   # Generate a random multilabel classification problem.
make_regression([n_samples, n_features, ...])     # Generate a random regression problem.
make_blobs([n_samples, n_features, centers, ...]) # Generate isotropic Gaussian blobs for clustering.
make_friedman1([n_samples, n_features, ...])      # Generate the "Friedman #1" regression problem.
make_friedman2([n_samples, noise, random_state])  # Generate the "Friedman #2" regression problem.
make_friedman3([n_samples, noise, random_state])  # Generate the "Friedman #3" regression problem.
make_hastie_10_2([n_samples, random_state])       # Generates data for binary classification used in.
make_low_rank_matrix([n_samples, ...])            # Generate a mostly low rank matrix with bell-shaped singular values.
make_sparse_coded_signal([n_samples, ...])        # Generate a signal as a sparse combination of dictionary elements.
make_sparse_uncorrelated([n_samples, ...])        # Generate a random regression problem with sparse uncorrelated design.
make_spd_matrix([dim, alpha, ...])               # Generate a symmetric, positive-definite matrix.
make_swiss_roll([n_samples, noise, random_state]) # Generate a swiss roll dataset.
make_s_curve([n_samples, noise, random_state])    # Generate an S curve dataset.
make_sparse_spd_matrix([dim, alpha, ...])        # Generate a sparse symmetric definite positive matrix.
```

1.7.5 Datasets in svmlight / libsvm format

scikit-learn includes utility functions for loading datasets in the svmlight / libsvm format. In this format, each line takes the form `<label> <feature-id>:<feature-value> <feature-id>:<feature-value> ...`. This format is especially suitable for sparse datasets. In this module, scipy sparse CSR matrices are used for X and numpy arrays are used for y.

You may load a dataset like as follows:

```
>>> from sklearn.datasets import load_svmlight_file
>>> X_train, y_train = load_svmlight_file("/path/to/train_dataset.txt")
...
```

You may also load two (or more) datasets at once:

```
>>> X_train, y_train, X_test, y_test = load_svmlight_files(
...   "/path/to/train_dataset.txt", "/path/to/test_dataset.txt")
...
```
In this case, \(X_{\text{train}}\) and \(X_{\text{test}}\) are guaranteed to have the same number of features. Another way to achieve the same result is to fix the number of features:

```python
>>> X_test, y_test = load_svmlight_file(
...   "path/to/test_dataset.txt", n_features=X_train.shape[1])
...```

1.7.6 The Olivetti faces dataset

This dataset contains a set of face images taken between April 1992 and April 1994 at AT&T Laboratories Cambridge. The website describing the original dataset is now defunct, but archived copies can be accessed through the Internet Archive’s Wayback Machine.

As described on the original website:

There are ten different images of each of 40 distinct subjects. For some subjects, the images were taken at different times, varying the lighting, facial expressions (open / closed eyes, smiling / not smiling) and facial details (glasses / no glasses). All the images were taken against a dark homogeneous background with the subjects in an upright, frontal position (with tolerance for some side movement).

The image is quantized to 256 grey levels and stored as unsigned 8-bit integers; the loader will convert these to floating point values on the interval \([0, 1]\), which are easier to work with for many algorithms.

The “target” for this database is an integer from 0 to 39 indicating the identity of the person pictured; however, with only 10 examples per class, this relatively small dataset is more interesting from an unsupervised or semi-supervised perspective.

The original dataset consisted of 92 x 112, while the version available here consists of 64x64 images.

When using these images, please give credit to AT&T Laboratories Cambridge.

1.7.7 The 20 newsgroups text dataset

The 20 newsgroups dataset comprises around 18000 newsgroups posts on 20 topics splitted in two subsets: one for training (or development) and the other one for testing (or for performance evaluation). The split between the train and test set is based upon a messages posted before and after a specific date.

This module contains two loaders. The first one, `sklearn.datasets.fetch_20newsgroups`, returns a list of the raw text files that can be fed to text feature extractors such as `sklearn.feature_extraction.text.TfidfVectorizer` with custom parameters so as to extract feature vectors. The second one, `sklearn.datasets.fetch_20newsgroups_vectorized`, returns ready-to-use features, i.e., it is not necessary to use a feature extractor.

Usage

The `sklearn.datasets.fetch_20newsgroups` function is a data fetching / caching functions that downloads the data archive from the original 20 newsgroups website, extracts the archive contents in the `~/scikit_learn_data/20news_home` folder and calls the `sklearn.datasets.load_file` on either the training or testing set folder, or both of them:
from sklearn.datasets import fetch_20newsgroups
newsgroups_train = fetch_20newsgroups(subset='train')
from pprint import pprint
pprint(list(newsgroups_train.target_names))


The real data lies in the filenames and target attributes. The target attribute is the integer index of the category:

newsgroups_train.filenames.shape
(11314,)
newsgroups_train.target.shape
(11314,)
newsgroups_train.target[:10]
array([12, 6, 9, 8, 6, 7, 9, 2, 13, 19])

It is possible to load only a sub-selection of the categories by passing the list of the categories to load to the fetch_20newsgroups function:

cats = ['alt.atheism', 'sci.space']
newsgroups_train = fetch_20newsgroups(subset='train', categories=cats)

list(newsgroups_train.target_names)
['alt.atheism', 'sci.space']
newsgroups_train.filenames.shape
(1073,)
newsgroups_train.target.shape
(1073,)
newsgroups_train.target[:10]
array([1, 1, 1, 0, 1, 0, 0, 1, 1, 1])

In order to feed predictive or clustering models with the text data, one first need to turn the text into vectors of numerical values suitable for statistical analysis. This can be achieved with the utilities of the sklearn.feature_extraction.text as demonstrated in the following example that extract TF-IDF vectors of unigram tokens:

from sklearn.feature_extraction.text import Vectorizer
documents = [open(f).read() for f in newsgroups_train.filenames]
vectorizer = Vectorizer()
vectors = vectorizer.fit_transform(documents)
The extracted TF-IDF vectors are very sparse with an average of 118 non zero components by sample in a more than 20000 dimensional space (less than 1% non zero features):

```
>>> vectors.nnz / vectors.shape[0]
118
```

sklearn.datasets.fetch_20newsgroups_vectorized is a function which returns ready-to-use tfidf features instead of file names.

Examples
- Sample pipeline for text feature extraction and evaluation
- Classification of text documents using sparse features

## 1.7.8 Downloading datasets from the mldata.org repository

mldata.org is a public repository for machine learning data, supported by the PASCAL network.

The sklearn.datasets package is able to directly download data sets from the repository using the function fetch_mldata(dataname).

For example, to download the MNIST digit recognition database:

```
>>> from sklearn.datasets import fetch_mldata
>>> mnist = fetch_mldata('MNIST original', data_home=custom_data_home)
```

The MNIST database contains a total of 70000 examples of handwritten digits of size 28x28 pixels, labeled from 0 to 9:

```
>>> mnist.data.shape
(70000, 784)
>>> mnist.target.shape
(70000,)
>>> np.unique(mnist.target)
array([ 0., 1., 2., 3., 4., 5., 6., 7., 8., 9.])
```

After the first download, the dataset is cached locally in the path specified by the data_home keyword argument, which defaults to ~/scikit_learn_data/:

```
>>> os.listdir(os.path.join(custom_data_home, 'mldata'))
['mnist-original.mat']
```

Data sets in mldata.org do not adhere to a strict naming or formatting convention. fetch_mldata is able to make sense of the most common cases, but allows to tailor the defaults to individual datasets:

- The data arrays in mldata.org are most often shaped as (n_features, n_samples). This is the opposite of the scikit-learn convention, so fetch_mldata transposes the matrix by default. The transpose_data keyword controls this behavior:

```
>>> iris = fetch_mldata('iris', data_home=custom_data_home)
>>> iris.data.shape
(150, 4)
>>> iris = fetch_mldata('iris', transpose_data=False, data_home=custom_data_home)
```
For datasets with multiple columns, fetch_mldata tries to identify the target and data columns and rename them to target and data. This is done by looking for arrays named label and data in the dataset, and failing that by choosing the first array to be target and the second to be data. This behavior can be changed with the target_name and data_name keywords, setting them to a specific name or index number (the name and order of the columns in the datasets can be found at its mldata.org under the tab “Data”):

```python
>>> iris2 = fetch_mldata('datasets-UCI iris', target_name=1, data_name=0,
... data_home=custom_data_home)
>>> iris3 = fetch_mldata('datasets-UCI iris', target_name='class',
... data_name='double0', data_home=custom_data_home)
```

### 1.7.9 The Labeled Faces in the Wild face recognition dataset

This dataset is a collection of JPEG pictures of famous people collected over the internet, all details are available on the official website:

http://vis-www.cs.umass.edu/lfw/

Each picture is centered on a single face. The typical task is called Face Verification: given a pair of two pictures, a binary classifier must predict whether the two images are from the same person.

An alternative task, Face Recognition or Face Identification is: given the picture of the face of an unknown person, identify the name of the person by referring to a gallery of previously seen pictures of identified persons.

Both Face Verification and Face Recognition are tasks that are typically performed on the output of a model trained to perform Face Detection. The most popular model for Face Detection is called Viola-Jones and is implemented in the OpenCV library. The LFW faces were extracted by this face detector from various online websites.

**Usage**

scikit-learn provides two loaders that will automatically download, cache, parse the metadata files, decode the jpeg and convert the interesting slices into memmapped numpy arrays. This dataset size is more than 200 MB. The first load typically takes more than a couple of minutes to fully decode the relevant part of the JPEG files into numpy arrays. If the dataset has been loaded once, the following times the loading times less than 200ms by using a memmapped version memoized on the disk in the ~/scikit_learn_data/lfw_home/ folder using joblib.

The first loader is used for the Face Identification task: a multi-class classification task (hence supervised learning):

```python
>>> from sklearn.datasets import fetch_lfw_people
>>> lfw_people = fetch_lfw_people(min_faces_per_person=70, resize=0.4)

>>> for name in lfw_people.target_names:
...     print name
...     ...
Ariel Sharon
Colin Powell
Donald Rumsfeld
George W Bush
Gerhard Schroeder
Hugo Chavez
Tony Blair
```

The default slice is a rectangular shape around the face, removing most of the background:
Each of the 1140 faces is assigned to a single person id in the target array:

```python
>>> lfw_people.target.shape
(1288,)
```

The second loader is typically used for the face verification task: each sample is a pair of two picture belonging or not to the same person:

```python
>>> from sklearn.datasets import fetch_lfw_pairs
>>> lfw_pairs_train = fetch_lfw_pairs(subset='train')
```

```python
>>> list(lfw_pairs_train.target_names)
['Different persons', 'Same person']
```

Both for the `fetch_lfw_people` and `fetch_lfw_pairs` function it is possible to get an additional dimension with the RGB color channels by passing `color=True`, in that case the shape will be `(2200, 2, 62, 47, 3)`.

The `fetch_lfw_pairs` datasets is subdived in 3 subsets: the development train set, the development test set and an evaluation 10_folds set meant to compute performance metrics using a 10-folds cross validation scheme.

References:


Examples

Faces recognition example using eigenfaces and SVMs

1.7. Dataset loading utilities
1.8 Reference

This is the class and function reference of scikit-learn. Please refer to the full user guide for further details, as the class and function raw specifications may not be enough to give full guidelines on their uses.

<table>
<thead>
<tr>
<th>List of modules</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>sklearn.cluster</strong>: Clustering</td>
</tr>
<tr>
<td>- Classes</td>
</tr>
<tr>
<td>- Functions</td>
</tr>
<tr>
<td><strong>sklearn.covariance</strong>: Covariance Estimators</td>
</tr>
<tr>
<td><strong>sklearn.cross_validation</strong>: Cross Validation</td>
</tr>
<tr>
<td><strong>sklearn.datasets</strong>: Datasets</td>
</tr>
<tr>
<td>- Loaders</td>
</tr>
<tr>
<td>- Samples generator</td>
</tr>
<tr>
<td><strong>sklearn.decomposition</strong>: Matrix Decomposition</td>
</tr>
<tr>
<td><strong>sklearn.ensemble</strong>: Ensemble Methods</td>
</tr>
<tr>
<td><strong>sklearn.feature_extraction</strong>: Feature Extraction</td>
</tr>
<tr>
<td>- From images</td>
</tr>
<tr>
<td>- From text</td>
</tr>
<tr>
<td><strong>sklearn.feature_selection</strong>: Feature Selection</td>
</tr>
<tr>
<td><strong>sklearn.gaussian_process</strong>: Gaussian Processes</td>
</tr>
<tr>
<td><strong>sklearn.grid_search</strong>: Grid Search</td>
</tr>
<tr>
<td><strong>sklearn.hmm</strong>: Hidden Markov Models</td>
</tr>
<tr>
<td><strong>sklearn.kernel_approximation</strong>: Kernel Approximation</td>
</tr>
<tr>
<td><strong>sklearn.semi_supervised</strong>: Semi-Supervised Learning</td>
</tr>
<tr>
<td><strong>sklearn.lda</strong>: Linear Discriminant Analysis</td>
</tr>
<tr>
<td><strong>sklearn.linear_model</strong>: Generalized Linear Models</td>
</tr>
<tr>
<td>- For dense data</td>
</tr>
<tr>
<td>- For sparse data</td>
</tr>
<tr>
<td><strong>sklearn.manifold</strong>: Manifold Learning</td>
</tr>
<tr>
<td><strong>sklearn.metrics</strong>: Metrics</td>
</tr>
<tr>
<td>- Classification metrics</td>
</tr>
<tr>
<td>- Regression metrics</td>
</tr>
<tr>
<td>- Clustering metrics</td>
</tr>
<tr>
<td>- Pairwise metrics</td>
</tr>
<tr>
<td><strong>sklearn.mixture</strong>: Gaussian Mixture Models</td>
</tr>
<tr>
<td><strong>sklearn.multiclass</strong>: Multiclass and multilabel classification</td>
</tr>
<tr>
<td>- Multiclass and multilabel classification strategies</td>
</tr>
<tr>
<td><strong>sklearn.naive_bayes</strong>: Naive Bayes</td>
</tr>
<tr>
<td><strong>sklearn.neighbors</strong>: Nearest Neighbors</td>
</tr>
<tr>
<td><strong>sklearn.pls</strong>: Partial Least Squares</td>
</tr>
<tr>
<td><strong>sklearn.pipeline</strong>: Pipeline</td>
</tr>
<tr>
<td><strong>sklearn.preprocessing</strong>: Preprocessing and Normalization</td>
</tr>
<tr>
<td><strong>sklearn.qda</strong>: Quadratic Discriminant Analysis</td>
</tr>
<tr>
<td><strong>sklearn.svm</strong>: Support Vector Machines</td>
</tr>
<tr>
<td>- Estimators</td>
</tr>
<tr>
<td>- Low-level methods</td>
</tr>
<tr>
<td><strong>sklearn.tree</strong>: Decision Trees</td>
</tr>
<tr>
<td><strong>sklearn.utils</strong>: Utilities</td>
</tr>
</tbody>
</table>
1.8.1 sklearn.cluster: Clustering

The sklearn.cluster module gathers popular unsupervised clustering algorithms.

User guide: See the Clustering section for further details.

Classes

- `cluster.AffinityPropagation([damping, ...])`: Perform Affinity Propagation Clustering of data
  - Parameters:
    - `damping`: float, optional
      - Damping factor
    - `max_iter`: int, optional
      - Maximum number of iterations
    - `convit`: int, optional
      - Number of iterations with no change in the number of estimated clusters that stops the convergence.
    - `copy`: boolean, optional
      - Make a copy of input data. True by default.

Notes

See examples/plot_affinity_propagation.py for an example.

The algorithmic complexity of affinity propagation is quadratic in the number of points.

References


Attributes

- `cluster_centers_indices_`: array, [n_clusters] - Indices of cluster centers
- `labels_`: array, [n_samples] - Labels of each point
Methods

<table>
<thead>
<tr>
<th>Method</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>fit(S[, p])</td>
<td>Compute affinity propagation clustering.</td>
</tr>
<tr>
<td>get_params([deep])</td>
<td>Get parameters for the estimator</td>
</tr>
<tr>
<td>set_params(**params)</td>
<td>Set the parameters of the estimator.</td>
</tr>
</tbody>
</table>

```
__init__(damping=0.5, max_iter=200, convit=30, copy=True)
```

```
fit (S, p=None)
Compute affinity propagation clustering.
```

**Parameters**

- **S**: array [n_points, n_points]
  - Matrix of similarities between points
- **p**: array [n_points,] or float, optional
  - Preferences for each point - points with larger values of preferences are more likely to be chosen as exemplars. The number of exemplars, ie of clusters, is influenced by the input preferences value. If the preferences are not passed as arguments, they will be set to the median of the input similarities.
- **damping**: float, optional
  - Damping factor
- **copy**: boolean, optional
  - If copy is False, the affinity matrix is modified in-place by the algorithm, for memory efficiency

```
get_params (deep=True)
Get parameters for the estimator
```

**Parameters**

- **deep**: boolean, optional
  - If True, will return the parameters for this estimator and contained subobjects that are estimators.

```
set_params (**params)
Set the parameters of the estimator.
```

- The method works on simple estimators as well as on nested objects (such as pipelines). The former have parameters of the form `<component>__<parameter>` so that it's possible to update each component of a nested object.

Returns **self**:

```
sklearn.cluster.DBSCAN
```

```
class sklearn.cluster.DBSCAN (eps=0.5, min_samples=5, metric='euclidean', random_state=None)
Perform DBSCAN clustering from vector array or distance matrix.
```

DBSCAN - Density-Based Spatial Clustering of Applications with Noise. Finds core samples of high density and expands clusters from them. Good for data which contains clusters of similar density.

**Parameters**

- **eps**: float, optional
  - The maximum distance between two samples for them to be considered as in the same neighborhood.
**min_samples**: int, optional

The number of samples in a neighborhood for a point to be considered as a core point.

**metric**: string, or callable

The metric to use when calculating distance between instances in a feature array. If metric is a string or callable, it must be one of the options allowed by metrics.pairwise.calculate_distance for its metric parameter. If metric is “precomputed”, X is assumed to be a distance matrix and must be square.

**random_state**: numpy.RandomState, optional

The generator used to initialize the centers. Defaults to numpy.random.

**Notes**

See examples/plot_dbscan.py for an example.

**References**


**Attributes**

| core_sample_indices array, shape = [n_core_samples] | Indices of core samples. |
| components_ array, shape = [n_core_samples, n_features] | Copy of each core sample found by training. |
| labels_ array, shape = [n_samples] | Cluster labels for each point in the dataset given to fit(). Noisy samples are given the label -1. |

**Methods**

| fit(X, **params) | Perform DBSCAN clustering from vector array or distance matrix. |
| set_params(deep) | Get parameters for the estimator |
| set_params(**params) | Set the parameters of the estimator. |

**__init__** (eps=0.5, min_samples=5, metric='euclidean', random_state=None)

**fit** (X, **params)

Perform DBSCAN clustering from vector array or distance matrix.

**Parameters** X: array [n_samples, n_samples] or [n_samples, n_features]

Array of distances between samples, or a feature array. The array is treated as a feature array unless the metric is given as ‘precomputed’.

**params**: dict

Overwrite keywords from __init__.
get_params (deep=True)
Get parameters for the estimator

Parameters deep: boolean, optional:
If True, will return the parameters for this estimator and contained subobjects that are
estimators.

set_params (**params)
Set the parameters of the estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The former have
parameters of the form <component>__<parameter> so that it’s possible to update each component
of a nested object.

Returns self:

sklearn.cluster.KMeans

class sklearn.cluster.KMeans (n_clusters=8, init='k-means++', n_init=10, max_iter=300,
tol=0.0001, precompute_distances=True, verbose=0, random_state=None, copy_x=True, n_jobs=1, k=None)
K-Means clustering

Parameters n_clusters: int, optional, default: 8
The number of clusters to form as well as the number of centroids to generate.

max_iter: int
Maximum number of iterations of the k-means algorithm for a single run.

n_init: int, optional, default: 10:
Number of time the k-means algorithm will be run with different centroid seeds. The
final results will be the best output of n_init consecutive runs in terms of inertia.

init: {‘k-means++’, ‘random’ or an ndarray}
Method for initialization, defaults to ‘k-means++’:
‘k-means++’ : selects initial cluster centers for k-mean clustering in a smart way to
speed up convergence. See section Notes in k_init for more details.
‘random’: choose k observations (rows) at random from data for the initial centroids.
if init is an 2d array, it is used as a seed for the centroids

precompute_distances: boolean
Precompute distances (faster but takes more memory).

tol: float, optional default: 1e-4:
Relative tolerance w.r.t. inertia to declare convergence

n_jobs: int:
The number of jobs to use for the computation. This works by breaking down the
pairwise matrix into n_jobs even slices and computing them in parallel.
If -1 all CPUs are used. If 1 is given, no parallel computing code is used at all, which
is useful for debugging. For n_jobs below -1, (n_cpus + 1 - n_jobs) are used. Thus for
n_jobs = -2, all CPUs but one are used.
random_state: integer or numpy.RandomState, optional:
The generator used to initialize the centers. If an integer is given, it fixes the seed.
Defaults to the global numpy random number generator.

See Also:

MiniBatchKMeans: Alternative online implementation that does incremental updates of the centers positions using mini-batches. For large scale learning (say n_samples > 10k) MiniBatchKMeans is probably much faster to than the default batch implementation.

Notes

The k-means problem is solved using Lloyd’s algorithm.
The average complexity is given by O(k n T), were n is the number of samples and T is the number of iteration.
The worst case complexity is given by O(n^(k+2/p)) with n = n_samples, p = n_features. (D. Arthur and S. Vassilvitskii, ‘How slow is the k-means method?’ SoCG2006)

In practice, the k-means algorithm is very fast (one of the fastest clustering algorithms available), but it falls in local minima. That’s why it can be useful to restart it several times.

Attributes

| cluster_centers_: array, [n_clusters, n_features] | Coordinates of cluster centers |
| labels_: | Labels of each point |
| inertia_: float | The value of the inertia criterion associated with the chosen partition. |

Methods

- fit(X[, y]) Compute k-means
- fit_predict(X) Compute cluster centers and predict cluster index for each sample.
- get_params([deep]) Get parameters for the estimator
- predict(X) Predict the closest cluster each sample in X belongs to.
- score(X) Opposite of the value of X on the K-means objective.
- set_params(**params) Set the parameters of the estimator.
- transform(X[, y]) Transform the data to a cluster-distance space

__init__ (n_clusters=8, init='k-means++', n_init=10, max_iter=300, tol=0.0001, precompute_distances=True, verbose=0, random_state=None, copy_x=True, n_jobs=1, k=None)

fit (X, y=None)
Compute k-means

fit_predict (X)
Compute cluster centers and predict cluster index for each sample.
Convenience method; equivalent to calling fit(X) followed by predict(X).

get_params (deep=True)
Get parameters for the estimator
.. _{}:

scikit-learn user guide, Release 0.12-git

.. currentmodule:: sklearn.cluster

.. class:: MiniBatchKMeans

   Mini-Batch K-Means clustering

   Parameters **n_clusters** : int, optional, default: 8

   The number of clusters to form as well as the number of centroids to generate.

   **init** : \{'k-means++\', \'random\'}

   Initialization of centroids.

   **max_iter** : int, default: 100

   The number of iterations before stopping the algorithm.

   **batch_size** : int, default: 100

   The number of samples to process at once.

   **compute_labels** : bool, default: True

   Whether to compute the labels for all samples. If set to False, only the new index for each point will be returned.

   **random_state** : int, RandomState instance or None, default: None

   Initial random state for batch generation.

   **tol** : float, default: 0.0

   Relative tolerance w.r.t. betweens-set distances stop criterion.

   **max_no_improvement** : int, default: 10

   Maximum number of iterations without improvement.

   **init_size** : int, default: None

   Number of samples to incrementally select when initializing centroids.

   **n_init** : int, default: 3

   Number of initializations.

   **chunk_size** : int, default: None

   Number of samples per new cluster.

   **k** : int, default: None

   Number of initial clusters.
**max_iter**: int, optional

Maximum number of iterations over the complete dataset before stopping independently of any early stopping criterion heuristics.

**max_no_improvement**: int, optional

Control early stopping based on the consecutive number of mini batches that does not yield an improvement on the smoothed inertia.

To disable convergence detection based on inertia, set `max_no_improvement` to `None`.

**tol**: float, optional

Control early stopping based on the relative center changes as measured by a smoothed, variance-normalized of the mean center squared position changes. This early stopping heuristics is closer to the one used for the batch variant of the algorithms but induces a slight computational and memory overhead over the inertia heuristic.

To disable convergence detection based on normalized center change, set `tol` to `0.0` (default).

**batch_size**: int, optional, default: 100:

Size of the mini batches.

**init_size**: int, optional, default: `3 * batch_size`:

Number of samples to randomly sample for speeding up the initialization (sometimes at the expense of accuracy): the only algorithm is initialized by running a batch KMeans on a random subset of the data. This needs to be larger than k.

**init**: {'k-means++', 'random' or an ndarray}

Method for initialization, defaults to ‘k-means++’:

‘k-means++’ : selects initial cluster centers for k-mean clustering in a smart way to speed up convergence. See section Notes in `k_init` for more details.

‘random’ : choose k observations (rows) at random from data for the initial centroids.

if init is an 2d array, it is used as a seed for the centroids

**compute_labels**: boolean:

Compute label assignments and inertia for the complete dataset once the minibatch optimization has converged in fit.

**random_state**: integer or numpy.RandomState, optional:

The generator used to initialize the centers. If an integer is given, it fixes the seed. Defaults to the global numpy random number generator.

**Notes**

Attributes

<table>
<thead>
<tr>
<th>attribute</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>cluster_centers_</code></td>
<td>Coordinates of cluster centers</td>
</tr>
<tr>
<td><code>labels_</code></td>
<td>Labels of each point (if <code>compute_labels</code> is set to True).</td>
</tr>
<tr>
<td><code>inertia_</code></td>
<td>The value of the inertia criterion associated with the chosen partition (if <code>compute_labels</code> is set to True). The inertia is defined as the sum of square distances of samples to their nearest neighbor.</td>
</tr>
</tbody>
</table>

Methods

- **fit**(X[, y])
  Compute the centroids on X by chunking it into mini-batches.

- **fit_predict**(X)
  Compute cluster centers and predict cluster index for each sample.

- **get_params**(deep=False)
  Get parameters for the estimator

- **partial_fit**(X[, y])
  Update k means estimate on a single mini-batch X.

- **predict**(X)
  Predict the closest cluster each sample in X belongs to.

- **score**(X)
  Opposite of the value of X on the K-means objective.

- **set_params**(**params)
  Set the parameters of the estimator.

- **transform**(X[, y])
  Transform the data to a cluster-distance space

**__init__**(n_clusters=8, init='k-means++', max_iter=100, batch_size=100, verbose=0, compute_labels=True, random_state=None, tol=0.0, max_no_improvement=10, init_size=None, n_init=3, chunk_size=None, k=None)

**fit**(X, y=None)
Compute the centroids on X by chunking it into mini-batches.

Parameters X: array-like, shape = [n_samples, n_features] :
Coordinates of the data points to cluster

**fit_predict**(X)
Compute cluster centers and predict cluster index for each sample.

Convenience method; equivalent to calling fit(X) followed by predict(X).

**get_params**(deep=True)
Get parameters for the estimator

Parameters deep: boolean, optional :
If True, will return the parameters for this estimator and contained subobjects that are estimators.

**partial_fit**(X, y=None)
Update k means estimate on a single mini-batch X.

Parameters X: array-like, shape = [n_samples, n_features] :
Coordinates of the data points to cluster.

**predict**(X)
Predict the closest cluster each sample in X belongs to.
In the vector quantization literature, `cluster_centers_` is called the code book and each value returned by `predict` is the index of the closest code in the code book.
Parameters $X$: {array-like, sparse matrix}, shape = [n_samples, n_features] :

New data to predict.

Returns $Y$ : array, shape [n_samples,]

Index of the closest center each sample belongs to.

$score(X)$

Opposite of the value of $X$ on the K-means objective.

Parameters $X$: {array-like, sparse matrix}, shape = [n_samples, n_features] :

New data.

Returns score: float :

Opposite of the value of $X$ on the K-means objective.

$set\_params(**params)$

Set the parameters of the estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The former have parameters of the form \texttt{<component>\_<parameter> so that it's possible to update each component of a nested object.

Returns self :

$transform(X, y=None)$

Transform the data to a cluster-distance space

In the new space, each dimension is the distance to the cluster centers. Note that even if $X$ is sparse, the array returned by $transform$ will typically be dense.

Parameters $X$: {array-like, sparse matrix}, shape = [n_samples, n_features] :

New data to transform.

Returns $X\_new$ : array, shape [n_samples, k]

$X$ transformed in the new space.

sklearn.cluster.MeanShift

class sklearn.cluster.MeanShift ($bandwidth=None$, $seeds=None$, $bin\_seeding=False$, $cluster\_all=True$)

MeanShift clustering

Parameters bandwidth: float, optional :

Bandwith used in the RBF kernel If not set, the bandwidth is estimated. See clustering.estimate_bandwidth

seeds: array [n_samples, n_features], optional :

Seeds used to initialize kernels. If not set, the seeds are calculated by clustering.get_bin_seeds with bandwidth as the grid size and default values for other parameters.

cluster_all: boolean, default True :

If true, then all points are clustered, even those orphans that are not within any kernel. Orphans are assigned to the nearest kernel. If false, then orphans are given cluster label -1.
Notes

Scalability:

Because this implementation uses a flat kernel and a Ball Tree to look up members of each kernel, the complexity will is to $O(T*n^2 \log(n))$ in lower dimensions, with $n$ the number of samples and $T$ the number of points. In higher dimensions the complexity will tend towards $O(T*n^2)$.

Scalability can be boosted by using fewer seeds, for example by using a higher value of min_bin_freq in the get_bin_seeds function.

Note that the estimate_bandwidth function is much less scalable than the mean shift algorithm and will be the bottleneck if it is used.

References


Attributes

<table>
<thead>
<tr>
<th>attribute</th>
<th>description</th>
</tr>
</thead>
<tbody>
<tr>
<td>cluster_centers_</td>
<td>array, [n_clusters, n_features]</td>
</tr>
<tr>
<td>labels_</td>
<td>labels of each point</td>
</tr>
</tbody>
</table>

Methods

<table>
<thead>
<tr>
<th>method</th>
<th>description</th>
</tr>
</thead>
<tbody>
<tr>
<td>fit(X)</td>
<td>Compute MeanShift</td>
</tr>
<tr>
<td>get_params([deep])</td>
<td>Get parameters for the estimator</td>
</tr>
<tr>
<td>set_params(**params)</td>
<td>Set the parameters of the estimator.</td>
</tr>
</tbody>
</table>

__init__ (bandwidth=None, seeds=None, bin_seeding=False, cluster_all=True)

fit (X)

Compute MeanShift

Parameters X : array [n_samples, n_features]

Input points

get_params (deep=True)

Get parameters for the estimator

Parameters deep: boolean, optional :

If True, will return the parameters for this estimator and contained subobjects that are estimators.

set_params (**params)

Set the parameters of the estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The former have parameters of the form <component>__.<parameter> so that it’s possible to update each component of a nested object.

Returns self :
sklearn.cluster.SpectralClustering

class sklearn.cluster.SpectralClustering (n_clusters=8, mode=None, random_state=None, n_init=10, k=None)

Apply k-means to a projection to the normalized laplacian
In practice Spectral Clustering is very useful when the structure of the individual clusters is highly non-convex
or more generally when a measure of the center and spread of the cluster is not a suitable description of the
complete cluster. For instance when clusters are nested circles on the 2D plan.
If affinity is the adjacency matrix of a graph, this method can be used to find normalized graph cuts.

Parameters

n_clusters : integer, optional
The dimension of the projection subspace.

mode : {None, ‘arpack’ or ‘amg’}
The eigenvalue decomposition strategy to use. AMG requires pyamg to be installed. It
can be faster on very large, sparse problems, but may also lead to instabilities

random_state : int seed, RandomState instance, or None (default)
A pseudo random number generator used for the initialization of the lobpcg eigen vec-
tors decomposition when mode == ‘amg’ and by the K-Means initialization.

n_init : int, optional, default: 10
Number of time the k-means algorithm will be run with different centroid seeds. The
final results will be the best output of n_init consecutive runs in terms of inertia.

References

•Normalized cuts and image segmentation, 2000 Jianbo Shi, Jitendra Malik
http://citeseer.ist.psu.edu/viewdoc/summary?doi=10.1.1.160.2324

•A Tutorial on Spectral Clustering, 2007 Ulrike von Luxburg
http://citeseerx.ist.psu.edu/viewdoc/summary?doi=10.1.1.165.9323

Attributes

labels_ : Labels of each point

Methods

fit(X) Compute the spectral clustering from the affinity matrix

get_params([deep]) Get parameters for the estimator

set_params(**params) Set the parameters of the estimator.

__init__  (n_clusters=8, mode=None, random_state=None, n_init=10, k=None)

fit(X)
Compute the spectral clustering from the affinity matrix

Parameters

X: array-like or sparse matrix, shape: (n_samples, n_samples) :
An affinity matrix describing the pairwise similarity of the data. If can also be an adjacency matrix of the graph to embed. X must be symmetric and its entries must be positive or zero. Zero means that elements have nothing in common, whereas high values mean that elements are strongly similar.

Notes

If you have an affinity matrix, such as a distance matrix, for which 0 means identical elements, and high values mean very dissimilar elements, it can be transformed in a similarity matrix that is well suited for the algorithm by applying the gaussian (heat) kernel:

\[
\exp\left(- \frac{X \times 2}{2 \times \delta \times 2}\right)
\]

Another alternative is to take a symmetric version of the k nearest neighbors connectivity matrix of the points.

If the pyamg package is installed, it is used: this greatly speeds up computation.

get_params (deep=True)

Get parameters for the estimator

Parameters deep: boolean, optional :

If True, will return the parameters for this estimator and contained subobjects that are estimators.

set_params (**params)

Set the parameters of the estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The former have parameters of the form <component>__<parameter> so that it’s possible to update each component of a nested object.

Returns self :

sklearn.cluster.Ward

class sklearn.cluster.Ward(n_clusters=2, memory=Memory(cachedir=None), connectivity=None, copy=True, n_components=None)

Ward hierarchical clustering: constructs a tree and cuts it.

Parameters n_clusters : int or ndarray

The number of clusters to find.

connectivity : sparse matrix.

Connectivity matrix. Defines for each sample the neighboring samples following a given structure of the data. Default is None, i.e, the hierarchical clustering algorithm is unstructured.

memory : Instance of joblib.Memory or string

Used to cache the output of the computation of the tree. By default, no caching is done. If a string is given, it is the path to the caching directory.

copy : bool

Copy the connectivity matrix or work inplace.

n_components : int (optional)
The number of connected components in the graph defined by the connectivity matrix. If not set, it is estimated.

**Attributes**

<table>
<thead>
<tr>
<th>name</th>
<th>description</th>
</tr>
</thead>
<tbody>
<tr>
<td>labels_</td>
<td>array [n_points]</td>
</tr>
<tr>
<td>n_leaves_</td>
<td>int</td>
</tr>
</tbody>
</table>

**Methods**

<table>
<thead>
<tr>
<th>method</th>
<th>description</th>
</tr>
</thead>
<tbody>
<tr>
<td>fit(X)</td>
<td>Fit the hierarchical clustering on the data</td>
</tr>
<tr>
<td>get_params([deep])</td>
<td>Get parameters for the estimator</td>
</tr>
<tr>
<td>set_params(**params)</td>
<td>Set the parameters of the estimator.</td>
</tr>
</tbody>
</table>

**__init__**(n_clusters=2, memory=Memory(cachedir=None), connectivity=None, copy=True, n_components=None)

**fit**(X)
Fit the hierarchical clustering on the data

Parameters **X** : array-like, shape = [n_samples, n_features]
The samples a.k.a. observations.

Returns **self** :

**get_params**(deep=True)
Get parameters for the estimator

Parameters deep: boolean, optional:
If True, will return the parameters for this estimator and contained subobjects that are estimators.

**set_params**(**params)
Set the parameters of the estimator.
The method works on simple estimators as well as on nested objects (such as pipelines). The former have parameters of the form <component>__<parameter> so that it’s possible to update each component of a nested object.

Returns **self** :

**Functions**

<table>
<thead>
<tr>
<th>function</th>
<th>description</th>
</tr>
</thead>
<tbody>
<tr>
<td>cluster.estimate_bandwidth(X[, quantile, ...])</td>
<td>Estimate the bandwidth to use with MeanShift algorithm</td>
</tr>
<tr>
<td>cluster.k_means(X, n_clusters[, init, ...])</td>
<td>K-means clustering algorithm.</td>
</tr>
<tr>
<td>cluster.ward_tree(X[, connectivity, ...])</td>
<td>Ward clustering based on a Feature matrix.</td>
</tr>
<tr>
<td>cluster.affinity_propagation(S[, p, convit, ...])</td>
<td>Perform Affinity Propagation Clustering of data</td>
</tr>
<tr>
<td>cluster.dbscan(X[, eps, min_samples, ...])</td>
<td>Perform DBSCAN clustering from vector array or distance matrix.</td>
</tr>
<tr>
<td>cluster.mean_shift(X[, bandwidth, seeds, ...])</td>
<td>Perform MeanShift Clustering of data using a flat kernel</td>
</tr>
<tr>
<td>cluster.spectral_clustering(affinity[, ...])</td>
<td>Apply k-means to a projection to the normalized laplacian</td>
</tr>
</tbody>
</table>

1.8. Reference
sklearn.cluster.estimate_bandwidth

**sklearn.cluster.estimate_bandwidth** *(X, quantile=0.3, n_samples=None, random_state=0)*

Estimate the bandwith to use with MeanShift algorithm

**Parameters**

- **X**: array [n_samples, n_features]
  - Input points
- **quantile**: float, default 0.3
  - should be between [0, 1] 0.5 means that the median is all pairwise distances is used
- **n_samples**: int
  - The number of samples to use. If None, all samples are used.
- **random_state**: int or RandomState
  - Pseudo number generator state used for random sampling.

**Returns**

- **bandwidth**: float
  - The bandwidth parameter

sklearn.cluster.k_means

**sklearn.cluster.k_means** *(X, n_clusters, init='k-means++', precompute_distances=True, n_init=10, max_iter=300, verbose=False, tol=0.0001, random_state=None, copy_x=True, n_jobs=1, k=None)*

K-means clustering algorithm.

**Parameters**

- **X**: array-like of floats, shape (n_samples, n_features)
  - The observations to cluster.
- **n_clusters**: int
  - The number of clusters to form as well as the number of centroids to generate.
- **max_iter**: int, optional, default 300
  - Maximum number of iterations of the k-means algorithm to run.
- **n_init**: int, optional, default: 10
  - Number of time the k-means algorithm will be run with different centroid seeds. The final results will be the best output of n_init consecutive runs in terms of inertia.
- **init**: {'k-means++', 'random', or ndarray, or a callable}, optional
  - Method for initialization, default to ‘k-means++’:
    - ‘k-means++’ : selects initial cluster centers for k-mean clustering in a smart way to speed up convergence. See section Notes in k_init for more details.
    - ‘random’ : generate k centroids from a Gaussian with mean and variance estimated from the data.
      - If an ndarray is passed, it should be of shape (k, p) and gives the initial centers.
      - If a callable is passed, it should take arguments X, k and a random state and return an initialization.
- **tol**: float, optional
The relative increment in the results before declaring convergence.

**verbose**: boolean, optional:
Verbosity mode

**random_state**: integer or numpy.RandomState, optional:
The generator used to initialize the centers. If an integer is given, it fixes the seed. Defaults to the global numpy random number generator.

**copy_x**: boolean, optional:
When pre-computing distances it is more numerically accurate to center the data first. If copy_x is True, then the original data is not modified. If False, the original data is modified, and put back before the function returns, but small numerical differences may be introduced by subtracting and then adding the data mean.

**n_jobs**: int:
The number of jobs to use for the computation. This works by breaking down the pairwise matrix into n_jobs even slices and computing them in parallel.
If -1 all CPUs are used. If 1 is given, no parallel computing code is used at all, which is useful for debugging. For n_jobs below -1, (n cpus + 1 - n_jobs) are used. Thus for n_jobs = -2, all CPUs but one are used.

**Returns centroid**: float ndarray with shape (k, n_features):
Centroids found at the last iteration of k-means.

**label**: integer ndarray with shape (n_samples,):
label[i] is the code or index of the centroid the i’th observation is closest to.

**inertia**: float:
The final value of the inertia criterion (sum of squared distances to the closest centroid for all observations in the training set).

---

`sklearn.cluster.ward_tree`

`sklearn.cluster.ward_tree(X, connectivity=None, n_components=None, copy=True)`
Ward clustering based on a Feature matrix.
The inertia matrix uses a Heapq-based representation.
This is the structured version, that takes into account a some topological structure between samples.

**Parameters**

- **X**: array of shape (n_samples, n_features)
  Feature matrix representing n_samples samples to be clustered.

- **connectivity**: sparse matrix.
  Connectivity matrix. Defines for each sample the neighboring samples following a given structure of the data. The matrix is assumed to be symmetric and only the upper triangular half is used. Default is None, i.e., the Ward algorithm is unstructured.

- **n_components**: int (optional)
  Number of connected components. If None the number of connected components is estimated from the connectivity matrix.

- **copy**: bool (optional)
Make a copy of connectivity or work inplace. If connectivity is not of LIL type there will be a copy in any case.

**Returns children** : list of pairs. Length of n_nodes

list of the children of each nodes. Leaves of the tree have empty list of children.

**n_components** : sparse matrix.

The number of connected components in the graph.

**n_leaves** : int

The number of leaves in the tree

```
scikit-learn.cluster.affinity_propagation
```

```
sklearn.cluster.affinity_propagation(S, p=None, convit=30, max_iter=200, damping=0.5, copy=True, verbose=False)
```

Perform Affinity Propagation Clustering of data

**Parameters**

**S**: array [n_points, n_points] :

Matrix of similarities between points

**p**: array [n_points,] or float, optional :

Preferences for each point - points with larger values of preferences are more likely to be chosen as exemplars. The number of exemplars, ie of clusters, is influenced by the input preferences value. If the preferences are not passed as arguments, they will be set to the median of the input similarities (resulting in a moderate number of clusters). For a smaller amount of clusters, this can be set to the minimum value of the similarities.

**damping** : float, optional

Damping factor

**copy** : boolean, optional :

If copy is False, the affinity matrix is modified inplace by the algorithm, for memory efficiency

**verbose** : boolean, optional :

The verbosity level

**Returns**

**cluster_centers_indices** : array [n_clusters] :

index of clusters centers

**labels** : array [n_points]

cluster labels for each point

**Notes**

See examples/plot_affinity_propagation.py for an example.

**References**

sklearn.cluster.dbSCAN

sklearn.cluster.dbSCAN (X, eps=0.5, min_samples=5, metric='euclidean', random_state=None)
Perform DBSCAN clustering from vector array or distance matrix.

Parameters X: array [n_samples, n_samples] or [n_samples, n_features]
   Array of distances between samples, or a feature array. The array is treated as a feature
   array unless the metric is given as ‘precomputed’.
eps: float, optional
   The maximum distance between two samples for them to be considered as in the same
   neighborhood.
min_samples: int, optional
   The number of samples in a neighborhood for a point to be considered as a core point.
metric: string, or callable
   The metric to use when calculating distance between instances in a feature array.
   If metric is a string or callable, it must be one of the options allowed by met-
   rics.pairwise.calculate_distance for its metric parameter. If metric is “precomputed”,
   X is assumed to be a distance matrix and must be square.
random_state: numpy.RandomState, optional
   The generator used to initialize the centers. Defaults to numpy.random.

Returns core_samples: array [n_core_samples]
   Indices of core samples.
labels : array [n_samples]
   Cluster labels for each point. Noisy samples are given the label -1.

Notes

See examples/plot_dbSCAN.py for an example.

References

Ester, M., H. P. Kriegel, J. Sander, and X. Xu, “A Density-Based Algorithm for Discovering Clusters in Large
Spatial Databases with Noise”. In: Proceedings of the 2nd International Conference on Knowledge Discovery
and Data Mining, Portland, OR, AAAI Press, pp. 226–231. 1996

sklearn.cluster.mean_shift

sklearn.cluster.mean_shift (X, bandwidth=None, seeds=None, bin_seeding=False, cluster_all=True, max_iterations=300)
Perform MeanShift Clustering of data using a flat kernel.

Seed using a binning technique for scalability.

Parameters X : array [n_samples, n_features]
   Input points
bandwidth : float, optional

kernel bandwidth If bandwidth is not defined, it is set using a heuristic given by the
median of all pairwise distances

seeds: array [n_seeds, n_features] :

point used as initial kernel locations

bin_seeding: boolean :

If true, initial kernel locations are not locations of all points, but rather the location of
the discretized version of points, where points are binned onto a grid whose coarseness
corresponds to the bandwidth. Setting this option to True will speed up the algorithm
because fewer seeds will be initialized. default value: False Ignored if seeds argument
is not None

min_bin_freq: int, optional :

To speed up the algorithm, accept only those bins with at least min_bin_freq points as
seeds. If not defined, set to 1.

Returns

cluster_centers : array [n_clusters, n_features]

Coordinates of cluster centers

labels : array [n_samples]

cluster labels for each point

Notes

See examples/plot_meanshift.py for an example.

sklearn.cluster.spectral_clustering

sklearn.cluster.spectral_clustering(affinity, n_clusters=8, n_components=None,
mode=None, random_state=None, n_init=10, k=None)

Apply k-means to a projection to the normalized laplacian

In practice Spectral Clustering is very useful when the structure of the individual clusters is highly non-convex
or more generally when a measure of the center and spread of the cluster is not a suitable description of the
complete cluster. For instance when clusters are nested circles on the 2D plan.

If affinity is the adjacency matrix of a graph, this method can be used to find normalized graph cuts.

Parameters

affinity: array-like or sparse matrix, shape: (n_samples, n_samples) :

The affinity matrix describing the relationship of the samples to embed. Must be sym-
metric.

Possible examples:

• adjacency matrix of a graph,
• heat kernel of the pairwise distance matrix of the samples,
• symmetric k-nearest neighbours connectivity matrix of the samples.

n_clusters: integer, optional :

Number of clusters to extract.
n_components: integer, optional, default is k:
Number of eigen vectors to use for the spectral embedding

mode: {None, ‘arpack’ or ‘amg’}:
The eigenvalue decomposition strategy to use. AMG requires pyamg to be installed. It can be faster on very large, sparse problems, but may also lead to instabilities

random_state: int seed, RandomState instance, or None (default):
A pseudo random number generator used for the initialization of the lobpcg eigen vectors decomposition when mode == ‘amg’ and by the K-Means initialization.

n_init: int, optional, default: 10:
Number of time the k-means algorithm will be run with different centroid seeds. The final results will be the best output of n_init consecutive runs in terms of inertia.

Returns labels: array of integers, shape: n_samples:
The labels of the clusters.

centers: array of integers, shape: k:
The indices of the cluster centers

Notes
The graph should contain only one connect component, elsewhere the results make little sense.
This algorithm solves the normalized cut for k=2: it is a normalized spectral clustering.

References

•Normalized cuts and image segmentation, 2000 Jianbo Shi, Jitendra Malik
http://citeseer.ist.psu.edu/viewdoc/summary?doi=10.1.1.160.2324
•A Tutorial on Spectral Clustering, 2007 Ulrike von Luxburg
http://citeseerx.ist.psu.edu/viewdoc/summary?doi=10.1.1.165.9323

1.8.2 sklearn.covariance: Covariance Estimators

The sklearn.covariance module includes methods and algorithms to robustly estimate the covariance of features given a set of points. The precision matrix defined as the inverse of the covariance is also estimated. Covariance estimation is closely related to the theory of Gaussian Graphical Models.

User guide: See the Covariance estimation section for further details.

<table>
<thead>
<tr>
<th>Method</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>covariance.EmpiricalCovariance(...)</td>
<td>Maximum likelihood covariance estimator</td>
</tr>
<tr>
<td>covariance.EllipticEnvelope(...)</td>
<td>An object for detecting outliers in a Gaussian distributed dataset.</td>
</tr>
<tr>
<td>covariance.GraphLasso((alpha, mode, tol, ...))</td>
<td>Sparse inverse covariance estimation with an H-penalized estimator.</td>
</tr>
<tr>
<td>covariance.GraphLassoCV((alphas, ...))</td>
<td>Sparse inverse covariance w/ cross-validated choice of the H penalty</td>
</tr>
<tr>
<td>covariance.LedoitWolf((store_precision, ...))</td>
<td>LedoitWolf Estimator</td>
</tr>
<tr>
<td>covariance.MinCovDet((store_precision, ...))</td>
<td>Minimum Covariance Determinant (MCD): robust estimator of covariance</td>
</tr>
<tr>
<td>covariance.OAS((store_precision, ...))</td>
<td>Oracle Approximating Shrinkage Estimator</td>
</tr>
<tr>
<td>covariance.ShrunkenCovariance(...)</td>
<td>Covariance estimator with shrinkage</td>
</tr>
</tbody>
</table>
Maximum likelihood covariance estimator

**Parameters**

- store_precision : bool
  Specifies if the estimated precision is stored

**Attributes**

<table>
<thead>
<tr>
<th>Attribute</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>covariance_</td>
<td>2D ndarray, shape (n_features, n_features) Estimated covariance matrix</td>
</tr>
<tr>
<td>precision_</td>
<td>2D ndarray, shape (n_features, n_features) Estimated pseudo-inverse matrix. (stored only if store_precision is True)</td>
</tr>
</tbody>
</table>

**Methods**

- error_norm(comp_cov[, norm, scaling, squared]) Computes the Mean Squared Error between two covariance estimators.
- fit(X) Fits the Maximum Likelihood Estimator covariance model
- get_params([deep]) Get parameters for the estimator
- mahalanobis(observations) Computes the mahalanobis distances of given observations.
- score(X_test[, assume_centered]) Computes the log-likelihood of a gaussian data set with self.covariance_ as an estimate of its covariance matrix.
- set_params(**params) Set the parameters of the estimator.

**__init__** (store_precision=True, assume_centered=False)

**Parameters**

- store_precision: bool
  Specify if the estimated precision is stored
- assume_centered: Boolean
  If True, data are not centered before computation. Useful when working with data whose mean is almost, but not exactly zero. If False, data are centered before computation.

**error_norm** (comp_cov, norm='frobenius', scaling=True, squared=True)

Computes the Mean Squared Error between two covariance estimators. (In the sense of the Frobenius norm)

**Parameters**

- comp_cov: array-like, shape = [n_features, n_features]
The covariance to compare with.
- norm: str
  The type of norm used to compute the error. Available error types: - ‘frobenius’ (default): sqrt(tr(A^t.A)) - ‘spectral’: sqrt(max(eigenvalues(A^t.A)) where A is the error (comp_cov - self.covariance_).
- scaling: bool
  If True (default), the squared error norm is divided by n_features. If False, the squared error norm is not rescaled.
squared: bool:

Whether to compute the squared error norm or the error norm. If True (default), the
squared error norm is returned. If False, the error norm is returned.

Returns The Mean Squared Error (in the sense of the Frobenius norm) between:

'self' and 'comp_cov' covariance estimators.

fit(X)

Fits the Maximum Likelihood Estimator covariance model according to the given training data and parameters.

Parameters X: array-like, shape = [n_samples, n_features]

Training data, where n_samples is the number of samples and n_features is the number
of features.

Returns self: object

Returns self.

get_params(deep=True)

Get parameters for the estimator

Parameters deep: boolean, optional:

If True, will return the parameters for this estimator and contained subobjects that are estimators.

mahalanobis(observations)

Computes the mahalanobis distances of given observations.

The provided observations are assumed to be centered. One may want to center them using a location
estimate first.

Parameters observations: array-like, shape = [n_observations, n_features]:

The observations, the Mahalanobis distances of which we compute.

Returns mahalanobis_distance: array, shape = [n_observations,]

Mahalanobis distances of the observations.

score(X_test, assume_centered=False)

Computes the log-likelihood of a gaussian data set with self.covariance_ as an estimator of its covariance
matrix.

Parameters X_test: array-like, shape = [n_samples, n_features]

Test data of which we compute the likelihood, where n_samples is the number of sam-
ples and n_features is the number of features.

Returns res: float

The likelihood of the data set with self.covariance_ as an estimator of its covariance
matrix.

set_params(**params)

Set the parameters of the estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The former have parameters of the form <component>__<parameter> so that it’s possible to update each component
of a nested object.

Returns self:
sklearn.covariance.EllipticEnvelope

**class sklearn.covariance.EllipticEnvelope**

An object for detecting outliers in a Gaussian distributed dataset.

**Parameters**

- **store_precision**: bool
  Specify if the estimated precision is stored

- **assume_centered**: Boolean
  If True, the support of robust location and covariance estimates is computed, and a covariance estimate is recomputed from it, without centering the data. Useful to work with data whose mean is significantly equal to zero but is not exactly zero. If False, the robust location and covariance are directly computed with the FastMCD algorithm without additional treatment.

- **support_fraction**: float, 0 < support_fraction < 1
  The proportion of points to be included in the support of the raw MCD estimate. Default is None, which implies that the minimum value of support_fraction will be used within the algorithm: \([n_{\text{sample}} + n_{\text{features}} + 1] / 2\)

- **contamination**: float, 0. < contamination < 0.5
  The amount of contamination of the data set, i.e. the proportion of outliers in the data set.

**See Also:**

EmpiricalCovariance, MinCovDet

**Notes**

Outlier detection from covariance estimation may break or not perform well in high-dimensional settings. In particular, one will always take care to work with \(n_{\text{samples}} > n_{\text{features}} \times 2\).

**References**

**Attributes**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>contamination: float, 0. &lt; contamination &lt; 0.5</td>
<td>The amount of contamination of the data set, i.e. the proportion of outliers in the data set.</td>
</tr>
<tr>
<td>location_: array-like, shape (n_features,)</td>
<td>Estimated robust location</td>
</tr>
<tr>
<td>covariance_: array-like, shape (n_features, n_features)</td>
<td>Estimated robust covariance matrix</td>
</tr>
<tr>
<td>precision_: array-like, shape (n_features, n_features)</td>
<td>Estimated pseudo inverse matrix. (stored only if store_precision is True)</td>
</tr>
<tr>
<td>support_: array-like, shape (n_samples,)</td>
<td>A mask of the observations that have been used to compute the robust estimates of location and shape.</td>
</tr>
</tbody>
</table>

**Methods**
correct_covariance(data) Apply a correction to raw Minimum Covariance Determinant estimates.

decision_function(X[, raw_mahalanobis]) Compute the decision function of the given observations.

error_norm(comp_cov[, norm, scaling, squared]) Computes the Mean Squared Error between two covariance estimators.

fit(X)

get_params([deep]) Get parameters for the estimator

mahalanobis(observations) Computes the mahalanobis distances of given observations.

predict(X) Outlyingness of observations in X according to the fitted model.

reweight_covariance(data) Reweight raw Minimum Covariance Determinant estimates.

score(X, y) Returns the mean accuracy on the given test data and labels.

set_params(**params) Set the parameters of the estimator.

__init__(store_precision=True, assume_centered=False, support_fraction=None, contamination=0.1)

correct_covariance(data)
Apply a correction to raw Minimum Covariance Determinant estimates.

Correction using the empirical correction factor suggested by Rousseeuw and Van Driessen in [Rouseeuw1984].

Parameters data: array-like, shape (n_samples, n_features):

The data matrix, with p features and n samples. The data set must be the one which was used to compute the raw estimates.

Returns covariance_corrected: array-like, shape (n_features, n_features):

Corrected robust covariance estimate.

decision_function(X, raw_mahalanobis=False)
Compute the decision function of the given observations.

Parameters X: array-like, shape (n_samples, n_features):

raw_mahalanobis: bool:

Whether or not to consider raw Mahalanobis distances as the decision function. Must be False (default) for compatibility with the others outlier detection tools.

Returns decision: array-like, shape (n_samples, ):

The values of the decision function for each observations. It is equal to the Mahalanobis distances if raw_mahalanobis is True. By default (raw_mahalanobis=True), it is equal to the cubic root of the shifted Mahalanobis distances. In that case, the threshold for being an outlier is 0, which ensures a compatibility with other outlier detection tools such as the One-Class SVM.

error_norm(comp_cov[, norm='frobenius', scaling=True, squared=True])
Computes the Mean Squared Error between two covariance estimators. (In the sense of the Frobenius norm)

Parameters comp_cov: array-like, shape = [n_features, n_features]:

The covariance to compare with.

norm: str:

The type of norm used to compute the error. Available error types: - ‘frobenius’ (default): sqrt(tr(A^t.A)) - ‘spectral’: sqrt(max(eigenvalues(A^t.A)) where A is the error (comp_cov - self.covariance_).
scaling: bool :
    If True (default), the squared error norm is divided by n_features. If False, the squared
    error norm is not rescaled.

squared: bool :
    Whether to compute the squared error norm or the error norm. If True (default), the
    squared error norm is returned. If False, the error norm is returned.

Returns The Mean Squared Error (in the sense of the Frobenius norm) between :

'self' and 'comp_cov' covariance estimators. :

fit (X)

get_params (deep=True)
    Get parameters for the estimator

Parameters deep: boolean, optional :
    If True, will return the parameters for this estimator and contained subobjects that are
    estimators.

mahalanobis (observations)
    Computes the mahalanobis distances of given observations.

The provided observations are assumed to be centered. One may want to center them using a location
estimate first.

Parameters observations: array-like, shape = [n_observations, n_features] :
    The observations, the Mahalanobis distances of the which we compute.

Returns mahalanobis_distance: array, shape = [n_observations,] :
    Mahalanobis distances of the observations.

predict (X)
    Outlyingness of observations in X according to the fitted model.

Parameters X: array-like, shape = (n_samples, n_features) :

Returns is_outliers: array, shape = (n_samples,), dtype = bool :
    For each observations, tells whether or not it should be considered as an outlier accord-
    ing to the fitted model.

threshold: float, :
    The values of the less outlying point’s decision function.

reweight_covariance (data)
    Reweight raw Minimum Covariance Determinant estimates.

Reweight observations using Rousseeuw’s method (equivalent to deleting outlying observations from the
data set before computing location and covariance estimates). [Rousseeuw1984]

Parameters data: array-like, shape (n_samples, n_features) :
    The data matrix, with p features and n samples. The data set must be the one which was
    used to compute the raw estimates.

Returns location_reweighted: array-like, shape (n_features, ) :
    Reweighted robust location estimate.

covariance_reweighted: array-like, shape (n_features, n_features) :

Reweighted robust covariance estimate.

**support_reweighted**: array-like, type boolean, shape (n_samples,):
A mask of the observations that have been used to compute the reweighted robust location and covariance estimates.

**score** (X, y)
Returns the mean accuracy on the given test data and labels.

**Parameters**

- **X**: array-like, shape = [n_samples, n_features]
  - Training set.
- **y**: array-like, shape = [n_samples]
  - Labels for X.

**Returns**

- **z**: float

**set_params** (**params**)
Set the parameters of the estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The former have parameters of the form `<component>__<parameter>` so that it’s possible to update each component of a nested object.

**Returns**

- **self**

---

**sklearn.covariance.GraphLasso**

class **sklearn.covariance.GraphLasso** (alpha=0.01, mode='cd', tol=0.0001, max_iter=100, verbose=False)
Sparse inverse covariance estimation with an l1-penalized estimator.

**Parameters**

- **alpha**: positive float, optional
  - The regularization parameter: the higher alpha, the more regularization, the sparser the inverse covariance

- **cov_init**: 2D array (n_features, n_features), optional
  - The initial guess for the covariance

- **mode**: {'cd', 'lars'}
  - The Lasso solver to use: coordinate descent or LARS. Use LARS for very sparse underlying graphs, where p > n. Elsewhere prefer cd which is more numerically stable.

- **tol**: positive float, optional
  - The tolerance to declare convergence: if the dual gap goes below this value, iterations are stopped

- **max_iter**: integer, optional
  - The maximum number of iterations

- **verbose**: boolean, optional
  - If verbose is True, the objective function and dual gap are plotted at each iteration

**See Also**

- `graph_lasso`, `GraphLassoCV`

---

1.8. Reference

251
Attributes

| covariance_ | array-like, shape (n_features, n_features) | Estimated covariance matrix |
| precision_  | array-like, shape (n_features, n_features) | Estimated pseudo inverse matrix |

Methods

- `error_norm(comp_cov[, norm, scaling, squared])`: Computes the Mean Squared Error between two covariance estimators. (In the sense of the Frobenius norm)

  Parameters:
  - `comp_cov`: array-like, shape = [n_features, n_features]: The covariance to compare with.
  - `norm`: str: The type of norm used to compute the error. Available error types: 'frobenius' (default): sqrt(tr(A^t.A)) - 'spectral': sqrt(max(eigenvalues(A^t.A))) where A is the error (comp_cov - self.covariance_).
  - `scaling`: bool: If True (default), the squared error norm is divided by n_features. If False, the squared error norm is not rescaled.
  - `squared`: bool: Whether to compute the squared error norm or the error norm. If True (default), the squared error norm is returned. If False, the error norm is returned.

  Returns: The Mean Squared Error (in the sense of the Frobenius norm) between:
  - 'self' and 'comp_cov' covariance estimators.

- `fit(X[, y])`: Get parameters for the estimator

- `mahalanobis(observations)`: Computes the mahalanobis distances of given observations.

  Parameters:
  - `observations`: array-like, shape = [n_observations, n_features]: The provided observations are assumed to be centered. One may want to center them using a location estimate first.

- `score(X_test[, assume_centered])`: Computes the log-likelihood of a gaussian data set with self.covariance_ as an estimator.

- `set_params(**params)`: Set the parameters of the estimator.

__init__(alpha=0.01, mode='cd', tol=0.0001, max_iter=100, verbose=False)

- `error_norm(comp_cov, norm='frobenius', scaling=True, squared=True)`: Computes the Mean Squared Error between two covariance estimators (in the sense of the Frobenius norm)
The observations, the Mahalanobis distances of the which we compute.

Returns mahalanobis_distance: array, shape = [n_observations,] :
    Mahalanobis distances of the observations.

score (X_test, assume_centered=False)
    Computes the log-likelihood of a gaussian data set with self.covariance_ as an estimator of its covariance matrix.

    Parameters X_test: array-like, shape = [n_samples, n_features]
        Test data of which we compute the likelihood, where n_samples is the number of samples and n_features is the number of features.

    Returns res: float
        The likelihood of the data set with self.covariance_ as an estimator of its covariance matrix.

set_params (**params)
    Set the parameters of the estimator.

    The method works on simple estimators as well as on nested objects (such as pipelines). The former have parameters of the form <component>__<parameter> so that it's possible to update each component of a nested object.

    Returns self:

sklearn.covariance.GraphLassoCV

class sklearn.covariance.GraphLassoCV(alphas=4, n_refinements=4, cv=None, tol=0.0001, max_iter=100, mode='cd', n_jobs=1, verbose=False)
    Sparse inverse covariance w/ cross-validated choice of the l1 penalty

    Parameters alphas: integer, or list positive float, optional:
        If an integer is given, it fixes the number of points on the grids of alpha to be used. If a list is given, it gives the grid to be used. See the notes in the class docstring for more details.

    n_refinements: strictly positive integer:
        The number of time the grid is refined. Not used if explicit values of alphas are passed.

    cv: crossvalidation generator, optional
        see sklearn.cross_validation module. If None is passed, default to a 3-fold strategy

    tol: positive float, optional:
        The tolerance to declare convergence: if the dual gap goes below this value, iterations are stopped

    max_iter: integer, optional:
        The maximum number of iterations

    mode: {'cd', 'lars'}:
        The Lasso solver to use: coordinate descent or LARS. Use LARS for very sparse underlying graphs, where p > n. Elsewhere prefer cd which is more numerically stable.

    n_jobs: int, optional:
number of jobs to run in parallel (default 1)

verbose: boolean, optional :

If verbose is True, the objective function and dual gap are print at each iteration

See Also:

graph_lasso, GraphLasso

Notes

The search for the optimal alpha is done on an iteratively refined grid: first the cross-validated scores on a grid
are computed, then a new refined grid is center around the maximum...

One of the challenges that we have to face is that the solvers can fail to converge to a well-conditioned estimate.
The corresponding values of alpha then come out as missing values, but the optimum may be close to these
missing values.

Attributes

covariance_ array-like, shape (n_features, n_features)  Estimated covariance matrix

precision_ array-like, shape (n_features, n_features)  Estimated precision matrix (inverse covariance).

alpha_ float  Penalization parameter selected

cv_alphas_ list of float  All the penalization parameters explored

cv_scores_ 2D array (n alphas, n_folds)  The log-likelihood score on left-out data across the folds.

Methods

error_norm(comp_cov[, norm, scaling, squared])  Computes the Mean Squared Error between two covariance estimators.

fit(X[, y])  Get parameters for the estimator

get_params([deep])  Computes the mahalanobis distances of given observations.

mahalanobis(observations)  Computes the log-likelihood of a gaussian data set with self.covariance_ as an

score(X_test[, assume_centered])  Set the parameters of the estimator.

set_params(**params)

__init__(alphas=4, n_refinements=4, cv=None, tol=0.0001, max_iter=100, mode='cd', n_jobs=1,
verbose=False)

error_norm(comp_c cov, norm='frobenius', scaling=True, squared=True)

Computes the Mean Squared Error between two covariance estimators. (In the sense of the Frobenius norm)

Parameters comp_c cov: array-like, shape = [n_features, n_features] :

The covariance to compare with.

norm: str :

The type of norm used to compute the error. Available error types: - ‘frobenius’ (default): sqrt(tr(A^t.A)) - ‘spectral’: sqrt(max(eigenvalues(A^t.A)) where A is the error
(comp_c cov - self.covariance_).
scaling: bool:
If True (default), the squared error norm is divided by n_features. If False, the squared error norm is not rescaled.

squared: bool:
Whether to compute the squared error norm or the error norm. If True (default), the squared error norm is returned. If False, the error norm is returned.

Returns The Mean Squared Error (in the sense of the Frobenius norm) between:
'self' and 'comp_cov' covariance estimators.

get_params (deep=True)
Get parameters for the estimator

Parameters deep: boolean, optional:
If True, will return the parameters for this estimator and contained subobjects that are estimators.

mahalanobis (observations)
Computes the mahalanobis distances of given observations.
The provided observations are assumed to be centered. One may want to center them using a location estimate first.

Parameters observations: array-like, shape = [n_observations, n_features]:
The observations, the Mahalanobis distances of the which we compute.

Returns mahalanobis_distance: array, shape = [n_observations,]:
Mahalanobis distances of the observations.

score (X_test, assume_centered=False)
Computes the log-likelihood of a gaussian data set with self.covariance_ as an estimator of its covariance matrix.

Parameters X_test: array-like, shape = [n_samples, n_features]
Test data of which we compute the likelihood, where n_samples is the number of samples and n_features is the number of features.

Returns res: float
The likelihood of the data set with self.covariance_ as an estimator of its covariance matrix.

set_params (**params)
Set the parameters of the estimator.
The method works on simple estimators as well as on nested objects (such as pipelines). The former have parameters of the form <component>__<parameter> so that it’s possible to update each component of a nested object.

Returns self:

sklearn.covariance.LedoitWolf

class sklearn.covariance.LedoitWolf (store_precision=True, assume_centered=False)
LedoitWolf Estimator

1.8. Reference 255

**Parameters**

```
store_precision : bool
```

Specify if the estimated precision is stored

**Notes**

The regularised covariance is:

\[(1 - \text{shrinkage}) \cdot \text{cov} + \text{shrinkage} \cdot \mu \cdot \text{np.identity(n_features)}\]

where \(\mu = \text{trace(cov)} / \text{n_features}\) and shrinkage is given by the Ledoit and Wolf formula (see References)

**References**


**Attributes**

| covariance_ | array-like, shape (n_features, n_features) | Estimated covariance matrix |
| precision_  | array-like, shape (n_features, n_features) | Estimated pseudo inverse matrix. (stored only if store_precision is True) |
| shrinkage_  | float, 0 <= shrinkage <= 1 | coefficient in the convex combination used for the computation of the shrunk estimate. |

**Methods**

- **error_norm**(comp_cov[, norm, scaling, squared]) Computes the Mean Squared Error between two covariance estimators.
- **fit**(X[, assume_centered]) Fits the Ledoit-Wolf shrunk covariance model.
- **get_params**([deep]) Get parameters for the estimator.
- **mahalanobis**(observations) Computes the mahalanobis distances of given observations.
- **score**(X_test[, assume_centered]) Computes the log-likelihood of a gaussian data set with self.covariance_ as an estimate.
- **set_params**(**params) Set the parameters of the estimator.

**__init__**(store_precision=True, assume_centered=False)

```
Parameters store_precision: bool :

    Specify if the estimated precision is stored
```

```
assume_centered: Boolean :

    If True, data are not centered before computation. Useful when working with data whose mean is almost, but not exactly zero. If False, data are centered before computation.
```

```
error_norm (comp_cov, norm='frobenius', scaling=True, squared=True)
```
Computes the Mean Squared Error between two covariance estimators. (In the sense of the Frobenius norm)

**Parameters** comp_cov: array-like, shape = [n_features, n_features] :

The covariance to compare with.

**norm**: str :

The type of norm used to compute the error. Available error types: - ‘frobenius’ (default): sqrt(tr(A^t.A)) - ‘spectral’: sqrt(max(eigenvalues(A^t.A))) where A is the error (comp_cov - self.covariance_).

**scaling**: bool :

If True (default), the squared error norm is divided by n_features. If False, the squared error norm is not rescaled.

**squared**: bool :

Whether to compute the squared error norm or the error norm. If True (default), the squared error norm is returned. If False, the error norm is returned.

**Returns** The Mean Squared Error (in the sense of the Frobenius norm) between :

‘self’ and ‘comp_cov’ covariance estimators.

**fit** (X, assume_centered=False)

Fits the Ledoit-Wolf shrunk covariance model according to the given training data and parameters.

**Parameters** X : array-like, shape = [n_samples, n_features]

Training data, where n_samples is the number of samples and n_features is the number of features.

**assume_centered**: Boolean :

If True, data are not centered before computation. Useful work with data whose mean is significantly equal to zero but is not exactly zero. If False, data are centered before computation.

**Returns** self : object

Returns self.

**get_params** (deep=True)

Get parameters for the estimator

**Parameters** deep: boolean, optional :

If True, will return the parameters for this estimator and contained subobjects that are estimators.

**mahalanobis** (observations)

Computes the mahalanobis distances of given observations.

The provided observations are assumed to be centered. One may want to center them using a location estimate first.

**Parameters** observations: array-like, shape = [n_observations, n_features] :

The observations, the Mahalanobis distances of which we compute.

**Returns** mahalanobis_distance: array, shape = [n_observations,] :

Mahalanobis distances of the observations.
score(X_test, assume_centered=False)
Computes the log-likelihood of a gaussian data set with self.covariance_ as an estimator of its covariance matrix.

Parameters X_test : array-like, shape = [n_samples, n_features]
Test data of which we compute the likelihood, where n_samples is the number of samples and n_features is the number of features.

Returns res : float
The likelihood of the data set with self.covariance_ as an estimator of its covariance matrix.

set_params(**params)
Set the parameters of the estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The former have parameters of the form <component>__<parameter> so that it’s possible to update each component of a nested object.

Returns self:

sklearn.covariance.MinCovDet

class sklearn.covariance.MinCovDet(store_precision=True, assume_centered=False, support_fraction=None, random_state=None)
Minimum Covariance Determinant (MCD): robust estimator of covariance

Parameters store_precision: bool :
Specify if the estimated precision is stored

assume_centered: Boolean :
If True, the support of robust location and covariance estimates is computed, and a covariance estimate is recomputed from it, without centering the data. Useful to work with data whose mean is significantly equal to zero but is not exactly zero. If False, the robust location and covariance are directly computed with the FastMCD algorithm without additional treatment.

support_fraction: float, 0 < support_fraction < 1 :
The proportion of points to be included in the support of the raw MCD estimate. Default is None, which implies that the minimum value of support_fraction will be used within the algorithm: [n_sample + n_features + 1] / 2

random_state: integer or numpy.RandomState, optional :
The random generator used. If an integer is given, it fixes the seed. Defaults to the global numpy random number generator.

References

[Rouseeuw1984], [Rouseeuw1999], [Butler1993]
Attributes

<table>
<thead>
<tr>
<th>Attribute</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>raw_location_</td>
<td>The raw robust estimated location before correction and reweighting</td>
</tr>
<tr>
<td>raw_covariance_</td>
<td>The raw robust estimated covariance before correction and reweighting</td>
</tr>
<tr>
<td>raw_support_</td>
<td>A mask of the observations that have been used to compute the raw robust</td>
</tr>
<tr>
<td>location_</td>
<td>Estimated robust location</td>
</tr>
<tr>
<td>covariance_</td>
<td>Estimated robust covariance matrix</td>
</tr>
<tr>
<td>precision_</td>
<td>Estimated pseudo inverse matrix. (stored only if store_precision is True)</td>
</tr>
<tr>
<td>support_</td>
<td>A mask of the observations that have been used to compute the robust</td>
</tr>
</tbody>
</table>

Methods

- **correct_covariance**(data) - Apply a correction to raw Minimum Covariance Determinant estimates.
- **error_norm**(comp_cov[, norm, scaling, squared]) - Computes the Mean Squared Error between two covariance estimators.
- **fit**(X) - Fits a Minimum Covariance Determinant with the FastMCD algorithm.
- **get_params**(deep) - Get parameters for the estimator.
- **mahalanobis**(observations) - Computes the mahalanobis distances of given observations.
- **reweight_covariance**(data) - Reweight raw Minimum Covariance Determinant estimates.
- **score**(X_test[, assume_centered]) - Computes the log-likelihood of a gaussian data set with self.covariance_ as an
- **set_params**(**params) - Set the parameters of the estimator.

**__init__**(store_precision=True, assume_centered=False, support_fraction=None, random_state=None)

**correct_covariance**(data) - Apply a correction to raw Minimum Covariance Determinant estimates.

Correction using the empirical correction factor suggested by Rousseeuw and Van Driessen in [Rousseeuw1984].

**Parameters**

data: array-like, shape (n_samples, n_features)

The data matrix, with p features and n samples. The data set must be the one which was used to compute the raw estimates.

**Returns**

covariance_corrected: array-like, shape (n_features, n_features)

Corrected robust covariance estimate.

**error_norm**(comp_cov, norm='frobenius', scaling=True, squared=True) - Computes the Mean Squared Error between two covariance estimators. (In the sense of the Frobenius norm)

**Parameters**

comp_cov: array-like, shape = [n_features, n_features]
The covariance to compare with.

norm: str :
The type of norm used to compute the error. Available error types: - ‘frobenius’ (default): $\sqrt{\text{tr}(A^t.A)}$ - ‘spectral’: $\sqrt{\text{max}(\text{eigenvalues}(A^t.A))}$ where $A$ is the error (comp_cov - self.covariance_).

**scaling**: bool

If True (default), the squared error norm is divided by n_features. If False, the squared error norm is not rescaled.

**squared**: bool

Whether to compute the squared error norm or the error norm. If True (default), the squared error norm is returned. If False, the error norm is returned.

**Returns**

The Mean Squared Error (in the sense of the Frobenius norm) between:

- self
- comp_cov

### fit(X)

Fits a Minimum Covariance Determinant with the FastMCD algorithm.

**Parameters**

- **X**: array-like, shape = [n_samples, n_features]

  Training data, where n_samples is the number of samples and n_features is the number of features.

**Returns**

- **self**: object

  Returns self.

### get_params(deep=True)

Get parameters for the estimator

**Parameters**

- **deep**: boolean, optional

  If True, will return the parameters for this estimator and contained subobjects that are estimators.

### mahalanobis(observations)

Computes the mahalanobis distances of given observations.

The provided observations are assumed to be centered. One may want to center them using a location estimate first.

**Parameters**

- **observations**: array-like, shape = [n_observations, n_features]

  The observations, the Mahalanobis distances of the which we compute.

**Returns**

- **mahalanobis_distance**: array, shape = [n_observations,]

  Mahalanobis distances of the observations.

### reweight_covariance(data)

Reweight raw Minimum Covariance Determinant estimates.

Reweight observations using Rousseeuw’s method (equivalent to deleting outlying observations from the data set before computing location and covariance estimates). [Rousseeuw1984]

**Parameters**

- **data**: array-like, shape (n_samples, n_features)

  The data matrix, with p features and n samples. The data set must be the one which was used to compute the raw estimates.

**Returns**

- **location_reweighted**: array-like, shape (n_features, )

  Reweighted robust location estimate.
**covariance_reweighted**: array-like, shape (n_features, n_features)

Reweighted robust covariance estimate.

**support_reweighted**: array-like, type boolean, shape (n_samples,)

A mask of the observations that have been used to compute the reweighted robust location and covariance estimates.

**score**(X_test, assume_centered=False)

Computes the log-likelihood of a gaussian data set with self.covariance_ as an estimator of its covariance matrix.

**Parameters**

- **X_test**: array-like, shape = [n_samples, n_features]
  Test data of which we compute the likelihood, where n_samples is the number of samples and n_features is the number of features.

**Returns**

- **res**: float
  The likelihood of the data set with self.covariance_ as an estimator of its covariance matrix.

**set_params(**params**)

Set the parameters of the estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The former have parameters of the form <component>__<parameter> so that it’s possible to update each component of a nested object.

**Returns**

- **self**: 

**sklearn.covariance.OAS**

class sklearn.covariance.OAS(store_precision=True, assume_centered=False)

Oracle Approximating Shrinkage Estimator


The formula used here does not correspond to the one given in the article. It has been taken from the matlab programm available from the authors webpage (https://tbayes.eecs.umich.edu/yilun/covestimation).

**Parameters**

- **store_precision**: bool
  Specify if the estimated precision is stored

**Notes**

The regularised covariance is:

\[(1 - \text{shrinkage}) \times \text{cov} + \text{shrinkage} \times \mu \times \text{np.identity(n_features)}\]

where \(\mu = \text{trace(cov)} / \text{n_features}\) and shrinkage is given by the OAS formula (see References)

**References**

Attributes

<table>
<thead>
<tr>
<th>Attribute</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>covariance_</td>
<td>Estimated covariance matrix</td>
</tr>
<tr>
<td>precision_</td>
<td>Estimated pseudo inverse matrix. (stored only if store_precision is True)</td>
</tr>
<tr>
<td>shrinkage_</td>
<td>coefficient in the convex combination used for the computation of the shrunk estimate.</td>
</tr>
</tbody>
</table>

Methods

- error_norm(comp_cov[, norm, scaling, squared]) Computes the Mean Squared Error between two covariance estimators.
- fit(X[, assume_centered]) Fits the Oracle Approximating Shrinkage covariance model.
- get_params([deep]) Get parameters for the estimator.
- mahalanobis(observations) Computes the mahalanobis distances of given observations.
- score(X_test[, assume_centered]) Computes the log-likelihood of a gaussian data set with self.covariance_ as an estimator.
- set_params(**params) Set the parameters of the estimator.

__init__(store_precision=True, assume_centered=False)

Parameters
- store_precision: bool
  Specify if the estimated precision is stored.
- assume_centered: Boolean
  If True, data are not centered before computation. Useful when working with data whose mean is almost, but not exactly zero. If False, data are centered before computation.

error_norm(comp_cov[, norm='frobenius', scaling=True, squared=True]) Computes the Mean Squared Error between two covariance estimators. (In the sense of the Frobenius norm)

Parameters
- comp_cov: array-like, shape = [n_features, n_features]
  The covariance to compare with.
- norm: str
  The type of norm used to compute the error. Available error types: - ‘frobenius’ (default): sqrt(tr(A^t.A)) - ‘spectral’: sqrt(max(eigenvalues(A^t.A)) where A is the error (comp_cov - self.covariance_).
- scaling: bool
  If True (default), the squared error norm is divided by n_features. If False, the squared error norm is not rescaled.
- squared: bool
  Whether to compute the squared error norm or the error norm. If True (default), the squared error norm is returned. If False, the error norm is returned.

Returns
The Mean Squared Error (in the sense of the Frobenius norm) between:
- ‘self’ and ‘comp_cov’ covariance estimators.
**fit** *(X, assume_centered=False)*

Fits the Oracle Approximating Shrinkage covariance model according to the given training data and parameters.

**Parameters**

- **X**: array-like, shape = [n_samples, n_features]
  - Training data, where n_samples is the number of samples and n_features is the number of features.
- **assume_centered**: boolean
  - If True, data are not centered before computation. Useful to work with data whose mean is significantly equal to zero but is not exactly zero. If False, data are centered before computation.

**Returns**

- **self**: object

**get_params** *(deep=True)*

Get parameters for the estimator.

**Parameters**

- **deep**: boolean, optional
  - If True, will return the parameters for this estimator and contained subobjects that are estimators.

**mahalanobis** *(observations)*

Computes the mahalanobis distances of given observations.

The provided observations are assumed to be centered. One may want to center them using a location estimate first.

**Parameters**

- **observations**: array-like, shape = [n_observations, n_features]
  - The observations, the Mahalanobis distances of which we compute.

**Returns**

- **mahalanobis_distance**: array, shape = [n_observations,]
  - Mahalanobis distances of the observations.

**score** *(X_test, assume_centered=False)*

Computes the log-likelihood of a gaussian data set with `self.covariance_` as an estimator of its covariance matrix.

**Parameters**

- **X_test**: array-like, shape = [n_samples, n_features]
  - Test data of which we compute the likelihood, where n_samples is the number of samples and n_features is the number of features.

**Returns**

- **res**: float
  - The likelihood of the data set with `self.covariance_` as an estimator of its covariance matrix.

**set_params** *(**params)*

Set the parameters of the estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The former have parameters of the form `<component>__<parameter>` so that it’s possible to update each component of a nested object.

**Returns**

- **self**: object
sklearn.covariance.ShrunkenCovariance

class sklearn.covariance.ShrunkenCovariance (store_precision=True, shrinkage=0.1)

Covariance estimator with shrinkage

Parameters store_precision : bool

Specify if the estimated precision is stored

shrinkage: float, 0 <= shrinkage <= 1 :

coefficient in the convex combination used for the computation of the shrunk estimate.

Notes

The regularized covariance is given by

\[(1 - \text{shrinkage}) \cdot \text{cov} - \text{shrinkage} \cdot \mu \cdot \text{np.identity(n_features)}\]

where \(\mu = \text{trace} (\text{cov}) / \text{n_features}\)

Attributes

covariance_ : array-like, shape (n_features, n_features)
Estimated covariance matrix

precision_ : array-like, shape (n_features, n_features)
Estimated pseudo inverse matrix. (stored only if store_precision is True)

shrinkage: float, 0 <= shrinkage <= 1
coefficient in the convex combination used for the computation of the shrunk estimate.

Methods

error_norm (comp_cov[, norm, scaling, squared])

Computes the Mean Squared Error between two covariance estimators. (In the sense of the Frobenius norm)

Parameters comp_cov: array-like, shape = [n_features, n_features] :

The covariance to compare with.

norm: str :

The type of norm used to compute the error. Available error types: - ‘frobenius’ (default): \(\sqrt{\text{tr}(A^t.A)}\) - ‘spectral’: \(\sqrt{\text{max(eigenvalues}(A^t.A)}\) where A is the error

264 Chapter 1. User Guide
(comp_cov - self.covariance_).

scaling: bool :
If True (default), the squared error norm is divided by n_features. If False, the squared error norm is not rescaled.

squared: bool :
Whether to compute the squared error norm or the error norm. If True (default), the squared error norm is returned. If False, the error norm is returned.

Returns The Mean Squared Error (in the sense of the Frobenius norm) between :
'self' and 'comp_cov' covariance estimators. :

fit (X, assume_centered=False)
Fits the shrunk covariance model according to the given training data and parameters.

Parameters X : array-like, shape = [n_samples, n_features]
Training data, where n_samples is the number of samples and n_features is the number of features.

assume_centered: Boolean :
If True, data are not centered before computation. Useful to work with data whose mean is significantly equal to zero but is not exactly zero. If False, data are centered before computation.

Returns self : object
Returns self.

get_params (deep=True)
Get parameters for the estimator

Parameters deep: boolean, optional :
If True, will return the parameters for this estimator and contained subobjects that are estimators.

mahalanobis (observations)
Computes the mahalanobis distances of given observations.
The provided observations are assumed to be centered. One may want to center them using a location estimate first.

Parameters observations: array-like, shape = [n_observations, n_features] :
The observations, the Mahalanobis distances of the which we compute.

Returns mahalanobis_distance: array, shape = [n_observations,] :
Mahalanobis distances of the observations.

score (X_test, assume_centered=False)
Computes the log-likelihood of a gaussian data set with self.covariance_ as an estimator of its covariance matrix.

Parameters X_test : array-like, shape = [n_samples, n_features]
Test data of which we compute the likelihood, where n_samples is the number of samples and n_features is the number of features.

Returns res : float
The likelihood of the data set with `self.covariance_` as an estimator of its covariance matrix.

**set_params(**params**)
Set the parameters of the estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The former have parameters of the form `<component>__<parameter>` so that it’s possible to update each component of a nested object.

Returns `self`:

- `covariance.empirical_covariance(X[, ...])`: Computes the Maximum likelihood covariance estimator
- `covariance.ledoit_wolf(X[, assume_centered])`: Estimates the shrunk Ledoit-Wolf covariance matrix.
- `covariance.shrunk_covariance(emp_cov[, ...])`: Calculates a covariance matrix shrunk on the diagonal
- `covariance.oas(X[, assume_centered])`: Estimate covariance with the Oracle Approximating Shrinkage algorithm.
- `covariance.graph_lasso(emp_cov, alpha[, ...])`: L1-penalized covariance estimator

### sklearn.covariance.empirical_covariance

**sklearn.covariance.empirical_covariance** *(X, assume_centered=False)*
Computes the Maximum likelihood covariance estimator

**Parameters** X: 2D ndarray, shape (n_samples, n_features)
Data from which to compute the covariance estimate

**assume_centered**: Boolean
If True, data are not centered before computation. Useful when working with data whose mean is almost, but not exactly zero. If False, data are centered before computation.

**Returns** covariance: 2D ndarray, shape (n_features, n_features)
Empirical covariance (Maximum Likelihood Estimator)

### sklearn.covariance.ledoit_wolf

**sklearn.covariance.ledoit_wolf** *(X, assume_centered=False)*
Estimates the shrunk Ledoit-Wolf covariance matrix.

**Parameters** X: array-like, shape (n_samples, n_features)
Data from which to compute the covariance estimate

**assume_centered**: Boolean
If True, data are not centered before computation. Usefull to work with data whose mean is significantly equal to zero but is not exactly zero. If False, data are centered before computation.

**Returns** shrunk_cov: array-like, shape (n_features, n_features)
Shrunk covariance

**shrinkage**: float
Coefficient in the convex combination used for the computation of the shrunk estimate.
Notes

The regularised (shrunk) covariance is:

\[(1 - \text{shrinkage}) \cdot \text{cov} \]

\[\cdot \text{shrinkage} \cdot \mu \cdot \text{n}.\text{identity}(n_{\text{features}})\]

where \(\mu = \text{trace(cov)} / n_{\text{features}}\)

**sklearn.covariance.shrunk_covariance**

Calculates a covariance matrix shrunk on the diagonal.

Parameters

- **emp_cov**: array-like, shape (n_features, n_features)
  - Covariance matrix to be shrunk
- **shrinkage**: float, 0 <= shrinkage <= 1
  - Coefficient in the convex combination used for the computation of the shrunk estimate.

Returns

- **shrunk_cov**: array-like
  - Shrunk covariance

Notes

The regularized (shrunk) covariance is given by

\[(1 - \text{shrinkage}) \cdot \text{cov} \]

\[\cdot \text{shrinkage} \cdot \mu \cdot \text{n}.\text{identity}(n_{\text{features}})\]

where \(\mu = \text{trace(cov)} / n_{\text{features}}\)

**sklearn.covariance.oas**

Estimate covariance with the Oracle Approximating Shrinkage algorithm.

Parameters

- **X**: array-like, shape (n_samples, n_features)
  - Data from which to compute the covariance estimate
- **assume_centered**: boolean
  - If True, data are not centered before computation. Useful to work with data whose mean is significantly equal to zero but is not exactly zero. If False, data are centered before computation.

Returns

- **shrunk_cov**: array-like, shape (n_features, n_features)
  - Shrunken covariance
- **shrinkage**: float
  - Coefficient in the convex combination used for the computation of the shrunk estimate.
Notes

The regularised (shrunk) covariance is:

$$(1 - \text{shrinkage}) \cdot \text{cov}$$

$\cdot \text{shrinkage} \cdot \mu \cdot \text{np.identity(n_features)}$

where $\mu = \text{trace(cov)} / \text{n\_features}$

sklearn.covariance.graph_lasso

sklearn.covariance.graph_lasso(emp_cov, alpha, cov_init=None, mode='cd', tol=0.0001, max_iter=100, verbose=False, return_costs=False, eps=2.2204460492503131e-16)

l1-penalized covariance estimator

Parameters emp_cov: 2D ndarray, shape (n_features, n_features):

Empirical covariance from which to compute the covariance estimate

alpha: positive float:

The regularization parameter: the higher alpha, the more regularization, the sparser the inverse covariance

cov_init: 2D array (n_features, n_features), optional:

The initial guess for the covariance

mode: {'cd', 'lars'}:

The Lasso solver to use: coordinate descent or LARS. Use LARS for very sparse underlying graphs, where $p > n$. Elsewhere prefer cd which is more numerically stable.

tol: positive float, optional:

The tolerance to declare convergence: if the dual gap goes below this value, iterations are stopped

max_iter: integer, optional:

The maximum number of iterations

verbose: boolean, optional:

If verbose is True, the objective function and dual gap are printed at each iteration

return_costs: boolean, optional:

If return_costs is True, the objective function and dual gap at each iteration are returned

eps: float, optional:

The machine-precision regularization in the computation of the Cholesky diagonal factors. Increase this for very ill-conditioned systems.

Returns covariance: 2D ndarray, shape (n_features, n_features)

The estimated covariance matrix

precision: 2D ndarray, shape (n_features, n_features)

The estimated (sparse) precision matrix

costs: list of (objective, dual_gap) pairs
The list of values of the objective function and the dual gap at each iteration. Returned only if return_costs is True

See Also:
GraphLasso, GraphLassoCV

Notes

The algorithm employed to solve this problem is the GLasso algorithm, from the Friedman 2008 Biostatistics paper. It is the same algorithm as in the R glasso package.

One possible difference with the glasso R package is that the diagonal coefficients are not penalized.

1.8.3 sklearn.cross_validation: Cross Validation

The sklearn.cross_validation module includes utilities for cross-validation and performance evaluation.

User guide: See the Cross-Validation: evaluating estimator performance section for further details.

<table>
<thead>
<tr>
<th>Function</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>cross_validation.Bootstrap</td>
<td>Random sampling with replacement cross-validation iterator</td>
</tr>
<tr>
<td>cross_validation.KFold</td>
<td>K-Folds cross validation iterator</td>
</tr>
<tr>
<td>cross_validation.LeaveOneLabelOut</td>
<td>Leave-One-Label_Out cross-validation iterator</td>
</tr>
<tr>
<td>cross_validation.LeaveOneOut</td>
<td>Leave-One-Out cross validation iterator</td>
</tr>
<tr>
<td>cross_validation.LeavePLabelOut</td>
<td>Leave-P-Label_Out cross-validation iterator</td>
</tr>
<tr>
<td>cross_validation.LeavePOut</td>
<td>Leave-P-Out cross validation iterator</td>
</tr>
<tr>
<td>cross_validation.StratifiedKFold</td>
<td>Stratified K-Folds cross validation iterator</td>
</tr>
<tr>
<td>cross_validation.ShuffleSplit</td>
<td>Random permutation cross-validation iterator</td>
</tr>
<tr>
<td>cross_validation.StratifiedShuffleSplit</td>
<td>Stratified ShuffleSplit cross validation iterator</td>
</tr>
</tbody>
</table>

sklearn.cross_validation.Bootstrap

class sklearn.cross_validation.Bootstrap (n, n_bootstraps=3, train_size=0.5, test_size=None, n_train=None, n_test=None, random_state=None)

Random sampling with replacement cross-validation iterator

Provides train/test indices to split data in train test sets while resampling the input n_bootstraps times: each time a new random split of the data is performed and then samples are drawn (with replacement) on each side of the split to build the training and test sets.

Note: contrary to other cross-validation strategies, bootstrapping will allow some samples to occur several times in each splits. However a sample that occurs in the train split will never occur in the test split and vice-versa.

If you want each sample to occur at most once you should probably use ShuffleSplit cross validation instead.

Parameters

- **n**: int
  Total number of elements in the dataset.

- **n_bootstraps**: int (default is 3)
  Number of bootstrapping iterations

- **train_size**: int or float (default is 0.5)
  If int, number of samples to include in the training split (should be smaller than the total number of samples passed in the dataset).
If float, should be between 0.0 and 1.0 and represent the proportion of the dataset to include in the train split.

test_size : int or float or None (default is None)

If int, number of samples to include in the training set (should be smaller than the total number of samples passed in the dataset).

If float, should be between 0.0 and 1.0 and represent the proportion of the dataset to include in the test split.

If None, n_test is set as the complement of n_train.

random_state : int or RandomState

Pseudo number generator state used for random sampling.

See Also:

ShuffleSplit cross validation using random permutations.

Examples

>>> from sklearn import cross_validation
>>> bs = cross_validation.Bootstrap(9, random_state=0)
>>> len(bs)
3

>>> print(bs)
Bootstrap(9, n_bootstraps=3, train_size=5, test_size=4, random_state=0)

>>> for train_index, test_index in bs:
... print("TRAIN: %s TEST: %s" % (train_index, test_index))
... 
TRAIN: [1 8 7 7 8] TEST: [0 3 0 5]
TRAIN: [5 4 2 4 2] TEST: [6 7 1 0]
TRAIN: [4 7 0 1 1] TEST: [5 3 6 5]

__init__(n, n_bootstraps=3, train_size=0.5, test_size=None, n_train=None, n_test=None, random_state=None)

sklearn.cross_validation.KFold

class sklearn.cross_validation.KFold(n, k, indices=True, shuffle=False, random_state=None)

K-Folds cross validation iterator

Provides train/test indices to split data in train test sets. Split dataset into k consecutive folds (without shuffling).

Each fold is then used a validation set once while the k - 1 remaining fold form the training set.

Parameters n: int :

Total number of elements

k: int :

Number of folds

indices: boolean, optional (default True) :

Return train/test split as arrays of indices, rather than a boolean mask array. Integer indices are required when dealing with sparse matrices, since those cannot be indexed by boolean masks.
shuffle: boolean, optional:
whether to shuffle the data before splitting into batches

random_state: int or RandomState:
Pseudo number generator state used for random sampling.

See Also:

StratifiedKFold take label information into account to avoid building folds, classification

Notes
All the folds have size trunc(n_samples / n_folds), the last one has the complementary.

Examples

```python
>>> from sklearn import cross_validation
>>> X = np.array([[1, 2], [3, 4], [1, 2], [3, 4]])
>>> y = np.array([1, 2, 3, 4])
>>> kf = cross_validation.KFold(4, k=2)
>>> len(kf)
2
>>> for train_index, test_index in kf:
...     print("TRAIN: \%s TEST: \%s" % (train_index, test_index))
...     X_train, X_test = X[train_index], X[test_index]
...     y_train, y_test = y[train_index], y[test_index]
TRAIN: [2 3] TEST: [0 1]
TRAIN: [0 1] TEST: [2 3]
```

__init__ (n, k, indices=True, shuffle=False, random_state=None)

sklearn.cross_validation.LeaveOneLabelOut

class sklearn.cross_validation.LeaveOneLabelOut (labels, indices=True)

Leave-One-Label_Out cross-validation iterator

Provides train/test indices to split data according to a third-party provided label. This label information can be used to encode arbitrary domain specific stratifications of the samples as integers.

For instance the labels could be the year of collection of the samples and thus allow for cross-validation against time-based splits.

Parameters labels : array-like of int with shape (n_samples,)
Arbitrary domain-specific stratification of the data to be used to draw the splits.

indices: boolean, optional (default True):
Return train/test split as arrays of indices, rather than a boolean mask array. Integer indices are required when dealing with sparse matrices, since those cannot be indexed by boolean masks.
Examples

```python
>>> from sklearn import cross_validation
>>> X = np.array([[1, 2], [3, 4], [5, 6], [7, 8]])
>>> y = np.array([1, 2, 1, 2])
>>> labels = np.array([1, 2, 1, 2])
>>> lol = cross_validation.LeaveOneLabelOut(labels)
>>> len(lol)
2
>>> print(lol)
sklearn.cross_validation.LeaveOneLabelOut(labels=[1 1 2 2])
>>> for train_index, test_index in lol:
...     print("TRAIN: \[%s\] TEST: \[%s\]\) % (train_index, test_index))

TRAIN: [2 3] TEST: [0 1]
[[5 6]
 [7 8]] [1 2]
[3 4] [1 2] [1 2]
TRAIN: [0 1] TEST: [2 3]
[[1 2]
 [3 4]] [1 2]
[5 6] [7 8] [1 2] [1 2]

__init__(labels, indices=True)
```

**sklearn.cross_validation.LeaveOneOut**

class sklearn.cross_validation.LeaveOneOut(n, indices=True)

Leaves-One-Out cross validation iterator.

Provides train/test indices to split data in train test sets. Each sample is used once as a test set (singleton) while the remaining samples form the training set.

Due to the high number of test sets (which is the same as the number of samples) this cross validation method can be very costly. For large datasets one should favor KFold, StratifiedKFold or ShuffleSplit.

Parameters:

- `n`: int
  - Total number of elements

- `indices`: boolean, optional (default True)
  - Return train/test split as arrays of indices, rather than a boolean mask array. Integer indices are required when dealing with sparse matrices, since those cannot be indexed by boolean masks.

**See Also:**

LeaveOneLabelOut, domain-specific

Examples

```python
>>> from sklearn import cross_validation
>>> X = np.array([[1, 2], [3, 4]])
>>> y = np.array([1, 2])
>>> l0o = cross_validation.LeaveOneOut(2)
```
>>> len(loo)
2
>>> print(loo)
sklearn.cross_validation.LeaveOneOut(n=2)

>>> for train_index, test_index in loo:
...     print("TRAIN: %s TEST: %s" % (train_index, test_index))
...     X_train, X_test = X[train_index], X[test_index]
...     y_train, y_test = y[train_index], y[test_index]
...     print("$s \& $s \& $s\& $s" % (X_train, X_test, y_train, y_test))
TRAIN: [1] TEST: [0]
[[3 4]] [[1 2]] [2] [1]
TRAIN: [0] TEST: [1]
[[1 2]] [[3 4]] [1] [2]

__init__ (n, indices=True)

sklearn.cross_validation.LeavePLabelOut

class sklearn.cross_validation.LeavePLabelOut (labels, p, indices=True)

Leave-P-Label_Out cross-validation iterator

Provides train/test indices to split data according to a third-party provided label. This label information can be
used to encode arbitrary domain specific stratifications of the samples as integers.

For instance the labels could be the year of collection of the samples and thus allow for cross-validation against
time-based splits.

The difference between LeavePLabelOut and LeaveOneLabelOut is that the former builds the test sets with all
the samples assigned to p different values of the labels while the latter uses samples all assigned the same labels.

Parameters labels : array-like of int with shape (n_samples,)

Arbitrary domain-specific stratification of the data to be used to draw the splits.

p : int

Number of samples to leave out in the test split.

indices: boolean, optional (default True) :

Return train/test split as arrays of indices, rather than a boolean mask array. Integer
indices are required when dealing with sparse matrices, since those cannot be indexed
by boolean masks.

Examples

```python
>>> from sklearn import cross_validation
>>> X = np.array([[1, 2], [3, 4], [5, 6]])
>>> y = np.array([1, 2, 1])
>>> labels = np.array([1, 2, 3])
>>> lpl = cross_validation.LeavePLabelOut(labels, p=2)
>>> len(lpl)
3
>>> print(lpl)
sklearn.cross_validation.LeavePLabelOut (labels=[1 2 3], p=2)

>>> for train_index, test_index in lpl:
...     print("TRAIN: %s TEST: %s" % (train_index, test_index))
...     X_train, X_test = X[train_index], X[test_index]
```

1.8. Reference
... y_train, y_test = y[train_index], y[test_index]
... print("%s %s %s %s" % (X_train, X_test, y_train, y_test))

TRAIN: [2] TEST: [0 1]
[[5 6]] [[1 2]]
[3 4] [1] [1 2]
TRAIN: [1] TEST: [0 2]
[[3 4]] [[1 2]]
[5 6] [2] [1 1]
TRAIN: [0] TEST: [1 2]
[[1 2]] [[3 4]]
[5 6] [1] [2 1]

__init__(labels, p, indices=True)

sklearn.cross_validation.LeavePOut

class sklearn.cross_validation.LeavePOut(n, p, indices=True)
Leave-P-Out cross validation iterator

Provides train/test indices to split data in train test sets. The test set is built using p samples while the remaining samples form the training set.

Due to the high number of iterations which grows with the number of samples this cross validation method can be very costly. For large datasets one should favor KFold, StratifiedKFold or ShuffleSplit.

Parameters

n: int:
Total number of elements

p: int:
Size of the test sets

indices: boolean, optional (default True):
Return train/test split as arrays of indices, rather than a boolean mask array. Integer indices are required when dealing with sparse matrices, since those cannot be indexed by boolean masks.

Examples

>>> from sklearn import cross_validation
>>> X = np.array([[1, 2], [3, 4], [5, 6], [7, 8]])
>>> y = np.array([1, 2, 3, 4])
>>> lpo = cross_validation.LeavePOut(4, 2)
>>> len(lpo)
6
>>> print(list(lpo))
sklearn.cross_validation.LeavePOut(n=4, p=2)
>>> for train_index, test_index in lpo:
...    print("TRAIN: %s TEST: %s" % (train_index, test_index))
...    X_train, X_test = X[train_index], X[test_index]
...    y_train, y_test = y[train_index], y[test_index]
TRAIN: [2 3] TEST: [0 1]
TRAIN: [1 3] TEST: [0 2]
TRAIN: [1 2] TEST: [0 3]
TRAIN: [0 3] TEST: [1 2]
TRAIN: [0 2] TEST: [1 3]
TRAIN: [0 1] TEST: [2 3]

__init__(n, p, indices=True)

**sklearn.cross_validation.StratifiedKFold**

class sklearn.cross_validation.StratifiedKFold(y, k, indices=True)

Stratified K-Folds cross validation iterator

Provides train/test indices to split data in train test sets.

This cross-validation object is a variation of KFold, which returns stratified folds. The folds are made by preserving the percentage of samples for each class.

**Parameters y:** array, [n_samples]

Samples to split in K folds

**k:** int

Number of folds

**indices:** boolean, optional (default True)

Return train/test split as arrays of indices, rather than a boolean mask array. Integer indices are required when dealing with sparse matrices, since those cannot be indexed by boolean masks.

**Notes**

All the folds have size trunc(n_samples / n_folds), the last one has the complementary.

**Examples**

```python
>>> from sklearn import cross_validation
>>> X = np.array([[1, 2], [3, 4], [1, 2], [3, 4]])
>>> y = np.array([0, 0, 1, 1])
>>> skf = cross_validation.StratifiedKFold(y, k=2)
>>> len(skf)
2
>>> print(skf)
sklearn.cross_validation.StratifiedKFold(labels=[0 0 1 1], k=2)
>>> for train_index, test_index in skf:
...     print("TRAIN: %s TEST: %s" % (train_index, test_index))
...     X_train, X_test = X[train_index], X[test_index]
...     y_train, y_test = y[train_index], y[test_index]
TRAIN: [1 3] TEST: [0 2]
TRAIN: [0 2] TEST: [1 3]

__init__(y, k, indices=True)
```
Random permutation cross-validation iterator.

Yields indices to split data into training and test sets.

Note: contrary to other cross-validation strategies, random splits do not guarantee that all folds will be different, although this is still very likely for sizeable datasets.

Parameters

- **n**: int
  - Total number of elements in the dataset.
- **n_iterations**: int (default 10)
  - Number of re-shuffling & splitting iterations.
- **test_size**: float (default 0.1) or int
  - If float, should be between 0.0 and 1.0 and represent the proportion of the dataset to include in the test split. If int, represents the absolute number of test samples.
- **train_size**: float, int, or None (default is None)
  - If float, should be between 0.0 and 1.0 and represent the proportion of the dataset to include in the train split. If int, represents the absolute number of train samples. If None, the value is automatically set to the complement of the test fraction.
- **indices**: boolean, optional (default True)
  - Return train/test split as arrays of indices, rather than a boolean mask array. Integer indices are required when dealing with sparse matrices, since those cannot be indexed by boolean masks.
- **random_state**: int or RandomState
  - Pseudo-random number generator state used for random sampling.

See Also:

- **Bootstrap**: cross-validation using re-sampling with replacement.

Examples

```python
>>> from sklearn import cross_validation
>>> rs = cross_validation.ShuffleSplit(4, n_iterations=3, 
   ...   test_size=.25, random_state=0)
>>> len(rs)
3
>>> print(rs)
ShuffleSplit(4, n_iterations=3, test_size=0.25, random_state=0)
>>> for train_index, test_index in rs:
...   print("TRAIN: %s TEST: %s" % (train_index, test_index))
... TRAIN: [3 1 0] TEST: [2]
```
TRAIN: [2 1 3] TEST: [0]
TRAIN: [0 2 1] TEST: [3]

>>> rs = cross_validation.ShuffleSplit(4, n_iterations=3,
...    train_size=0.5, test_size=.25, random_state=0)
>>> for train_index, test_index in rs:
...   print("TRAIN: {} TEST: {}" % (train_index, test_index))
... 
TRAIN: [3 1] TEST: [2]
TRAIN: [2 1] TEST: [0]
TRAIN: [0 2] TEST: [3]

__init__(n, n_iterations=10, test_size=0.1, train_size=None, indices=True, random_state=None, test_fraction=None, train_fraction=None)

sklearn.cross_validation.StratifiedShuffleSplit

class sklearn.cross_validation.StratifiedShuffleSplit (y, n_iterations=10, test_size=0.1, train_size=None, indices=True, random_state=None)

Stratified ShuffleSplit cross validation iterator

Provides train/test indices to split data in train test sets.

This cross-validation object is a merge of StratifiedKFold and ShuffleSplit, which returns stratified randomized folds. The folds are made by preserving the percentage of samples for each class.

Note: like the ShuffleSplit strategy, stratified random splits do not guarantee that all folds will be different, although this is still very likely for sizeable datasets.

Parameters y: array, [n_samples] :

Labels of samples.

n_iterations : int (default 10)

Number of re-shuffling & splitting iterations.

test_size : float (default 0.1) or int

If float, should be between 0.0 and 1.0 and represent the proportion of the dataset to include in the test split. If int, represents the absolute number of test samples.

train_size : float, int, or None (default is None)

If float, should be between 0.0 and 1.0 and represent the proportion of the dataset to include in the train split. If int, represents the absolute number of train samples. If None, the value is automatically set to the complement of the test fraction.

indices: boolean, optional (default True) :

Return train/test split as arrays of indices, rather than a boolean mask array. Integer indices are required when dealing with sparse matrices, since those cannot be indexed by boolean masks.

Examples
```python
>>> from sklearn.cross_validation import StratifiedShuffleSplit
>>> X = np.array([[1, 2], [3, 4], [1, 2], [3, 4]])
>>> y = np.array([0, 0, 1, 1])
>>> sss = StratifiedShuffleSplit(y, 3, test_size=0.5, random_state=0)
>>> len(sss)
3
>>> print(sss)
StratifiedShuffleSplit(labels=[0 0 1 1], n_iterations=3, ...)
>>> for train_index, test_index in sss:
...     print("TRAIN: %s TEST: %s" % (train_index, test_index))
...     X_train, X_test = X[train_index], X[test_index]
...     y_train, y_test = y[train_index], y[test_index]
TRAIN: [0 3] TEST: [1 2]
TRAIN: [0 2] TEST: [1 3]
TRAIN: [1 2] TEST: [0 3]
```

__init__(y, n_iterations=10, test_size=0.1, train_size=None, indices=True, random_state=None)
```
>>> import numpy as np
>>> from sklearn.cross_validation import train_test_split
>>> a, b = np.arange(10).reshape((5, 2)), range(5)
>>> a
array([[0, 1],
[2, 3],
[4, 5],
[6, 7],
[8, 9]])
>>> list(b)
[0, 1, 2, 3, 4]

>>> a_train, a_test, b_train, b_test = train_test_split(
...     a, b, test_size=0.33, random_state=42)
...

>>> a_train
array([[4, 5],
[0, 1],
[6, 7]])
>>> b_train
array([2, 0, 3])

>>> a_test
array([[2, 3],
[8, 9]])
>>> b_test
array([1, 4])

sklearn.cross_validation.cross_val_score

sklearn.cross_validation.cross_val_score(estimator, X, y=None, score_func=None,
cv=None, n_jobs=1, verbose=0)

Evaluate a score by cross-validation

Parameters estimator: estimator object implementing `fit`

The object to use to fit the data

X: array-like of shape at least 2D

The data to fit.

y: array-like, optional

The target variable to try to predict in the case of supervised learning.

score_func: callable, optional

callable, has priority over the score function in the estimator. In a non-supervised setting, where y is None, it takes the test data (X_test) as its only argument. In a supervised setting it takes the test target (y_true) and the test prediction (y_pred) as arguments.

cv: cross-validation generator, optional

A cross-validation generator. If None, a 3-fold cross validation is used or 3-fold stratified cross-validation when y is supplied and estimator is a classifier.

n_jobs: integer, optional

The number of CPUs to use to do the computation. -1 means ‘all CPUs’.

verbose: integer, optional

1.8. Reference
The verbosity level

sklearn.cross_validation.permutation_test_score

Evaluate the significance of a cross-validated score with permutations

**Parameters**

*estimator* : estimator object implementing ‘fit’ :

The object to use to fit the data

*X* : array-like of shape at least 2D :

The data to fit.

*y* : array-like :

The target variable to try to predict in the case of supervised learning.

*score_func* : callable :

Callable taking as arguments the test targets (y_test) and the predicted targets (y_pred) and returns a float. The score functions are expected to return a bigger value for a better result otherwise the returned value does not correspond to a p-value (see Returns below for further details).

*cv* : integer or crossvalidation generator, optional

If an integer is passed, it is the number of fold (default 3). Specific crossvalidation objects can be passed, see sklearn.cross_validation module for the list of possible objects

*n_jobs* : integer, optional :

The number of CPUs to use to do the computation. -1 means ‘all CPUs’.

*labels* : array-like of shape [n_samples] (optional) :

Labels constrain the permutation among groups of samples with a same label.

*random_state* : RandomState or an int seed (0 by default) :

A random number generator instance to define the state of the random permutations generator.

*verbose* : integer, optional :

The verbosity level

**Returns**

*score* : float :

The true score without permuting targets.

*permutation_scores* : array, shape = [n_permutations]

The scores obtained for each permutations.

*pvalue* : float :

The returned value equals p-value if score_func returns bigger numbers for better scores (e.g., zero_one). If score_func is rather a loss function (i.e. when lower is better such as with mean_squared_error) then this is actually the complement of the p-value: 1 - p-value.
Notes

This function implements Test 1 in:


**sklearn.cross_validation.check_cv**

**sklearn.cross_validation.check_cv** *(cv=None, X=None, y=None, classifier=False)*

Input checker utility for building a CV in a user friendly way.

**Parameters**

- **cv**: an integer, a cv generator instance, or None
  
  The input specifying which cv generator to use. It can be an integer, in which case it is the number of folds in a KFold, None, in which case 3 fold is used, or another object, that will then be used as a cv generator.

- **X**: 2D ndarray
  
  the data the cross-val object will be applied on

- **y**: 1D ndarray
  
  the target variable for a supervised learning problem

- **classifier**: boolean optional
  
  whether the task is a classification task, in which case stratified KFold will be used.

### 1.8.4 sklearn.datasets: Datasets

The **sklearn.datasets** module includes utilities to load datasets, including methods to load and fetch popular reference datasets. It also features some artificial data generators.

**User guide**: See the [Dataset loading utilities](#) section for further details.

**Loaders**

<table>
<thead>
<tr>
<th>Dataset Function</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>datasets.load_20newsgroups(*args, **kwargs)</td>
<td>DEPRECATED: Use fetch_20newsgroups instead with download_if_missing=True</td>
</tr>
<tr>
<td>datasets.fetch_20newsgroups([data_home, ...])</td>
<td>Load the filenames of the 20 newsgroups dataset.</td>
</tr>
<tr>
<td>datasets.fetch_20newsgroups_vectorized([...])</td>
<td>Load the 20 newsgroups dataset and transform it into tf-idf vectors.</td>
</tr>
<tr>
<td>datasets.load_boston()</td>
<td>Load and return the boston house-prices dataset (regression).</td>
</tr>
<tr>
<td>datasets.load_diabetes()</td>
<td>Load and return the diabetes dataset (regression).</td>
</tr>
<tr>
<td>datasets.load_digits([n_class])</td>
<td>Load and return the digits dataset (classification).</td>
</tr>
<tr>
<td>datasets.load_files(container_path[, ...])</td>
<td>Load text files with categories as subfolder names.</td>
</tr>
<tr>
<td>datasets.load_iris()</td>
<td>Load and return the iris dataset (classification).</td>
</tr>
<tr>
<td>datasets.load_lfw_pairs([download_if_missing])</td>
<td>Alias for fetch_lfw_pairs(download_if_missing=False)</td>
</tr>
<tr>
<td>datasets.fetch_lfw_pairs([subset, ...])</td>
<td>Loader for the Labeled Faces in the Wild (LFW) pairs dataset</td>
</tr>
<tr>
<td>datasets.load_lfw_people([download_if_missing])</td>
<td>Alias for fetch_lfw_people(download_if_missing=False)</td>
</tr>
<tr>
<td>datasets.fetch_lfw_people([data_home, ...])</td>
<td>Loader for the Labeled Faces in the Wild (LFW) people dataset</td>
</tr>
<tr>
<td>datasets.load_linnerud()</td>
<td>Load and return the linnerud dataset (multivariate regression).</td>
</tr>
<tr>
<td>datasets.fetch_olivetti_faces([data_home, ...])</td>
<td>Loader for the Olivetti faces data-set from AT&amp;T.</td>
</tr>
<tr>
<td>datasets.load_sample_image(image_name)</td>
<td>Load the numpy array of a single sample image</td>
</tr>
</tbody>
</table>
### sklearn.datasets.load_20newsgroups

**sklearn.datasets.load_20newsgroups** (*args, **kwargs*)

DEPRECATED: Use fetch_20newsgroups instead with download_if_missing=False

Alias for fetch_20newsgroups(download_if_missing=False).

See fetch_20newsgroups.__doc__ for documentation and parameter list.

### sklearn.datasets.fetch_20newsgroups

**sklearn.datasets.fetch_20newsgroups** (data_home=None, subset='train', categories=None, shuffle=True, random_state=42, download_if_missing=True)

Load the filenames of the 20 newsgroups dataset.

**Parameters**

- **subset**: ‘train’ or ‘test’, ‘all’, optional:
  - Select the dataset to load: ‘train’ for the training set, ‘test’ for the test set, ‘all’ for both, with shuffled ordering.

- **data_home**: optional, default: None:
  - Specify an download and cache folder for the datasets. If None, all scikit-learn data is stored in ‘~/scikit_learn_data’ subfolders.

- **categories**: None or collection of string or unicode:
  - If None (default), load all the categories. If not None, list of category names to load (other categories ignored).

- **shuffle**: bool, optional:
  - Whether or not to shuffle the data: might be important for models that make the assumption that the samples are independent and identically distributed (i.i.d.), such as stochastic gradient descent.

- **random_state**: numpy random number generator or seed integer:
  - Used to shuffle the dataset.

- **download_if_missing**: optional, True by default:
  - If False, raise an IOError if the data is not locally available instead of trying to download the data from the source site.

### sklearn.datasets.fetch_20newsgroups_vectorized

**sklearn.datasets.fetch_20newsgroups_vectorized** (subset='train', data_home=None)

Load the 20 newsgroups dataset and transform it into tf-idf vectors.

This is a convenience function; the tf-idf transformation is done using the default settings for sklearn.feature_extraction.textVectorizer. For more advanced usage (stopword filtering, n-gram extraction, etc.), combine fetch_20newsgroups with a custom Vectorizer or CountVectorizer.
Parameters subset: ‘train’ or ‘test’, ‘all’, optional:
  Select the dataset to load: ‘train’ for the training set, ‘test’ for the test set, ‘all’ for both,
  with shuffled ordering.

data_home: optional, default: None:
  Specify an download and cache folder for the datasets. If None, all scikit-learn data is
  stored in ‘~/scikit_learn_data’ subfolders.

Returns bunch: Bunch object
  bunch.data: sparse matrix, shape [n_samples, n_features] bunch.target: array, shape
  [n_samples] bunch.target_names: list, length [n_classes]

```
from sklearn.datasets import load_boston
>>> boston = load_boston()
>>> boston.data.shape
(506, 13)
```

```
from sklearn.datasets import load_diabetes
>>> diabetes = load_diabetes()
>>> diabetes.data.shape
(442, 10)
```

1.8. Reference
sklearn.datasets.load_digits

sklearn.datasets.load_digits(n_class=10)

Load and return the digits dataset (classification).

Each datapoint is a 8x8 image of a digit.

<table>
<thead>
<tr>
<th>Classes</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Samples per class</td>
<td>~180</td>
</tr>
<tr>
<td>Samples total</td>
<td>1797</td>
</tr>
<tr>
<td>Dimensionality</td>
<td>64</td>
</tr>
<tr>
<td>Features</td>
<td>integers 0-16</td>
</tr>
</tbody>
</table>

**Parameters** n_class : integer, between 0 and 10, optional (default=10)

The number of classes to return.

**Returns**

Dictionary-like object, the interesting attributes are: ‘data’, the data to learn, ‘images’, the images corresponding to each sample, ‘target’, the classification labels for each sample, ‘target_names’, the meaning of the labels, and ‘DESCR’, the full description of the dataset.

**Examples**

To load the data and visualize the images:

```python
>>> from sklearn.datasets import load_digits
>>> digits = load_digits()
>>> digits.data.shape
(1797, 64)
>>> import matplotlib.pyplot as plt
>>> plt.gray()
>>> plt.matshow(digits.images[0])
>>> plt.show()
```

sklearn.datasets.load_files

sklearn.datasets.load_files(container_path, description=None, categories=None, load_content=True, shuffle=True, charset=None, charse_error='strict', random_state=0)

Load text files with categories as subfolder names.

Individual samples are assumed to be files stored in a two levels folder structure such as the following:

- **container_folder/**
  - **category_1_folder/**file_1.txt file_2.txt ... file_42.txt
  - **category_2_folder/**file_43.txt file_44.txt ... 

The folder names are used as supervised signal label names. The individual file names are not important.

This function does not try to extract features into a numpy array or scipy sparse matrix. In addition, if load_content is false it does not try to load the files in memory.

To use utf-8 text files in a scikit-learn classification or clustering algorithm you will first need to use the sklearn.feature_extraction module to build a feature extraction transformer that suits your problem.
Similar feature extractors should be build for other kind of unstructured data input such as images, audio, video, ...

**Parameters**

**container_path** : string or unicode

Path to the main folder holding one subfolder per category

**description** : string or unicode, optional (default=None)

A paragraph describing the characteristic of the dataset: its source, reference, etc.

**categories** : A collection of strings or None, optional (default=None)

If None (default), load all the categories. If not None, list of category names to load (other categories ignored).

**load_content** : boolean, optional (default=True)

Whether to load or not the content of the different files. If true a ‘data’ attribute containing the text information is present in the data structure returned. If not, a filenames attribute gives the path to the files.

**charset** : string or None (default is None)

If None, do not try to decode the content of the files (e.g. for images or other non-text content). If not None, charset to use to decode text files if load_content is True.

**charset_error** : {‘strict’, ‘ignore’, ‘replace’}

Instruction on what to do if a byte sequence is given to analyze that contains characters not of the given charset. By default, it is ‘strict’, meaning that a UnicodeDecodeError will be raised. Other values are ‘ignore’ and ‘replace’.

**shuffle** : bool, optional (default=True)

Whether or not to shuffle the data: might be important for models that make the assumption that the samples are independent and identically distributed (i.i.d.), such as stochastic gradient descent.

**random_state** : int, RandomState instance or None, optional (default=0)

If int, random_state is the seed used by the random number generator; If RandomState instance, random_state is the random number generator; If None, the random number generator is the RandomState instance used by np.random.

**Returns**

**data** : Bunch

Dictionary-like object, the interesting attributes are: either data, the raw text data to learn, or ‘filenames’, the files holding it, ‘target’, the classification labels (integer index), ‘target_names’, the meaning of the labels, and ‘DESCR’, the full description of the dataset.

```python
sklearn.datasets.load_iris

sklearn.datasets.load_iris()
```

Load and return the iris dataset (classification).

The iris dataset is a classic and very easy multi-class classification dataset.
Classes  |  3
---|---
Samples per class | 50
Samples total | 150
Dimensionality | 4
Features | real, positive

**Returns data**: Bunch

Dictionary-like object, the interesting attributes are: ‘data’, the data to learn, ‘target’, the classification labels, ‘target_names’, the meaning of the labels, ‘feature_names’, the meaning of the features, and ‘DESCR’, the full description of the dataset.

**Examples**

Let’s say you are interested in the samples 10, 25, and 50, and want to know their class name.

```python
>>> from sklearn.datasets import load_iris
>>> data = load_iris()
>>> data.target[[10, 25, 50]]
array([0, 0, 1])
```

```python
>>> list(data.target_names)
['setosa', 'versicolor', 'virginica']
```

**sklearn.datasets.load_lfw_pairs**

**sklearn.datasets.load_lfw_pairs** *(download_if_missing=False, **kwargs)*

Alias for `fetch_lfw_pairs(download_if_missing=False)`

Check `fetch_lfw_pairs.__doc__` for the documentation and parameter list.

**sklearn.datasets.fetch_lfw_pairs**

**sklearn.datasets.fetch_lfw_pairs** *(subset='train', data_home=None, funneled=True, resize=0.5, color=False, slice_=slice(70, 195, None), slice_=slice(78, 172, None), download_if_missing=True)*

Loader for the Labeled Faces in the Wild (LFW) pairs dataset

This dataset is a collection of JPEG pictures of famous people collected on the internet, all details are available on the official website:

http://vis-www.cs.umass.edu/lfw/

Each picture is centered on a single face. Each pixel of each channel (color in RGB) is encoded by a float in range 0.0 - 1.0.

The task is called Face Verification: given a pair of two pictures, a binary classifier must predict whether the two images are from the same person.

In the official README.txt this task is described as the “Restricted” task. As I am not sure as to implement the “Unrestricted” variant correctly, I left it as unsupported for now.

**Parameters**

- **subset**: optional, default: ‘train’ :
Select the dataset to load: ‘train’ for the development training set, ‘test’ for the development test set, and ‘10_folds’ for the official evaluation set that is meant to be used with a 10-folds cross validation.

**data_home: optional, default: None** :

Specify another download and cache folder for the datasets. By default all scikit learn data is stored in ‘~/scikit_learn_data’ subfolders.

**funneled: boolean, optional, default: True** :

Download and use the funneled variant of the dataset.

**resize: float, optional, default 0.5** :

Ratio used to resize the each face picture.

**color: boolean, optional, default False** :

Keep the 3 RGB channels instead of averaging them to a single gray level channel. If color is True the shape of the data has one more dimension than the shape with color = False.

**slice_: optional** :

Provide a custom 2D slice (height, width) to extract the ‘interesting’ part of the jpeg files and avoid use statistical correlation from the background.

**download_if_missing: optional, True by default** :

If False, raise a IOError if the data is not locally available instead of trying to download the data from the source site.

```python
sklearn.datasets.load_lfw_people
```

```python
sklearn.datasets.load_lfw_people(download_if_missing=False, **kwargs)
```

 Alias for fetch_lfw_people(download_if_missing=False)

Check fetch_lfw_people.__doc__ for the documentation and parameter list.

```python
sklearn.datasets.fetch_lfw_people
```

```python
sklearn.datasets.fetch_lfw_people(data_home=None, funneled=True, resize=0.5, min_faces_per_person=None, color=False, slice_=(slice(70, 195, None), slice(78, 172, None)), download_if_missing=True)
```

Loader for the Labeled Faces in the Wild (LFW) people dataset

This dataset is a collection of JPEG pictures of famous people collected on the internet, all details are available on the official website:


Each picture is centered on a single face. Each pixel of each channel (color in RGB) is encoded by a float in range 0.0 - 1.0.

The task is called Face Recognition (or Identification): given the picture of a face, find the name of the person given a training set (gallery).

**Parameters data_home: optional, default: None** :
Specify another download and cache folder for the datasets. By default all scikit learn data is stored in ‘~/scikit_learn_data’ subfolders.

**funneled**: boolean, optional, default: True

Download and use the funneled variant of the dataset.

**resize**: float, optional, default 0.5

Ratio used to resize the each face picture.

**min_faces_per_person**: int, optional, default None

The extracted dataset will only retain pictures of people that have at least \( \text{min\_faces\_per\_person} \) different pictures.

**color**: boolean, optional, default False

Keep the 3 RGB channels instead of averaging them to a single gray level channel. If color is True the shape of the data has one more dimension than than the shape with color = False.

**slice_**: optional

Provide a custom 2D slice (height, width) to extract the ‘interesting’ part of the jpeg files and avoid use statistical correlation from the background.

**download_if_missing**: optional, True by default

If False, raise a IOError if the data is not locally available instead of trying to download the data from the source site.

**sklearn.datasets.load_linnerud**

Load and return the linnerud dataset (multivariate regression).

Samples total: 20 Dimensionality: 3 for both data and targets Features: integer Targets: integer

**Returns**

Dictionary-like object, the interesting attributes are: ‘data’ and ‘targets’, the two multivariate datasets, with ‘data’ corresponding to the exercise and ‘targets’ corresponding to the physiological measurements, as well as ‘feature_names’ and ‘target_names’.

**sklearn.datasets.fetch_olivetti_faces**

Loader for the Olivetti faces data-set from AT&T.

**Parameters**

Specify another download and cache folder for the datasets. By default all scikit learn data is stored in ‘~/scikit_learn_data’ subfolders.

**shuffle**: boolean, optional

If True the order of the dataset is shuffled to avoid having images of the same person grouped.

**download_if_missing**: optional, True by default:
If False, raise a IOError if the data is not locally available instead of trying to download the data from the source site.

**random_state** : optional, integer or RandomState object

The seed or the random number generator used to shuffle the data.

**Notes**

This dataset consists of 10 pictures each of 40 individuals. The original database was available from (now defunct)

http://www.uk.research.att.com/facedatabase.html

The version retrieved here comes in MATLAB format from the personal web page of Sam Roweis:

http://www.cs.nyu.edu/~roweis/

### sklearn.datasets.load_sample_image

**sklearn.datasets.load_sample_image(image_name)**

Load the numpy array of a single sample image

**Parameters image_name**: ‘china.jpg’, ‘flower.jpg’:

The name of the sample image loaded

**Returns img**: 3D array:

The image as a numpy array: height x width x color

### Examples

```python
>>> from sklearn.datasets import load_sample_image
>>> china = load_sample_image('china.jpg')
>>> china.dtype
dtype('uint8')
>>> china.shape
(427, 640, 3)
>>> flower = load_sample_image('flower.jpg')
>>> flower.dtype
dtype('uint8')
>>> flower.shape
(427, 640, 3)
```

### sklearn.datasets.load_sample_images

**sklearn.datasets.load_sample_images()**

Load sample images for image manipulation. Loads both, china and flower.

**Returns data** : Bunch

Dictionary-like object with the following attributes: ‘images’, the two sample images, ‘filenames’, the file names for the images, and ‘DESCR’ the full description of the dataset.
Examples

To load the data and visualize the images:

```python
>>> from sklearn.datasets import load_sample_images
>>> dataset = load_sample_images()
>>> len(dataset.images)
2
>>> first_img_data = dataset.images[0]
>>> first_img_data.shape
(427, 640, 3)
>>> first_img_data.dtype
dtype('uint8')
```

**sklearn.datasets.load_svmlight_file**

`sklearn.datasets.load_svmlight_file(f, n_features=None, dtype=<type 'numpy.float64'>, multilabel=False, zero_based='auto')`

Load datasets in the svmlight / libsvm format into sparse CSR matrix

This format is a text-based format, with one sample per line. It does not store zero valued features hence is suitable for sparse dataset.

The first element of each line can be used to store a target variable to predict.

This format is used as the default format for both svmlight and the libsvm command line programs.

Parsing a text based source can be expensive. When working on repeatedly on the same dataset, it is recommended to wrap this loader with joblib.Memory.cache to store a memmapped backup of the CSR results of the first call and benefit from the near instantaneous loading of memmapped structures for the subsequent calls.

This implementation is naive: it does allocate too much memory and is slow since written in python. On large datasets it is recommended to use an optimized loader such as:

https://github.com/mblondel/svmlight-loader

**Parameters**

- **f**: str or file-like open in binary mode.
  - (Path to) a file to load.

- **n_features**: int or None
  - The number of features to use. If None, it will be inferred. This argument is useful to load several files that are subsets of a bigger sliced dataset: each subset might not have example of every feature, hence the inferred shape might vary from one slice to another.

- **multilabel**: boolean, optional
  - Samples may have several labels each (see http://www.csie.ntu.edu.tw/~cjlin/libsvmtools/datasets/multilabel.html)

- **zero_based**: boolean or “auto”, optional
  - Whether column indices in f are zero-based (True) or one-based (False). If set to “auto”, a heuristic check is applied to determine this from the file contents. Both kinds of files occur “in the wild”, but they are unfortunately not self-identifying. Using “auto” or True should always be safe.

**Returns**

- **(X, y)**:
  - where X is a scipy.sparse matrix of shape (n_samples, n_features),
y is a ndarray of shape (n_samples,), or, in the multilabel case, a list of tuples of length n_samples.

See Also:

load_svmlight_files similar function for loading multiple files in this format, enforcing

Samples generator

<table>
<thead>
<tr>
<th>Function</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>datasets.make_blobs</td>
<td>Generate isotropic Gaussian blobs for clustering.</td>
</tr>
<tr>
<td>datasets.make_classification</td>
<td>Generate a random n-class classification problem.</td>
</tr>
<tr>
<td>datasets.make_circles</td>
<td>Make a large circle containing a smaller circle in 2d</td>
</tr>
<tr>
<td>datasets.make_friedman1</td>
<td>Generate the “Friedman #1” regression problem</td>
</tr>
<tr>
<td>datasets.make_friedman2</td>
<td>Generate the “Friedman #2” regression problem</td>
</tr>
<tr>
<td>datasets.make_friedman3</td>
<td>Generate the “Friedman #3” regression problem</td>
</tr>
<tr>
<td>datasets.make_hastie_10_2</td>
<td>Generates data for binary classification used in</td>
</tr>
<tr>
<td>datasets.make_lows_rank_matrix</td>
<td>Generate a mostly low rank matrix with bell-shaped singular values</td>
</tr>
<tr>
<td>datasets.make_moons</td>
<td>Make two interleaving half circles</td>
</tr>
<tr>
<td>datasets.make_multilabel_classification</td>
<td>Generate a random multilabel classification problem.</td>
</tr>
<tr>
<td>datasets.make_regression</td>
<td>Generate a random regression problem.</td>
</tr>
<tr>
<td>datasets.make_s_curve</td>
<td>Generate an S curve dataset.</td>
</tr>
<tr>
<td>datasets.make_sparse_coded_signal</td>
<td>Generate a signal as a sparse combination of dictionary elements.</td>
</tr>
<tr>
<td>datasets.make_sparse_spd_matrix</td>
<td>Generate a sparse symmetric definite positive matrix.</td>
</tr>
<tr>
<td>datasets.make_uncorrelated</td>
<td>Generate a random regression problem with sparse uncorrelated design.</td>
</tr>
<tr>
<td>datasets.make_spd_matrix</td>
<td>Generate a random symmetric, positive-definite matrix.</td>
</tr>
<tr>
<td>datasets.make_s_curve</td>
<td>Generate a swiss roll dataset.</td>
</tr>
</tbody>
</table>

sklearn.datasets.make_blobs

sklearn.datasets.make_blobs (n_samples=100, n_features=2, centers=3, cluster_std=1.0, center_box=(-10.0, 10.0), shuffle=True, random_state=None)

Generate isotropic Gaussian blobs for clustering.

**Parameters**

- **n_samples**: int, optional (default=100)
  
The total number of points equally divided among clusters.

- **n_features**: int, optional (default=2)
  
The number of features for each sample.

- **centers**: int or array of shape [n_centers, n_features], optional
  
  (default=3) The number of centers to generate, or the fixed center locations.

- **cluster_std**: float or sequence of floats, optional (default=1.0)
  
  The standard deviation of the clusters.

- **center_box**: pair of floats (min, max), optional (default=(-10.0, 10.0))
  
  The bounding box for each cluster center when centers are generated at random.

- **shuffle**: boolean, optional (default=True)
Shuffle the samples.

**random_state** : int, RandomState instance or None, optional (default=None)

If int, random_state is the seed used by the random number generator; If RandomState instance, random_state is the random number generator; If None, the random number generator is the RandomState instance used by np.random.

**Returns**

X : array of shape [n_samples, n_features]

The generated samples.

y : array of shape [n_samples]

The integer labels for cluster membership of each sample.

**Examples**

```python
>>> from sklearn.datasets.samples_generator import make_blobs
>>> X, y = make_blobs(n_samples=10, centers=3, n_features=2,
...                    random_state=0)
>>> X.shape
(10, 2)
>>> y
array([0, 0, 1, 0, 2, 2, 2, 1, 1, 0])
```

**sklearn.datasets.make_classification**

sklearn.datasets.make_classification(n_samples=100, n_features=20, n_informative=2, n_redundant=2, n_repeated=0, n_clusters_per_class=2, weights=None, flip_y=0.01, class_sep=1.0, hypercube=True, shift=0.0, scale=1.0, shuffle=True, random_state=None)

Generate a random n-class classification problem.

**Parameters**

- **n_samples** : int, optional (default=100)
  The number of samples.

- **n_features** : int, optional (default=20)
  The total number of features. These comprise `n_informative` informative features, `n_redundant` redundant features, `n_repeated` duplicated features and `n_features-n_informative-n_redundant-n_repeated` useless features drawn at random.

- **n_informative** : int, optional (default=2)
  The number of informative features. Each class is composed of a number of gaussian clusters each located around the vertices of a hypercube in a subspace of dimension `n_informative`. For each cluster, informative features are drawn independently from N(0, 1) and then randomly linearly combined in order to add covariance. The clusters are then placed on the vertices of the hypercube.

- **n_redundant** : int, optional (default=2)
  The number of redundant features. These features are generated as random linear combinations of the informative features.

- **n_repeated** : int, optional (default=2)
  The number of repeated features. These features are generated as random linear combinations of the informative features.
The number of duplicated features, drawn randomly from the informative and the redundant features.

**n_classes**: int, optional (default=2)

The number of classes (or labels) of the classification problem.

**n_clusters_per_class**: int, optional (default=2)

The number of clusters per class.

**weights**: list of floats or None (default=None)

The proportions of samples assigned to each class. If None, then classes are balanced. Note that if \( \text{len(weights)} = n\_classes - 1 \), then the last class weight is automatically inferred.

**flip_y**: float, optional (default=0.01)

The fraction of samples whose class are randomly exchanged.

**class_sep**: float, optional (default=1.0)

The factor multiplying the hypercube dimension.

**hypercube**: boolean, optional (default=True)

If True, the clusters are put on the vertices of a hypercube. If False, the clusters are put on the vertices of a random polytope.

**shift**: float or None, optional (default=0.0)

Shift all features by the specified value. If None, then features are shifted by a random value drawn in \([-\text{class\_sep}, \text{class\_sep}]\).

**scale**: float or None, optional (default=1.0)

Multiply all features by the specified value. If None, then features are scaled by a random value drawn in \([1, 100]\). Note that scaling happens after shifting.

**shuffle**: boolean, optional (default=True)

Shuffle the samples and the features.

**random_state**: int, RandomState instance or None, optional (default=None)

If int, random_state is the seed used by the random number generator; If RandomState instance, random_state is the random number generator; If None, the random number generator is the RandomState instance used by \( \text{np.random} \).

**Returns**

**X**: array of shape \([\text{n\_samples}, \text{n\_features}]\)

The generated samples.

**y**: array of shape \([\text{n\_samples}]\)

The integer labels for class membership of each sample.

**Notes**

The algorithm is adapted from Guyon [1] and was designed to generate the “Madelon” dataset.
References

[R48]

sklearn.datasets.make_circles

sklearn.datasets.make_circles(n_samples=100, shuffle=True, noise=None, random_state=None, factor=0.8)

Make a large circle containing a smaller circle in 2d

A simple toy dataset to visualize clustering and classification algorithms.

Parameters

**n_samples** : int, optional (default=100)

The total number of points generated.

**shuffle** : bool, optional (default=True)

Whether to shuffle the samples.

**noise** : double or None (default=None)

Standard deviation of Gaussian noise added to the data.

**factor** : double < 1 (default=.8)

Scale factor between inner and outer circle.

sklearn.datasets.make_friedman1

sklearn.datasets.make_friedman1(n_samples=100, n_features=10, noise=0.0, random_state=None)

Generate the “Friedman #1” regression problem

This dataset is described in Friedman [1] and Breiman [2].

Inputs X are independent features uniformly distributed on the interval [0, 1]. The output y is created according to the formula:

\[ y(X) = 10 \cdot \sin(\pi \cdot X[:, 0] \cdot X[:, 1]) + 20 \cdot (X[:, 2] - 0.5)^2 + 10 \cdot X[:, 3] + 5 \cdot X[:, 4] + \text{noise} \cdot N(0, 1). \]

Out of the n_features features, only 5 are actually used to compute y. The remaining features are independent of y.

The number of features has to be >= 5.

Parameters

**n_samples** : int, optional (default=100)

The number of samples.

**n_features** : int, optional (default=10)

The number of features. Should be at least 5.

**noise** : float, optional (default=0.0)

The standard deviation of the gaussian noise applied to the output.

**random_state** : int, RandomState instance or None, optional (default=None)

If int, random_state is the seed used by the random number generator; If RandomState instance, random_state is the random number generator; If None, the random number generator is the RandomState instance used by np.random.
**Returns** $X$ : array of shape [n_samples, n_features]

The input samples.

$y$ : array of shape [n_samples]

The output values.

**References**

[R49], [R50]

**sklearn.datasets.make_friedman2**

`sklearn.datasets.make_friedman2(n_samples=100, noise=0.0, random_state=None)`

Generate the “Friedman #2” regression problem

This dataset is described in Friedman [1] and Breiman [2].

Inputs $X$ are 4 independent features uniformly distributed on the intervals:

\[
\begin{align*}
0 & \leq X[:, 0] \leq 100, \\
40 \times \pi & \leq X[:, 1] \leq 560 \times \pi, \\
0 & \leq X[:, 2] \leq 1, \\
1 & \leq X[:, 3] \leq 11.
\end{align*}
\]

The output $y$ is created according to the formula:

\[
y(X) = (X[:, 0] ** 2 + (X[:, 1] * X[:, 2] - 1 / (X[:, 1] * X[:, 3]))) ** 2) ** 0.5 + \text{noise} \cdot N(0, 1).
\]

**Parameters**

- **n_samples** : int, optional (default=100)
  The number of samples.

- **noise** : float, optional (default=0.0)
  The standard deviation of the gaussian noise applied to the output.

- **random_state** : int, RandomState instance or None, optional (default=None)
  If int, random_state is the seed used by the random number generator; If RandomState instance, random_state is the random number generator; If None, the random number generator is the RandomState instance used by `np.random`.

**Returns** $X$ : array of shape [n_samples, 4]

The input samples.

$y$ : array of shape [n_samples]

The output values.

**References**

[R51], [R52]
sklearn.datasets.make_friedman3

**sklearn.datasets.make_friedman3**(*n_samples=100, noise=0.0, random_state=None*)

Generate the “Friedman #3” regression problem

This dataset is described in Friedman [1] and Breiman [2].

Inputs X are 4 independent features uniformly distributed on the intervals:

\[
0 \leq X[:, 0] \leq 100, \\
40 \pi \leq X[:, 1] \leq 560 \pi, \\
0 \leq X[:, 2] \leq 1, \\
1 \leq X[:, 3] \leq 11.
\]

The output *y* is created according to the formula:

\[
y(X) = \arctan((X[:, 1] \times X[:, 2] - 1 / (X[:, 1] \times X[:, 3])) / X[:, 0]) + \text{noise} \times N(0, 1).
\]

**Parameters**

- **n_samples** : int, optional (default=100)
  The number of samples.
- **noise** : float, optional (default=0.0)
  The standard deviation of the gaussian noise applied to the output.
- **random_state** : int, RandomState instance or None, optional (default=None)
  If int, random_state is the seed used by the random number generator; If RandomState instance, random_state is the random number generator; If None, the random number generator is the RandomState instance used by np.random.

**Returns**

- **X** : array of shape [n_samples, 4]
  The input samples.
- **y** : array of shape [n_samples]
  The output values.

**References**

[R53], [R54]

sklearn.datasets.make_hastie_10_2

**sklearn.datasets.make_hastie_10_2**(*n_samples=12000, random_state=None*)

Generates data for binary classification used in Hastie et al. 2009, Example 10.2.

The ten features are standard independent Gaussian and the target *y* is defined by:

\[
y[i] = 1 \text{ if } np.sum(X[i]) > 9.34 \text{ else } -1
\]

**Parameters**

- **n_samples** : int, optional (default=12000)
  The number of samples.
- **random_state** : int, RandomState instance or None, optional (default=None)
If int, random_state is the seed used by the random number generator; If RandomState instance, random_state is the random number generator; If None, the random number generator is the RandomState instance used by np.random.

Returns X : array of shape [n_samples, 10]

The input samples.

y : array of shape [n_samples]

The output values.

**References:**


sklearn.datasets.make_low_rank_matrix

sklearn.datasets.make_low_rank_matrix(n_samples=100, n_features=100, effective_rank=10, tail_strength=0.5, random_state=None)

Generate a mostly low rank matrix with bell-shaped singular values

Most of the variance can be explained by a bell-shaped curve of width effective_rank: the low rank part of the singular values profile is:

\[(1 - \text{tail_strength}) \times \exp(-1.0 \times (i / \text{effective_rank})^2)\]

The remaining singular values’ tail is fat, decreasing as:

\[\text{tail_strength} \times \exp(-0.1 \times i / \text{effective_rank}).\]

The low rank part of the profile can be considered the structured signal part of the data while the tail can be considered the noisy part of the data that cannot be summarized by a low number of linear components (singular vectors).

This kind of singular profiles is often seen in practice, for instance:

- gray level pictures of faces
- TF-IDF vectors of text documents crawled from the web

Parameters n_samples : int, optional (default=100)

The number of samples.

n_features : int, optional (default=100)

The number of features.

effective_rank : int, optional (default=10)

The approximate number of singular vectors required to explain most of the data by linear combinations.

tail_strength : float between 0.0 and 1.0, optional (default=0.5)

The relative importance of the fat noisy tail of the singular values profile.

random_state : int, RandomState instance or None, optional (default=None)
If int, random_state is the seed used by the random number generator; If RandomState instance, random_state is the random number generator; If None, the random number generator is the RandomState instance used by np.random.

Returns X : array of shape [n_samples, n_features]
The matrix.

sklearn.datasets.make_moons

sklearn.datasets.make_moons (n_samples=100, shuffle=True, noise=None, random_state=None)
Make two interleaving half circles
A simple toy dataset to visualize clustering and classification algorithms.

Parameters n_samples : int, optional (default=100)
The total number of points generated.
shuffle : bool, optional (default=True)
Whether to shuffle the samples.
noise : double or None (default=None)
Standard deviation of Gaussian noise added to the data.

sklearn.datasets.make_multilabel_classification

sklearn.datasets.make_multilabel_classification (n_samples=100, n_features=20, n_classes=5, n_labels=2, length=50, allow_unlabeled=True, random_state=None)
Generate a random multilabel classification problem.

For each sample, the generative process is:
• pick the number of labels: n ~ Poisson(n_labels)
• n times, choose a class c: c ~ Multinomial(theta)
• pick the document length: k ~ Poisson(length)
• k times, choose a word: w ~ Multinomial(theta_c)

In the above process, rejection sampling is used to make sure that n is never zero or more than n_classes, and that the document length is never zero. Likewise, we reject classes which have already been chosen.

Parameters n_samples : int, optional (default=100)
The number of samples.

n_features : int, optional (default=20)
The total number of features.
n_classes : int, optional (default=5)
The number of classes of the classification problem.
n_labels : int, optional (default=2)
The average number of labels per instance. Number of labels follows a Poisson distribution that never takes the value 0.
length : int, optional (default=50)

Sum of the features (number of words if documents).

allow_unlabeled : bool, optional (default=True)

If True, some instances might not belong to any class.

random_state : int, RandomState instance or None, optional (default=None)

If int, random_state is the seed used by the random number generator; If RandomState instance, random_state is the random number generator; If None, the random number generator is the RandomState instance used by np.random.

Returns X : array of shape [n_samples, n_features]

The generated samples.

Y : list of tuples

The label sets.

**sklearn.datasets.make_regression**

```python
sklearn.datasets.make_regression(n_samples=100, n_features=100, n_informative=10, bias=0.0, effective_rank=None, tail_strength=0.5, noise=0.0, shuffle=True, coef=False, random_state=None)
```

Generate a random regression problem.

The input set can either be well conditioned (by default) or have a low rank-fat tail singular profile. See the **make_low_rank_matrix** for more details.

The output is generated by applying a (potentially biased) random linear regression model with n_informative nonzero regressors to the previously generated input and some gaussian centered noise with some adjustable scale.

**Parameters**

- **n_samples** : int, optional (default=100)
  
The number of samples.

- **n_features** : int, optional (default=100)
  
The number of features.

- **n_informative** : int, optional (default=10)
  
The number of informative features, i.e., the number of features used to build the linear model used to generate the output.

- **bias** : float, optional (default=0.0)
  
The bias term in the underlying linear model.

- **effective_rank** : int or None, optional (default=None)
  
  if not None: The approximate number of singular vectors required to explain most of the input data by linear combinations. Using this kind of singular spectrum in the input allows the generator to reproduce the correlations often observed in practice.
  
  if None: The input set is well conditioned, centered and gaussian with unit variance.

- **tail_strength** : float between 0.0 and 1.0, optional (default=0.5)
  
The relative importance of the fat noisy tail of the singular values profile if effective_rank is not None.
**noise** : float, optional (default=0.0)

The standard deviation of the gaussian noise applied to the output.

**shuffle** : boolean, optional (default=True)

Shuffle the samples and the features.

**coef** : boolean, optional (default=False)

If True, the coefficients of the underlying linear model are returned.

**random_state** : int, RandomState instance or None, optional (default=None)

If int, random_state is the seed used by the random number generator; If RandomState instance, random_state is the random number generator; If None, the random number generator is the RandomState instance used by `np.random`.

**Returns**

**X** : array of shape [n_samples, n_features]

The input samples.

**y** : array of shape [n_samples]

The output values.

**coef** : array of shape [n_features], optional

The coefficient of the underlying linear model. It is returned only if coef is True.

---

**sklearn.datasets.make_s_curve**

**sklearn.datasets.make_s_curve(n_samples=100, noise=0.0, random_state=None)**

Generate an S curve dataset.

**Parameters**

**n_samples** : int, optional (default=100)

The number of sample points on the S curve.

**noise** : float, optional (default=0.0)

The standard deviation of the gaussian noise.

**random_state** : int, RandomState instance or None, optional (default=None)

If int, random_state is the seed used by the random number generator; If RandomState instance, random_state is the random number generator; If None, the random number generator is the RandomState instance used by `np.random`.

**Returns**

**X** : array of shape [n_samples, 3]

The points.

**t** : array of shape [n_samples]

The univariate position of the sample according to the main dimension of the points in the manifold.

---

**sklearn.datasets.make_sparse_coded_signal**

**sklearn.datasets.make_sparse_coded_signal(n_samples, n_components, n_features, n_nonzero_coefs, random_state=None)**

Generate a signal as a sparse combination of dictionary elements.
Returns a matrix $Y = DX$, such as $D$ is (n_features, n_components), $X$ is (n_components, n_samples) and each column of $X$ has exactly n_nonzero_coefs non-zero elements.

**Parameters**

**n_samples : int**

number of samples to generate

**n_components : int,**

number of components in the dictionary

**n_features : int**

number of features of the dataset to generate

**n_nonzero_coefs : int**

number of active (non-zero) coefficients in each sample

**random_state : int or RandomState instance, optional (default=None)**

seed used by the pseudo random number generator

**Returns**

**data : array of shape [n_features, n_samples]**

The encoded signal ($Y$).

**dictionary : array of shape [n_features, n_components]**

The dictionary with normalized components ($D$).

**code : array of shape [n_components, n_samples]**

The sparse code such that each column of this matrix has exactly n_nonzero_coefs non-zero items ($X$).

---

**sklearn.datasets.make_sparse_spd_matrix**

**sklearn.datasets.make_sparse_spd_matrix**(dim=1, alpha=0.95, norm_diag=False, smallest_coef=0.1, largest_coef=0.9, random_state=None)

Generate a sparse symmetric definite positive matrix.

**Parameters**

**dim : integer, optional (default=1)**

The size of the random (matrix to generate.

**alpha : float between 0 and 1, optional (default=0.95)**

The probability that a coefficient is non zero (see notes).

**random_state : int, RandomState instance or None, optional (default=None)**

If int, random_state is the seed used by the random number generator; If RandomState instance, random_state is the random number generator; If None, the random number generator is the RandomState instance used by np.random.

**Returns**

**prec : array of shape = [dim, dim]**

Notes

The sparsity is actually imposed on the cholesky factor of the matrix. Thus alpha does not translate directly into the filling fraction of the matrix itself.
sklearn.datasets.make_sparse_uncorrelated

```python
sklearn.datasets.make_sparse_uncorrelated(n_samples=100, n_features=10, random_state=None)
```

Generate a random regression problem with sparse uncorrelated design

This dataset is described in Celeux et al [1] as:

\[
X \sim N(0, 1) \\
y(X) = X[:, 0] + 2 \times X[:, 1] - 2 \times X[:, 2] - 1.5 \times X[:, 3]
\]

Only the first 4 features are informative. The remaining features are useless.

**Parameters**

- **n_samples** : int, optional (default=100)
  - The number of samples.
- **n_features** : int, optional (default=10)
  - The number of features.
- **random_state** : int, RandomState instance or None, optional (default=None)
  - If int, random_state is the seed used by the random number generator; If RandomState instance, random_state is the random number generator; If None, the random number generator is the RandomState instance used by np.random.

**Returns**

- **X** : array of shape [n_samples, n_features]
  - The input samples.
- **y** : array of shape [n_samples]
  - The output values.

**References**

[R55]

sklearn.datasets.make_spd_matrix

```python
sklearn.datasets.make_spd_matrix(n_dim, random_state=None)
```

Generate a random symmetric, positive-definite matrix.

**Parameters**

- **n_dim** : int
  - The matrix dimension.
- **random_state** : int, RandomState instance or None, optional (default=None)
  - If int, random_state is the seed used by the random number generator; If RandomState instance, random_state is the random number generator; If None, the random number generator is the RandomState instance used by np.random.

**Returns**

- **X** : array of shape [n_dim, n_dim]
  - The random symmetric, positive-definite matrix.
sklearn.datasets.make_swiss_roll

Generate a swiss roll dataset.

**Parameters**

- `n_samples` : int, optional (default=100)
  The number of sample points on the S curve.
- `noise` : float, optional (default=0.0)
  The standard deviation of the gaussian noise.
- `random_state` : int, RandomState instance or None, optional (default=None)
  If int, random_state is the seed used by the random number generator; If RandomState instance, random_state is the random number generator; If None, the random number generator is the RandomState instance used by `np.random`.

**Returns**

- `X` : array of shape [n_samples, 3]
  The points.
- `t` : array of shape [n_samples]
  The univariate position of the sample according to the main dimension of the points in the manifold.

**Notes**

The algorithm is from Marsland [1].

**References**

[R56]

### 1.8.5 sklearn.decomposition: Matrix Decomposition

The `sklearn.decomposition` module includes matrix decomposition algorithms, including among others PCA, NMF or ICA. Most of the algorithms of this module can be regarded as dimensionality reduction techniques.

**User guide:** See the *Decomposing signals in components (matrix factorization problems)* section for further details.

<table>
<thead>
<tr>
<th>Function</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>decomposition.PCA</code></td>
<td>Principal component analysis (PCA)</td>
</tr>
<tr>
<td><code>decomposition.ProbabilisticPCA</code></td>
<td>Additional layer on top of PCA that adds a probabilistic evaluation</td>
</tr>
<tr>
<td><code>decompositionProjectedGradientNMF</code></td>
<td>Non-Negative matrix factorization by Projected Gradient (NMF)</td>
</tr>
<tr>
<td><code>decomposition.RandomizedPCA</code></td>
<td>Principal component analysis (PCA) using randomized SVD</td>
</tr>
<tr>
<td><code>decompositionKernelPCA</code></td>
<td>Kernel Principal component analysis (KPCA)</td>
</tr>
<tr>
<td><code>decompositionFastICA</code></td>
<td>FastICA: a fast algorithm for Independent Component Analysis</td>
</tr>
<tr>
<td><code>decomposition.NMF</code></td>
<td>Non-Negative matrix factorization by Projected Gradient (NMF)</td>
</tr>
<tr>
<td><code>decompositionSparsePCA</code></td>
<td>Sparse Principal Components Analysis (SparsePCA)</td>
</tr>
<tr>
<td><code>decompositionMiniBatchSparsePCA</code></td>
<td>Mini-batch Sparse Principal Components Analysis</td>
</tr>
<tr>
<td><code>decompositionSparseCoder</code></td>
<td>Sparse coding</td>
</tr>
<tr>
<td><code>decompositionDictionaryLearning</code></td>
<td>Dictionary learning</td>
</tr>
<tr>
<td><code>decompositionMiniBatchDictionaryLearning</code></td>
<td>Mini-batch dictionary learning</td>
</tr>
</tbody>
</table>
Principal component analysis (PCA)

Linear dimensionality reduction using Singular Value Decomposition of the data and keeping only the most significant singular vectors to project the data to a lower dimensional space.

This implementation uses the scipy.linalg implementation of the singular value decomposition. It only works for dense arrays and is not scalable to large dimensional data.

The time complexity of this implementation is $O(n^{**3})$ assuming $n \sim n_{samples} \sim n_{features}$.

Parameters

- **n_components**: int, None or string
  Number of components to keep. If n_components is not set all components are kept:
  
  $n_{components} = \min(n_{samples}, n_{features})$
  
  if n_components == 'mle', Minka's MLE is used to guess the dimension if $0 < n_{components} < 1$, select the number of components such that the amount of variance that needs to be explained is greater than the percentage specified by n_components

- **copy**: bool
  If False, data passed to fit are overwritten

- **whiten**: bool, optional
  When True (False by default) the components_. vectors are divided by $n_{samples}$ times singular values to ensure uncorrelated outputs with unit component-wise variances.

  Whitening will remove some information from the transformed signal (the relative variance scales of the components) but can sometimes improve the predictive accuracy of the downstream estimators by making there data respect some hard-wired assumptions.

See Also:

ProbabilisticPCA, RandomizedPCA, KernelPCA, SparsePCA

Notes

For n_components='mle', this class uses the method of Thomas P. Minka: Automatic Choice of Dimensionality for PCA. NIPS 2000: 598-604

Due to implementation subtleties of the Singular Value Decomposition (SVD), which is used in this implementation, running fit twice on the same matrix can lead to principal components with signs flipped (change in direction). For this reason, it is important to always use the same estimator object to transform data in a consistent fashion.

Examples

```python
>>> import numpy as np
>>> from sklearn.decomposition import PCA
>>> X = np.array([[-1, -1], [-2, -1], [-3, -2], [1, 1], [2, 1], [3, 2]])
>>> pca = PCA(n_components=2)
>>> pca.fit(X)
PCA(copy=True, n_components=2, whiten=False)
```
>>> print(pca.explained_variance_ratio_)
[ 0.99244...  0.00755...]

Attributes

| components_ | array, [n_components, n_features] | Components with maximum variance. |
| explanaed_variance_ | array, [n_components] | Percentage of variance explained by each of the selected components. k is not set then all components are stored and the sum of explained variances is equal to 1.0 |

Methods

| fit(X, y) | Fit the model with X. |
| fit_transform(X, y) | Fit the model with X and apply the dimensionality reduction on X. |
| get_params([deep]) | Get parameters for the estimator |
| inverse_transform(X) | Transform data back to its original space, i.e., |
| set_params(**params) | Set the parameters of the estimator. |
| transform(X) | Apply the dimensionality reduction on X. |

__init__(n_components=None, copy=True, whiten=False)

fit (X, y=None, **params)
Fit the model with X.

Parameters X: array-like, shape (n_samples, n_features) :
Training data, where n_samples in the number of samples and n_features is the number of features.

Returns self : object
Returns the instance itself.

fit_transform (X, y=None, **params)
Fit the model with X and apply the dimensionality reduction on X.

Parameters X : array-like, shape (n_samples, n_features)
Training data, where n_samples in the number of samples and n_features is the number of features.

Returns X_new : array-like, shape (n_samples, n_components)

get_params (deep=True)
Get parameters for the estimator

Parameters deep: boolean, optional :
If True, will return the parameters for this estimator and contained subobjects that are estimators.

inverse_transform (X)
Transform data back to its original space, i.e., return an input X_original whose transform would be X

Parameters X : array-like, shape (n_samples, n_components)
New data, where n_samples in the number of samples and n_components is the number of components.

Returns X_original array-like, shape (n_samples, n_features):

Notes

If whitening is enabled, inverse_transform does not compute the exact inverse operation as transform.

set_params (**params)
Set the parameters of the estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The former have parameters of the form <component>__<parameter> so that it’s possible to update each component of a nested object.

Returns self:

transform (X)
Apply the dimensionality reduction on X.

Parameters X : array-like, shape (n_samples, n_features)
New data, where n_samples in the number of samples and n_features is the number of features.

Returns X_new : array-like, shape (n_samples, n_components)

sklearn.decomposition.ProbabilisticPCA

class sklearn.decomposition.ProbabilisticPCA (n_components=None, copy=True, whiten=False)

Additional layer on top of PCA that adds a probabilistic evaluationPrincipal component analysis (PCA)
Linear dimensionality reduction using Singular Value Decomposition of the data and keeping only the most significant singular vectors to project the data to a lower dimensional space.
This implementation uses the scipy.linalg implementation of the singular value decomposition. It only works for dense arrays and is not scalable to large dimensional data.
The time complexity of this implementation is $O(n \times 3)$ assuming $n \sim n_{samples} \sim n_{features}$.

Parameters n_components : int, None or string
Number of components to keep. if n_components is not set all components are kept:

n_components == min(n_samples, n_features)

if n_components == ‘mle’, Minka’s MLE is used to guess the dimension if $0 < n_{components} < 1$, select the number of components such that the amount of variance that needs to be explained is greater than the percentage specified by n_components

copy : bool
If False, data passed to fit are overwritten

whiten : bool, optional
When True (False by default) the components_ vectors are divided by $n_{samples}$ times singular values to ensure uncorrelated outputs with unit component-wise variances.
Whitening will remove some information from the transformed signal (the relative variance scales of the components) but can sometime improve the predictive accuracy of the downstream estimators by making there data respect some hard-wired assumptions.

See Also:
ProbabilisticPCA, RandomizedPCA, KernelPCA, SparsePCA

Notes

For n_components='mle', this class uses the method of Thomas P. Minka: Automatic Choice of Dimensionality for PCA. NIPS 2000: 598-604

Due to implementation subtleties of the Singular Value Decomposition (SVD), which is used in this implementation, running fit twice on the same matrix can lead to principal components with signs flipped (change in direction). For this reason, it is important to always use the same estimator object to transform data in a consistent fashion.

Examples

```python
>>> import numpy as np
>>> from sklearn.decomposition import PCA

>>> X = np.array([[[-1, -1], [-2, -1], [-3, -2], [1, 1], [2, 1], [3, 2]]])
>>> pca = PCA(n_components=2)
>>> pca.fit(X)
PCA(copy=True, n_components=2, whiten=False)
>>> print(pca.explained_variance_ratio_)
[ 0.99244... 0.00755...]
```

Attributes

<table>
<thead>
<tr>
<th>components_</th>
<th>array, [n_components, n_features]</th>
<th>Components with maximum variance.</th>
</tr>
</thead>
<tbody>
<tr>
<td>explained_variance_</td>
<td>array, [n_components]</td>
<td>Percentage of variance explained by each of the selected components. k is not set then all components are stored and the sum of explained variances is equal to 1.0</td>
</tr>
</tbody>
</table>

Methods

- `fit(X[, y, homoscedastic])`: Additionally to PCA.fit, learns a covariance model
- `fit_transform(X[, y])`: Fit the model with X and apply the dimensionality reduction on X.
- `get_params([deep])`: Get parameters for the estimator
- `inverse_transform(X)`: Transform data back to its original space, i.e.,
- `score(X[, y])`: Return a score associated to new data
- `set_params(**params)`: Set the parameters of the estimator.
- `transform(X)`: Apply the dimensionality reduction on X.

```
__init__ (n_components=None, copy=True, whiten=False)
fit (X, y=None, homoscedastic=True)
```
Additionally to PCA.fit, learns a covariance model

**Parameters**

- **X**: array of shape(n_samples, n_dim)
  
  The data to fit

- **homoscedastic**: bool, optional,
  
  If True, average variance across remaining dimensions

**fit_transform**(X, y=None, **params)

Fit the model with X and apply the dimensionality reduction on X.

**Parameters**

- **X**: array-like, shape (n_samples, n_features)

  Training data, where n_samples in the number of samples and n_features is the number of features.

**Returns**

- **X_new**: array-like, shape (n_samples, n_components)

**get_params**(deep=True)

Get parameters for the estimator

**Parameters**

- **deep**: boolean, optional

  If True, will return the parameters for this estimator and contained subobjects that are estimators.

**inverse_transform**(X)

Transform data back to its original space, i.e., return an input X_original whose transform would be X

**Parameters**

- **X**: array-like, shape (n_samples, n_components)

  New data, where n_samples in the number of samples and n_components is the number of components.

**Returns**

- **X_original**: array-like, shape (n_samples, n_features)

**Notes**

If whitening is enabled, inverse_transform does not compute the exact inverse operation as transform.

**score**(X, y=None)

Return a score associated to new data

**Parameters**

- **X**: array of shape(n_samples, n_dim)

  The data to test

**Returns**

- **ll**: array of shape (n_samples),

  log-likelihood of each row of X under the current model

**set_params**(**params)

Set the parameters of the estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The former have parameters of the form `<component>__<parameter>` so that it's possible to update each component of a nested object.

**Returns**

- **self**

**transform**(X)

Apply the dimensionality reduction on X.
**Parameters** X : array-like, shape (n_samples, n_features)

New data, where n_samples in the number of samples and n_features is the number of features.

**Returns** X_new : array-like, shape (n_samples, n_components)

### sklearn.decompositionProjectedGradientNMF

**class** sklearn.decomposition.ProjectedGradientNMF (n_components=None, init='nndsvdar', sparseness=None, beta=1, eta=0.1, tol=0.0001, max_iter=200, nls_max_iter=2000)

Non-Negative matrix factorization by Projected Gradient (NMF)

**Parameters** X: {array-like, sparse matrix}, shape = [n_samples, n_features] :

Data the model will be fit to.

**n_components**: int or None :

Number of components, if n_components is not set all components are kept

**init**: ‘nndsvd’ | ‘nndsvda’ | ‘nndsvdar’ | int | RandomState :

Method used to initialize the procedure. Default: ‘nndsvdar’. Valid options:

‘nndsvd’: Nonnegative Double Singular Value Decomposition (NNDSVD) initialization (better for sparseness)

‘nndsvda’: NNDSVD with zeros filled with the average of X (better when sparsity is not desired)

‘nndsvdar’: NNDSVD with zeros filled with small random values (generally faster, less accurate alternative to NNDSVDa for when sparsity is not desired)

int seed or RandomState: non-negative random matrices

**sparseness**: ‘data’ | ‘components’ | None, default: None :

Where to enforce sparsity in the model.

**beta**: double, default: 1 :

Degree of sparseness, if sparseness is not None. Larger values mean more sparseness.

**eta**: double, default: 0.1 :

Degree of correctness to maintain, if sparsity is not None. Smaller values mean larger error.

**tol**: double, default: 1e-4 :

Tolerance value used in stopping conditions.

**max_iter**: int, default: 200 :

Number of iterations to compute.

**nls_max_iter**: int, default: 2000 :

Number of iterations in NLS subproblem.
Notes

This implements


NNDSVD is introduced in


Examples

```python
>>> import numpy as np
>>> X = np.array([[1,1], [2, 1], [3, 1.2], [4, 1], [5, 0.8], [6, 1]])
>>> from sklearn.decomposition import ProjectedGradientNMF
>>> model = ProjectedGradientNMF(n_components=2, init=0)
>>> model.fit(X)
ProjectedGradientNMF(beta=1, eta=0.1, init=0, max_iter=200, n_components=2, nls_max_iter=2000, sparseness=None, tol=0.0001)
>>> model.components_
array([[ 0.77032744, 0.11118662],
        [ 0.38526873, 0.38228063]])
>>> model.reconstruction_err_
0.00746...
>>> model = ProjectedGradientNMF(n_components=2, init=0, sparseness='components')
>>> model.fit(X)
ProjectedGradientNMF(beta=1, eta=0.1, init=0, max_iter=200, n_components=2, nls_max_iter=2000, sparseness='components', tol=0.0001)
>>> model.components_
array([[ 1.67481991, 0.29614922],
        [-0. , 0.4681982 ]])
>>> model.reconstruction_err_
0.513...
```

Attributes

| components_ | array, [n_components, n_features] | Non-negative components of the data |
| reconstruc- | number | Frobenius norm of the matrix difference between the training data and the reconstructed data from the fit produced by the model. $||X - WH||_2^2$ Not computed for sparse input matrices because it is too expensive in terms of memory. |

Methods

| fit(X[, y]) | Learn a NMF model for the data X. | Continued on next page |
Table 1.47 – continued from previous page

<table>
<thead>
<tr>
<th>Method</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>fit_transform</td>
<td>Learn a NMF model for the data X and returns the transformed data.</td>
</tr>
<tr>
<td>get_params</td>
<td>Get parameters for the estimator</td>
</tr>
<tr>
<td>set_params</td>
<td>Set the parameters of the estimator.</td>
</tr>
<tr>
<td>transform</td>
<td>Transform the data X according to the fitted NMF model.</td>
</tr>
</tbody>
</table>

__init__ (n_components=None, init='nndsvdar', sparseness=None, beta=1, eta=0.1, tol=0.0001, max_iter=200, nls_max_iter=2000)

fit (X, y=None, **params)
Learn a NMF model for the data X.

Parameters X: [array-like, sparse matrix], shape = [n_samples, n_features]:
Data matrix to be decomposed

Returns self:

fit_transform (X, y=None)
Learn a NMF model for the data X and returns the transformed data.

This is more efficient than calling fit followed by transform.

Parameters X: [array-like, sparse matrix], shape = [n_samples, n_features]:
Data matrix to be decomposed

Returns data: array, [n_samples, n_components]:
Transformed data

get_params (deep=True)
Get parameters for the estimator

Parameters deep: boolean, optional:
If True, will return the parameters for this estimator and contained subobjects that are estimators.

set_params (**params)
Set the parameters of the estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The former have parameters of the form <component>.__<parameter> so that it’s possible to update each component of a nested object.

Returns self:

transform (X)
Transform the data X according to the fitted NMF model

Parameters X: [array-like, sparse matrix], shape = [n_samples, n_features]:
Data matrix to be transformed by the model

Returns data: array, [n_samples, n_components]:
Transformed data

sklearn.decomposition.RandomizedPCA

class sklearn.decomposition.RandomizedPCA (n_components, copy=True, iterated_power=3, whiten=False, random_state=None)

Principal component analysis (PCA) using randomized SVD

1.8. Reference
Linear dimensionality reduction using approximated Singular Value Decomposition of the data and keeping only the most significant singular vectors to project the data to a lower dimensional space.

This implementation uses a randomized SVD implementation and can handle both scipy.sparse and numpy dense arrays as input.

**Parameters**

- **n_components**: int
  
  Maximum number of components to keep: default is 50.

- **copy**: bool
  
  If False, data passed to fit are overwritten.

- **iterated_power**: int, optional
  
  Number of iteration for the power method. 3 by default.

- **whiten**: bool, optional
  
  When True (False by default) the components_vectors are divided by the singular values to ensure uncorrelated outputs with unit component-wise variances.

  Whitening will remove some information from the transformed signal (the relative variance scales of the components) but can sometime improve the predictive accuracy of the downstream estimators by making there data respect some hard-wired assumptions.

- **random_state**: int or RandomState instance or None (default)
  
  Pseudo Random Number generator seed control. If None, use the numpy.random singleton.

See Also:

- PCA, ProbabilisticPCA

References

[Halko2009], [MRT]

Examples

```python
>>> import numpy as np
>>> from sklearn.decomposition import RandomizedPCA
>>> X = np.array([[-1, -1], [-2, -1], [-3, -2], [1, 1], [2, 1], [3, 2]])
>>> pca = RandomizedPCA(n_components=2)
>>> pca.fit(X)
RandomizedPCA(copy=True, iterated_power=3, n_components=2,
              random_state=<mtrand.RandomState object at 0x...>, whiten=False)
>>> print(pca.explained_variance_ratio_)
[ 0.99244... 0.00755...]
```
Attributes

<table>
<thead>
<tr>
<th>components_</th>
<th>array, [n_components, n_features]</th>
<th>Components with maximum variance.</th>
</tr>
</thead>
<tbody>
<tr>
<td>explained_variance_</td>
<td>array, [n_components]</td>
<td>Percentage of variance explained by each of the selected components. k is not set then all components are stored and the sum of explained variances is equal to 1.0</td>
</tr>
</tbody>
</table>

Methods

| __init__ | (n_components, copy=True, iterated_power=3, whiten=False, random_state=None) |  |
| fit | (X, y=None) | Fit the model to the data X. |
| fit_transform | (X, y) | Fit to data, then transform it |
| get_params | ([deep]) | Get parameters for the estimator |
| inverse_transform | (X) | Transform data back to its original space, i.e., |
| set_params | (**params) | Set the parameters of the estimator. |
| transform | (X) | Apply the dimensionality reduction on X. |

fit (X, y=None)

Fit the model to the data X.

Parameters X: array-like or scipy.sparse matrix, shape (n_samples, n_features):

Training vector, where n_samples in the number of samples and n_features is the number of features.

Returns self:

Returns the instance itself.

fit_transform (X, y=None, **fit_params)

Fit to data, then transform it

Fits transformer to X and y with optional parameters fit_params and returns a transformed version of X.

Parameters X: numpy array of shape [n_samples, n_features]

Training set.

y: numpy array of shape [n_samples]

Target values.

Returns X_new: numpy array of shape [n_samples, n_features_new]

Transformed array.

Notes

This method just calls fit and transform consecutively, i.e., it is not an optimized implementation of fit_transform, unlike other transformers such as PCA.

get_params (deep=True)

Get parameters for the estimator
Parameters deep: boolean, optional:
If True, will return the parameters for this estimator and contained subobjects that are estimators.

inverse_transform(X)
Transform data back to its original space, i.e., return an input X_original whose transform would be X

Parameters X: array-like or scipy.sparse matrix, shape (n_samples, n_components)
New data, where n_samples in the number of samples and n_components is the number of components.

Returns X_original array-like, shape (n_samples, n_features):

Notes
If whitening is enabled, inverse_transform does not compute the exact inverse operation as transform.

set_params(**params)
Set the parameters of the estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The former have parameters of the form <component>__<parameter> so that it’s possible to update each component of a nested object.

Returns self:

transform(X)
Apply the dimensionality reduction on X.

Parameters X: array-like or scipy.sparse matrix, shape (n_samples, n_features)
New data, where n_samples in the number of samples and n_features is the number of features.

Returns X_new : array-like, shape (n_samples, n_components)

sklearn.decomposition.KernelPCA

class sklearn.decomposition.KernelPCA(n_components=None, kernel='linear', gamma=0, degree=3, coef0=1, alpha=1.0, fit_inverse_transform=False, eigen_solver='auto', tol=0, max_iter=None)

Kernel Principal component analysis (KPCA)
Non-linear dimensionality reduction through the use of kernels.

Parameters n_components: int or None:
Number of components. If None, all non-zero components are kept.

kernel: “linear” | “poly” | “rbf” | “sigmoid” | “precomputed”:
Kernel. Default: “linear”

degree : int, optional
Degree for poly, rbf and sigmoid kernels. Default: 3.

gamma : float, optional
Kernel coefficient for rbf and poly kernels. Default: 1/n_features.
**coef0**: float, optional

Independent term in poly and sigmoid kernels.

**alpha**: int

Hyperparameter of the ridge regression that learns the inverse transform (when fit_inverse_transform=True). Default: 1.0

**fit_inverse_transform**: bool

Learn the inverse transform for non-precomputed kernels. (i.e. learn to find the pre-image of a point) Default: False

**eigen_solver**: string ['auto'|'dense'|'arpack']

Select eigensolver to use. If n_components is much less than the number of training samples, arpack may be more efficient than the dense eigensolver.

**tol**: float

Convergence tolerance for arpack. Default: 0 (optimal value will be chosen by arpack)

**max_iter**: int

Maximum number of iterations for arpack Default: None (optimal value will be chosen by arpack)

### References


### Attributes

- **lambdas_, alphas_**: Eigenvalues and eigenvectors of the centered kernel matrix
- **dual_coef_**: Inverse transform matrix
- **X_transformed_fit_**: Projection of the fitted data on the kernel principal components

### Methods

- **fit(X[, y])**: Fit the model from data in X.
- **fit_transform(X[, y])**: Fit the model from data in X and transform X.
- **get_params([deep])**: Get parameters for the estimator
- **inverse_transform(X)**: Transform X back to original space.
- **set_params(**params)**: Set the parameters of the estimator.
- **transform(X)**: Transform X.

### __init__

__init__(n_components=None, kernel='linear', gamma=0, degree=3, coef0=1, alpha=1.0, fit_inverse_transform=False, eigen_solver='auto', tol=0, max_iter=None)

**fit**(X, y=None)

Fit the model from data in X.

Parameters X: array-like, shape (n_samples, n_features)
Training vector, where n_samples is the number of samples and n_features is the number of features.

Returns self : object

Returns the instance itself.

**fit_transform** *(X, y=None, **params)*

Fit the model from data in X and transform X.

Parameters X: array-like, shape (n_samples, n_features):

Training vector, where n_samples is the number of samples and n_features is the number of features.

Returns X_new: array-like, shape (n_samples, n_components):

**get_params** *(deep=True)*

Get parameters for the estimator

Parameters deep: boolean, optional:

If True, will return the parameters for this estimator and contained subobjects that are estimators.

**inverse_transform** *(X)*

Transform X back to original space.

Parameters X: array-like, shape (n_samples, n_components):

Returns X_new: array-like, shape (n_samples, n_features):

**References**


**set_params** *(**params)*

Set the parameters of the estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The former have parameters of the form `<component>__<parameter>` so that it's possible to update each component of a nested object.

Returns self :

**transform** *(X)*

Transform X.

Parameters X: array-like, shape (n_samples, n_features):

Returns X_new: array-like, shape (n_samples, n_components):

**sklearn.decomposition.FastICA**

class sklearn.decomposition.FastICA *(n_components=None, algorithm='parallel', whiten=True, fun='logcosh', fun_prime='', fun_args=None, max_iter=200, tol=0.0001, w_init=None, random_state=None)*

FastICA; a fast algorithm for Independent Component Analysis

Parameters n_components : int, optional

Number of components to use. If none is passed, all are used.
algorithm : {'parallel', 'deflation'}

Apply parallel or deflational algorithm for FastICA

whiten : boolean, optional

If whiten is false, the data is already considered to be whitened, and no whitening is performed.

fun : {'logcosh', 'exp', or 'cube'}, or a callable

The non-linear function used in the FastICA loop to approximate negentropy. If a function is passed, its derivative should be passed as the 'fun_prime' argument.

fun_prime : None or a callable

The derivative of the non-linearity used.

max_iter : int, optional

Maximum number of iterations during fit

tol : float, optional

Tolerance on update at each iteration

w_init : None of an (n_components, n_components) ndarray

The mixing matrix to be used to initialize the algorithm.

random_state: int or RandomState :

Pseudo number generator state used for random sampling.

Notes


Attributes

| components_ | 2D array, [n_components, n_features] | The unmixing matrix |
| sources_: | 2D array, [n_samples, n_components] | The estimated latent sources of the data. |

Methods

fit(X)

generate_random_state()

get_mixing_matrix() Compute the mixing matrix

get_params([deep]) Get parameters for the estimator

set_params(**params) Set the parameters of the estimator.

transform(X) Apply un-mixing matrix “W” to X to recover the sources

__init__(n_components=None, algorithm='parallel', whiten=True, fun='logcosh', fun_prime='', fun_args=None, max_iter=200, tol=0.0001, w_init=None, random_state=None)

generate_random_state()
Compute the mixing matrix

**get_params** *(deep=True)*

Get parameters for the estimator

**Parameters**

deep: boolean, optional

If True, will return the parameters for this estimator and contained subobjects that are estimators.

**set_params** (**params**)

Set the parameters of the estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The former have parameters of the form `<component>__<parameter>` so that it’s possible to update each component of a nested object.

**Returns**

self:

**transform** *(X)*

Apply un-mixing matrix “W” to X to recover the sources

\[ S = X \cdot W^T \]

**unmixing_matrix**

DEPRECATED: Renamed to `components_`

---

**sklearn.decomposition.NMF**

class sklearn.decomposition.NMF *(n_components=None, init='nndsvdar', sparseness=None, beta=1, eta=0.1, tol=0.0001, max_iter=200, nls_max_iter=2000)*

Non-Negative matrix factorization by Projected Gradient (NMF)

**Parameters**

X: {array-like, sparse matrix}, shape = [n_samples, n_features]

Data the model will be fit to.

n_components: int or None

Number of components, if n_components is not set all components are kept

init: ‘nndsvd’ | ‘nndsvda’ | ‘nndsvdar’ | int | RandomState

Method used to initialize the procedure. Default: ‘nndsvdar’ Valid options:

‘nndsvd’: Nonnegative Double Singular Value Decomposition (NNDSDV) initialization (better for sparseness)
‘nndsvda’: NNDSVD with zeros filled with the average of X (better when sparsity is not desired)
‘nndsvdar’: NNDSVD with zeros filled with small random values (generally faster, less accurate alternative to NNDSVDa for when sparsity is not desired)

int seed or RandomState: non-negative random matrices

sparseness: ‘data’ | ‘components’ | None, default: None

Where to enforce sparsity in the model.

beta: double, default: 1

Degree of sparseness, if sparseness is not None. Larger values mean more sparseness.

eta: double, default: 0.1
Degree of correctness to maintain, if sparsity is not None. Smaller values mean larger
error.

**tol**: double, default: 1e-4 :

tolerance value used in stopping conditions.

**max_iter**: int, default: 200 :

number of iterations to compute.

**nls_max_iter**: int, default: 2000 :

number of iterations in NLS subproblem.

Notes

This implements


NNDSVD is introduced in

C. Boutsidis, E. Gallopoulos: SVD based initialization: A head start for nonnegative matrix factorization -

Examples

```python
>>> import numpy as np
>>> X = np.array([[1,1], [2, 1], [3, 1.2], [4, 1], [5, 0.8], [6, 1]])
>>> from sklearn.decomposition import ProjectedGradientNMF
>>> model = ProjectedGradientNMF(n_components=2, init=0)
>>> model.fit(X)
ProjectedGradientNMF(beta=1, eta=0.1, init=0, max_iter=200, n_components=2,
                   nls_max_iter=2000, sparseness=None, tol=0.0001)
>>> model.components_
array([[ 0.77032744, 0.11118662],
        [ 0.38526873, 0.38228063]])
>>> model.reconstruction_err_
0.00746...
>>> model = ProjectedGradientNMF(n_components=2, init=0,
                   sparseness='components')
>>> model.fit(X)
ProjectedGradientNMF(beta=1, eta=0.1, init=0, max_iter=200, n_components=2,
                   nls_max_iter=2000, sparseness='components', tol=0.0001)
>>> model.components_
array([[ 1.67481991, 0.29614922],
        [-0. , 0.4681982 ]])
>>> model.reconstruction_err_
0.513...
```
Attributes

<table>
<thead>
<tr>
<th>components_</th>
<th>array, [n_components, n_features]</th>
<th>Non-negative components of the data</th>
</tr>
</thead>
<tbody>
<tr>
<td>reconstruction_err_</td>
<td>number</td>
<td>Frobenius norm of the matrix difference between the training data and the reconstructed data from the fit produced by the model. ( | X - WH |_2 )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Not computed for sparse input matrices because it is too expensive in terms of memory.</td>
</tr>
</tbody>
</table>

Methods

```python
fit(X[, y])
Learn a NMF model for the data X.

fit_transform(X[, y])
Learn a NMF model for the data X and returns the transformed data.

get_params([deep])
Get parameters for the estimator

set_params(**params)
Set the parameters of the estimator.

transform(X)
Transform the data X according to the fitted NMF model
```

```python
__init__(n_components=None, init='nndsvdar', sparseness=None, beta=1, eta=0.1, tol=0.0001, max_iter=200, nls_max_iter=2000)

fit(X, y=None, **params)
Learn a NMF model for the data X.

Parameters X: {array-like, sparse matrix}, shape = [n_samples, n_features] :
Data matrix to be decomposed

Returns self :

fit_transform(X, y=None)
Learn a NMF model for the data X and returns the transformed data.

This is more efficient than calling fit followed by transform.

Parameters X: {array-like, sparse matrix}, shape = [n_samples, n_features] :
Data matrix to be decomposed

Returns data: array, [n_samples, n_components] :
Transformed data

get_params(deep=True)
Get parameters for the estimator

Parameters deep: boolean, optional :
If True, will return the parameters for this estimator and contained subobjects that are estimators.

set_params(**params)
Set the parameters of the estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The former have parameters of the form <component>__<parameter> so that it's possible to update each component of a nested object.

Returns self :
```
**transform**($X$)

Transform the data $X$ according to the fitted NMF model

**Parameters**

$X$: {array-like, sparse matrix}, shape = [n_samples, n_features]

Data matrix to be transformed by the model

**Returns**

$data$: array, [n_samples, n_components]

Transformed data

**sklearn.decomposition.SparsePCA**

**class** `sklearn.decomposition.SparsePCA`(n_components, alpha=1, ridge_alpha=0.01, max_iter=1000, tol=1e-08, method='lars', n_jobs=1, U_init=None, V_init=None, verbose=False, random_state=None)

Sparse Principal Components Analysis (SparsePCA)

Finds the set of sparse components that can optimally reconstruct the data. The amount of sparseness is controllable by the coefficient of the L1 penalty, given by the parameter alpha.

**Parameters**

**n_components** : int,

Number of sparse atoms to extract.

**alpha** : float,

Sparsity controlling parameter. Higher values lead to sparser components.

**ridge_alpha** : float,

Amount of ridge shrinkage to apply in order to improve conditioning when calling the transform method.

**max_iter** : int,

Maximum number of iterations to perform.

**tol** : float,

Tolerance for the stopping condition.

**method** : {'lars', 'cd'}

lars: uses the least angle regression method to solve the lasso problem (linear_model.lars_path) cd: uses the coordinate descent method to compute the Lasso solution (linear_model.Lasso). Lars will be faster if the estimated components are sparse.

**n_jobs** : int,

Number of parallel jobs to run.

**U_init** : array of shape (n_samples, n_atoms),

Initial values for the loadings for warm restart scenarios.

**V_init** : array of shape (n_atoms, n_features),

Initial values for the components for warm restart scenarios.

**verbose** : 

Degree of verbosity of the printed output.

**random_state** : int or RandomState
Pseudo number generator state used for random sampling.

See Also:

PCA, MiniBatchSparsePCA, DictionaryLearning

Attributes

<table>
<thead>
<tr>
<th>components_</th>
<th>array, [n_components, n_features]</th>
<th>Sparse components extracted from the data.</th>
</tr>
</thead>
<tbody>
<tr>
<td>error_</td>
<td>array</td>
<td>Vector of errors at each iteration.</td>
</tr>
</tbody>
</table>

Methods

- **fit** *(X, y=None)*
  Fit the model from data in X.

- **fit_transform** *(X, y=None, **fit_params)*
  Fits transformer to X and y with optional parameters fit_params and returns a transformed version of X.

__init__ *(n_components, alpha=1, ridge_alpha=0.01, max_iter=1000, tol=1e-08, method='lars', n_jobs=1, U_init=None, V_init=None, verbose=False, random_state=None)*

- **fit** *(X, y=None)*
  Fit the model from data in X.

  Parameters:
  - **X**: array-like, shape (n_samples, n_features)
    Training vector, where n_samples is the number of samples and n_features is the number of features.

  Returns:
  - **self**: object
    Returns the instance itself.

- **fit_transform** *(X, y=None, **fit_params)*
  Fit to data, then transform it

  Fits transformer to X and y with optional parameters fit_params and returns a transformed version of X.

  Parameters:
  - **X**: numpy array of shape [n_samples, n_features]
    Training set.
  - **y**: numpy array of shape [n_samples]
    Target values.

  Returns:
  - **X_new**: numpy array of shape [n_samples, n_features_new]
    Transformed array.

Notes

This method just calls fit and transform consecutively, i.e., it is not an optimized implementation of fit_transform, unlike other transformers such as PCA.

- **get_params** *(deep=True)*
  Get parameters for the estimator
Parameters deep: boolean, optional:

If True, will return the parameters for this estimator and contained subobjects that are estimators.

set_params (**params)

Set the parameters of the estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The former have parameters of the form <component>__<parameter> so that it’s possible to update each component of a nested object.

Returns self:

transform(X, ridge_alpha=None)

Least Squares projection of the data onto the sparse components.

To avoid instability issues in case the system is under-determined, regularization can be applied (Ridge regression) via the ridge_alpha parameter.

Note that Sparse PCA components orthogonality is not enforced as in PCA hence one cannot use a simple linear projection.

Parameters X: array of shape (n_samples, n_features):

Test data to be transformed, must have the same number of features as the data used to train the model.

ridge_alpha: float, default: 0.01:

Amount of ridge shrinkage to apply in order to improve conditioning.

Returns X_new array, shape (n_samples, n_components):

Transformed data.

sklearn.decomposition.MiniBatchSparsePCA

class sklearn.decomposition.MiniBatchSparsePCA(n_components, alpha=1, ridge_alpha=0.01, n_iter=100, callback=None, chunk_size=3, verbose=False, shuffle=True, n_jobs=1, method='lars', random_state=None)

Mini-batch Sparse Principal Components Analysis

Finds the set of sparse components that can optimally reconstruct the data. The amount of sparseness is controllable by the coefficient of the L1 penalty, given by the parameter alpha.

Parameters n_components: int,

number of sparse atoms to extract

alpha: int,

Sparsity controlling parameter. Higher values lead to sparser components.

ridge_alpha: float,

Amount of ridge shrinkage to apply in order to improve conditioning when calling the transform method.

n_iter: int,

number of iterations to perform for each mini batch
callback : callable,
callable that gets invoked every five iterations

chunk_size : int,
the number of features to take in each mini batch

verbose : :
degree of output the procedure will print

shuffle : boolean,
whether to shuffle the data before splitting it in batches

n_jobs : int,
number of parallel jobs to run, or -1 to autodetect.

method : {'lars', 'cd'}
lars: uses the least angle regression method to solve the lasso problem (linear_model.lars_path) cd: uses the coordinate descent method to compute the Lasso solution (linear_model.Lasso). Lars will be faster if the estimated components are sparse.

random_state : int or RandomState
Pseudo number generator state used for random sampling.

See Also:
PCA, SparsePCA, DictionaryLearning

Attributes

|(components_, array, [n_components, n_features]) | Sparse components extracted from the data. |
|error_ | array | Vector of errors at each iteration. |

Methods

| fit(X, y) | Fit the model from data in X. |
| fit_transform(X, y) | Fit to data, then transform it |
| get_params([deep]) | Get parameters for the estimator |
| set_params(**params) | Set the parameters of the estimator. |
| transform(X[, ridge_alpha]) | Least Squares projection of the data onto the sparse components. |

__init__ (n_components, alpha=1, ridge_alpha=0.01, n_iter=100, callback=None, chunk_size=3, verbose=False, shuffle=True, n_jobs=1, method='lars', random_state=None)

fit (X, y=None)
Fit the model from data in X.

Parameters X: array-like, shape (n_samples, n_features)
Training vector, where n_samples in the number of samples and n_features is the number of features.

Returns self : object
Returns the instance itself.
fit_transform \((X, y=None, **fit_params)\)

Fit to data, then transform it

Fits transformer to X and y with optional parameters fit_params and returns a transformed version of X.

Parameters

- **X**: numpy array of shape \([n_{samples}, n_{features}]\)
  
  Training set.

- **y**: numpy array of shape \([n_{samples}]\)
  
  Target values.

Returns **X_new**: numpy array of shape \([n_{samples}, n_{features\_new}]\)

Transformed array.

Notes

This method just calls fit and transform consecutively, i.e., it is not an optimized implementation of fit_transform, unlike other transformers such as PCA.

get_params \((deep=True)\)

Get parameters for the estimator

Parameters

- **deep**: boolean, optional

  If True, will return the parameters for this estimator and contained subobjects that are estimators.

set_params \(**params\)

Set the parameters of the estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The former have parameters of the form \(<\text{component}>__<\text{parameter}>\) so that it's possible to update each component of a nested object.

Returns **self**

transform \((X, ridge\_alpha=None)\)

Least Squares projection of the data onto the sparse components.

To avoid instability issues in case the system is under-determined, regularization can be applied (Ridge regression) via the ridge_alpha parameter.

Note that Sparse PCA components orthogonality is not enforced as in PCA hence one cannot use a simple linear projection.

Parameters

- **X**: array of shape \((n_{samples}, n_{features})\)

  Test data to be transformed, must have the same number of features as the data used to train the model.

- **ridge_alpha**: float, default: 0.01

  Amount of ridge shrinkage to apply in order to improve conditioning.

Returns **X_new** array, shape \((n_{samples}, n_{components})\)

Transformed data.
sklearn.decomposition.SparseCoder

class sklearn.decomposition.SparseCoder(dictionary, transform_algorithm='omp',
transform_n_nonzero_coefs=None, transform_alpha=None, split_sign=False, n_jobs=1)

Sparse coding

Finds a sparse representation of data against a fixed, precomputed dictionary.

Each row of the result is the solution to a sparse coding problem. The goal is to find a sparse array `code` such that:

\[ X \approx \text{code} \times \text{dictionary} \]

**Parameters**

- **dictionary**: array, [n_atoms, n_features]
  - The dictionary atoms used for sparse coding. Lines are assumed to be normalized to unit norm.

- **transform_algorithm**: {'lasso_lars', 'lasso_cd', 'lars', 'omp', 'threshold'}
  - Algorithm used to transform the data: lars: uses the least angle regression method (linear_model.lars_path) lasso_lars: uses Lars to compute the Lasso solution lasso_cd: uses the coordinate descent method to compute the Lasso solution (linear_model.Lasso). lasso_lars will be faster if the estimated components are sparse. omp: uses orthogonal matching pursuit to estimate the sparse solution threshold: squashes to zero all coefficients less than alpha from the projection `dictionary \times X'`

- **transform_n_nonzero_coefs**: int, 0.1 * n_features by default
  - Number of nonzero coefficients to target in each column of the solution. This is only used by `algorithm='lars'` and `algorithm='omp'` and is overridden by `alpha` in the `omp` case.

- **transform_alpha**: float, 1. by default
  - If `algorithm='lasso_lars'` or `algorithm='lasso_cd'`, `alpha` is the penalty applied to the L1 norm. If `algorithm='threshold'`, `alpha` is the absolute value of the threshold below which coefficients will be squashed to zero. If `algorithm='omp'`, `alpha` is the tolerance parameter: the value of the reconstruction error targeted. In this case, it overrides `n_nonzero_coefs`.

- **split_sign**: bool, False by default
  - Whether to split the sparse feature vector into the concatenation of its negative part and its positive part. This can improve the performance of downstream classifiers.

- **n_jobs**: int
  - number of parallel jobs to run

**See Also:**

DictionaryLearning, MiniBatchDictionaryLearning, SparsePCA, MiniBatchSparsePCA, sparse_encode

**Attributes**

- **components_**: array, [n_atoms, n_features]
  - The unchanged dictionary atoms
Methods

<table>
<thead>
<tr>
<th>Method</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>fit(X[, y])</td>
<td>Do nothing and return the estimator unchanged</td>
</tr>
<tr>
<td>fit_transform(X[, y])</td>
<td>Fit to data, then transform it</td>
</tr>
<tr>
<td>get_params([deep])</td>
<td>Get parameters for the estimator</td>
</tr>
<tr>
<td>set_params(**params)</td>
<td>Set the parameters of the estimator.</td>
</tr>
<tr>
<td>transform(X[, y])</td>
<td>Encode the data as a sparse combination of the dictionary atoms.</td>
</tr>
</tbody>
</table>

__init__(dictionary, transform_algorithm='omp', transform_n_nonzero_coefs=None, transform_alpha=None, split_sign=False, n_jobs=1)

fit (X, y=None)
Do nothing and return the estimator unchanged
This method is just there to implement the usual API and hence work in pipelines.

fit_transform (X, y=None, **fit_params)
Fit to data, then transform it
Fits transformer to X and y with optional parameters fit_params and returns a transformed version of X.

Parameters X : numpy array of shape [n_samples, n_features]
Training set.
y : numpy array of shape [n_samples]
Target values.

Returns X_new : numpy array of shape [n_samples, n_features_new]
Transformed array.

Notes

This method just calls fit and transform consecutively, i.e., it is not an optimized implementation of fit_transform, unlike other transformers such as PCA.

get_params (deep=True)
Get parameters for the estimator

Parameters deep: boolean, optional :
If True, will return the parameters for this estimator and contained subobjects that are estimators.

set_params (**params)
Set the parameters of the estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The former have parameters of the form <component>__<parameter> so that it's possible to update each component of a nested object.

Returns self :

transform (X, y=None)
Encode the data as a sparse combination of the dictionary atoms.

Coding method is determined by the object parameter transform_algorithm.

Parameters X : array of shape (n_samples, n_features)
Test data to be transformed, must have the same number of features as the data used to train the model.

Returns X_new : array, shape (n_samples, n_components)
Transformed data

sklearn.decomposition.DictionaryLearning

class sklearn.decomposition.DictionaryLearning(n_atoms=1, alpha=1, max_iter=1000, tol=1e-08, fit_algorithm='lars', transform_algorithm='omp', transform_n_nonzero_coefs=None, transform_alpha=None, n_jobs=1, code_init=None, dict_init=None, verbose=False, split_sign=False, random_state=None)

Dictionary learning

Finds a dictionary (a set of atoms) that can best be used to represent data using a sparse code.

Solves the optimization problem:

\[(U^*, V^*) = \arg\min_{(U, V)} 0.5 \| Y - U V \|_2^2 + \alpha \| U \|_1\]

with \( \| V_k \|_2 = 1 \) for all \( 0 \leq k < n\_atoms \)

Parameters n_atoms : int,
number of dictionary elements to extract

alpha : int,
sparsity controlling parameter

max_iter : int,
maximum number of iterations to perform

tol : float,
tolerance for numerical error

fit_algorithm : ['lars', 'cd']

lars: uses the least angle regression method to solve the lasso problem (linear_model.lars_path) cd: uses the coordinate descent method to compute the Lasso solution (linear_model.Lasso). Lars will be faster if the estimated components are sparse.

transform_algorithm : ['lasso_lars', 'lasso_cd', 'lars', 'omp', 'threshold']

Algorithm used to transform the data lars: uses the least angle regression method (linear_model.lars_path) lasso_lars: uses Lars to compute the Lasso solution lasso_cd: uses the coordinate descent method to compute the Lasso solution (linear_model.Lasso). lasso_lars will be faster if the estimated components are sparse. omp: uses orthogonal matching pursuit to estimate the sparse solution threshold: squashes to zero all coefficients less than alpha from the projection dictionary * X

transform_n_nonzero_coefs : int, 0.1 * n_features by default
Number of nonzero coefficients to target in each column of the solution. This is only used by algorithm='lars' and algorithm='omp' and is overridden by alpha in the omp case.

**transform_alpha** : float, 1. by default

If algorithm='lasso_lars' or algorithm='lasso_cd', alpha is the penalty applied to the L1 norm. If algorithm='threshold', alpha is the absolute value of the threshold below which coefficients will be squashed to zero. If algorithm='omp', alpha is the tolerance parameter: the value of the reconstruction error targeted. In this case, it overrides n_nonzero_coefs.

**split_sign** : bool, False by default

Whether to split the sparse feature vector into the concatenation of its negative part and its positive part. This can improve the performance of downstream classifiers.

**n_jobs** : int,

number of parallel jobs to run

**code_init** : array of shape (n_samples, n_atoms),

initial value for the code, for warm restart

**dict_init** : array of shape (n_atoms, n_features),

initial values for the dictionary, for warm restart

**verbose** : :

degree of verbosity of the printed output

**random_state** : int or RandomState

Pseudo number generator state used for random sampling.

See Also:

SparseCoder, MiniBatchDictionaryLearning, SparsePCA, MiniBatchSparsePCA

Notes

References:


Attributes

| components | array, [n_atoms, n_features] | dictionary atoms extracted from the data |
| error_ | array | vector of errors at each iteration |

Methods

```python
fit(X[, y]) Fit the model from data in X.
fit_transform(X[, y]) Fit to data, then transform it
```

Continued on next page
Table 1.55 – continued from previous page

<table>
<thead>
<tr>
<th>Method</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>get_params([deep])</code></td>
<td>Get parameters for the estimator</td>
</tr>
<tr>
<td><code>set_params(**params)</code></td>
<td>Set the parameters of the estimator.</td>
</tr>
<tr>
<td><code>transform(X[, y])</code></td>
<td>Encode the data as a sparse combination of the dictionary atoms.</td>
</tr>
</tbody>
</table>

__init__(n_atoms, alpha=1, max_iter=1000, tol=1e-08, fit_algorithm='lars', transform_algorithm='omp', transform_n_nonzero_coefs=None, transform_alpha=None, n_jobs=1, code_init=None, dict_init=None, verbose=False, split_sign=False, random_state=None)

`fit(X, y=None)`
Fit the model from data in X.

**Parameters**
- **X**: array-like, shape (n_samples, n_features)
  - Training vector, where n_samples is the number of samples and n_features is the number of features.

**Returns**
- **self**: object
  - Returns the object itself

`fit_transform(X, y=None, **fit_params)`
Fit to data, then transform it
Fits transformer to X and y with optional parameters fit_params and returns a transformed version of X.

**Parameters**
- **X**: numpy array of shape [n_samples, n_features]
  - Training set.
- **y**: numpy array of shape [n_samples]
  - Target values.

**Returns**
- **X_new**: numpy array of shape [n_samples, n_features_new]
  - Transformed array.

**Notes**

This method just calls fit and transform consecutively, i.e., it is not an optimized implementation of `fit_transform`, unlike other transformers such as PCA.

`get_params (deep=True)`
Get parameters for the estimator

**Parameters**
- **deep**: boolean, optional
  - If True, will return the parameters for this estimator and contained subobjects that are estimators.

`set_params(**params)`
Set the parameters of the estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The former have parameters of the form `<component>__<parameter>` so that it’s possible to update each component of a nested object.

**Returns**
- **self**: object
**transform** *(X, y=None)*

Encode the data as a sparse combination of the dictionary atoms.

Coding method is determined by the object parameter `transform_algorithm`.

**Parameters**

- **X**: array of shape (n_samples, n_features)
  - Test data to be transformed, must have the same number of features as the data used to train the model.

**Returns**

- **X_new**: array, shape (n_samples, n_components)
  - Transformed data

---

### sklearn.decomposition.MiniBatchDictionaryLearning

**class sklearn.decomposition.MiniBatchDictionaryLearning** *(n_atoms, alpha=1, n_iter=1000, fit_algorithm='lars', n_jobs=1, chunk_size=3, shuffle=True, dict_init=None, transform_algorithm='omp', transform_n_nonzero_coefs=None, transform_alpha=None, verbose=False, split_sign=False, random_state=None)*

Mini-batch dictionary learning

Finds a dictionary (a set of atoms) that can best be used to represent data using a sparse code.

Solves the optimization problem:

\[
(U^*, V^*) = \arg\min_{(U, V)} 0.5 \| Y - U V \|_2^2 + \alpha \| U \|_1 \\
\text{with} \quad \| V_k \|_2 = 1 \text{ for all } 0 \leq k < n_{\text{atoms}}
\]

**Parameters**

- **n_atoms**: int,
  - number of dictionary elements to extract

- **alpha**: int,
  - sparsity controlling parameter

- **n_iter**: int,
  - total number of iterations to perform

- **fit_algorithm**: ['lars', 'cd']
  - lars: uses the least angle regression method to solve the lasso problem (linear_model.lars_path) cd: uses the coordinate descent method to compute the Lasso solution (linear_model.Lasso). Lars will be faster if the estimated components are sparse.

- **transform_algorithm**: ['lasso_lars', 'lasso_cd', 'lars', 'omp', 'threshold']
  - Algorithm used to transform the data. lars: uses the least angle regression method (linear_model.lars_path) lasso_lars: uses Lars to compute the Lasso solution lasso_cd: uses the coordinate descent method to compute the Lasso solution (linear_model.Lasso). lasso_lars will be faster if the estimated components are sparse. omp: uses orthogonal...
matching pursuit to estimate the sparse solution threshold: squashes to zero all coefficients less than alpha from the projection dictionary \( * X' \)

**transform_n_nonzero_coefs**: int, 0.1 * n_features by default

Number of nonzero coefficients to target in each column of the solution. This is only used by algorithm='lars' and algorithm='omp' and is overridden by alpha in the omp case.

**transform_alpha**: float, 1. by default

If algorithm='lasso_lars' or algorithm='lasso_cd', alpha is the penalty applied to the L1 norm. If algorithm='threshold', alpha is the absolute value of the threshold below which coefficients will be squashed to zero. If algorithm='omp', alpha is the tolerance parameter: the value of the reconstruction error targeted. In this case, it overrides n_nonzero_coefs.

**split_sign**: bool, False by default

Whether to split the sparse feature vector into the concatenation of its negative part and its positive part. This can improve the performance of downstream classifiers.

**n_jobs**: int,

number of parallel jobs to run

**dict_init**: array of shape (n_atoms, n_features),

initial value of the dictionary for warm restart scenarios

**verbose**: :

degree of verbosity of the printed output

**chunk_size**: int,

number of samples in each mini-batch

**shuffle**: bool,

whether to shuffle the samples before forming batches

**random_state**: int or RandomState

Pseudo number generator state used for random sampling.

See Also:

SparseCoder, DictionaryLearning, SparsePCA, MiniBatchSparsePCA

Notes

References:


Attributes

| components_ | array, [n_atoms, n_features] | components extracted from the data |
Methods

<table>
<thead>
<tr>
<th>Method</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>fit(X[, y])</code></td>
<td>Fit the model from data in X.</td>
</tr>
<tr>
<td><code>fit_transform(X[, y])</code></td>
<td>Fit to data, then transform it</td>
</tr>
<tr>
<td><code>get_params([deep])</code></td>
<td>Get parameters for the estimator</td>
</tr>
<tr>
<td><code>partial_fit(X[, y, iter_offset])</code></td>
<td>Updates the model using the data in X as a mini-batch.</td>
</tr>
<tr>
<td><code>set_params(**params)</code></td>
<td>Set the parameters of the estimator.</td>
</tr>
<tr>
<td><code>transform(X[, y])</code></td>
<td>Encode the data as a sparse combination of the dictionary atoms.</td>
</tr>
</tbody>
</table>

__init__ (``n_atoms, alpha=1, n_iter=1000, fit_algorithm='lars', n_jobs=1, chunk_size=3, shuffle=True, dict_init=None, transform_algorithm='omp', transform_n_nonzero_coefs=None, transform_alpha=None, verbose=False, split_sign=False, random_state=None``)

**fit (``X=\text{None}``)**

Fit the model from data in X.

Parameters **X**: array-like, shape (``n_samples, n_features``)

Training vector, where n_samples in the number of samples and n_features is the number of features.

Returns **self** : object

Returns the instance itself.

**fit_transform (``X=\text{None}, **fit_params``)**

Fit to data, then transform it

Fits transformer to X and y with optional parameters fit_params and returns a transformed version of X.

Parameters **X** : numpy array of shape [``n_samples, n_features``]

Training set.

``y`` : numpy array of shape [``n_samples``]

Target values.

Returns **X_new** : numpy array of shape [``n_samples, n_features_new``]

Transformed array.

Notes

This method just calls fit and transform consecutively, i.e., it is not an optimized implementation of fit_transform, unlike other transformers such as PCA.

**get_params (``deep=True``)**

Get parameters for the estimator

Parameters **deep**: boolean, optional

If True, will return the parameters for this estimator and contained subobjects that are estimators.

**partial_fit (``X=\text{None}, iter_offset=0``)**

Updates the model using the data in X as a mini-batch.

Parameters **X**: array-like, shape (``n_samples, n_features``)
Training vector, where n_samples in the number of samples and n_features is the number of features.

**Returns self**: object

Returns the instance itself.

```python
set_params (**params)
```

Set the parameters of the estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The former have parameters of the form `<component>__<parameter>` so that it’s possible to update each component of a nested object.

**Returns self**

```python
transform (X, y=None)
```

Encode the data as a sparse combination of the dictionary atoms.

Coding method is determined by the object parameter `transform_algorithm`.

**Parameters**

- **X**: array of shape (n_samples, n_features)
  
  Test data to be transformed, must have the same number of features as the data used to train the model.

**Returns**

- **X_new**: array, shape (n_samples, n_components)
  
  Transformed data

<table>
<thead>
<tr>
<th>Function</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>decomposition.fastica(X[, n_components, ...])</td>
<td>Perform Fast Independent Component Analysis.</td>
</tr>
<tr>
<td>decomposition.dict_learning(X, n_atoms, alpha)</td>
<td>Solves a dictionary learning matrix factorization problem.</td>
</tr>
<tr>
<td>decomposition.dict_learning_online(X, ...[, ...])</td>
<td>Solves a dictionary learning matrix factorization problem online.</td>
</tr>
<tr>
<td>decomposition.sparse_encode(X, dictionary[, ...])</td>
<td>Sparse coding</td>
</tr>
</tbody>
</table>

**sklearn.decomposition.fastica**

```python
sklearn.decomposition.fastica(X, n_components=None, algorithm='parallel', whiten=True, fun='logcosh', fun_prime='', fun_args={}, max_iter=200, tol=0.0001, w_init=None, random_state=None)
```

Perform Fast Independent Component Analysis.

**Parameters**

- **X**: array-like, shape = [n_samples, n_features]
  
  Training vector, where n_samples is the number of samples and n_features is the number of features.

- **n_components**: int, optional
  
  Number of components to extract. If None no dimension reduction is performed.

- **algorithm**: {'parallel', 'deflation'}, optional
  
  Apply a parallel or deflational FASTICA algorithm.

- **whiten**: boolean, optional
  
  If True perform an initial whitening of the data. If False, the data is assumed to have already been preprocessed: it should be centered, normed and white. Otherwise you will get incorrect results. In this case the parameter `n_components` will be ignored.

- **fun**: string or function, optional
  
  The function used to measure the quality of the solution.

The functional form of the G function used in the approximation to neg-entropy. Could be either ‘logcosh’, ‘exp’, or ‘cube’. You can also provide your own function but in this case, its derivative should be provided via argument fun_prime

fun_prime : empty string ("") or function, optional
See fun.

fun_args: dictionary, optional :
If empty and if fun='logcosh', fun_args will take value {'alpha': 1.0}

max_iter: int, optional :
Maximum number of iterations to perform

tol: float, optional :
A positive scalar giving the tolerance at which the un-mixing matrix is considered to have converged

w_init: (n_components, n_components) array, optional :
Initial un-mixing array of dimension (n.comp,n.comp). If None (default) then an array of normal r.v.'s is used

source_only: boolean, optional :
if True, only the sources matrix is returned

random_state: int or RandomState :
Pseudo number generator state used for random sampling.

Returns K: (n_components, p) array or None. :
If whiten is ‘True’, K is the pre-whitening matrix that projects data onto the first n.comp principal components. If whiten is ‘False’, K is ‘None’.

W: (n_components, n_components) array :
estimated un-mixing matrix The mixing matrix can be obtained by:

\[ w = \text{np.dot}(\overline{W}, K.T) \]
\[ A = w.T * (w * w.T).I \]

S: (n_components, n) array :
estimated source matrix

Notes

The data matrix X is considered to be a linear combination of non-Gaussian (independent) components i.e. X = AS where columns of S contain the independent components and A is a linear mixing matrix. In short ICA attempts to un-mix the data by estimating an un-mixing matrix W where ‘S = W K X.’

This implementation was originally made for data of shape [n_features, n_samples]. Now the input is transposed before the algorithm is applied. This makes it slightly faster for Fortran-ordered input.

Solves a dictionary learning matrix factorization problem.

Finds the best dictionary and the corresponding sparse code for approximating the data matrix $X$ by solving:

$$(U^*, V^*) = \arg\min_{(U,V)} 0.5 \| X - U V \|_2^2 + \alpha \| U \|_1$$

with $\| V_k \|_2 = 1$ for all $0 \leq k < n_{\text{atoms}}$

where $V$ is the dictionary and $U$ is the sparse code.

**Parameters**

- **X**: array of shape (n_samples, n_features):
  - Data matrix.
- **n_atoms**: int:
  - Number of dictionary atoms to extract.
- **alpha**: int:
  - Sparsity controlling parameter.
- **max_iter**: int:
  - Maximum number of iterations to perform.
- **tol**: float:
  - Tolerance for the stopping condition.
- **method**: {'lars', 'cd'}:
  - lars: uses the least angle regression method to solve the lasso problem (linear_model.lars_path) cd: uses the coordinate descent method to compute the Lasso solution (linear_model.Lasso). Lars will be faster if the estimated components are sparse.
- **n_jobs**: int:
  - Number of parallel jobs to run, or -1 to autodetect.
- **dict_init**: array of shape (n_atoms, n_features):
  - Initial value for the dictionary for warm restart scenarios.
- **code_init**: array of shape (n_samples, n_atoms):
  - Initial value for the sparse code for warm restart scenarios.
- **callback**: callable
  - Callable that gets invoked every five iterations.
- **verbose**: bool
  - Degree of output the procedure will print.
- **random_state**: int or RandomState:
  - Pseudo number generator state used for random sampling.

**Returns**

- **code**: array of shape (n_samples, n_atoms):
  -
The sparse code factor in the matrix factorization.

**dictionary**: array of shape (n_atoms, n_features), :
The dictionary factor in the matrix factorization.

**errors**: array :
Vector of errors at each iteration.

See Also:

*dict_learning_online*, *DictionaryLearning*, *MiniBatchDictionaryLearning*, *SparsePCA*, *MiniBatchSparsePCA*

```python
sklearn.decomposition.dict_learning_online
```

Solves a dictionary learning matrix factorization problem online.

Finds the best dictionary and the corresponding sparse code for approximating the data matrix X by solving:

$$(U^*, V^*) = \arg\min_{U, V} 0.5 \| X - U V \|_2^2 + \alpha \| U \|_1$$

with \( \| V_k \|_2 = 1 \) for all \( 0 \leq k < n_{atoms} \)

where V is the dictionary and U is the sparse code. This is accomplished by repeatedly iterating over mini-batches by slicing the input data.

**Parameters**

* X: array of shape (n_samples, n_features) :
  data matrix

* n_atoms: int, :
  number of dictionary atoms to extract

* alpha: int, :
  sparsity controlling parameter

* n_iter: int, :
  number of iterations to perform

* return_code: boolean, :
  whether to also return the code U or just the dictionary V

* dict_init: array of shape (n_atoms, n_features), :
  initial value for the dictionary for warm restart scenarios

* callback: :
  callable that gets invoked every five iterations

* chunk_size: int, :
  the number of samples to take in each batch

* verbose: :
degree of output the procedure will print

shuffle: boolean,

whether to shuffle the data before splitting it in batches

n_jobs: int,

number of parallel jobs to run, or -1 to autodetect.

method: {'lars', 'cd'}:

lars: uses the least angle regression method to solve the lasso problem (linear_model.lars_path) cd: uses the coordinate descent method to compute the Lasso solution (linear_model.Lasso). Lars will be faster if the estimated components are sparse.

iter_offset: int, default 0:

number of previous iterations completed on the dictionary used for initialization

random_state: int or RandomState:

Pseudo number generator state used for random sampling.

Returns code: array of shape (n_samples, n_atoms),:

the sparse code (only returned if return_code=True)

dictionary: array of shape (n_atoms, n_features),:

the solutions to the dictionary learning problem

See Also:
dict_learning, DictionaryLearning, MiniBatchDictionaryLearning, SparsePCA, MiniBatchSparsePCA

sklearn.decomposition.sparse_encode

sklearn.decomposition.sparse_encode(X, dictionary, gram=None, cov=None, algorithm='lasso_lars', n_nonzero_coefs=None, alpha=None, copy_gram=None, copy_cov=True, init=None, max_iter=1000, n_jobs=1)

Sparse coding

Each row of the result is the solution to a sparse coding problem. The goal is to find a sparse array code such that:

\[ X \approx \text{code} \times \text{dictionary} \]

Parameters X: array of shape (n_samples, n_features):

Data matrix

dictionary: array of shape (n_atoms, n_features):

The dictionary matrix against which to solve the sparse coding of the data. Some of the algorithms assume normalized rows for meaningful output.

gram: array, shape=(n_atoms, n_atoms):

Precomputed Gram matrix, dictionary \times dictionary'

cov: array, shape=(n_atoms, n_samples):

...
Precomputed covariance, dictionary’ * X


  lars: uses the least angle regression method (linear_model.lars_path) lasso_lars: uses LARS to compute the Lasso solution lasso_cd: uses the coordinate descent method to compute the Lasso solution (linear_model.Lasso). lasso_lars will be faster if the estimated components are sparse. omp: uses orthogonal matching pursuit to estimate the sparse solution threshold: squashes to zero all coefficients less than alpha from the projection dictionary’ * X’

n_nonzero_coefs: int, 0.1 * n_features by default:

  Number of nonzero coefficients to target in each column of the solution. This is only used by algorithm=’lars’ and algorithm=’omp’ and is overridden by alpha in the omp case.

alpha: float, 1. by default:

  If algorithm=’lasso_lars’ or algorithm=’lasso_cd’, alpha is the penalty applied to the L1 norm. If algorithm=’threshold’, alpha is the absolute value of the threshold below which coefficients will be squashed to zero. If algorithm=’omp’, alpha is the tolerance parameter: the value of the reconstruction error targeted. In this case, it overrides n_nonzero_coefs.

init: array of shape (n_samples, n_atoms):

  Initialization value of the sparse codes. Only used if algorithm=’lasso_cd’.

max_iter: int, 1000 by default:

  Maximum number of iterations to perform if algorithm=’lasso_cd’.

copy_cov: boolean, optional:

  Whether to copy the precomputed covariance matrix; if False, it may be overwritten.

n_jobs: int, optional:

  Number of parallel jobs to run.

Returns code: array of shape (n_samples, n_atoms):

  The sparse codes

See Also:

  sklearn.linear_model.lars_path, sklearn.linear_model.orthogonal_mp, sklearn.linear_model.Lasso, SparseCoder

1.8.6 sklearn.ensemble: Ensemble Methods

The sklearn.ensemble module includes ensemble-based methods for classification and regression.

User guide: See the Ensemble methods section for further details.

<table>
<thead>
<tr>
<th>Class</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>ensemble.RandomForestClassifier(...)</td>
<td>A random forest classifier.</td>
</tr>
<tr>
<td>ensemble.RandomForestRegressor(...)</td>
<td>A random forest regressor.</td>
</tr>
<tr>
<td>ensemble.ExtraTreesClassifier(...)</td>
<td>An extra-trees classifier.</td>
</tr>
<tr>
<td>ensemble.GradientBoostingClassifier(...)</td>
<td>Gradient Boosting for classification.</td>
</tr>
</tbody>
</table>
**ensemble.GradientBoostingRegressor**

Gradient Boosting for regression.

**sklearn.ensemble.RandomForestClassifier**

```python
class sklearn.ensemble.RandomForestClassifier(n_estimators=10, criterion='gini', max_depth=None, min_samples_split=1, min_samples_leaf=1, min_density=0.1, max_features='auto', bootstrap=True, compute_importances=False, oob_score=False, n_jobs=1, random_state=None, verbose=0)
```

A random forest classifier.

A random forest is a meta estimator that fits a number of classifical decision trees on various sub-samples of the dataset and use averaging to improve the predictive accuracy and control over-fitting.

**Parameters**

- **n_estimators**: integer, optional (default=10)
  - The number of trees in the forest.

- **criterion**: string, optional (default="gini")
  - The function to measure the quality of a split. Supported criteria are “gini” for the Gini impurity and “entropy” for the information gain. Note: this parameter is tree-specific.

- **max_depth**: integer or None, optional (default=None)
  - The maximum depth of the tree. If None, then nodes are expanded until all leaves are pure or until all leaves contain less than min_samples_split samples. Note: this parameter is tree-specific.

- **min_samples_split**: integer, optional (default=1)
  - The minimum number of samples required to split an internal node. Note: this parameter is tree-specific.

- **min_samples_leaf**: integer, optional (default=1)
  - The minimum number of samples in newly created leaves. A split is discarded if after the split, one of the leaves would contain less then min_samples_leaf samples. Note: this parameter is tree-specific.

- **min_density**: float, optional (default=0.1)
  - This parameter controls a trade-off in an optimization heuristic. It controls the minimum density of the sample_mask (i.e. the fraction of samples in the mask). If the density falls below this threshold the mask is recomputed and the input data is packed which results in data copying. If min_density equals to one, the partitions are always represented as copies of the original data. Otherwise, partitions are represented as bit masks (aka sample masks). Note: this parameter is tree-specific.

- **max_features**: int, string or None, optional (default="auto")
  - The number of features to consider when looking for the best split:
    - If “auto”, then max_features=sqrt(n_features) on classification tasks and max_features=n_features on regression problems.
    - If “sqrt”, then max_features=sqrt(n_features).
    - If “log2”, then max_features=log2(n_features).
    - If None, then max_features=n_features.
Note: this parameter is tree-specific.

**bootstrap**: boolean, optional (default=True)

Whether bootstrap samples are used when building trees.

**compute_importances**: boolean, optional (default=True)

Whether feature importances are computed and stored into the `feature_importances_` attribute when calling `fit`.

**oob_score**: bool

Whether to use out-of-bag samples to estimate the generalization error.

**n_jobs**: integer, optional (default=1)

The number of jobs to run in parallel. If -1, then the number of jobs is set to the number of cores.

**random_state**: int, RandomState instance or None, optional (default=None)

If int, `random_state` is the seed used by the random number generator; If `RandomState` instance, `random_state` is the random number generator; If None, the random number generator is the `RandomState` instance used by `np.random`.

**verbose**: int, optional (default=0)

Controls the verbosity of the tree building process.

**See Also:**

`DecisionTreeClassifier`, `ExtraTreesClassifier`

**References**

[R59]

**Attributes**

<table>
<thead>
<tr>
<th>feature_importances_</th>
<th>array, shape = [n_features]</th>
<th>The feature importances (the higher, the more important the feature).</th>
</tr>
</thead>
<tbody>
<tr>
<td>oob_score_</td>
<td>float</td>
<td>Score of the training dataset obtained using an out-of-bag estimate.</td>
</tr>
<tr>
<td>oob_decision_function_</td>
<td>array, shape = [n_samples, n_classes]</td>
<td>Decision function computed with out-of-bag estimate on the training set.</td>
</tr>
</tbody>
</table>

**Methods**

<table>
<thead>
<tr>
<th>fit(X, y)</th>
<th>Build a forest of trees from the training set (X, y).</th>
</tr>
</thead>
<tbody>
<tr>
<td>fit_transform(X, y)</td>
<td>Fit to data, then transform it</td>
</tr>
<tr>
<td>get_params([deep])</td>
<td>Get parameters for the estimator</td>
</tr>
<tr>
<td>predict(X)</td>
<td>Predict class for X.</td>
</tr>
<tr>
<td>predict_log_proba(X)</td>
<td>Predict class log-probabilities for X.</td>
</tr>
<tr>
<td>predict_proba(X)</td>
<td>Predict class probabilities for X.</td>
</tr>
<tr>
<td>score(X, y)</td>
<td>Returns the mean accuracy on the given test data and labels.</td>
</tr>
</tbody>
</table>
Table 1.59 – continued from previous page

<table>
<thead>
<tr>
<th>Method</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>set_params(**params)</code></td>
<td>Set the parameters of the estimator.</td>
</tr>
<tr>
<td><code>transform(X[, threshold])</code></td>
<td>Reduce X to its most important features.</td>
</tr>
</tbody>
</table>

```
__init__ (n_estimators=10, criterion='gini', max_depth=None, min_samples_split=1,
min_samples_leaf=1, min_density=0.1, max_features='auto', bootstrap=True,
compute_importances=False, oob_score=False, n_jobs=1, random_state=None,
verbose=0)
```

```
fit (X, y)
```
Build a forest of trees from the training set (X, y).

Parameters

**X**: array-like of shape = [n_samples, n_features]
The training input samples.

**y**: array-like, shape = [n_samples]
The target values (integers that correspond to classes in classification, real numbers in regression).

Returns

**self**: object

```
fit_transform (X, y=None, **fit_params)
```
Fit to data, then transform it

Fits transformer to X and y with optional parameters fit_params and returns a transformed version of X.

Parameters

**X**: numpy array of shape [n_samples, n_features]
Training set.

**y**: numpy array of shape [n_samples]
Target values.

Returns

**X_new**: numpy array of shape [n_samples, n_features_new]
Transformed array.

Notes

This method just calls fit and transform consecutively, i.e., it is not an optimized implementation of fit_transform, unlike other transformers such as PCA.

```
get_params (deep=True)
```
Get parameters for the estimator

Parameters

**deep**: boolean, optional
If True, will return the parameters for this estimator and contained subobjects that are estimators.

```
predict (X)
```
Predict class for X.

The predicted class of an input sample is computed as the majority prediction of the trees in the forest.

Parameters

**X**: array-like of shape = [n_samples, n_features]
The input samples.

Returns

**y**: array of shape = [n_samples]
The predicted classes.

**predict_log_proba** (*X*)
Predict class log-probabilities for *X*.

The predicted class log-probabilities of an input sample is computed as the mean predicted class log-probabilities of the trees in the forest.

**Parameters**

- **X**: array-like of shape = [n_samples, n_features]
  - The input samples.

**Returns**

- **p**: array of shape = [n_samples]
  - The class log-probabilities of the input samples. Classes are ordered by arithmetical order.

**predict_proba** (*X*)
Predict class probabilities for *X*.

The predicted class probabilities of an input sample is computed as the mean predicted class probabilities of the trees in the forest.

**Parameters**

- **X**: array-like of shape = [n_samples, n_features]
  - The input samples.

**Returns**

- **p**: array of shape = [n_samples]
  - The class probabilities of the input samples. Classes are ordered by arithmetical order.

**score** (*X, y*)
Returns the mean accuracy on the given test data and labels.

**Parameters**

- **X**: array-like, shape = [n_samples, n_features]
  - Training set.
- **y**: array-like, shape = [n_samples]
  - Labels for *X*.

**Returns**

- **z**: float

**set_params** (**params**)
Set the parameters of the estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The former have parameters of the form `<component>__<parameter>` so that it’s possible to update each component of a nested object.

**Returns**

- **self**: 

**transform** (*X, threshold=None*)
Reduce *X* to its most important features.

**Parameters**

- **X**: array or scipy sparse matrix of shape [n_samples, n_features]
  - The input samples.
- **threshold**: string, float or None, optional (default=None)
  - The threshold value to use for feature selection. Features whose importance is greater or equal are kept while the others are discarded. If “median” (resp. “mean”), then the threshold value is the median (resp. the mean) of the feature importances. A scaling factor (e.g., “1.25*mean”) may also be used. If None and if available, the object attribute `threshold` is used. Otherwise, “mean” is used by default.
**Returns** $X_r$ : array of shape [n_samples, n_selected_features]

The input samples with only the selected features.

```python
sklearn.ensemble.RandomForestRegressor
```

class sklearn.ensemble.RandomForestRegressor (n_estimators=10, criterion='mse', max_depth=None, min_samples_split=1, min_samples_leaf=1, min_density=0.1, max_features='auto', bootstrap=True, compute_importances=False, oob_score=False, n_jobs=1, random_state=None, verbose=0)

A random forest regressor.

A random forest is a meta estimator that fits a number of classification decision trees on various sub-samples of the dataset and uses averaging to improve the predictive accuracy and control over-fitting.

**Parameters**

**n_estimators** : integer, optional (default=10)

The number of trees in the forest.

**criterion** : string, optional (default="mse")

The function to measure the quality of a split. The only supported criterion is “mse” for the mean squared error. Note: this parameter is tree-specific.

**max_depth** : integer or None, optional (default=None)

The maximum depth of the tree. If None, then nodes are expanded until all leaves are pure or until all leaves contain less than min_samples_split samples. Note: this parameter is tree-specific.

**min_samples_split** : integer, optional (default=1)

The minimum number of samples required to split an internal node. Note: this parameter is tree-specific.

**min_samples_leaf** : integer, optional (default=1)

The minimum number of samples in newly created leaves. A split is discarded if after the split, one of the leaves would contain less then min_samples_leaf samples. Note: this parameter is tree-specific.

**min_density** : float, optional (default=0.1)

This parameter controls a trade-off in an optimization heuristic. It controls the minimum density of the sample_mask (i.e. the fraction of samples in the mask). If the density falls below this threshold the mask is recomputed and the input data is packed which results in data copying. If min_density equals to one, the partitions are always represented as copies of the original data. Otherwise, partitions are represented as bit masks (aka sample masks). Note: this parameter is tree-specific.

**max_features** : int, string or None, optional (default="auto")

The number of features to consider when looking for the best split:

- If “auto”, then max_features=sqrt(n_features) on classification tasks and max_features=n_features on regression problems.
- If “sqrt”, then max_features=sqrt(n_features).
- If “log2”, then max_features=log2(n_features).
If None, then *max_features=n_features*.

Note: this parameter is tree-specific.

**bootstrap** : boolean, optional (default=True)

Whether bootstrap samples are used when building trees.

**compute_importances** : boolean, optional (default=True)

Whether feature importances are computed and stored into the *feature_importances_* attribute when calling *fit*.

**oob_score** : bool

whether to use out-of-bag samples to estimate the generalization error.

**n_jobs** : integer, optional (default=1)

The number of jobs to run in parallel. If -1, then the number of jobs is set to the number of cores.

**random_state** : int, RandomState instance or None, optional (default=None)

If int, random_state is the seed used by the random number generator; If RandomState instance, random_state is the random number generator; If None, the random number generator is the RandomState instance used by *np.random*.

**verbose** : int, optional (default=0)

Controlls the verbosity of the tree building process.

**See Also:**

*DecisionTreeRegressor, ExtraTreesRegressor*

**References**

[R60]

**Attributes**

<table>
<thead>
<tr>
<th>feature_importances_</th>
<th>array of shape = [n_features]</th>
<th>The feature importances (the higher, the more important the feature).</th>
</tr>
</thead>
<tbody>
<tr>
<td>oob_score_</td>
<td>float</td>
<td>Score of the training dataset obtained using an out-of-bag estimate.</td>
</tr>
<tr>
<td>oob_prediction_</td>
<td>array, shape = [n_samples]</td>
<td>Prediction computed with out-of-bag estimate on the training set.</td>
</tr>
</tbody>
</table>

**Methods**

<table>
<thead>
<tr>
<th>fit(X, y)</th>
<th>Build a forest of trees from the training set (X, y).</th>
</tr>
</thead>
<tbody>
<tr>
<td>fit_transform(X[, y])</td>
<td>Fit to data, then transform it</td>
</tr>
<tr>
<td>get_params([deep])</td>
<td>Get parameters for the estimator</td>
</tr>
<tr>
<td>predict(X)</td>
<td>Predict regression target for X.</td>
</tr>
<tr>
<td>score(X, y)</td>
<td>Returns the coefficient of determination $R^2$ of the prediction.</td>
</tr>
</tbody>
</table>
Table 1.60 – continued from previous page

<table>
<thead>
<tr>
<th>Method</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>set_params(**params)</td>
<td>Set the parameters of the estimator.</td>
</tr>
<tr>
<td>transform(X[, threshold])</td>
<td>Reduce X to its most important features.</td>
</tr>
</tbody>
</table>

```python
__init__(n_estimators=10, criterion='mse', max_depth=None, min_samples_split=1,
        min_samples_leaf=1, min_density=0.1, max_features='auto', bootstrap=True,
        compute_importances=False, oob_score=False, n_jobs=1, random_state=None, verbose=0)
```

```python
fit (X, y)
```
Build a forest of trees from the training set (X, y).

- **Parameters**
  - X : array-like of shape = [n_samples, n_features]
    The training input samples.
  - y : array-like, shape = [n_samples]
    The target values (integers that correspond to classes in classification, real numbers in regression).

- **Returns**
  - self : object
    Returns self.

```python
fit_transform (X, y=None, **fit_params)
```
Fits transformer to X and y with optional parameters fit_params and returns a transformed version of X.

- **Parameters**
  - X : numpy array of shape [n_samples, n_features]
    Training set.
  - y : numpy array of shape [n_samples]
    Target values.

- **Returns**
  - X_new : numpy array of shape [n_samples, n_features_new]
    Transformed array.

**Notes**
This method just calls fit and transform consecutively, i.e., it is not an optimized implementation of fit_transform, unlike other transformers such as PCA.

```python
get_params (deep=True)
```
Get parameters for the estimator

- **Parameters**
  - deep : boolean, optional
    If True, will return the parameters for this estimator and contained subobjects that are estimators.

```python
predict (X)
```
Predict regression target for X.

- **Parameters**
  - X : array-like of shape = [n_samples, n_features]
    The input samples.
Returns y: array of shape = [n_samples] :

The predicted values.

score (X, y)

Returns the coefficient of determination $R^2$ of the prediction.

The coefficient $R^2$ is defined as $(1 - u/v)$, where $u$ is the regression sum of squares $((y - y\_pred) ** 2).sum()$ and $v$ is the residual sum of squares $((y\_true - y\_true.mean()) ** 2).sum()$. Best possible score is 1.0, lower values are worse.

Parameters X : array-like, shape = [n_samples, n_features]

Training set.

y : array-like, shape = [n_samples]

Returns z : float

set_params (**params)

Set the parameters of the estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The former have parameters of the form <component>__<parameter> so that it’s possible to update each component of a nested object.

Returns self :

transform (X, threshold=None)

Reduce X to its most important features.

Parameters X : array or scipy sparse matrix of shape [n_samples, n_features]

The input samples.

threshold : string, float or None, optional (default=None)

The threshold value to use for feature selection. Features whose importance is greater or equal are kept while the others are discarded. If “median” (resp. “mean”), then the threshold value is the median (resp. the mean) of the feature importances. A scaling factor (e.g., “1.25*mean”) may also be used. If None and if available, the object attribute threshold is used. Otherwise, “mean” is used by default.

Returns X_r : array of shape [n_samples, n_selected_features]

The input samples with only the selected features.

sklearn.ensemble.ExtraTreesClassifier

class sklearn.ensemble.ExtraTreesClassifier (n_estimators=10, criterion='gini', max_depth=None, min_samples_split=1, min_samples_leaf=1, min_density=0.1, max_features='auto', bootstrap=False, compute_importances=False, oob_score=False, n_jobs=1, random_state=None, verbose=0)

An extra-trees classifier.

This class implements a meta estimator that fits a number of randomized decision trees (a.k.a. extra-trees) on various sub-samples of the dataset and use averaging to improve the predictive accuracy and control over-fitting.

Parameters n_estimators : integer, optional (default=10)

The number of trees in the forest.
**criterion**: string, optional (default=”gini”)

The function to measure the quality of a split. Supported criteria are “gini” for the Gini impurity and “entropy” for the information gain. Note: this parameter is tree-specific.

**max_depth**: integer or None, optional (default=None)

The maximum depth of the tree. If None, then nodes are expanded until all leaves are pure or until all leaves contain less than min_samples_split samples. Note: this parameter is tree-specific.

**min_samples_split**: integer, optional (default=1)

The minimum number of samples required to split an internal node. Note: this parameter is tree-specific.

**min_samples_leaf**: integer, optional (default=1)

The minimum number of samples in newly created leaves. A split is discarded if after the split, one of the leaves would contain less than min_samples_leaf samples. Note: this parameter is tree-specific.

**min_density**: float, optional (default=0.1)

This parameter controls a trade-off in an optimization heuristic. It controls the minimum density of the sample_mask (i.e. the fraction of samples in the mask). If the density falls below this threshold the mask is recomputed and the input data is packed which results in data copying. If min_density equals to one, the partitions are always represented as copies of the original data. Otherwise, partitions are represented as bit masks (aka sample masks). Note: this parameter is tree-specific.

**max_features**: int, string or None, optional (default=”auto”)

The number of features to consider when looking for the best split.

- If “auto”, then max_features=sqrt(n_features) on classification tasks and max_features=n_features on regression problems.
- If “sqrt”, then max_features=sqrt(n_features).
- If “log2”, then max_features=log2(n_features).
- If None, then max_features=n_features.

Note: this parameter is tree-specific.

**bootstrap**: boolean, optional (default=False)

Whether bootstrap samples are used when building trees.

**compute_importances**: boolean, optional (default=True)

Whether feature importances are computed and stored into the feature_importances_ attribute when calling fit.

**oob_score**: bool

Whether to use out-of-bag samples to estimate the generalization error.

**n_jobs**: integer, optional (default=1)

The number of jobs to run in parallel. If -1, then the number of jobs is set to the number of cores.

**random_state**: int, RandomState instance or None, optional (default=None)
If int, random_state is the seed used by the random number generator; If RandomState instance, random_state is the random number generator; If None, the random number generator is the RandomState instance used by np.random.

verbose : int, optional (default=0)

Controls the verbosity of the tree building process.

See Also:

sklearn.tree.ExtraTreeClassifierBase classifier for this ensemble.
RandomForestClassifierEnsemble Classifier based on trees with optimal splits.

References

[R57]

Attributes

<table>
<thead>
<tr>
<th>feature_importances_</th>
<th>array of shape = [n_features]</th>
<th>The feature importances (the higher, the more important the feature).</th>
</tr>
</thead>
<tbody>
<tr>
<td>oob_score_</td>
<td>float</td>
<td>Score of the training dataset obtained using an out-of-bag estimate.</td>
</tr>
<tr>
<td>oob_decision_function_</td>
<td>array, shape = [n_samples, n_classes]</td>
<td>Decision function computed with out-of-bag estimate on the training set.</td>
</tr>
</tbody>
</table>

Methods

- **fit**(X, y) Build a forest of trees from the training set (X, y).
- **fit_transform**(X[ , y]) Fit to data, then transform it
- **get_params**(deep) Get parameters for the estimator
- **predict**(X) Predict class for X.
- **predict_log_proba**(X) Predict class log-probabilities for X.
- **predict_proba**(X) Predict class probabilities for X.
- **score**(X, y) Returns the mean accuracy on the given test data and labels.
- **set_params**(**params**) Set the parameters of the estimator.
- **transform**(X[, threshold]) Reduce X to its most important features.

__init__ (n_estimators=10, criterion='gini', max_depth=None, min_samples_split=1, min_samples_leaf=1, min_density=0.1, max_features='auto', compute_importances=False, oob_score=False, n_jobs=1, random_state=None, verbose=0)

**fit**(X, y)

Build a forest of trees from the training set (X, y).

**Parameters**

X : array-like of shape = [n_samples, n_features]

The training input samples.

y : array-like, shape = [n_samples]

The target values (integers that correspond to classes in classification, real numbers in regression).
Returns self : object

Returns self.

**fit_transform** *(X, y=None, **fit_params)*

Fit to data, then transform it

Fits transformer to X and y with optional parameters fit_params and returns a transformed version of X.

**Parameters**

- **X** : numpy array of shape [n_samples, n_features]
  
  Training set.

- **y** : numpy array of shape [n_samples]
  
  Target values.

**Returns**

- **X_new** : numpy array of shape [n_samples, n_features_new]
  
  Transformed array.

**Notes**

This method just calls fit and transform consecutively, i.e., it is not an optimized implementation of fit_transform, unlike other transformers such as PCA.

**get_params** *(deep=True)*

Get parameters for the estimator

**Parameters**

- **deep** : boolean, optional
  
  If True, will return the parameters for this estimator and contained subobjects that are estimators.

**predict** *(X)*

Predict class for X.

The predicted class of an input sample is computed as the majority prediction of the trees in the forest.

**Parameters**

- **X** : array-like of shape = [n_samples, n_features]
  
  The input samples.

**Returns**

- **y** : array of shape = [n_samples]
  
  The predicted classes.

**predict_log_proba** *(X)*

Predict class log-probabilities for X.

The predicted class log-probabilities of an input sample is computed as the mean predicted class log-probabilities of the trees in the forest.

**Parameters**

- **X** : array-like of shape = [n_samples, n_features]
  
  The input samples.

**Returns**

- **p** : array of shape = [n_samples]
  
  The class log-probabilities of the input samples. Classes are ordered by arithmetical order.

**predict_proba** *(X)*

Predict class probabilities for X.
The predicted class probabilities of an input sample is computed as the mean predicted class probabilities of the trees in the forest.

**Parameters**

- **X**: array-like of shape = [n_samples, n_features]
  - The input samples.

**Returns**

- **p**: array of shape = [n_samples]
  - The class probabilities of the input samples. Classes are ordered by arithmetical order.

**score** *(X, y)*

Returns the mean accuracy on the given test data and labels.

**Parameters**

- **X**: array-like, shape = [n_samples, n_features]
  - Training set.
- **y**: array-like, shape = [n_samples]
  - Labels for X.

**Returns**

- **z**: float

**set_params** (**params**)

Set the parameters of the estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The former have parameters of the form `<component>__<parameter>` so that it’s possible to update each component of a nested object.

**Returns**

- **self**:

**transform** *(X, threshold=None)*

Reduce X to its most important features.

**Parameters**

- **X**: array or scipy sparse matrix of shape [n_samples, n_features]
  - The input samples.
- **threshold**: string, float or None, optional (default=None)
  - The threshold value to use for feature selection. Features whose importance is greater or equal are kept while the others are discarded. If “median” (resp. “mean”), then the threshold value is the median (resp. the mean) of the feature importances. A scaling factor (e.g., “1.25*mean”) may also be used. If None and if available, the object attribute `threshold` is used. Otherwise, “mean” is used by default.

**Returns**

- **X_r**: array of shape [n_samples, n_selected_features]
  - The input samples with only the selected features.

**sklearn.ensemble.ExtraTreesRegressor**

An extra-trees regressor.

This class implements a meta estimator that fits a number of randomized decision trees (a.k.a. extra-trees) on various sub-samples of the dataset and use averaging to improve the predictive accuracy and control over-fitting.
Parameters n_estimators : integer, optional (default=10)

The number of trees in the forest.

criterion : string, optional (default="mse")

The function to measure the quality of a split. The only supported criterion is “mse” for the mean squared error. Note: this parameter is tree-specific.

max_depth : integer or None, optional (default=None)

The maximum depth of the tree. If None, then nodes are expanded until all leaves are pure or until all leaves contain less than min_samples_split samples. Note: this parameter is tree-specific.

min_samples_split : integer, optional (default=1)

The minimum number of samples required to split an internal node. Note: this parameter is tree-specific.

min_samples_leaf : integer, optional (default=1)

The minimum number of samples in newly created leaves. A split is discarded if after the split, one of the leaves would contain less than min_samples_leaf samples. Note: this parameter is tree-specific.

min_density : float, optional (default=0.1)

This parameter controls a trade-off in an optimization heuristic. It controls the minimum density of the sample_mask (i.e. the fraction of samples in the mask). If the density falls below this threshold the mask is recomputed and the input data is packed which results in data copying. If min_density equals to one, the partitions are always represented as copies of the original data. Otherwise, partitions are represented as bit masks (aka sample masks). Note: this parameter is tree-specific.

max_features : int, string or None, optional (default=”auto”)

The number of features to consider when looking for the best split:

- If “auto”, then max_features=sqrt(n_features) on classification tasks and max_features=n_features on regression problems.
- If “sqrt”, then max_features=sqrt(n_features).
- If “log2”, then max_features=log2(n_features).
- If None, then max_features=n_features.

Note: this parameter is tree-specific.

bootstrap : boolean, optional (default=False)

Whether bootstrap samples are used when building trees. Note: this parameter is tree-specific.

compute_importances : boolean, optional (default=True)

Whether feature importances are computed and stored into the feature_importances_ attribute when calling fit.

oob_score : bool

Whether to use out-of-bag samples to estimate the generalization error.

n_jobs : integer, optional (default=1)
The number of jobs to run in parallel. If -1, then the number of jobs is set to the number of cores.

**random_state** : int, RandomState instance or None, optional (default=None)

If int, random_state is the seed used by the random number generator; If RandomState instance, random_state is the random number generator; If None, the random number generator is the RandomState instance used by `np.random`.

**verbose** : int, optional (default=0)

Controls the verbosity of the tree building process.

**See Also:**

* `sklearn.tree.ExtraTreeRegressor` Base estimator for this ensemble.

* `RandomForestRegressor` Ensemble regressor using trees with optimal splits.

**References**

[R58]

**Attributes**

<table>
<thead>
<tr>
<th>Feature</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>feature_importances_</td>
<td>array of shape = [n_features] The feature importances (the higher, the more important the feature).</td>
</tr>
<tr>
<td>oob_score_</td>
<td>float Score of the training dataset obtained using an out-of-bag estimate.</td>
</tr>
<tr>
<td>oob_prediction_</td>
<td>array, shape = [n_samples] Prediction computed with out-of-bag estimate on the training set.</td>
</tr>
</tbody>
</table>

**Methods**

<table>
<thead>
<tr>
<th>Method</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>fit(X, y)</td>
<td>Build a forest of trees from the training set (X, y).</td>
</tr>
<tr>
<td>fit_transform(X[, y])</td>
<td>Fit to data, then transform it</td>
</tr>
<tr>
<td>get_params([deep])</td>
<td>Get parameters for the estimator</td>
</tr>
<tr>
<td>predict(X)</td>
<td>Predict regression target for X</td>
</tr>
<tr>
<td>score(X, y)</td>
<td>Returns the coefficient of determination R^2 of the prediction.</td>
</tr>
<tr>
<td>set_params(**params)</td>
<td>Set the parameters of the estimator.</td>
</tr>
<tr>
<td>transform(X[, threshold])</td>
<td>Reduce X to its most important features.</td>
</tr>
</tbody>
</table>

**init**

*(n_estimators=10, criterion='mse', max_depth=None, min_samples_split=1, min_samples_leaf=1, min_density=0.1, max_features='auto', bootstrap=False, compute_importances=False, oob_score=False, n_jobs=1, random_state=None, verbose=0)*

**fit** (X, y)

Build a forest of trees from the training set (X, y).

**Parameters**

* **X** : array-like of shape = [n_samples, n_features]

  The training input samples.

* **y** : array-like, shape = [n_samples]
The target values (integers that correspond to classes in classification, real numbers in regression).

**Returns self**: object

Returns self.

**fit_transform** *(X, y=None, **fit_params)*

Fit to data, then transform it

Fits transformer to X and y with optional parameters fit_params and returns a transformed version of X.

**Parameters**

- **X**: numpy array of shape [n_samples, n_features]
  
  Training set.

- **y**: numpy array of shape [n_samples]
  
  Target values.

**Returns**

- **X_new**: numpy array of shape [n_samples, n_features_new]
  
  Transformed array.

**Notes**

This method just calls fit and transform consecutively, i.e., it is not an optimized implementation of fit_transform, unlike other transformers such as PCA.

**get_params** *(deep=True)*

Get parameters for the estimator

**Parameters**

- **deep**: boolean, optional
  
  If True, will return the parameters for this estimator and contained subobjects that are estimators.

**predict** *(X)*

Predict regression target for X.

The predicted regression target of an input sample is computed as the mean predicted regression targets of the trees in the forest.

**Parameters**

- **X**: array-like of shape = [n_samples, n_features]
  
  The input samples.

**Returns**

- **y**: array of shape = [n_samples]
  
  The predicted values.

**score** *(X, y)*

Returns the coefficient of determination $R^2$ of the prediction.

The coefficient $R^2$ is defined as $(1 - u/v)$, where $u$ is the regression sum of squares $((y - y\_pred) ** 2).sum()$ and $v$ is the residual sum of squares $((y\_true - y\_true.mean()) ** 2).sum()$. Best possible score is 1.0, lower values are worse.

**Parameters**

- **X**: array-like, shape = [n_samples, n_features]
  
  Training set.

- **y**: array-like, shape = [n_samples]

**Returns**

- **z**: float
**set_params**(**params**)  
Set the parameters of the estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The former have parameters of the form `<component>__<parameter>` so that it’s possible to update each component of a nested object.

**Returns self**:

**transform**(X, threshold=None)  
Reduce X to its most important features.

**Parameters**

- **X**: array or scipy sparse matrix of shape [n_samples, n_features]  
The input samples.

- **threshold**: string, float or None, optional (default=None)  
The threshold value to use for feature selection. Features whose importance is greater or equal are kept while the others are discarded. If “median” (resp. “mean”), then the threshold value is the median (resp. the mean) of the feature importances. A scaling factor (e.g., “1.25*mean”) may also be used. If None and if available, the object attribute threshold is used. Otherwise, “mean” is used by default.

**Returns**

- **X_r**: array of shape [n_samples, n_selected_features]  
The input samples with only the selected features.

### sklearn.ensemble.GradientBoostingClassifier

**class sklearn.ensemble.GradientBoostingClassifier**(loss='deviance', learn_rate=0.1, n_estimators=100, subsample=1.0, min_samples_split=1, min_samples_leaf=1, max_depth=3, init=None, random_state=None)

Gradient Boosting for classification.

GB builds an additive model in a forward stage-wise fashion; it allows for the optimization of arbitrary differentiable loss functions. In each stage n_classes_ regression trees are fit on the negative gradient of the binomial or multinomial deviance loss function. Binary classification is a special case where only a single regression tree is induced.

**Parameters**

- **loss**: {'deviance', 'ls'}, optional (default='deviance')  
  loss function to be optimized. ‘deviance’ refers to deviance (= logistic regression) for classification with probabilistic outputs. ‘ls’ refers to least squares regression.

- **learn_rate**: float, optional (default=0.1)  
  learning rate shrinks the contribution of each tree by learn_rate. There is a trade-off between learn_rate and n_estimators.

- **n_estimators**: int (default=100)  
  The number of boosting stages to perform. Gradient boosting is fairly robust to overfitting so a large number usually results in better performance.

- **max_depth**: integer, optional (default=3)  
  maximum depth of the individual regression estimators. The maximum depth limits the number of nodes in the tree. Tune this parameter for best performance; the best value depends on the interaction of the input variables.
**min_samples_split**: integer, optional (default=1)

The minimum number of samples required to split an internal node.

**min_samples_leaf**: integer, optional (default=1)

The minimum number of samples required to be at a leaf node.

**subsample**: float, optional (default=1.0)

The fraction of samples to be used for fitting the individual base learners. If smaller than 1.0 this results in Stochastic Gradient Boosting. *subsample* interacts with the parameter *n_estimators*.

See Also:

`sckn.tree.DecisionTreeClassifier, RandomForestClassifier`

References


10. Friedman, Stochastic Gradient Boosting, 1999


Examples

```python
>>> samples = [[0, 0, 2], [1, 0, 0]]
>>> labels = [0, 1]
>>> from sklearn.ensemble import GradientBoostingClassifier
>>> gb = GradientBoostingClassifier().fit(samples, labels)
>>> print gb.predict([[0.5, 0, 0]])
[0]
```

Methods

```python
<table>
<thead>
<tr>
<th>Method</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>fit(X, y)</td>
<td>Fit the gradient boosting model.</td>
</tr>
<tr>
<td>fit_stage(i, X, X_argsorted, y, y_pred, ...)</td>
<td>Fit another stage of <em>n_classes</em> _trees to the boosting model.</td>
</tr>
<tr>
<td>get_params([deep])</td>
<td>Get parameters for the estimator</td>
</tr>
<tr>
<td>predict(X)</td>
<td>Predict class for X.</td>
</tr>
<tr>
<td>predict_proba(X)</td>
<td>Predict class probabilities for X.</td>
</tr>
<tr>
<td>score(X, y)</td>
<td>Returns the mean accuracy on the given test data and labels.</td>
</tr>
<tr>
<td>set_params(**params)</td>
<td>Set the parameters of the estimator.</td>
</tr>
<tr>
<td>staged_decision_function(X)</td>
<td>Compute decision function for X.</td>
</tr>
</tbody>
</table>
```

```python
__init__(loss='deviance', learn_rate=0.1, n_estimators=100, subsample=1.0, min_samples_split=1, min_samples_leaf=1, max_depth=3, init=None, random_state=None)
```

**fit(X, y)**

Fit the gradient boosting model.

**Parameters**

- **X**: array-like, shape = [n_samples, n_features]

Training vectors, where n_samples is the number of samples and n_features is the num-
ber of features. Use fortran-style to avoid memory copies.

\[ y : \text{array-like, shape} = [n_{\text{samples}}] \]

Target values (integers in classification, real numbers in regression) For classification, labels must correspond to classes 0, 1, ..., n_classes_-1

Returns self : object

Returns self.

\textbf{fit\_stage} (i, X, X\_argsorted, y, y\_pred, sample\_mask)

Fit another stage of n_classes_ trees to the boosting model.

\textbf{get\_params} (deep=True)

Get parameters for the estimator

Parameters deep: boolean, optional :

If True, will return the parameters for this estimator and contained subobjects that are estimators.

\textbf{predict} (X)

Predict class for X.

Parameters X : array-like of shape = [n_samples, n_features]

The input samples.

Returns y : array of shape = [n_samples]

The predicted classes.

\textbf{predict\_proba} (X)

Predict class probabilities for X.

Parameters X : array-like of shape = [n_samples, n_features]

The input samples.

Returns p : array of shape = [n_samples]

The class probabilities of the input samples. Classes are ordered by arithmetical order.

\textbf{score} (X, y)

Returns the mean accuracy on the given test data and labels.

Parameters X : array-like, shape = [n_samples, n_features]

Training set.

y : array-like, shape = [n_samples]

Labels for X.

Returns z : float

\textbf{set\_params} (**params)

Set the parameters of the estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The former have parameters of the form <component>__<parameter> so that it’s possible to update each component of a nested object.

Returns self :
**staged_decision_function** *(X)*  
Compute decision function for X.

This method allows monitoring (i.e. determine error on testing set) after each stage.

**Parameters**  
* X : array-like of shape = [n_samples, n_features]  
The input samples.

**Returns**  
* f : array of shape = [n_samples, n_classes]  
The decision function of the input samples. Classes are ordered by arithmetical order. Regression and binary classification are special cases with n_classes == 1.

**sklearn.ensemble.GradientBoostingRegressor**

class **sklearn.ensemble.GradientBoostingRegressor** *(loss='ls', learn_rate=0.1, n_estimators=100, subsample=1.0, min_samples_split=1, min_samples_leaf=1, max_depth=3, init=None, random_state=None)*

Gradient Boosting for regression.

GB builds an additive model in a forward stage-wise fashion; it allows for the optimization of arbitrary differentiable loss functions. In each stage a regression tree is fit on the negative gradient of the given loss function.

**Parameters**  
* loss : {'ls', 'lad'}, optional (default='ls')  
loss function to be optimized. 'ls' refers to least squares regression. 'lad' (least absolute deviation) is a highly robust loss function solely based on order information of the input variables.

* learn_rate : float, optional (default=0.1)  
learning rate shrinks the contribution of each tree by learn_rate. There is a trade-off between learn_rate and n_estimators.

* n_estimators : int (default=100)  
The number of boosting stages to perform. Gradient boosting is fairly robust to overfitting so a large number usually results in better performance.

* max_depth : integer, optional (default=3)  
maximum depth of the individual regression estimators. The maximum depth limits the number of nodes in the tree. Tune this parameter for best performance; the best value depends on the interaction of the input variables.

* min_samples_split : integer, optional (default=1)  
The minimum number of samples required to split an internal node.

* min_samples_leaf : integer, optional (default=1)  
The minimum number of samples required to be at a leaf node.

* subsample : float, optional (default=1.0)  
The fraction of samples to be used for fitting the individual base learners. If smaller than 1.0 this results in Stochastic Gradient Boosting. subsample interacts with the parameter n_estimators.
See Also:

sklearn.tree.DecisionTreeRegressor, RandomForestRegressor

References


10. Friedman, Stochastic Gradient Boosting, 1999


Examples

```python
>>> samples = [[0, 0, 2], [1, 0, 0]]
>>> labels = [0, 1]
>>> from sklearn.ensemble import GradientBoostingRegressor
>>> gb = GradientBoostingRegressor().fit(samples, labels)
>>> print gb.predict([[0, 0, 0]])
[ 1.32806997e-05]
```

Attributes

<table>
<thead>
<tr>
<th>feature_importances_</th>
<th>array, shape=[n_features]</th>
<th>The feature importances (the higher, the more important the feature).</th>
</tr>
</thead>
<tbody>
<tr>
<td>oob_score_</td>
<td>array, shape=[n_estimators]</td>
<td>Score of the training dataset obtained using an out-of-bag estimate. The i-th score oob_score_[i] is the deviance (= loss) of the model at iteration i on the out-of-bag sample.</td>
</tr>
<tr>
<td>train_score_</td>
<td>array, shape=[n_estimators]</td>
<td>The i-th score train_score_[i] is the deviance (= loss) of the model at iteration i on the in-bag sample. If subsample == 1 this is the deviance on the training data.</td>
</tr>
</tbody>
</table>

Methods

<table>
<thead>
<tr>
<th>fit(X, y)</th>
<th>Fit the gradient boosting model.</th>
</tr>
</thead>
<tbody>
<tr>
<td>fit_stage(i, X, X_argsorted, y, y_pred...)</td>
<td>Fit another stage of n_classes_trees to the boosting model.</td>
</tr>
<tr>
<td>get_params([deep])</td>
<td>Get parameters for the estimator</td>
</tr>
<tr>
<td>predict(X)</td>
<td>Predict regression target for X.</td>
</tr>
<tr>
<td>score(X, y)</td>
<td>Returns the coefficient of determination R^2 of the prediction.</td>
</tr>
<tr>
<td>set_params(**params)</td>
<td>Set the parameters of the estimator.</td>
</tr>
<tr>
<td>staged_decision_function(X)</td>
<td>Compute decision function for X.</td>
</tr>
<tr>
<td>staged_predict(X)</td>
<td>Predict regression target at each stage for X.</td>
</tr>
</tbody>
</table>

```python
__init__(loss='ls', learn_rate=0.1, n_estimators=100, subsample=1.0, min_samples_split=1, min_samples_leaf=1, max_depth=3, init=None, random_state=None)
```

```python
fit(X, y)
```

Fit the gradient boosting model.
Parameters $X$ : array-like, shape = [n_samples, n_features]

Training vectors, where n_samples is the number of samples and n_features is the number of features. Use fortran-style to avoid memory copies.

$y$ : array-like, shape = [n_samples]

Target values (integers in classification, real numbers in regression) For classification, labels must correspond to classes 0, 1, ..., n_classes - 1

Returns self : object

Returns self.

$\text{fit\_stage} (i, X, X_{\text{argsorted}}, y, y_{\text{pred}}, \text{sample\_mask})$

Fit another stage of n_classes trees to the boosting model.

$\text{get\_params} (\text{deep}=\text{True})$

Get parameters for the estimator

Parameters deep : boolean, optional :

If True, will return the parameters for this estimator and contained subobjects that are estimators.

$\text{predict} (X)$

Predict regression target for X.

Parameters $X$ : array-like of shape = [n_samples, n_features]

The input samples.

Returns $y$ : array of shape = [n_samples] :

The predicted values.

$\text{score} (X, y)$

Returns the coefficient of determination $R^2$ of the prediction.

The coefficient $R^2$ is defined as $(1 - u/v)$, where $u$ is the regression sum of squares (($y - y_{\text{pred}}) ** 2).sum()$ and $v$ is the residual sum of squares (($y_{\text{true}} - y_{\text{true}}.mean()) ** 2).sum()$. Best possible score is 1.0, lower values are worse.

Parameters $X$ : array-like, shape = [n_samples, n_features]

Training set.

$y$ : array-like, shape = [n_samples]

Returns $z$ : float

$\text{set\_params} (**\text{params})$

Set the parameters of the estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The former have parameters of the form $<\text{component}>._<\text{parameter}>$ so that it’s possible to update each component of a nested object.

Returns self :

$\text{staged\_decision\_function} (X)$

Compute decision function for X.

This method allows monitoring (i.e. determine error on testing set) after each stage.

Parameters $X$ : array-like of shape = [n_samples, n_features]
The input samples.

**Returns f**: array of shape = [n_samples, n_classes]

The decision function of the input samples. Classes are ordered by arithmetical order. Regression and binary classification are special cases with n_classes == 1.

**staged_predict** *(X)*

Predict regression target at each stage for X.

This method allows monitoring (i.e. determine error on testing set) after each stage.

**Parameters X**: array-like of shape = [n_samples, n_features]

The input samples.

**Returns y**: array of shape = [n_samples]

The predicted value of the input samples.

---

1.8.7 **sklearn.feature_extraction**: Feature Extraction

The `sklearn.feature_extraction` module deals with feature extraction from raw data. It currently includes methods to extract features from text and images.

**User guide**: See the Feature extraction section for further details.

**`feature_extraction.DictVectorizer([dtype, ...])`**: Transforms lists of feature-value mappings to vectors.

**sklearn.feature_extraction.DictVectorizer**

**class sklearn.feature_extraction.DictVectorizer** *(dtype=< type 'numpy.float64'>, separator='=', sparse=True)*

Transforms lists of feature-value mappings to vectors.

This transformer turns lists of mappings (dict-like objects) of feature names to feature values into Numpy arrays or scipy.sparse matrices for use with scikit-learn estimators.

When feature values are strings, this transformer will do a binary one-hot (aka one-of-K) coding: one boolean-valued feature is constructed for each of the possible string values that the feature can take on. For instance, a feature “f” that can take on the values “ham” and “spam” will become two features in the output, one signifying “f=ham”, the other “f=spam”.

Features that do not occur in a sample (mapping) will have a zero value in the resulting array/matrix.

**Parameters dtype**: callable, optional

The type of feature values. Passed to Numpy/scipy.sparse matrix constructors as the dtype argument.

**separator**: string, optional :

Separator string used when constructing new features for one-hot coding.

**sparse**: boolean, optional : 

Whether transform should produce scipy.sparse matrices. True by default.
Examples

```python
>>> from sklearn.feature_extraction import DictVectorizer
>>> v = DictVectorizer(sparse=False)
>>> D = [{'foo': 1, 'bar': 2}, {'foo': 3, 'baz': 1}]
>>> X = v.fit_transform(D)
>>> X
array([[ 2., 0., 1.],
       [ 0., 1., 3.]])
>>> v.inverse_transform(X) == [{'bar': 2.0, 'foo': 1.0}, {'baz': 1.0, 'foo': 3.0}]
True
>>> v.transform({'foo': 4, 'unseen_feature': 3})
array([[ 0., 0., 4.]])
```

Methods

- `fit(X, y=None)`: Learn a list of feature name -> indices mappings.
- `fit_transform(X, y=None)`: Learn a list of feature name -> indices mappings and transform X.
- `get_feature_names()`: Returns a list of feature names, ordered by their indices.
- `get_params([deep])`: Get parameters for the estimator.
- `inverse_transform(X, dict_type)`: Transform array or sparse matrix X back to feature mappings.
- `restrict(support[, indices])`: Restrict the features to those in support.
- `set_params(**params)`: Set the parameters of the estimator.
- `transform(X, y=None)`: Transform feature->value dicts to array or sparse matrix.

```python
__init__(dtype=<type 'numpy.float64'>, separator='=', sparse=True)
```

`fit(X, y=None)`: Learn a list of feature name -> indices mappings.

**Parameters** X : Mapping or iterable over Mappings

Dict(s) or Mapping(s) from feature names (arbitrary Python objects) to feature values (strings or convertible to dtype).

y : (ignored)

**Returns** self :

`fit_transform(X, y=None)`: Learn a list of feature name -> indices mappings and transform X.

Like fit(X) followed by transform(X).

**Parameters** X : Mapping or iterable over Mappings

Dict(s) or Mapping(s) from feature names (arbitrary Python objects) to feature values (strings or convertible to dtype).

y : (ignored)

**Returns** Xa : {array, sparse matrix}

Feature vectors; always 2-d.

`get_feature_names()`: Returns a list of feature names, ordered by their indices.
If one-of-K coding is applied to categorical features, this will include the constructed feature names but not the original ones.

**get_params** *(deep=True)*

Get parameters for the estimator

**Parameters**

- **deep**: boolean, optional
  
  If True, will return the parameters for this estimator and contained subobjects that are estimators.

**inverse_transform** *(X, dict_type=<type 'dict'>)*

Transform array or sparse matrix X back to feature mappings.

X must have been produced by this DictVectorizer’s transform or fit_transform method; it may only have passed through transformers that preserve the number of features and their order.

In the case of one-hot/one-of-K coding, the constructed feature names and values are returned rather than the original ones.

**Parameters**

- **X**: {array-like, sparse matrix}, shape = [n_samples, n_features]
  
  Sample matrix.

- **dict_type**: callable, optional
  
  Constructor for feature mappings. Must conform to the collections.Mapping API.

**Returns**

- **D**: list of dict_type objects, length = n_samples
  
  Feature mappings for the samples in X.

**restrict** *(support, indices=False)*

Restrict the features to those in support.

**Parameters**

- **support**: array-like
  
  Boolean mask or list of indices (as returned by the get_support member of feature selectors).

- **indices**: boolean, optional
  
  Whether support is a list of indices.

**set_params** (**params**)

Set the parameters of the estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The former have parameters of the form `<component>__<parameter>` so that it's possible to update each component of a nested object.

**Returns**

- **self**: 

**transform** *(X, y=None)*

Transform feature->value dicts to array or sparse matrix.

Named features not encountered during fit or fit_transform will be silently ignored.

**Parameters**

- **X**: Mapping or iterable over Mappings, length = n_samples
  
  Dict(s) or Mapping(s) from feature names (arbitrary Python objects) to feature values (strings or convertible to dtype).

- **y**: (ignored)

**Returns**

- **Xa**: {array, sparse matrix}
Feature vectors; always 2-d.

From images

The sklearn.feature_extraction.image submodule gathers utilities to extract features from images.

<table>
<thead>
<tr>
<th>Function</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>sklearn.feature_extraction.image.img_to_graph</td>
<td>Graph of the pixel-to-pixel gradient connections</td>
</tr>
<tr>
<td>sklearn.feature_extraction.image.grid_to_graph</td>
<td>Graph of the pixel-to-pixel connections</td>
</tr>
<tr>
<td>sklearn.feature_extraction.image.extract_patches_2d</td>
<td>Reshape a 2D image into a collection of patches</td>
</tr>
<tr>
<td>sklearn.feature_extraction.image.reconstruct_from_patches_2d</td>
<td>Reconstruct the image from all of its patches.</td>
</tr>
<tr>
<td>sklearn.feature_extraction.image.PatchExtractor</td>
<td>Extracts patches from a collection of images</td>
</tr>
</tbody>
</table>

**sklearn.feature_extraction.image.img_to_graph**

Graph of the pixel-to-pixel gradient connections

Edges are weighted with the gradient values.

**Parameters**

- **img**: ndarray, 2D or 3D
  
  2D or 3D image
  
  - **mask**: ndarray of booleans, optional
    
    An optional mask of the image, to consider only part of the pixels.
  
  - **return_as**: np.ndarray or a sparse matrix class, optional
    
    The class to use to build the returned adjacency matrix.
  
  - **dtype**: None or dtype, optional
    
    The data of the returned sparse matrix. By default it is the dtype of img

**sklearn.feature_extraction.image.grid_to_graph**

Graph of the pixel-to-pixel connections

Edges exist if 2 voxels are connected.

**Parameters**

- **n_x**: int
  
  Dimension in x axis
  
- **n_y**: int
  
  Dimension in y axis
  
- **n_z**: int, optional, default 1
  
  Dimension in z axis
  
- **mask**: ndarray of booleans, optional
An optional mask of the image, to consider only part of the pixels.

**return_as:** np.ndarray or a sparse matrix class, optional

The class to use to build the returned adjacency matrix.

**dtype:** dtype, optional, default int

The data of the returned sparse matrix. By default it is int

```
from sklearn.feature_extraction.image import extract_patches_2d

extract_patches_2d(image, patch_size, max_patches=None, random_state=None)
```

Reshape a 2D image into a collection of patches

The resulting patches are allocated in a dedicated array.

**Parameters**

- **image:** array, shape = (image_height, image_width) or (image_height, image_width, n_channels) The original image data. For color images, the last dimension specifies the channel: a RGB image would have n_channels=3.
- **patch_size:** tuple of ints (patch_height, patch_width) the dimensions of one patch
- **max_patches:** integer or float, optional default is None
  The maximum number of patches to extract. If max_patches is a float between 0 and 1, it is taken to be a proportion of the total number of patches.
- **random_state:** int or RandomState
  Pseudo number generator state used for random sampling to use if max_patches is not None.

**Returns**

- **patches:** array, shape = (n_patches, patch_height, patch_width) or (n_patches, patch_height, patch_width, n_channels) The collection of patches extracted from the image, where n_patches is either max_patches or the total number of patches that can be extracted.

**Examples**

```python
>>> from sklearn.feature_extraction import image
>>> one_image = np.arange(16).reshape((4, 4))
>>> one_image
array([[ 0,  1,  2,  3],
       [ 4,  5,  6,  7],
       [ 8,  9, 10, 11],
       [12, 13, 14, 15]])
>>> patches = image.extract_patches_2d(one_image, (2, 2))
>>> patches.shape
(9, 2, 2)
```

```python
>>> patches[0]
array([[0, 1],
       [4, 5]])
```

```python
>>> patches[1]
array([[1, 2],
       [8, 9]])
```
```
>>> patches[8]
array([[10, 11],
       [14, 15]])
```

**sklearn.feature_extraction.image.reconstruct_from_patches_2d**

```
sklearn.feature_extraction.image.reconstruct_from_patches_2d(patches, image_size)
```

Reconstruct the image from all of its patches.

Patches are assumed to overlap and the image is constructed by filling in the patches from left to right, top to bottom, averaging the overlapping regions.

**Parameters**

- `patches`: array, shape = (n_patches, patch_height, patch_width) or:
  - (n_patches, patch_height, patch_width, n_channels) The complete set of patches. If the patches contain colour information, channels are indexed along the last dimension: RGB patches would have `n_channels`=3.
- `image_size`: tuple of ints (image_height, image_width) or:
  - (image_height, image_width, n_channels) the size of the image that will be reconstructed

**Returns**

- `image`: array, shape = `image_size`:
  - the reconstructed image

**sklearn.feature_extraction.image.PatchExtractor**

```
class sklearn.feature_extraction.image.PatchExtractor
```

**Parameters**

- `patch_size`: tuple of ints (patch_height, patch_width):
  - the dimensions of one patch
- `max_patches`: integer or float, optional default is None:
  - The maximum number of patches per image to extract. If `max_patches` is a float in (0, 1), it is taken to mean a proportion of the total number of patches.
- `random_state`: int or RandomState:
  - Pseudo number generator state used for random sampling.

**Methods**

- `fit(X, y)`:
  - Do nothing and return the estimator unchanged
- `get_params([deep])`:
  - Get parameters for the estimator
- `set_params(**params)`:
  - Set the parameters of the estimator.
- `transform(X)`:
  - Transforms the image samples in X into a matrix of patch data.

```
__init__(patch_size, max_patches=None, random_state=None)
```
**fit** (*X*, *y=None*)
Do nothing and return the estimator unchanged
This method is just there to implement the usual API and hence work in pipelines.

**get_params** (*deep=True*)
Get parameters for the estimator

**Parameters**

**deep**: boolean, optional
- If True, will return the parameters for this estimator and contained subobjects that are estimators.

**set_params** (**params**)
Set the parameters of the estimator.
The method works on simple estimators as well as on nested objects (such as pipelines). The former have parameters of the form `<component>__<parameter>` so that it’s possible to update each component of a nested object.

**Returns**

**self**

**transform** (*X*)
Transforms the image samples in X into a matrix of patch data.

**Parameters**

**X** : array, shape = (n_samples, image_height, image_width) or
(n_samples, image_height, image_width, n_channels) Array of images from which to extract patches. For color images, the last dimension specifies the channel: a RGB image would have `n_channels=3`.

**Returns**

**patches** : array, shape = (n_patches, patch_height, patch_width) or :
(n_patches, patch_height, patch_width, n_channels) The collection of patches extracted from the images, where `n_patches` is either `n_samples * max_patches` or the total number of patches that can be extracted.

---

**From text**

The `sklearn.feature_extraction.text` submodule gathers utilities to build feature vectors from text documents.

| **feature_extraction.text.CountVectorizer([...])** | Convert a collection of raw documents to a matrix of token counts. |
| **feature_extraction.text.TfidfTransformer([...])** | Transform a count matrix to a normalized tf or tf–idf representation. |
| **feature_extraction.text.TfidfVectorizer([...])** | Convert a collection of raw documents to a matrix of TF-IDF features. |
Convert a collection of raw documents to a matrix of token counts

This implementation produces a sparse representation of the counts using scipy.sparse.coo_matrix.

If you do not provide an a-priori dictionary and you do not use an analyzer that does some kind of feature selection then the number of features will be equal to the vocabulary size found by analysing the data. The default analyzer does simple stop word filtering for English.

**Parameters**

**input**: string {'filename', 'file', 'content'}:
- If filename, the sequence passed as an argument to fit is expected to be a list of filenames that need reading to fetch the raw content to analyze.
- If 'file', the sequence items must have 'read' method (file-like object) it is called to fetch the bytes in memory.
- Otherwise the input is expected to be the sequence strings or bytes items are expected to be analyzed directly.

**charset**: string, ‘utf-8’ by default.:
- If bytes or files are given to analyze, this charset is used to decode.

**charset_error**: {'strict', 'ignore', 'replace'}:
- Instruction on what to do if a byte sequence is given to analyze that contains characters not of the given charset. By default, it is ‘strict’, meaning that a UnicodeDecodeError will be raised. Other values are ‘ignore’ and ‘replace’.

**strip_accents**: {'ascii', 'unicode', None}:
- Remove accents during the preprocessing step. ‘ascii’ is a fast method that only works on characters that have an direct ASCII mapping. ‘unicode’ is a slightly slower method that works on any characters. None (default) does nothing.

**analyzer**: string, {'word', 'char'} or callable:
- Whether the feature should be made of word or character n-grams.
- If a callable is passed it is used to extract the sequence of features out of the raw, unprocessed input.

**preprocessor**: callable or None (default):
- Override the preprocessing (string transformation) stage while preserving the tokenizing and n-grams generation steps.

**tokenizer**: callable or None (default):
Override the string tokenization step while preserving the preprocessing and n-grams generation steps.

**min_n**: integer

The lower boundary of the range of n-values for different n-grams to be extracted.

**max_n**: integer

The upper boundary of the range of n-values for different n-grams to be extracted. All values of n such that \( \text{min}_n \leq n \leq \text{max}_n \) will be used.

**stop_words**: string {'english'}, list, or None (default)

If a string, it is passed to _check_stop_list and the appropriate stop list is returned is currently the only supported string value.

If a list, that list is assumed to contain stop words, all of which will be removed from the resulting tokens.

If None, no stop words will be used. max_df can be set to a value in the range \([0.7, 1.0)\) to automatically detect and filter stop words based on intra corpus document frequency of terms.

**token_pattern**: string

Regular expression denoting what constitutes a “token”, only used if tokenize == ‘word’. The default regexp select tokens of 2 or more letters characters (punctuation is completely ignored and always treated as a token separator).

**max_df**: float in range \([0.0, 1.0]\), optional, 1.0 by default

When building the vocabulary ignore terms that have a term frequency strictly higher than the given threshold (corpus specific stop words).

This parameter is ignored if vocabulary is not None.

**max_features**: optional, None by default

If not None, build a vocabulary that only consider the top max_features ordered by term frequency across the corpus.

This parameter is ignored if vocabulary is not None.

**binary**: boolean, False by default

If True, all non zero counts are set to 1. This is useful for discrete probabilistic models that model binary events rather than integer counts.

**dtype**: type, optional

Type of the matrix returned by fit_transform() or transform().

### Methods

- **build_analyzer()**
  
  Return a callable that handles preprocessing and tokenization

- **build_preprocessor()**
  
  Return a function to preprocess the text before tokenization

- **build_tokenizer()**
  
  Return a function that split a string in sequence of tokens

- **decode(doc)**
  
  Decode the input into a string of unicode symbols

- **fit(raw_documents[, y])**
  
  Learn a vocabulary dictionary of all tokens in the raw documents

- **fit_transform(raw_documents[, y])**
  
  Learn the vocabulary dictionary and return the count vectors
Table 1.71 – continued from previous page

<table>
<thead>
<tr>
<th>Method</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>get_feature_names()</td>
<td>Array mapping from feature integer index to feature name</td>
</tr>
<tr>
<td>get_params([deep])</td>
<td>Get parameters for the estimator</td>
</tr>
<tr>
<td>get_stop_words()</td>
<td>Build or fetch the effective stop words list</td>
</tr>
<tr>
<td>inverse_transform(X)</td>
<td>Return terms per document with nonzero entries in X.</td>
</tr>
<tr>
<td>set_params(**params)</td>
<td>Set the parameters of the estimator.</td>
</tr>
<tr>
<td>transform(raw_documents)</td>
<td>Extract token counts out of raw text documents using the vocabulary fitted</td>
</tr>
<tr>
<td></td>
<td>with fit or the one provided in the constructor.</td>
</tr>
</tbody>
</table>

__init__ (input='content', charset='utf-8', charset_error='strict', strip_accents=None, lowercase=True, preprocessor=None, tokenizer=None, stop_words=None, token_pattern=u'\b\w\w+\b', min_n=1, max_n=1, analyzer='word', max_df=1.0, max_features=None, vocabulary=None, binary=False, dtype=<type 'long'>)

build_analyzer()  
Return a callable that handles preprocessing and tokenization

build_preprocessor()  
Return a function to preprocess the text before tokenization

build_tokenizer()  
Return a function that split a string in sequence of tokens

decode (doc)  
Decode the input into a string of unicode symbols

The decoding strategy depends on the vectorizer parameters.

fit (raw_documents, y=None)  
Learn a vocabulary dictionary of all tokens in the raw documents

Parameters raw_documents: iterable:
  an iterable which yields either str, unicode or file objects

Returns self:

fit_transform (raw_documents, y=None)  
Learn the vocabulary dictionary and return the count vectors

This is more efficient than calling fit followed by transform.

Parameters raw_documents: iterable:
  an iterable which yields either str, unicode or file objects

Returns vectors: array, [n_samples, n_features]:

get_feature_names ()  
Array mapping from feature integer index to feature name

get_params (deep=True)  
Get parameters for the estimator

Parameters deep: boolean, optional:
  If True, will return the parameters for this estimator and contained subobjects that are estimators.

get_stop_words ()  
Build or fetch the effective stop words list

inverse_transform (X)  
Return terms per document with nonzero entries in X.
Parameters $X$ : {array, sparse matrix}, shape = [n_samples, n_features]

Returns $X_{\text{inv}}$ : list of arrays, len = n_samples

List of arrays of terms.

**set_params** (**params**)

Set the parameters of the estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The former have parameters of the form `<component>__<parameter>` so that it’s possible to update each component of a nested object.

Returns self :

**transform** (**raw_documents**)

Extract token counts out of raw text documents using the vocabulary fitted with fit or the one provided in the constructor.

Parameters raw_documents: iterable :

an iterable which yields either str, unicode or file objects

Returns vectors: sparse matrix, [n_samples, n_features]:

sklearn.feature_extraction.text.TfidfTransformer

class sklearn.feature_extraction.text.TfidfTransformer (norm='l2', use_idf=True, smooth_idf=True, sublinear_tf=False)

Transform a count matrix to a normalized tf or tf–idf representation

Tf means term-frequency while tf–idf means term-frequency times inverse document-frequency. This is a common term weighting scheme in information retrieval, that has also found good use in document classification.

The goal of using tf–idf instead of the raw frequencies of occurrence of a token in a given document is to scale down the impact of tokens that occur very frequently in a given corpus and that are hence empirically less informative than features that occur in a small fraction of the training corpus.

In the SMART notation used in IR, this class implements several tf–idf variants. Tf is always “n” (natural), idf is “t” iff use_idf is given, “n” otherwise, and normalization is “c” iff norm='l2', “n” iff norm=None.

Parameters norm : ‘11’, ‘12’ or None, optional

Norm used to normalize term vectors. None for no normalization.

use_idf : boolean, optional

Enable inverse-document-frequency reweighting.

smooth_idf : boolean, optional

Smooth idf weights by adding one to document frequencies, as if an extra document was seen containing every term in the collection exactly once. Prevents zero divisions.

sublinear_tf : boolean, optional

Apply sublinear tf scaling, i.e. replace tf with $1 + \log(tf)$.

References

[Yates2011], [MSR2008]
Methods

<table>
<thead>
<tr>
<th>Method</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>fit(X, y)</code></td>
<td>Learn the idf vector (global term weights)</td>
</tr>
<tr>
<td><code>fit_transform(X[, y])</code></td>
<td>Fit to data, then transform it</td>
</tr>
<tr>
<td><code>get_params([deep])</code></td>
<td>Get parameters for the estimator</td>
</tr>
<tr>
<td><code>set_params(**params)</code></td>
<td>Set the parameters of the estimator.</td>
</tr>
<tr>
<td><code>transform(X[, copy])</code></td>
<td>Transform a count matrix to a tf or tf-idf representation</td>
</tr>
</tbody>
</table>

`__init__` (`norm='l2', use_idf=True, smooth_idf=True, sublinear_tf=False`)

`fit (X, y=None)`
Learn the idf vector (global term weights)

Parameters

- **X**: sparse matrix, [n_samples, n_features]: a matrix of term/token counts

`fit_transform (X, y=None, **fit_params)`
Fit to data, then transform it

Fits transformer to X and y with optional parameters fit_params and returns a transformed version of X.

Parameters

- **X**: numpy array of shape [n_samples, n_features]: Training set.
- **y**: numpy array of shape [n_samples]: Target values.

Returns

- **X_new**: numpy array of shape [n_samples, n_features_new]: Transformed array.

Notes

This method just calls fit and transform consecutively, i.e., it is not an optimized implementation of fit_transform, unlike other transformers such as PCA.

`get_params (deep=True)`
Get parameters for the estimator

Parameters

- deep: boolean, optional:
  If True, will return the parameters for this estimator and contained subobjects that are estimators.

`set_params (**params)`
Set the parameters of the estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The former have parameters of the form `<component>__<parameter>` so that it's possible to update each component of a nested object.

Returns **self**:

`transform (X, copy=True)`
Transform a count matrix to a tf or tf-idf representation

Parameters

- **X**: sparse matrix, [n_samples, n_features]:
Convert a collection of raw documents to a matrix of TF-IDF features.
Equivalent to CountVectorizer followed by TfidfTransformer.

See Also:

CountVectorizer Tokenize the documents and count the occurrences of token and return them as a sparse matrix

TfidfTransformer Apply Term Frequency Inverse Document Frequency normalization to a sparse matrix of occurrence counts.

Methods

- `build_analyzer()` Return a callable that handles preprocessing and tokenization
- `build_preprocessor()` Return a function to preprocess the text before tokenization
- `build_tokenizer()` Return a function that split a string in sequence of tokens
- `decode(doc)` Decode the input into a string of unicode symbols
- `fit(raw_documents)` Learn a conversion law from documents to array data
- `fit_transform(raw_documents[, y])` Learn the representation and return the vectors.
- `get_feature_names()` Array mapping from feature integer indicex to feature name
- `get_params([deep])` Get parameters for the estimator
- `get_stop_words()` Build or fetch the effective stop words list
- `inverse_transform(X)` Return terms per document with nonzero entries in X.
- `set_params(**params)` Set the parameters of the estimator.
- `transform(raw_documents[, copy])` Transform raw text documents to tf–idf vectors

__init__ (input='content', charset='utf-8', charset_error='strict', strip_accents=None, lowercase=True, preprocessor=None, tokenizer=None, analyzer='word', stop_words=None, token_pattern=u'\b\w\w+\b', min_n=1, max_n=1, max_df=1.0, max_features=None, vocabulary=None, binary=False, dtype=<type 'long'>, norm='l2', use_idf=True, smooth_idf=True, sublinear_tf=False)
**build_analyzer()**
Return a callable that handles preprocessing and tokenization

**build_preprocessor()**
Return a function to preprocess the text before tokenization

**build_tokenizer()**
Return a function that split a string in sequence of tokens

**decode(doc)**
Decode the input into a string of unicode symbols

The decoding strategy depends on the vectorizer parameters.

**fit(raw_documents)**
Learn a conversion law from documents to array data

**fit_transform(raw_documents, y=None)**
Learn the representation and return the vectors.

Parameters raw_documents: iterable
an iterable which yields either str, unicode or file objects

Returns vectors: array, [n_samples, n_features]

**get_feature_names()**
Array mapping from feature integer indicex to feature name

**get_params(deep=True)**
Get parameters for the estimator

Parameters deep: boolean, optional
If True, will return the parameters for this estimator and contained subobjects that are estimators.

**get_stop_words()**
Build or fetch the effective stop words list

**inverse_transform(X)**
Return terms per document with nonzero entries in X.

Parameters X : array, sparse matrix, shape = [n_samples, n_features]

Returns X_inv : list of arrays, len = n_samples
List of arrays of terms.

**set_params(**params)**
Set the parameters of the estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The former have parameters of the form <component>__<parameter> so that it’s possible to update each component of a nested object.

Returns self:

**transform(raw_documents, copy=True)**
Transform raw text documents to tf–idf vectors

Parameters raw_documents: iterable
an iterable which yields either str, unicode or file objects

Returns vectors: sparse matrix, [n_samples, n_features]
1.8.8 sklearn.feature_selection: Feature Selection

The sklearn.feature_selection module implements feature selection algorithms. It currently includes univariate filter selection methods and the recursive feature elimination algorithm.

User guide: See the Feature selection section for further details.

<table>
<thead>
<tr>
<th>Function</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>feature_selection.SelectPercentile(score_func)</td>
<td>Filter: Select the best percentile of the p_values</td>
</tr>
<tr>
<td>feature_selection.SelectKBest(score_func[, k])</td>
<td>Filter: Select the k lowest p-values.</td>
</tr>
<tr>
<td>feature_selection.SelectFpr(score_func[, alpha])</td>
<td>Filter: Select the pvalues below alpha based on a FPR test.</td>
</tr>
<tr>
<td>feature_selection.SelectFdr(score_func[, alpha])</td>
<td>Filter: Select the p-values for an estimated false discovery rate</td>
</tr>
<tr>
<td>feature_selection.SelectFwe(score_func[, alpha])</td>
<td>Filter: Select the p-values corresponding to Family-wise error rate</td>
</tr>
<tr>
<td>feature_selection.RFE(estimator, ...[, step])</td>
<td>Feature ranking with recursive feature elimination.</td>
</tr>
<tr>
<td>feature_selection.RFECV(estimator[, step, ...])</td>
<td>Feature ranking with recursive feature elimination and cross-validation</td>
</tr>
</tbody>
</table>

sklearn.feature_selection.SelectPercentile

class sklearn.feature_selection.SelectPercentile(score_func, percentile=10)

Filter: Select the best percentile of the p_values

Parameters score_func: callable :

function taking two arrays X and y, and returning 2 arrays: both scores and pvalues

percentile: int, optional :

percent of features to keep

Methods

<table>
<thead>
<tr>
<th>Method</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>fit(X, y)</td>
<td>Evaluate the function</td>
</tr>
<tr>
<td>fit_transform(X[, y])</td>
<td>Fit to data, then transform it</td>
</tr>
<tr>
<td>get_params([deep])</td>
<td>Get parameters for the estimator</td>
</tr>
<tr>
<td>get_support([indices])</td>
<td>Return a mask, or list, of the features/indices selected.</td>
</tr>
<tr>
<td>inverse_transform(X)</td>
<td>Transform a new matrix using the selected features</td>
</tr>
<tr>
<td>set_params(**params)</td>
<td>Set the parameters of the estimator.</td>
</tr>
<tr>
<td>transform(X)</td>
<td>Transform a new matrix using the selected features</td>
</tr>
</tbody>
</table>

__init__ (score_func, percentile=10)

fit (X, y)

Evaluate the function

fit_transform (X, y=None, **fit_params)

Fit to data, then transform it

Fits transformer to X and y with optional parameters fit_params and returns a transformed version of X.

Parameters X : numpy array of shape [n_samples, n_features]

Training set.

y : numpy array of shape [n_samples]

Target values.

Returns X_new : numpy array of shape [n_samples, n_features_new]
Transformed array.

Notes

This method just calls fit and transform consecutively, i.e., it is not an optimized implementation of fit_transform, unlike other transformers such as PCA.

**get_params** (*deep=True*)
Get parameters for the estimator

**Parameters**

- **deep**: boolean, optional
  If True, will return the parameters for this estimator and contained subobjects that are estimators.

**get_support** (*indices=False*)
Return a mask, or list, of the features/indices selected.

**inverse_transform** (*X*)
Transform a new matrix using the selected features

**set_params** (**params**) 
Set the parameters of the estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The former have parameters of the form `<component>__<parameter>` so that it’s possible to update each component of a nested object.

**Returns**

- **self**: 

**transform** (*X*)
Transform a new matrix using the selected features

**sklearn.feature_selection.SelectKBest**

class **sklearn.feature_selection.SelectKBest** (*score_func*, *k=10*)
Filter: Select the k lowest p-values.

**Parameters**

- **score_func**: callable
  Function taking two arrays X and y, and returning a pair of arrays (scores, pvalues).

- **k**: int, optional
  Number of top features to select.

Notes

Ties between features with equal p-values will be broken in an unspecified way.

Methods

<table>
<thead>
<tr>
<th>Method</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>fit</strong>(X, y)</td>
<td>Evaluate the function</td>
</tr>
<tr>
<td><strong>fit_transform</strong>(X[, y])</td>
<td>Fit to data, then transform it</td>
</tr>
<tr>
<td><strong>get_params</strong>([deep])</td>
<td>Get parameters for the estimator</td>
</tr>
</tbody>
</table>

Continued on next page
Table 1.76 – continued from previous page

<table>
<thead>
<tr>
<th>Method</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>get_support([indices])</td>
<td>Return a mask, or list, of the features/indices selected.</td>
</tr>
<tr>
<td>inverse_transform(X)</td>
<td>Transform a new matrix using the selected features</td>
</tr>
<tr>
<td>set_params(**params)</td>
<td>Set the parameters of the estimator.</td>
</tr>
<tr>
<td>transform(X)</td>
<td>Transform a new matrix using the selected features</td>
</tr>
</tbody>
</table>

___init__(score_func, k=10)

fit (X, y)
Evaluate the function

fit_transform (X, y=None, **fit_params)
Fit to data, then transform it
Fits transformer to X and y with optional parameters fit_params and returns a transformed version of X.

**Parameters**

- **X**: numpy array of shape [n_samples, n_features]
  - Training set.
- **y**: numpy array of shape [n_samples]
  - Target values.

**Returns**

- **X_new**: numpy array of shape [n_samples, n_features_new]
  - Transformed array.

**Notes**

This method just calls fit and transform consecutively, i.e., it is not an optimized implementation of fit_transform, unlike other transformers such as PCA.

get_params (deep=True)
Get parameters for the estimator

**Parameters**

- **deep**: boolean, optional
  - If True, will return the parameters for this estimator and contained subobjects that are estimators.

get_support (indices=False)
Return a mask, or list, of the features/indices selected.

inverse_transform(X)
Transform a new matrix using the selected features

set_params(**params)
Set the parameters of the estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The former have parameters of the form <component>__<parameter> so that it’s possible to update each component of a nested object.

**Returns**

- **self**: 
  - Transform a new matrix using the selected features
class sklearn.feature_selection.SelectFpr (score_func, alpha=0.05)
Filter: Select the pvalues below alpha based on a FPR test.

FPR test stands for False Positive Rate test. It controls the total amount of false detections.

Parameters score_func: callable :
function taking two arrays X and y, and returning 2 arrays: both scores and pvalues

alpha: float, optional :
the highest p-value for features to be kept

Methods

<table>
<thead>
<tr>
<th>Method</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>fit(X, y)</td>
<td>Evaluate the function</td>
</tr>
<tr>
<td>fit_transform(X[, y])</td>
<td>Fit to data, then transform it</td>
</tr>
<tr>
<td>get_params([deep])</td>
<td>Get parameters for the estimator</td>
</tr>
<tr>
<td>get_support([indices])</td>
<td>Return a mask, or list, of the features/indices selected.</td>
</tr>
<tr>
<td>inverse_transform(X)</td>
<td>Transform a new matrix using the selected features</td>
</tr>
<tr>
<td>set_params(**params)</td>
<td>Set the parameters of the estimator</td>
</tr>
<tr>
<td>transform(X)</td>
<td>Transform a new matrix using the selected features</td>
</tr>
</tbody>
</table>

__init__ (score_func, alpha=0.05)

fit (X, y)
Evaluate the function

fit_transform (X, y=None, **fit_params)
Fit to data, then transform it

Fits transformer to X and y with optional parameters fit_params and returns a transformed version of X.

Parameters X : numpy array of shape [n_samples, n_features]
Training set.

y : numpy array of shape [n_samples]
Target values.

Returns X_new : numpy array of shape [n_samples, n_features_new]
Transformed array.

Notes

This method just calls fit and transform consecutively, i.e., it is not an optimized implementation of fit_transform, unlike other transformers such as PCA.

get_params (deep=True)
Get parameters for the estimator

Parameters deep: boolean, optional :
If True, will return the parameters for this estimator and contained subobjects that are estimators.
get_support(indices=False)
    Return a mask, or list, of the features/indices selected.

inverse_transform(X)
    Transform a new matrix using the selected features

set_params(**params)
    Set the parameters of the estimator.

    The method works on simple estimators as well as on nested objects (such as pipelines). The former have
    parameters of the form <component>__<parameter> so that it’s possible to update each component
    of a nested object.

    Returns self :

transform(X)
    Transform a new matrix using the selected features

sklearn.feature_selection.SelectFdr

class sklearn.feature_selection.SelectFdr(score_func, alpha=0.05)
    Filter: Select the p-values for an estimated false discovery rate

    This uses the Benjamini-Hochberg procedure. alpha is the target false discovery rate.

    Parameters score_func: callable :
        function taking two arrays X and y, and returning 2 arrays: both scores and pvalues

    alpha: float, optional :
        the highest uncorrected p-value for features to keep

Methods

<table>
<thead>
<tr>
<th>Method</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>fit(x, y)</td>
<td>Evaluate the function</td>
</tr>
<tr>
<td>fit_transform(X, y)</td>
<td>Fit to data, then transform it</td>
</tr>
<tr>
<td>get_params()</td>
<td>Get parameters for the estimator</td>
</tr>
<tr>
<td>get_support()</td>
<td>Return a mask, or list, of the features/indices selected.</td>
</tr>
<tr>
<td>inverse_transform(X)</td>
<td>Transform a new matrix using the selected features</td>
</tr>
<tr>
<td>set_params(**params)</td>
<td>Set the parameters of the estimator.</td>
</tr>
<tr>
<td>transform(X)</td>
<td>Transform a new matrix using the selected features</td>
</tr>
</tbody>
</table>

__init__(score_func, alpha=0.05)

fit(X, y)
    Evaluate the function

fit_transform(X, y=None, **fit_params)
    Fit to data, then transform it

    Fits transformer to X and y with optional parameters fit_params and returns a transformed version of X.

    Parameters X : numpy array of shape [n_samples, n_features]
                    Training set.
    y : numpy array of shape [n_samples]
Target values.

**Returns X_new**: numpy array of shape [n_samples, n_features_new]

Transformed array.

**Notes**

This method just calls fit and transform consecutively, i.e., it is not an optimized implementation of fit_transform, unlike other transformers such as PCA.

**get_params**(deep=True)

Get parameters for the estimator

**Parameters deep**: boolean, optional:

If True, will return the parameters for this estimator and contained subobjects that are estimators.

**get_support**(indices=False)

Return a mask, or list, of the features/indices selected.

**inverse_transform**(X)

Transform a new matrix using the selected features

**set_params**(**params)

Set the parameters of the estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The former have parameters of the form `<component>__<parameter>` so that it’s possible to update each component of a nested object.

**Returns self**:

**transform**(X)

Transform a new matrix using the selected features

**sklearn.feature_selection.SelectFwe**

class **sklearn.feature_selection.SelectFwe**(score_func, alpha=0.05)

Filter: Select the p-values corresponding to Family-wise error rate

**Parameters score_func**: callable:

function taking two arrays X and y, and returning 2 arrays: both scores and pvalues

**alpha**: float, optional:

the highest uncorrected p-value for features to keep

**Methods**

<table>
<thead>
<tr>
<th>Method</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>fit(X, y)</td>
<td>Evaluate the function</td>
</tr>
<tr>
<td>fit_transform(X[, y])</td>
<td>Fit to data, then transform it</td>
</tr>
<tr>
<td>get_params([deep])</td>
<td>Get parameters for the estimator</td>
</tr>
<tr>
<td>get_support([indices])</td>
<td>Return a mask, or list, of the features/indices selected.</td>
</tr>
<tr>
<td>inverse_transform(X)</td>
<td>Transform a new matrix using the selected features</td>
</tr>
</tbody>
</table>
Table 1.79 – continued from previous page

<table>
<thead>
<tr>
<th>set_params(**params)</th>
<th>Set the parameters of the estimator.</th>
</tr>
</thead>
<tbody>
<tr>
<td>transform(X)</td>
<td>Transform a new matrix using the selected features</td>
</tr>
</tbody>
</table>

__init__(score_func, alpha=0.05)

fit (X, y)
Evaluate the function

fit_transform (X, y=None, **fit_params)
Fit to data, then transform it

Fits transformer to X and y with optional parameters fit_params and returns a transformed version of X.

**Parameters**

X : numpy array of shape [n_samples, n_features]
Training set.

y : numpy array of shape [n_samples]
Target values.

**Returns**

X_new : numpy array of shape [n_samples, n_features_new]
Transformed array.

**Notes**

This method just calls fit and transform consecutively, i.e., it is not an optimized implementation of fit_transform, unlike other transformers such as PCA.

get_params (deep=True)
Get parameters for the estimator

**Parameters**

depth: boolean, optional
If True, will return the parameters for this estimator and contained subobjects that are estimators.

get_support (indices=False)
Return a mask, or list, of the features/indices selected.

inverse_transform (X)
Transform a new matrix using the selected features

set_params (**params)
Set the parameters of the estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The former have parameters of the form `<component>__<parameter>` so that it’s possible to update each component of a nested object.

**Returns**

self :

transform (X)
Transform a new matrix using the selected features
sklearn.feature_selection.RFE

class sklearn.feature_selection.RFE(estimator, n_features_to_select, step=1)

Feature ranking with recursive feature elimination.

Given an external estimator that assigns weights to features (e.g., the coefficients of a linear model), the goal of recursive feature elimination (RFE) is to select features by recursively considering smaller and smaller sets of features. First, the estimator is trained on the initial set of features and weights are assigned to each one of them. Then, features whose absolute weights are the smallest are pruned from the current set features. That procedure is recursively repeated on the pruned set until the desired number of features to select is eventually reached.

Parameters

estimator : object
A supervised learning estimator with a fit method that updates a coef_ attribute that holds the fitted parameters. Important features must correspond to high absolute values in the coef_ array.

n_features_to_select : int
The number of features to select.

step : int or float, optional (default=1)
If greater than or equal to 1, then step corresponds to the (integer) number of features to remove at each iteration. If within (0.0, 1.0), then step corresponds to the percentage (rounded down) of features to remove at each iteration.

References

[R61]

Examples

The following example shows how to retrieve the 5 right informative features in the Friedman #1 dataset.

```python
>>> from sklearn.datasets import make_friedman1
>>> from sklearn.feature_selection import RFE
>>> from sklearn.svm import SVR

>>> X, y = make_friedman1(n_samples=50, n_features=10, random_state=0)
>>> estimator = SVR(kernel="linear")
>>> selector = RFE(estimator, 5, step=1)
>>> selector = selector.fit(X, y)
>>> selector.support_
array([True, True, True, True, True, False, False, False, False, False], dtype=bool)
>>> selector.ranking_
array([1, 1, 1, 1, 6, 4, 3, 2, 5])
```
Attributes

<table>
<thead>
<tr>
<th>Attribute</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>n_features_</code></td>
<td>The number of selected features.</td>
</tr>
<tr>
<td><code>support_</code></td>
<td>array of shape [n_features] The mask of selected features.</td>
</tr>
<tr>
<td><code>ranking_</code></td>
<td>array of shape [n_features] The feature ranking, such that ranking_[i]</td>
</tr>
<tr>
<td></td>
<td>corresponds to the ranking position of the i-th feature. Selected (i.e.,</td>
</tr>
<tr>
<td></td>
<td>estimated best) features are assigned rank 1.</td>
</tr>
</tbody>
</table>

Methods

```python
__init__(estimator, n_features_to_select, step=1)

fit(X, y)
Fit the RFE model and then the underlying estimator on the selected features.

Parameters X : array of shape [n_samples, n_features]
The training input samples.

y : array of shape [n_samples]
The target values.

get_params([deep])
Get parameters for the estimator

Parameters deep : boolean, optional:
If True, will return the parameters for this estimator and contained subobjects that are estimators.

predict(X)
Reduce X to the selected features and then predict using the underlying estimator.

Parameters X : array of shape [n_samples, n_features]
The input samples.

Returns y : array of shape [n_samples]
The predicted target values.

score(X, y)
Reduce X to the selected features and then return the score of the underlying estimator.

Parameters X : array of shape [n_samples, n_features]
The input samples.

y : array of shape [n_samples]
The target values.
```

1.8. Reference 383
**Parameters**

- `estimator` : object
  A supervised learning estimator with a `fit` method that updates a `coef_` attribute that holds the fitted parameters. Important features must correspond to high absolute values in the `coef_` array.

  For instance, this is the case for most supervised learning algorithms such as Support Vector Classifiers and Generalized Linear Models from the `svm` and `linear_model` modules.

- `step` : int or float, optional (default=1)
  If greater than or equal to 1, then `step` corresponds to the (integer) number of features to remove at each iteration. If within (0.0, 1.0), then `step` corresponds to the percentage (rounded down) of features to remove at each iteration.

- `cv` : int or cross-validation generator, optional (default=None)
  If int, it is the number of folds. If None, 3-fold cross-validation is performed by default. Specific cross-validation objects can also be passed, see `sklearn.cross_validation` module for details.

- `loss_function` : function, optional (default=None)
  The loss function to minimize by cross-validation. If None, then the score function of the estimator is maximized.

**References**

[R62]
**Examples**

The following example shows how to retrieve the a-priori not known 5 informative features in the Friedman #1 dataset.

```python
>>> from sklearn.datasets import make_friedman1
>>> from sklearn.feature_selection import RFE
>>> from sklearn.svm import SVR

>>> X, y = make_friedman1(n_samples=50, n_features=10, random_state=0)
>>> estimator = SVR(kernel="linear")
>>> selector = RFE(estimator, step=1, cv=5)
>>> selector = selector.fit(X, y)
>>> selector.support_
array([ True, True, True, True, True,
       False, False, False, False, False], dtype=bool)
>>> selector.ranking_
array([1, 1, 1, 1, 1, 6, 4, 3, 2, 5])
```

**Attributes**

<table>
<thead>
<tr>
<th>Attribute</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>n_features_</td>
<td>The number of selected features with cross-validation.</td>
</tr>
<tr>
<td>support_</td>
<td>The mask of selected features.</td>
</tr>
<tr>
<td>ranking_</td>
<td>The feature ranking, such that ranking_[i] corresponds to the ranking position of the i-th feature. Selected (i.e., estimated best) features are assigned rank 1.</td>
</tr>
<tr>
<td>cv_scores_</td>
<td>The cross-validation scores such that cv_scores_[i] corresponds to the CV score of the i-th subset of features.</td>
</tr>
</tbody>
</table>

**Methods**

- `fit(X, y)` Fit the RFE model and automatically tune the number of selected features.
- `get_params([deep])` Get parameters for the estimator
- `predict(X)` Reduce X to the selected features and then predict using the estimator.
- `score(X, y)` Reduce X to the selected features and then return the score of the estimator.
- `set_params(**params)` Set the parameters of the estimator.
- `transform(X)` Reduce X to the selected features during the elimination.

**__init__** *(estimator, step=1, cv=None, loss_func=None)*

- `fit(X, y)` Fit the RFE model and automatically tune the number of selected features.

**Parameters**

**X** : array of shape [n_samples, n_features]

Training vector, where `n_samples` is the number of samples and `n_features` is the total number of features.

**y** : array of shape [n_samples]

Target values (integers for classification, real numbers for regression).

**get_params** *(deep=True)*

Get parameters for the estimator
Parameters deep: boolean, optional:
    If True, will return the parameters for this estimator and contained subobjects that are
    estimators.

predict (X)
    Reduce X to the selected features and then predict using the underlying estimator.
    Parameters X : array of shape [n_samples, n_features]
        The input samples.
    Returns y : array of shape [n_samples]
        The predicted target values.

score (X, y)
    Reduce X to the selected features and then return the score of the underlying estimator.
    Parameters X : array of shape [n_samples, n_features]
        The input samples.
    y : array of shape [n_samples]
        The target values.

set_params (**params)
    Set the parameters of the estimator.
    The method works on simple estimators as well as on nested objects (such as pipelines). The former have
    parameters of the form <component>__<parameter> so that it's possible to update each component
    of a nested object.
    Returns self :

transform (X)
    Reduce X to the selected features during the elimination.
    Parameters X : array of shape [n_samples, n_features]
        The input samples.
    Returns X_r : array of shape [n_samples, n_selected_features]
        The input samples with only the features selected during the elimination.

feature_selection.chi2(X, y)
    Compute $\chi^2$ (chi-squared) statistic for each class/feature combination.

feature_selection.f_classif(X, y)
    Compute the Anova F-value for the provided sample

feature_selection.f_regression(X, y[, center])
    Univariate linear regression tests

sklearn.feature_selection.chi2

sklearn.feature_selection.chi2 (X, y)
    Compute $\chi^2$ (chi-squared) statistic for each class/feature combination.

    This transformer can be used to select the n_features features with the highest values for the $\chi^2$ (chi-square)
    statistic from either boolean or multinomially distributed data (e.g., term counts in document classification)
    relative to the classes.

    Recall that the $\chi^2$ statistic measures dependence between stochastic variables, so a transformer based on this
    function “weeds out” the features that are the most likely to be independent of class and therefore irrelevant for
    classification.
Parameters $X$ : {array-like, sparse matrix}, shape = [n_samples, n_features_in]

Sample vectors.

$y$ : array-like, shape = n_samples

Target vector (class labels).

Notes

Complexity of this algorithm is $O(n_{classes} \times n_{features})$.

sklearn.feature_selection.f_classif

sklearn.feature_selection.f_classif($X, y$)

Compute the Anova F-value for the provided sample

Parameters $X$ : array of shape (n_samples, n_features)

the set of regressors that will be tested sequentially

$y$ : array of shape(n_samples)

the data matrix

Returns $F$ : array of shape (m),

the set of F values

$pval$ : array of shape(m),

the set of p-values

sklearn.feature_selection.f_regression

sklearn.feature_selection.f_regression($X, y$, center=True)

Univariate linear regression tests

Quick linear model for testing the effect of a single regressor, sequentially for many regressors.

This is done in 3 steps: 1. the regressor of interest and the data are orthogonalized wrt constant regressors 2. the cross correlation between data and regressors is computed 3. it is converted to an F score then to a p-value

Parameters $X$ : array of shape (n_samples, n_features)

the set of regressors that will be tested sequentially

$y$ : array of shape(n_samples)

the data matrix

center : True, bool,

If true, $X$ and $y$ are centered

Returns $F$ : array of shape (m),

the set of F values

$pval$ : array of shape(m),

the set of p-values
1.8.9 sklearn.gaussian_process: Gaussian Processes

The sklearn.gaussian_process module implements scalar Gaussian Process based predictions.

User guide: See the Gaussian Processes section for further details.

gaussian_process.GaussianProcess([regr, ...])  The Gaussian Process model class.

sklearn.gaussian_process.GaussianProcess

class sklearn.gaussian_process.GaussianProcess(regr='constant', corr='squared_exponential', beta0=None, storage_mode='full', verbose=False, theta0=0.1, thetaL=None, thetaU=None, optimizer='fmin_cobyla', random_start=1, normalize=True, nugget=2.2204460492503131e-15, random_state=None)

The Gaussian Process model class.

Parameters regr : string or callable, optional

A regression function returning an array of outputs of the linear regression functional basis. The number of observations n_samples should be greater than the size p of this basis. Default assumes a simple constant regression trend. Available built-in regression models are:

'constant', 'linear', 'quadratic'

corr : string or callable, optional

A stationary autocorrelation function returning the autocorrelation between two points x and x'. Default assumes a squared-exponential autocorrelation model. Built-in correlation models are:

'absolute_exponential', 'squared_exponential', 'generalized_exponential', 'cubic', 'linear'

beta0 : double array_like, optional

The regression weight vector to perform Ordinary Kriging (OK). Default assumes Universal Kriging (UK) so that the vector beta of regression weights is estimated using the maximum likelihood principle.

storage_mode : string, optional

A string specifying whether the Cholesky decomposition of the correlation matrix should be stored in the class (storage_mode = ‘full’) or not (storage_mode = ‘light’). Default assumes storage_mode = ‘full’, so that the Cholesky decomposition of the correlation matrix is stored. This might be a useful parameter when one is not interested in the MSE and only plan to estimate the BLUP, for which the correlation matrix is not required.

verbose : boolean, optional

A boolean specifying the verbose level. Default is verbose = False.

theta0 : double array_like, optional
An array with shape \((n\_features, )\) or \((1, )\). The parameters in the autocorrelation model. If thetaL and thetaU are also specified, theta0 is considered as the starting point for the maximum likelihood estimation of the best set of parameters. Default assumes isotropic autocorrelation model with theta0 = 1e-1.

\textbf{thetaL} : double array_like, optional

An array with shape matching theta0’s. Lower bound on the autocorrelation parameters for maximum likelihood estimation. Default is None, so that it skips maximum likelihood estimation and it uses theta0.

\textbf{thetaU} : double array_like, optional

An array with shape matching theta0’s. Upper bound on the autocorrelation parameters for maximum likelihood estimation. Default is None, so that it skips maximum likelihood estimation and it uses theta0.

\textbf{normalize} : boolean, optional

Input X and observations y are centered and reduced wrt means and standard deviations estimated from the n_samples observations provided. Default is normalize = True so that data is normalized to ease maximum likelihood estimation.

\textbf{nugget} : double or ndarray, optional

Introduce a nugget effect to allow smooth predictions from noisy data. If nugget is an ndarray, it must be the same length as the number of data points used for the fit. The nugget is added to the diagonal of the assumed training covariance; in this way it acts as a Tikhonov regularization in the problem. In the special case of the squared exponential correlation function, the nugget mathematically represents the variance of the input values. Default assumes a nugget close to machine precision for the sake of robustness (nugget = 10. * MACHINE_EPSILON).

\textbf{optimizer} : string, optional

A string specifying the optimization algorithm to be used. Default uses ‘fmin_cobyla’ algorithm from scipy.optimize. Available optimizers are:

’fmin_cobyla’, ’Welch’

‘Welch’ optimizer is due to Welch et al., see reference [WBSWM1992]. It consists in iterating over several one-dimensional optimizations instead of running one single multi-dimensional optimization.

\textbf{random_start} : int, optional

The number of times the Maximum Likelihood Estimation should be performed from a random starting point. The first MLE always uses the specified starting point (theta0), the next starting points are picked at random according to an exponential distribution (log-uniform on [thetaL, thetaU]). Default does not use random starting point (random_start = 1).

\textbf{random_state: integer or numpy.RandomState, optional}:

The generator used to shuffle the sequence of coordinates of theta in the Welch optimizer. If an integer is given, it fixes the seed. Defaults to the global numpy random number generator.
Notes

The presentation implementation is based on a translation of the DACE Matlab toolbox, see reference [NLNS2002].

References

[NLNS2002], [WBSWM1992]

Examples

```python
>>> import numpy as np
>>> from sklearn.gaussian_process import GaussianProcess
>>> X = np.array([[1., 3., 5., 6., 7., 8.]]).T
>>> y = (X * np.sin(X)).ravel()
>>> gp = GaussianProcess(theta0=0.1, thetaL=.001, thetaU=1.)
>>> gp.fit(X, y)
GaussianProcess(beta0=None... ...
```

Attributes

| theta_ | array | Specified theta OR the best set of autocorrelation parameters (the sought maximizer of the reduced likelihood function). |
| reduced_likelihood_function_value_ | array | The optimal reduced likelihood function value. |

Methods

| arg_max_reduced_likelihood_function(*args, ...) | DEPRECATED: to be removed; access self.theta_ etc. directly after fit |
| fit(X, y) | The Gaussian Process model fitting method. |
| get_params([deep]) | Get parameters for the estimator |
| predict(X[, eval_MSE, batch_size]) | This function evaluates the Gaussian Process model at x. |
| reduced_likelihood_function([theta]) | This function determines the BLUP parameters and evaluates the reduced score |
| score(X, y) | Returns the coefficient of determination R^2 of the prediction. |
| set_params(**params) | Set the parameters of the estimator. |

```python
__init__(regr='constant', corr='squared_exponential', beta0=None, storage_mode='full', verbose=False, theta0=0.1, thetaL=None, thetaU=None, optimizer='fmin_cobyla', random_start=1, normalize=True, nugget=2.2204460492503131e-15, random_state=None)
```

arg_max_reduced_likelihood_function(*args, **kwargs)

DEPRECATED: to be removed; access self.theta_ etc. directly after fit

fit (X, y)

The Gaussian Process model fitting method.

Parameters X : double array_like
An array with shape (n_samples, n_features) with the input at which observations were made.

y : double array_like
An array with shape (n_features,) with the observations of the scalar output to be predicted.

Returns gp : self
A fitted Gaussian Process model object awaiting data to perform predictions.

get_params (deep=True)
Get parameters for the estimator

Parameters deep: boolean, optional:
If True, will return the parameters for this estimator and contained subobjects that are estimators.

predict (X, eval_MSE=False, batch_size=None)
This function evaluates the Gaussian Process model at x.

Parameters X : array_like
An array with shape (n_eval, n_features) giving the point(s) at which the prediction(s) should be made.

eval_MSE : boolean, optional
A boolean specifying whether the Mean Squared Error should be evaluated or not. Default assumes evalMSE = False and evaluates only the BLUP (mean prediction).

batch_size : integer, optional
An integer giving the maximum number of points that can be evaluated simultaneously (depending on the available memory). Default is None so that all given points are evaluated at the same time.

Returns y : array_like
An array with shape (n_eval,) with the Best Linear Unbiased Prediction at x.

MSE : array_like, optional (if eval_MSE == True)
An array with shape (n_eval,) with the Mean Squared Error at x.

reduced_likelihood_function (theta=None)
This function determines the BLUP parameters and evaluates the reduced likelihood function for the given autocorrelation parameters theta.

Maximizing this function wrt the autocorrelation parameters theta is equivalent to maximizing the likelihood of the assumed joint Gaussian distribution of the observations y evaluated onto the design of experiments X.

Parameters theta : array_like, optional
An array containing the autocorrelation parameters at which the Gaussian Process model parameters should be determined. Default uses the built-in autocorrelation parameters (ie theta = self.theta_).

Returns reduced_likelihood_function_value : double
The value of the reduced likelihood function associated to the given autocorrelation parameters theta.
par : dict

A dictionary containing the requested Gaussian Process model parameters:

- **sigma2** : Gaussian Process variance.
- **beta** : Generalized least-squares regression weights for Universal Kriging or given beta0 for Ordinary Kriging.
- **gamma** : Gaussian Process weights.
- **C** : Cholesky decomposition of the correlation matrix $[R]$.
- **Ft** : Solution of the linear equation system : $[R] \times Ft = F$.
- **G** : QR decomposition of the matrix Ft.

**reduced_likelihood_function_value**

DEPRECATED: reduced_likelihood_function_value is deprecated and will be removed;
please use reduced_likelihood_function_value_ instead.

**score** $(X, y)$

Returns the coefficient of determination $R^2$ of the prediction.

$R^2$ is defined as $(1 - u/v)$, where $u$ is the regression sum of squares $((y - y_{pred})^2).sum()$ and $v$ is the residual sum of squares $((y_{true} - y_{true}.mean())^2).sum()$. Best possible score is 1.0, lower values are worse.

**Parameters** $X$ : array-like, shape = [n_samples, n_features]

Training set.

$y$ : array-like, shape = [n_samples]

Returns $z$ : float

**set_params** (**params**)

Set the parameters of the estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The former have parameters of the form `<component>__<parameter>` so that it’s possible to update each component of a nested object.

Returns self :

**theta**

DEPRECATED: theta is deprecated and will be removed; please use theta_ instead.

<table>
<thead>
<tr>
<th>sklearn.gaussian_process.correlation_models.absolute_exponential(...)</th>
<th>Absolute exponential autocorrelation model. (Ornstein-Uhlenbeck stochastic process):</th>
</tr>
</thead>
<tbody>
<tr>
<td>sklearn.gaussian_process.correlation_models.squared_exponential(...)</td>
<td>Squared exponential correlation model.</td>
</tr>
<tr>
<td>sklearn.gaussian_process.correlation_models.generalized_exponential(...)</td>
<td>Generalized exponential correlation model.</td>
</tr>
<tr>
<td>sklearn.gaussian_process.correlation_models.pure_nugget(...)</td>
<td>Spatial independence correlation model.</td>
</tr>
<tr>
<td>sklearn.gaussian_process.correlation_models.cubic(...)</td>
<td>Cubic correlation model.</td>
</tr>
<tr>
<td>sklearn.gaussian_process.correlation_models.linear(...)</td>
<td>Linear correlation model.</td>
</tr>
<tr>
<td>sklearn.gaussian_process.regression_models.constant(x)</td>
<td>Zero order polynomial (constant, p = 1)</td>
</tr>
<tr>
<td>sklearn.gaussian_process.regression_models.linear(x)</td>
<td>First order polynomial (linear, p = n+1)</td>
</tr>
<tr>
<td>sklearn.gaussian_process.regression_models.quadratic(x)</td>
<td>Second order polynomial (quadratic, p = n(n+1)/2)</td>
</tr>
</tbody>
</table>
theta, dx --> r(\theta, dx) = \exp\left( \sum_{i=1}^{n} - \theta_i \times |dx_i| \right)

**Parameters theta** : array_like
An array with shape 1 (isotropic) or n (anisotropic) giving the autocorrelation parameter(s).

**dx** : array_like
An array with shape (n_eval, n_features) giving the componentwise distances between locations x and x’ at which the correlation model should be evaluated.

**Returns r** : array_like
An array with shape (n_eval, ) containing the values of the autocorrelation model.

**sklearn.gaussian_process.correlation_models.squared_exponential**

sklearn.gaussian_process.correlation_models.squared_exponential(\theta, d)
Squared exponential correlation model (Radial Basis Function). (Infinitely differentiable stochastic process, very smooth):

\begin{align*}
\theta, dx \rightarrow r(\theta, dx) &= \exp\left( \sum_{i=1}^{n} - \theta_i \times (dx_i)^2 \right)
\end{align*}

**Parameters theta** : array_like
An array with shape 1 (isotropic) or n (anisotropic) giving the autocorrelation parameter(s).

**dx** : array_like
An array with shape (n_eval, n_features) giving the componentwise distances between locations x and x’ at which the correlation model should be evaluated.

**Returns r** : array_like
An array with shape (n_eval, ) containing the values of the autocorrelation model.

**sklearn.gaussian_process.correlation_models.generalized_exponential**

sklearn.gaussian_process.correlation_models.generalized_exponential(\theta, d)
Generalized exponential correlation model. (Useful when one does not know the smoothness of the function to be predicted.):

\begin{align*}
\theta, dx \rightarrow r(\theta, dx) &= \exp\left( \sum_{i=1}^{n} - \theta_i \times |dx_i|^p \right)
\end{align*}

**Parameters theta** : array_like
An array with shape 1+1 (isotropic) or n+1 (anisotropic) giving the autocorrelation parameter(s) (\theta, p).

**dx** : array_like
An array with shape (n_eval, n_features) giving the componentwise distances between locations x and x’ at which the correlation model should be evaluated.

**Returns r**: array_like

An array with shape (n_eval, ) with the values of the autocorrelation model.

**sklearn.gaussian_process.correlation_models.pure_nugget**

sklearn.gaussian_process.correlation_models.pure_nugget(\(\theta, d\))

Spatial independence correlation model (pure nugget). (Useful when one wants to solve an ordinary least squares problem!):

\[
\theta, dx \rightarrow r(\theta, dx) = 1 \text{ if } \sum_{i=1}^{n} |dx_i| = 0 \\
0 \text{ otherwise}
\]

**Parameters \(\theta\)**: array_like

None.

\(dx\) : array_like

An array with shape (n_eval, n_features) giving the componentwise distances between locations x and x’ at which the correlation model should be evaluated.

**Returns r**: array_like

An array with shape (n_eval, ) with the values of the autocorrelation model.

**sklearn.gaussian_process.correlation_models.cubic**

sklearn.gaussian_process.correlation_models.cubic(\(\theta, d\))

Cubic correlation model:

\[
\theta, dx \rightarrow r(\theta, dx) = \prod_{j=1}^{n} \max(0, 1 - 3(\theta_j d_{ij})^2 + 2(\theta_j d_{ij})^3), \quad i = 1, \ldots, m
\]

**Parameters \(\theta\)**: array_like

An array with shape 1 (isotropic) or n (anisotropic) giving the autocorrelation parameter(s).

\(dx\) : array_like

An array with shape (n_eval, n_features) giving the componentwise distances between locations x and x’ at which the correlation model should be evaluated.

**Returns r**: array_like

An array with shape (n_eval, ) with the values of the autocorrelation model.
**sklearn.gaussian_process.correlation_models.linear**

**sklearn.gaussian_process.correlation_models.linear**(theta, d)

Linear correlation model:

\[ r(\theta, d) = \prod_{i=1}^{n} \max(0, 1 - \theta_j d_{ij}), \quad i = 1, \ldots, m \]

- **Parameters**
  - **theta**: array_like
    - An array with shape 1 (isotropic) or n (anisotropic) giving the autocorrelation parameter(s).
  - **dx**: array_like
    - An array with shape (n_eval, n_features) giving the componentwise distances between locations x and x' at which the correlation model should be evaluated.

- **Returns**
  - **r**: array_like
    - An array with shape (n_eval, ) with the values of the autocorrelation model.

**sklearn.gaussian_process.regression_models.constant**

**sklearn.gaussian_process.regression_models.constant**(x)

Zero order polynomial (constant, p = 1) regression model.

\[ f(x) = 1 \]

- **Parameters**
  - **x**: array_like
    - An array with shape (n_eval, n_features) giving the locations x at which the regression model should be evaluated.

- **Returns**
  - **f**: array_like
    - An array with shape (n_eval, p) with the values of the regression model.

**sklearn.gaussian_process.regression_models.linear**

**sklearn.gaussian_process.regression_models.linear**(x)

First order polynomial (linear, p = n+1) regression model.

\[ f(x) = [1, x_1, \ldots, x_n]^T \]

- **Parameters**
  - **x**: array_like
    - An array with shape (n_eval, n_features) giving the locations x at which the regression model should be evaluated.

- **Returns**
  - **f**: array_like
    - An array with shape (n_eval, p) with the values of the regression model.
sklearn.gaussian_process.regression_models.quadratic

sklearn.gaussian_process.regression_models.quadratic(x)
Second order polynomial (quadratic, \( p = n^2 + n + 1 \)) regression model.
\[
x \rightarrow f(x) = [1, \{x_i, i = 1, \ldots, n\}, \{x_i \cdot x_j, (i,j) = 1, \ldots, n\}].Ti > j
\]

Parameters x : array_like
   An array with shape (n_eval, n_features) giving the locations x at which the regression
model should be evaluated.

Returns f : array_like
   An array with shape (n_eval, p) with the values of the regression model.

1.8.10 sklearn.grid_search: Grid Search

The sklearn.grid_search includes utilities to fine-tune the parameters of an estimator.

User guide: See the Grid Search: setting estimator parameters section for further details.

    sklearn.grid_search.GridSearchCV(estimator, param_grid)  Grid search on the parameters of a classifier
    sklearn.grid_search.GridSearchCV(estimator, param_grid)  Grid search on the parameters of a classifier
    sklearn.grid_search.GridSearchCV(estimator, param_grid)  Grid search on the parameters of a classifier

sklearn.grid_search.GridSearchCV

class sklearn.grid_search.GridSearchCV(estimator, param_grid, loss_func=None, score_func=None, fit_params=None, n_jobs=1, iid=True, refit=True, cv=None, verbose=0, pre_dispatch='2*n_jobs')

Grid search on the parameters of a classifier

Important members are fit, predict.

GridSearchCV implements a “fit” method and a “predict” method like any classifier except that the parameters
of the classifier used to predict is optimized by cross-validation.

Parameters estimator: object type that implements the “fit” and “predict” methods :
   A object of that type is instantiated for each grid point.

    param_grid: dict :
       Dictionary with parameters names (string) as keys and lists of parameter settings to try
as values.

    loss_func: callable, optional :
       function that takes 2 arguments and compares them in order to evaluate the performance
of predicition (small is good) if None is passed, the score of the estimator is maximized

    score_func: callable, optional :
       A function that takes 2 arguments and compares them in order to evaluate the perform-
ance of prediction (high is good). If None is passed, the score of the estimator is maximized.

    fit_params : dict, optional
parameters to pass to the fit method

**n_jobs: int, optional**:

number of jobs to run in parallel (default 1)

**pre_dispatch: int, or string, optional**:

Controls the number of jobs that get dispatched during parallel execution. Reducing this number can be useful to avoid an explosion of memory consumption when more jobs get dispatched than CPUs can process. This parameter can be:

- None, in which case all the jobs are immediately created and spawned. Use this for lightweight and fast-running jobs, to avoid delays due to on-demand spawning of the jobs

- An int, giving the exact number of total jobs that are spawned

- A string, giving an expression as a function of n_jobs, as in `2*n_jobs`

**iid: boolean, optional**:

If True, the data is assumed to be identically distributed across the folds, and the loss minimized is the total loss per sample, and not the mean loss across the folds.

**cv**: integer or crossvalidation generator, optional

If an integer is passed, it is the number of fold (default 3). Specific crossvalidation objects can be passed, see sklearn.cross_validation module for the list of possible objects

**refit**: boolean

refit the best estimator with the entire dataset. If “False”, it is impossible to make predictions using this GridSearch instance after fitting.

**verbose**: integer

Controls the verbosity: the higher, the more messages.

See Also:

**IterGrid** generates all the combinations of a an hyperparameter grid.

**sklearn.cross_validation.train_test_split** utility function to split the data into a development set usable for fitting a GridSearchCV instance and an evaluation set for its final evaluation.

Notes

The parameters selected are those that maximize the score of the left out data, unless an explicit score_func is passed in which case it is used instead. If a loss function loss_func is passed, it overrides the score functions and is minimized.

If n_jobs was set to a value higher than one, the data is copied for each point in the grid (and not n_jobs times). This is done for efficiency reasons if individual jobs take very little time, but may raise errors if the dataset is large and not enough memory is available. A workaround in this case is to set pre_dispatch. Then, the memory is copied only pre_dispatch many times. A reasonable value for pre_dispatch is 2 * n_jobs.

Examples
>>> from sklearn import svm, grid_search, datasets
>>> iris = datasets.load_iris()
>>> parameters = {'kernel':('linear', 'rbf'), 'C':[1, 10]}
>>> svr = svm.SVC()
>>> clf = grid_search.GridSearchCV(svr, parameters)
>>> clf.fit(iris.data, iris.target)
...
GridSearchCV(cv=None,
estimator=SVC(C=1.0, cache_size=..., coef0=..., degree=..., gamma=..., kernel='rbf', probability=False,
shrinking=True, tol=...),
fit_params={}, iid=True, loss_func=None, n_jobs=1,
param_grid=...,
...)

Attributes

| grid_scores_ | dict of any to float | Contains scores for all parameter combinations in param_grid. |
| best_estimator | estimator | Estimator that was chosen by grid search, i.e. estimator which gave highest score (or smallest loss if specified) on the left out data. |
| best_score_ | float | score of best_estimator on the left out data. |
| best_params_ | dict | Parameter setting that gave the best results on the hold out data. |

Methods

| fit(X, y) | Run fit with all sets of parameters |
| get_params([deep]) | Get parameters for the estimator |
| score(X, y) | |
| set_params(**params) | Set the parameters of the estimator. |

__init__(estimator, param_grid, loss_func=None, score_func=None, fit_params=None, n_jobs=1, iid=True, refit=True, cv=None, verbose=0, pre_dispatch='2*n_jobs')

best_estimator
DEPRECATED: GridSearchCV.best_estimator is deprecated and will be removed in version 0.12. Please use GridSearchCV.best_estimator_ instead.

best_score
DEPRECATED: GridSearchCV.best_score is deprecated and will be removed in version 0.12. Please use GridSearchCV.best_score_ instead.

fit(X, y=None, **params)
Run fit with all sets of parameters

Returns the best classifier

Parameters X: array, [n_samples, n_features] :

Training vector, where n_samples in the number of samples and n_features is the number of features.

y: array-like, shape = [n_samples], optional :

Target vector relative to X for classification; None for unsupervised learning.
get_params (deep=True)
Get parameters for the estimator

Parameters deep: boolean, optional:
If True, will return the parameters for this estimator and contained subobjects that are
estimators.

set_params (**params)
Set the parameters of the estimator.
The method works on simple estimators as well as on nested objects (such as pipelines). The former have
parameters of the form <component>__<parameter> so that it’s possible to update each component
of a nested object.

Returns self:

sklearn.grid_search.IterGrid
class sklearn.grid_search.IterGrid (param_grid)
Generators on the combination of the various parameter lists given

Parameters param_grid: dict of string to sequence:
The parameter grid to explore, as a dictionary mapping estimator parameters to se-
quences of allowed values.

Returns params: dict of string to any:
Yields dictionaries mapping each estimator parameter to one of its allowed values.

See Also:

GridSearchCV uses IterGrid to perform a full parallelized grid search.

Examples

>>> from sklearn.grid_search import IterGrid
>>> param_grid = {'a':[1, 2], 'b':[True, False]}
>>> list(IterGrid(param_grid))
[{'a': 1, 'b': True}, {'a': 1, 'b': False},
 {'a': 2, 'b': True}, {'a': 2, 'b': False}]

__init__ (param_grid)

1.8.11 sklearn.hmm: Hidden Markov Models

The sklearn.hmm module implements hidden Markov models.

Warning: sklearn.hmm is orphaned, undocumented and has known numerical stability issues. If nobody volun-
teers to write documentation and make it more stable, this module will be removed in version 0.11.

User guide: See the Hidden Markov Models section for further details.

<table>
<thead>
<tr>
<th>hmm.GaussianHMM([n_components, ...])</th>
<th>Hidden Markov Model with Gaussian emissions</th>
</tr>
</thead>
<tbody>
<tr>
<td>hmm.MultinomialHMM([n_components, ...])</td>
<td>Hidden Markov Model with multinomial (discrete) emissions</td>
</tr>
<tr>
<td>hmm.GMMHMM([n_components, n_mix, startprob, ...])</td>
<td>Hidden Markov Model with Gaussian mixture emissions</td>
</tr>
</tbody>
</table>

1.8. Reference
Hidden Markov Model with Gaussian emissions

Representation of a hidden Markov model probability distribution. This class allows for easy evaluation of, sampling from, and maximum-likelihood estimation of the parameters of a HMM.

**Parameters**

- `n_components`: int
  Number of states.

- `covariance_type`: string
  String describing the type of covariance parameters to use. Must be one of ‘spherical’, ‘tied’, ‘diag’, ‘full’. Defaults to ‘diag’.

**See Also:**

GMM: Gaussian mixture model

**Examples**

```python
>>> from sklearn.hmm import GaussianHMM
>>> GaussianHMM(n_components=2)
... GaussianHMM(algorithm='viterbi',...```
Attributes

<table>
<thead>
<tr>
<th>Attribute</th>
<th>Type</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>_covariance_type</td>
<td>string</td>
<td>String describing the type of covariance parameters used by the model. Must be one of ‘spherical’, ‘tied’, ‘diag’, ‘full’.</td>
</tr>
<tr>
<td>n_features</td>
<td>int</td>
<td>Dimensionality of the Gaussian emissions.</td>
</tr>
<tr>
<td>n_components</td>
<td>int</td>
<td>Number of states in the model.</td>
</tr>
<tr>
<td>transmat</td>
<td>array, shape (n_components, n_components)</td>
<td>Matrix of transition probabilities between states.</td>
</tr>
<tr>
<td>startprob</td>
<td>array, shape (‘n_components’,)</td>
<td>Initial state occupation distribution.</td>
</tr>
<tr>
<td>means</td>
<td>array, shape (n_components, n_features)</td>
<td>Mean parameters for each state.</td>
</tr>
<tr>
<td>covars</td>
<td>array</td>
<td>Covariance parameters for each state. The shape depends on _covariance_type:</td>
</tr>
<tr>
<td></td>
<td></td>
<td>('n_components',)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>('n_features', 'n_features')</td>
</tr>
<tr>
<td></td>
<td></td>
<td>('n_components', 'n_features')</td>
</tr>
<tr>
<td></td>
<td></td>
<td>('n_components', 'n_features', 'n_features')</td>
</tr>
</tbody>
</table>

random_state: RandomState or an int seed (0 by default) | A random number generator instance |

n_iter | int, optional | Number of iterations to perform. |

thresh | float, optional | Convergence threshold. |

params | string, optional | Controls which parameters are updated in the training process. Can contain any combination of ‘s’ for startprob, ‘t’ for transmat, ‘m’ for means, and ‘c’ for covars, etc. Defaults to all parameters. |

init_params | string, optional | Controls which parameters are initialized prior to training. Can contain any combination of ‘s’ for startprob, ‘t’ for transmat, ‘m’ for means, and ‘c’ for covars, etc. Defaults to all parameters. |

Methods

**decode**(obs[, algorithm]) Find most likely state sequence corresponding to obs.

**eval**(obs) Compute the log probability under the model and compute posteriors.

**fit**(obs, **kwargs) Estimate model parameters.

**get_params**(deep) Get parameters for the estimator.

**predict**(obs[, algorithm]) Find most likely state sequence corresponding to obs.

**predict_proba**(obs) Compute the posterior probability for each state in the model.

**rvs**(args, **kwargs) DEPRECATED: rvs is deprecated in 0.11 will be removed in 0.13: use sample instead.

**sample**([n, random_state]) Generate random samples from the model.

**score**(obs) Compute the log probability under the model.

Continued on next page
Table 1.89 – continued from previous page

**set_params**(**params**) Set the parameters of the estimator.

```python
__init__(n_components=1, covariance_type='diag', startprob=None, trans- 
mat=None, startprob_prior=None, transmat_prior=None, algo-
rithm='viterbi', means_prior=None, means_weight=0, covars_prior=0.01, 
covars_weight=1, random_state=None, n_iter=10, thresh=0.01, 
params='abcdefghijklmnopqrstuvwxyzABCDEFGHIJKLMNOPQRSTUVWXYZ', 
init_params='abcdefghijklmnopqrstuvwxyzABCDEFGHIJKLMNOPQRSTUVWXYZ')
```

**algorithm**

decoder algorithm

**covariance_type**

Covariance type of the model.

Must be one of ‘spherical’, ‘tied’, ‘diag’, ‘full’.

**covars**

Return covars as a full matrix.

**decode** (obs, algorithm='viterbi')

Find most likely state sequence corresponding to `obs`. Uses the selected algorithm for decoding.

Parameters

- **obs**: array_like, shape (n, n_features)
  
  List of n_features-dimensional data points. Each row corresponds to a single data point.

- **algorithm**: string, one of the `decoder_algorithms`
  
  decoder algorithm to be used

Returns

- **logprob**: float
  
  Log probability of the maximum likelihood path through the HMM

- **state_sequence**: array_like, shape (n,)
  
  Index of the most likely states for each observation

See Also:

- `eval`
  
  Compute the log probability under the model and posteriors

- `score`
  
  Compute the log probability under the model

**eval** (obs)

Compute the log probability under the model and compute posteriors

Implements rank and beam pruning in the forward-backward algorithm to speed up inference in large models.

Parameters

- **obs**: array_like, shape (n, n_features)
  
  Sequence of n_features-dimensional data points. Each row corresponds to a single point in the sequence.

Returns

- **logprob**: float
  
  Log likelihood of the sequence `obs`

- **posteriors**: array_like, shape (n, n_components)
  
  Posterior probabilities of each state for each observation
See Also:

- `score` Compute the log probability under the model
- `decode` Find most likely state sequence corresponding to an observation

`fit` *(obs, **kwargs)*
Estimate model parameters.

An initialization step is performed before entering the EM algorithm. If you want to avoid this step, set the keyword argument init_params to the empty string `''`. Likewise, if you would like just to do an initialization, call this method with n_iter=0.

Parameters

- **obs**: list
  - List of array-like observation sequences (shape (n_i, n_features)).

Notes

In general, `logprob` should be non-decreasing unless aggressive pruning is used. Decreasing `logprob` is generally a sign of overfitting (e.g. a covariance parameter getting too small). You can fix this by getting more training data, or decreasing `covars_prior`.

Please note that setting parameters in the ‘fit‘ method is deprecated and will be removed in the next release. Set it on initialization instead.

`get_params` *(deep=True)*
Get parameters for the estimator

Parameters

- **deep**: boolean, optional
  - If True, will return the parameters for this estimator and contained subobjects that are estimators.

`means_`
Mean parameters for each state.

`predict` *(obs, algorithm='viterbi')*
Find most likely state sequence corresponding to an observation.

Parameters

- **obs**: array_like, shape (n, n_features)
  - List of n_features-dimensional data points. Each row corresponds to a single data point.

Returns

- **state_sequence**: array_like, shape (n,)
  - Index of the most likely states for each observation

`predict_proba` *(obs)*
Compute the posterior probability for each state in the model.

Parameters

- **obs**: array_like, shape (n, n_features)
  - List of n_features-dimensional data points. Each row corresponds to a single data point.

Returns

- **T**: array-like, shape (n, n_components)
  - Returns the probability of the sample for each state in the model.

`rvs` *(args, **kwargs)*
DEPRECATED: rvs is deprecated in 0.11 will be removed in 0.13: use sample instead
sample

Generate random samples from the model.

Parameters:

n : int
   Number of samples to generate.

random_state: RandomState or an int seed (0 by default):
   A random number generator instance. If None is given, the object’s random_state is used.

Returns:

(obs, hidden_states):

obs : array_like, length n
   List of samples

hidden_states : array_like, length n
   List of hidden states

score

Compute the log probability under the model.

Parameters:

obs : array_like, shape (n, n_features)
   Sequence of n_features-dimensional data points. Each row corresponds to a single data point.

Returns:

logprob : float
   Log likelihood of the obs

See Also:

- eval: Compute the log probability under the model and posteriors
- decode: Find most likely state sequence corresponding to a obs

set_params

Set the parameters of the estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The former have parameters of the form <component>__<parameter> so that it’s possible to update each component of a nested object.

Returns:

self

startprob_

Mixing startprob for each state.

transmat_

Matrix of transition probabilities.

sklearn.hmm.MultinomialHMM

class sklearn.hmm.MultinomialHMM(n_components=1, startprob=None, transmat=None, startprob_prior=None, transmat_prior=None, algorithm=’viterbi’, random_state=None, n_iter=10, thresh=0.01, params=’abcdefghijklmnopqrstuvwxyzABCDEFGHIJKLMNOPQRSTUVWXYZ’, init_params=’abcdefghijklmnopqrstuvwxyzABCDEFGHIJKLMNOPQRSTUVWXYZ’)

Hidden Markov Model with multinomial (discrete) emissions

See Also:

- GaussianHMM: HMM with Gaussian emissions
Examples

```python
>>> from sklearn.hmm import MultinomialHMM
>>> MultinomialHMM(n_components=2)
...
MultinomialHMM(algorithm='viterbi',...)
```

Attributes

<table>
<thead>
<tr>
<th>Attribute</th>
<th>Type</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>n_components</td>
<td>int</td>
<td>Number of states in the model.</td>
</tr>
<tr>
<td>n_symbols</td>
<td>int</td>
<td>Number of possible symbols emitted by the model (in the observations).</td>
</tr>
<tr>
<td>transmat</td>
<td>array, shape (n_components, n_components)</td>
<td>Matrix of transition probabilities between states.</td>
</tr>
<tr>
<td>startprob</td>
<td>array, shape ('n_components',)</td>
<td>Initial state occupation distribution.</td>
</tr>
<tr>
<td>emissionprob</td>
<td>array, shape ('n_components', 'n_symbols')</td>
<td>Probability of emitting a given symbol when in each state.</td>
</tr>
<tr>
<td>random_state</td>
<td>RandomState or an int seed (0 by default)</td>
<td>A random number generator instance</td>
</tr>
<tr>
<td>n_iter</td>
<td>int, optional</td>
<td>Number of iterations to perform.</td>
</tr>
<tr>
<td>thresh</td>
<td>float, optional</td>
<td>Convergence threshold.</td>
</tr>
<tr>
<td>params</td>
<td>string, optional</td>
<td>Controls which parameters are updated in the training process. Can contain any combination of ‘s’ for startprob, ‘t’ for transmat, ‘m’ for means, and ‘c’ for covars, etc. Defaults to all parameters.</td>
</tr>
<tr>
<td>init_params</td>
<td>string, optional</td>
<td>Controls which parameters are initialized prior to training. Can contain any combination of ‘s’ for startprob, ‘t’ for transmat, ‘m’ for means, and ‘c’ for covars, etc. Defaults to all parameters.</td>
</tr>
</tbody>
</table>

Methods

- `decode(obs[, algorithm])`: Find most likely state sequence corresponding to `obs`.
- `eval(obs)`: Compute the log probability under the model and compute posteriors.
- `fit(obs, **kwargs)`: Estimate model parameters.
- `get_params([deep])`: Get parameters for the estimator.
- `predict(obs[, algorithm])`: Find most likely state sequence corresponding to `obs`.
- `predict_proba(obs)`: Compute the posterior probability for each state in the model.
- `rvs(*args, **kwargs)`: DEPRECATED: rvs is deprecated in 0.11 will be removed in 0.13: use sample instead.
- `sample([n, random_state])`: Generate random samples from the model.
- `score(obs)`: Compute the log probability under the model.
- `set_params(**params)`: Set the parameters of the estimator.
__init__ (n_components=1, startprob=None, transmat=None, startprob_prior=None, transmat_prior=None, algorithm='viterbi', random_state=None, n_iter=10, thresh=0.01, params='abcdefghijklmnopqrstuvwxyzABCDEFGHIJKLMNOPQRSTUVWXYZ', init_params='abcdefghijklmnopqrstuvwxyzABCDEFGHIJKLMNOPQRSTUVWXYZ')

Create a hidden Markov model with multinomial emissions.

Parameters n_components : int
    Number of states.

algorithm : decoder algorithm

decode (obs, algorithm='viterbi')
    Find most likely state sequence corresponding to obs. Uses the selected algorithm for decoding.

Parameters obs : array_like, shape (n, n_features)
    List of n_features-dimensional data points. Each row corresponds to a single data point.

algorithm : string, one of the decoder_algorithms
    decoder algorithm to be used

Returns logprob : float
    Log probability of the maximum likelihood path through the HMM

state_sequence : array_like, shape (n,)
    Index of the most likely states for each observation

See Also:

eval : Compute the log probability under the model and posteriors
score : Compute the log probability under the model

emissionprob_ : Emission probability distribution for each state.
eval (obs)
    Compute the log probability under the model and compute posteriors

    Implements rank and beam pruning in the forward-backward algorithm to speed up inference in large models.

Parameters obs : array_like, shape (n, n_features)
    Sequence of n_features-dimensional data points. Each row corresponds to a single point in the sequence.

Returns logprob : float
    Log likelihood of the sequence obs

posteriors: array_like, shape (n, n_components):
    Posterior probabilities of each state for each observation

See Also:

score : Compute the log probability under the model
decode : Find most likely state sequence corresponding to a obs
**fit**(obs, **kwargs)

Estimate model parameters.

An initialization step is performed before entering the EM algorithm. If you want to avoid this step, set the keyword argument init_params to the empty string ''. Likewise, if you would like just to do an initialization, call this method with n_iter=0.

**Parameters obs**: list

List of array-like observation sequences (shape (n_i, n_features)).

**Notes**

In general, logprob should be non-decreasing unless aggressive pruning is used. Decreasing logprob is generally a sign of overfitting (e.g. a covariance parameter getting too small). You can fix this by getting more training data, or decreasing covars_prior.

Please note that setting parameters in the 'fit' method is deprecated and will be removed in the next release. Set it on initialization instead.

**get_params**(deep=True)

Get parameters for the estimator

**Parameters deep**: boolean, optional

If True, will return the parameters for this estimator and contained subobjects that are estimators.

**predict**(obs, algorithm='viterbi')

Find most likely state sequence corresponding to obs.

**Parameters obs**: array_like, shape (n, n_features)

List of n_features-dimensional data points. Each row corresponds to a single data point.

**Returns state_sequence**: array_like, shape (n,)

Index of the most likely states for each observation

**predict_proba**(obs)

Compute the posterior probability for each state in the model

**Parameters obs**: array_like, shape (n, n_features)

List of n_features-dimensional data points. Each row corresponds to a single data point.

**Returns T**: array-like, shape (n, n_components)

Returns the probability of the sample for each state in the model.

**rvs**(args, **kwargs)

DEPRECATED: rvs is deprecated in 0.11 will be removed in 0.13: use sample instead

**sample**(n=1, random_state=None)

Generate random samples from the model.

**Parameters n**: int

Number of samples to generate.

random_state: RandomState or an int seed (0 by default)

A random number generator instance. If None is given, the object’s random_state is used
Returns (obs, hidden_states):
    obs : array_like, length n List of samples
    hidden_states : array_like, length n List of hidden states

score(obs)
Compute the log probability under the model.

Parameters obs : array_like, shape (n, n_features)
    Sequence of n_features-dimensional data points. Each row corresponds to a single data
    point.

Returns logprob : float
    Log likelihood of the obs

See Also:
- eval: Compute the log probability under the model and posteriors
- decode: Find most likely state sequence corresponding to a obs

set_params(**params)
Set the parameters of the estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The former have
parameters of the form <component>__<parameter> so that it’s possible to update each component
of a nested object.

Returns self:

startprob_
    Mixing startprob for each state.

transmat_
    Matrix of transition probabilities.

sklearn.hmm.GMMHMM

class sklearn.hmm.GMMHMM(n_components=1, n_mix=1, startprob=None, trans-
    mat=None, startprob_prior=None, transmat_prior=None, al-
    gorithm='viterbi', gmms=None, covariance_type='diag', cov-
    ars_prior=0.01, random_state=None, n_iter=10, thresh=0.01,
    params='abcdefghijklmnopqrstuvwxyzABCDEFGHIJKLMNOPQRSTUVWXYZ',
    init_params='abcdefghijklmnopqrstuvwxyzABCDEFGHIJKLMNOPQRSTUVWXYZ')

Hidden Markov Model with Gaussian mixture emissions

See Also:
- GaussianHMM: HMM with Gaussian emissions

Examples

>>> from sklearn.hmm import GMMHMM
>>> GMMHMM(n_components=2, n_mix=10, covariance_type='diag')
...
scikit-learn user guide, Release 0.12-git

### Attributes

<table>
<thead>
<tr>
<th>init_params</th>
<th>string, optional</th>
<th>Controls which parameters are initialized prior to training. Can contain any combination of ’s’ for startprob, ’t’ for transmat, ‘m’ for means, and ‘c’ for covars, etc. Defaults to all parameters.</th>
</tr>
</thead>
<tbody>
<tr>
<td>params</td>
<td>string, optional</td>
<td>Controls which parameters are updated in the training process. Can contain any combination of ’s’ for startprob, ’t’ for transmat,’m’ for means, and ‘c’ for covars, etc. Defaults to all parameters.</td>
</tr>
<tr>
<td>n_components</td>
<td>int</td>
<td>Number of states in the model.</td>
</tr>
<tr>
<td>transmat</td>
<td>array, shape</td>
<td>Matrix of transition probabilities between states.</td>
</tr>
<tr>
<td>startprob</td>
<td>array, shape</td>
<td>Initial state occupation distribution.</td>
</tr>
<tr>
<td>gmms</td>
<td>array of GMM</td>
<td>GMM emission distributions for each state.</td>
</tr>
<tr>
<td>random_state</td>
<td>RandomState or an int seed (0 by default)</td>
<td>A random number generator instance</td>
</tr>
<tr>
<td>n_iter</td>
<td>int, optional</td>
<td>Number of iterations to perform.</td>
</tr>
<tr>
<td>thresh</td>
<td>float, optional</td>
<td>Convergence threshold.</td>
</tr>
</tbody>
</table>

### Methods

| decode(obs[, algorithm]) | Find most likely state sequence corresponding to obs. |
| eval(obs)               | Compute the log probability under the model and compute posteriors |
| fit(obs, **kwargs)      | Estimate model parameters. |
| get_params([deep])     | Get parameters for the estimator |
| predict(obs[, algorithm]) | Find most likely state sequence corresponding to obs. |
| predict_proba(obs)     | Compute the posterior probability for each state in the model |
| rvs(*args, **kwargs)   | DEPRECATED: rvs is deprecated in 0.11 will be removed in 0.13: use sample instead |
| sample(n, random_state) | Generate random samples from the model. |
| score(obs)             | Compute the log probability under the model. |
| set_params(**params)   | Set the parameters of the estimator. |

```
__init__(n_components=1, n_mix=1, startprob=None, transmat=None, startprob_prior=None, transmat_prior=None, algorithm='viterbi', gmms=None, covariance_type='diag', covars_prior=0.01, random_state=None, n_iter=10, thresh=0.01, params='abcdefghijklmnopqrstuvwxyzABCDEFGHIJKLMNOPQRSTUVWXYZ', init_params='abcdefghijklmnopqrstuvwxyzABCDEFGHIJKLMNOPQRSTUVWXYZ')
```

Create a hidden Markov model with GMM emissions.

**Parameters**

- **n_components** : int
  Number of states.

- **algorithm**
  Decoder algorithm

- **covariance_type**
  Covariance type of the model.
Must be one of ‘spherical’, ‘tied’, ‘diag’, ‘full’.

**decode**(obs, algorithm='viterbi')

Find most likely state sequence corresponding to obs. Uses the selected algorithm for decoding.

**Parameters**

- **obs**: array_like, shape (n, n_features)
  
  List of n_features-dimensional data points. Each row corresponds to a single data point.

- **algorithm**: string, one of the decoder_algorithms
  
  decoder algorithm to be used

- **Returns**

  - **logprob**: float
    
    Log probability of the maximum likelihood path through the HMM

  - **state_sequence**: array_like, shape (n,)
    
    Index of the most likely states for each observation

**See Also:**

- **eval**: Compute the log probability under the model and posteriors
- **score**: Compute the log probability under the model

**eval**(obs)

Compute the log probability under the model and compute posteriors

Implements rank and beam pruning in the forward-backward algorithm to speed up inference in large models.

**Parameters**

- **obs**: array_like, shape (n, n_features)
  
  Sequence of n_features-dimensional data points. Each row corresponds to a single point in the sequence.

- **Returns**

  - **logprob**: float
    
    Log likelihood of the sequence obs

  - **posteriors**: array_like, shape (n, n_components)
    
    Posterior probabilities of each state for each observation

**See Also:**

- **score**: Compute the log probability under the model
- **decode**: Find most likely state sequence corresponding to a obs

**fit**(obs, **kwargs)

Estimate model parameters.

An initialization step is performed before entering the EM algorithm. If you want to avoid this step, set the keyword argument init_params to the empty string ‘’. Likewise, if you would like just to do an initialization, call this method with n_iter=0.

**Parameters**

- **obs**: list
  
  List of array-like observation sequences (shape (n_i, n_features)).
Notes

In general, \textit{logprob} should be non-decreasing unless aggressive pruning is used. Decreasing \textit{logprob} is generally a sign of overfitting (e.g. a covariance parameter getting too small). You can fix this by getting more training data, or decreasing \textit{covars\_prior}.

Please note that setting parameters in the ‘fit’ method is deprecated and will be removed in the next release. Set it on initialization instead.

\textbf{get\_params} (\textit{deep=True})

Get parameters for the estimator

\begin{verbatim}
Parameters deep: boolean, optional :
If True, will return the parameters for this estimator and contained subobjects that are estimators.
\end{verbatim}

\textbf{predict} (\textit{obs, algorithm='viterbi'})

Find most likely state sequence corresponding to \textit{obs}.

\begin{verbatim}
Parameters obs : array_like, shape (n, n_features)
List of n_features-dimensional data points. Each row corresponds to a single data point.

Returns state_sequence : array_like, shape (n,)
Index of the most likely states for each observation
\end{verbatim}

\textbf{predict\_proba} (\textit{obs})

Compute the posterior probability for each state in the model

\begin{verbatim}
Parameters obs : array_like, shape (n, n_features)
List of n_features-dimensional data points. Each row corresponds to a single data point.

Returns T : array-like, shape (n, n_components)
Returns the probability of the sample for each state in the model.
\end{verbatim}

\textbf{rvs} (*\textit{args}, **\textit{kwargs})

DEPRECATED: rvs is deprecated in 0.11 will be removed in 0.13: use \textit{sample} instead

\textbf{sample} \((n=1, random\_state=None)\)

Generate random samples from the model.

\begin{verbatim}
Parameters n : int
Number of samples to generate.

random_state: RandomState or an int seed (0 by default) :
A random number generator instance. If None is given, the object’s random_state is used

Returns (obs, hidden\_states) :
obs : array_like, length n List of samples
hidden\_states : array_like, length n List of hidden states
\end{verbatim}

\textbf{score} (\textit{obs})

Compute the log probability under the model.

\begin{verbatim}
Parameters obs : array_like, shape (n, n_features)
Sequence of n_features-dimensional data points. Each row corresponds to a single data point.
\end{verbatim}
Returns logprob : float

Log likelihood of the obs

See Also:

eval: Compute the log probability under the model and posteriors
decode: Find most likely state sequence corresponding to a obs

set_params (**params)

Set the parameters of the estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The former have parameters of the form <component>__<parameter> so that it’s possible to update each component of a nested object.

Returns self :

startprob_

Mixing startprob for each state.

transmat_

Matrix of transition probabilities.

1.8.12 sklearn.kernel_approximation Kernel Approximation

The sklearn.kernel_approximation module implements several approximate kernel feature maps base on Fourier transforms.

User guide: See the Kernel Approximation section for further details.

kernel_approximation.RBFSampler([gamma,...]) Approximates feature map of an RBF kernel by Monte Carlo approximation of its Fourier transform.

class sklearn.kernel_approximation.RBFSampler(gamma=1.0, n_components=100.0, random_state=None) Approximates feature map of an RBF kernel by Monte Carlo approximation of its Fourier transform.

Parameters gamma : float:

parameter of RBF kernel: exp(-gamma * x**2)

n_components : int:

number of Monte Carlo samples per original feature. Equals the dimensionality of the computed feature space.

random_state : [int, RandomState], optional

If int, random_state is the seed used by the random number generator; if RandomState instance, random_state is the random number generator.
Notes

See “Random Features for Large-Scale Kernel Machines” by A. Rahimi and Benjamin Recht.

Methods

<table>
<thead>
<tr>
<th>Method</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>fit(X[, y])</td>
<td>Fit the model with X.</td>
</tr>
<tr>
<td>fit_transform(X[, y])</td>
<td>Fit to data, then transform it</td>
</tr>
<tr>
<td>get_params([deep])</td>
<td>Get parameters for the estimator</td>
</tr>
<tr>
<td>set_params(**params)</td>
<td>Set the parameters of the estimator.</td>
</tr>
<tr>
<td>transform(X[, y])</td>
<td>Apply the approximate feature map to X.</td>
</tr>
</tbody>
</table>

__init__(gamma=1.0, n_components=100.0, random_state=None)

fit (X, y=None)
Fit the model with X.

Samples random projection according to n_features.

Parameters X: {array-like, sparse matrix}, shape (n_samples, n_features) :
Training data, where n_samples is the number of samples and n_features is the number of features.

Returns self : object
Returns the transformer.

fit_transform (X, y=None, **fit_params)
Fit to data, then transform it

Fits transformer to X and y with optional parameters fit_params and returns a transformed version of X.

Parameters X : numpy array of shape [n_samples, n_features]
Training set.

y : numpy array of shape [n_samples]
Target values.

Returns X_new : numpy array of shape [n_samples, n_features_new]
Transformed array.

Notes

This method just calls fit and transform consecutively, i.e., it is not an optimized implementation of fit_transform, unlike other transformers such as PCA.

get_params (deep=True)
Get parameters for the estimator

Parameters deep: boolean, optional :
If True, will return the parameters for this estimator and contained subobjects that are estimators.
**set_params** (**params**)  
Set the parameters of the estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The former have parameters of the form `<component>__<parameter>` so that it's possible to update each component of a nested object.

**Returns self**:  

**transform** (*X*, *y=None*)  
Apply the approximate feature map to *X*.

**Parameters** *X*: {array-like, sparse matrix}, shape (n_samples, n_features) :

New data, where n_samples is the number of samples and n_features is the number of features.

**Returns** X_new: array-like, shape (n_samples, n_components) :

---

**sklearn.kernel_approximation.AdditiveChi2Sampler**

**class** `sklearn.kernel_approximation.AdditiveChi2Sampler` (*sample_steps=2*, *sample_interval=None*)

Approximate feature map for additive chi² kernel.

Uses sampling the fourier transform of the kernel characteristic at regular intervals.

Since the kernel that is to be approximated is additive, the components of the input vectors can be treated separately. Each entry in the original space is transformed into 2×sample_steps+1 features, where sample_steps is a parameter of the method. Typical values of n include 1, 2 and 3.

Optimal choices for the sampling interval for certain data ranges can be computed (see the reference). The default values should be reasonable.

**Parameters**

**sample_steps**: int, optional

Gives the number of (complex) sampling points.

**sample_interval**: float, optional

Sampling interval. Must be specified when sample_steps not in {1,2,3}.

**Notes**


**Methods**

- **fit** (*X[, y]*)  
  Set parameters.

- **fit_transform** (*X[, y]*)  
  Fit to data, then transform it

- **get_params** ([**deep**])  
  Get parameters for the estimator

- **set_params** (**params**)  
  Set the parameters of the estimator.

- **transform** (*X[, y]*)  
  Apply approximate feature map to *X*.

- **__init__** (*sample_steps=2*, *sample_interval=None*)
**fit** ($X, y=None$)
Set parameters.

**fit_transform** ($X, y=None, **fit_params$)
Fit to data, then transform it

Fits transformer to $X$ and $y$ with optional parameters fit_params and returns a transformed version of $X$.

- **Parameters**
  - $X$: numpy array of shape [n_samples, n_features]
    Training set.
  - $y$: numpy array of shape [n_samples]
    Target values.

- **Returns**
  - $X_{\text{new}}$: numpy array of shape [n_samples, n_features_new]
    Transformed array.

**Notes**

This method just calls fit and transform consecutively, i.e., it is not an optimized implementation of fit_transform, unlike other transformers such as PCA.

**get_params** ($deep=True$)
Get parameters for the estimator

- **Parameters**
  - deep: boolean, optional
    If True, will return the parameters for this estimator and contained subobjects that are estimators.

**set_params** ($**params$)
Set the parameters of the estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The former have parameters of the form `<component>__<parameter>` so that it’s possible to update each component of a nested object.

- **Returns** self:

**transform** ($X, y=None$)
Apply approximate feature map to $X$.

- **Parameters**
  - $X$: array-like, shape (n_samples, n_features):
  - $y$: None

- **Returns** $X_{\text{new}}$: array-like, shape (n_samples, n_features * (2n + 1)):

**sklearn.kernel_approximation.SkewedChi2Sampler**

class sklearn.kernel_approximation.SkewedChi2Sampler ($skewedness=1.0$, $n\_components=100$, random_state=None)

Approximates feature map of the “skewed chi-squared” kernel by Monte Carlo approximation of its Fourier transform.

- **Parameters**
  - skewedness: float
    “skewedness” parameter of the kernel. Needs to be cross-validated.
  - n\_components: int
    ...
number of Monte Carlo samples per original feature. Equals the dimensionality of the computed feature space.

**random_state** : {int, RandomState}, optional

If int, random_state is the seed used by the random number generator; if RandomState instance, random_state is the random number generator.

**Notes**

See “Random Fourier Approximations for Skewed Multiplicative Histogram Kernels” by Fuxin Li, Catalin Ionescu and Cristian Sminchisescu.

**Methods**

- `fit(X[, y])` Fit the model with X.
- `fit_transform(X[, y])` Fit to data, then transform it
- `get_params([deep])` Get parameters for the estimator
- `set_params(**params)` Set the parameters of the estimator.
- `transform(X[, y])` Apply the approximate feature map to X.

**__init__**(skewedness=1.0, n_components=100, random_state=None)

**fit**(X, y=None)

Fit the model with X.

Samples random projection according to n_features.

**Parameters**

- **X**: array-like, shape (n_samples, n_features):
  - Training data, where n_samples in the number of samples and n_features is the number of features.

**Returns**

- **self**: object
  - Returns the transformer.

**fit_transform**(X, y=None, **fit_params)

Fit to data, then transform it

Fits transformer to X and y with optional parameters fit_params and returns a transformed version of X.

**Parameters**

- **X**: numpy array of shape [n_samples, n_features]
  - Training set.
- **y**: numpy array of shape [n_samples]
  - Target values.

**Returns**

- **X_new**: numpy array of shape [n_samples, n_features_new]
  - Transformed array.

**Notes**

This method just calls fit and transform consecutively, i.e., it is not an optimized implementation of fit_transform, unlike other transformers such as PCA.
**get_params** (deep=True)
Get parameters for the estimator

**Parameters**

**deep**: boolean, optional:
If True, will return the parameters for this estimator and contained subobjects that are estimators.

**set_params** (**params**)
Set the parameters of the estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The former have parameters of the form `<component>__<parameter>` so that it’s possible to update each component of a nested object.

**Returns**

**self**:

**transform** (X, y=None)
Apply the approximate feature map to X.

**Parameters**

**X**: array-like, shape (n_samples, n_features):
New data, where n_samples in the number of samples and n_features is the number of features.

**Returns**

**X_new**: array-like, shape (n_samples, n_components):

### 1.8.13 sklearn.semi_supervised Semi-Supervised Learning

The `sklearn.semi_supervised` module implements semi-supervised learning algorithms. These algorithms utilized small amounts of labeled data and large amounts of unlabeled data for classification tasks. This module includes Label Propagation.

**User guide:** See the *Semi-Supervised* section for further details.

<table>
<thead>
<tr>
<th>Function</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>sklearn.semi_supervised.LabelPropagation([[kernel, ...]])</code></td>
<td>Label Propagation classifier</td>
</tr>
<tr>
<td><code>sklearn.semi_supervised.LabelSpreading([[kernel, ...]])</code></td>
<td>LabelSpreading model for semi-supervised learning</td>
</tr>
</tbody>
</table>

**sklearn.semi_supervised.LabelPropagation**

**class** `sklearn.semi_supervised.LabelPropagation`(`kernel='rbf', gamma=20, n_neighbors=7, alpha=1, max_iters=30, tol=0.001)`
Label Propagation classifier

**Parameters**

**kernel**: `{‘knn’, ‘rbf’}`
String identifier for kernel function to use. Only ‘rbf’ and ‘knn’ kernels are currently supported.

**gamma**: float
parameter for rbf kernel

**n_neighbors**: integer > 0
parameter for knn kernel

**alpha**: float
clamping factor

**max_iters**: float

1.8. Reference
change maximum number of iterations allowed

tol : float
Convergence tolerance: threshold to consider the system at steady state

See Also:

LabelSpreading Alternate label propagation strategy more robust to noise

References


Examples

```python
>>> from sklearn import datasets
>>> from sklearn.semi_supervised import LabelPropagation
>>> label_prop_model = LabelPropagation()
>>> iris = datasets.load_iris()
>>> random_unlabeled_points = np.where(np.random.random_integers(0, 1, ...
... size=len(iris.target))
>>> labels = np.copy(iris.target)
>>> labels[random_unlabeled_points] = -1
>>> label_prop_model.fit(iris.data, labels)
```

Methods

- `fit(X, y)` Fit a semi-supervised label propagation model based
- `get_params([deep])` Get parameters for the estimator
- `predict(X)` Performs inductive inference across the model.
- `predict_proba(X)` Predict probability for each possible outcome.
- `score(X, y)` Returns the mean accuracy on the given test data and labels.
- `set_params(**params)` Set the parameters of the estimator.

```
__init__ (kernel='rbf', gamma=20, n_neighbors=7, alpha=1, max_iters=30, tol=0.001)

fit (X, y)
Fit a semi-supervised label propagation model based
All the input data is provided matrix X (labeled and unlabeled) and corresponding label matrix y with a dedicated marker value for unlabeled samples.

Parameters X : array-like, shape = [n_samples, n_features]
A [n_samples by n_features] size matrix will be created from this

y : array_like, shape = [n_samples]
n_labeled_samples (unlabeled points are marked as -1) All unlabeled samples will be transductively assigned labels
Returns self : returns an instance of self.

get_params (deep=True)
Get parameters for the estimator

Parameters deep: boolean, optional :
If True, will return the parameters for this estimator and contained subobjects that are estimators.

predict (X)
Performs inductive inference across the model.

Parameters X : array_like, shape = [n_samples, n_features]
Returns y : array_like, shape = [n_samples]
Predictions for input data

predict_proba (X)
Predict probability for each possible outcome.

Parameters X : array_like, shape = [n_samples, n_features]
Returns probabilities : array, shape = [n_samples, n_classes]
Normalized probability distributions across class labels

score (X, y)
Returns the mean accuracy on the given test data and labels.

Parameters X : array-like, shape = [n_samples, n_features]
Training set.
y : array-like, shape = [n_samples]
Labels for X.
Returns z : float

set_params (**params)
Set the parameters of the estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The former have parameters of the form <component>__<parameter> so that it’s possible to update each component of a nested object.

Returns self :

sklearn.semi_supervised.LabelSpreading

class sklearn.semi_supervised.LabelSpreading (kernel='rbf', gamma=20, n_neighbors=7, alpha=0.2, max_iters=30, tol=0.001)
LabelSpreading model for semi-supervised learning
This model is similar to the basic Label Propagation algorithm, but uses affinity matrix based on the normalized graph Laplacian and soft clamping across the labels.

Parameters kernel : {‘knn’, ‘rbf’}
String identifier for kernel function to use. Only ‘rbf’ and ‘knn’ kernels are currently supported.

**gamma** : float
parameter for rbf kernel

**n_neighbors** : integer > 0
parameter for knn kernel

**alpha** : float
clamping factor

**max_iters** : float
maximum number of iterations allowed

**tol** : float
Convergence tolerance: threshold to consider the system at steady state

See Also:

*LabelPropagation* Unregularized graph based semi-supervised learning

References


Examples

```python
>>> from sklearn import datasets
>>> from sklearn.semi_supervised import LabelSpreading
>>> label_prop_model = LabelSpreading()
>>> iris = datasets.load_iris()
>>> random_unlabeled_points = np.where(np.random.random_integers(0, 1,
... size=len(iris.target)))
>>> labels = np.copy(iris.target)
>>> labels[random_unlabeled_points] = -1
>>> label_prop_model.fit(iris.data, labels)
... LabelSpreading(...)
```

Methods

<table>
<thead>
<tr>
<th>Method</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>fit</strong>(X, y)</td>
<td>Fit a semi-supervised label propagation model based</td>
</tr>
<tr>
<td><strong>get_params</strong>(deep)</td>
<td>Get parameters for the estimator</td>
</tr>
<tr>
<td><strong>predict</strong>(X)</td>
<td>Performs inductive inference across the model.</td>
</tr>
<tr>
<td><strong>predict_proba</strong>(X)</td>
<td>Predict probability for each possible outcome.</td>
</tr>
<tr>
<td><strong>score</strong>(X, y)</td>
<td>Returns the mean accuracy on the given test data and labels.</td>
</tr>
<tr>
<td><strong>set_params</strong>(<strong>params</strong>)</td>
<td>Set the parameters of the estimator.</td>
</tr>
</tbody>
</table>

```python
__init__(kernel='rbf', gamma=20, n_neighbors=7, alpha=0.2, max_iters=30, tol=0.001)
```
**fit** ($X, y$)

Fit a semi-supervised label propagation model based

All the input data is provided matrix $X$ (labeled and unlabeled) and corresponding label matrix $y$ with a dedicated marker value for unlabeled samples.

**Parameters** $X$ : array-like, shape = [n_samples, n_features]

A {n_samples by n_samples} size matrix will be created from this

$y$ : array_like, shape = [n_samples]

n_labeled_samples (unlabeled points are marked as -1) All unlabeled samples will be transductively assigned labels

**Returns** self : returns an instance of self.

**get_params** ($deep=True$)

Get parameters for the estimator

**Parameters** deep: boolean, optional :

If True, will return the parameters for this estimator and contained subobjects that are estimators.

**predict** ($X$)

Performs inductive inference across the model.

**Parameters** $X$ : array_like, shape = [n_samples, n_features]

**Returns** $y$ : array_like, shape = [n_samples]

Predictions for input data

**predict_proba** ($X$)

Predict probability for each possible outcome.

Compute the probability estimates for each single sample in $X$ and each possible outcome seen during training (categorical distribution).

**Parameters** $X$ : array_like, shape = [n_samples, n_features]

**Returns** probabilities : array, shape = [n_samples, n_classes]

Normalized probability distributions across class labels

**score** ($X, y$)

Returns the mean accuracy on the given test data and labels.

**Parameters** $X$ : array-like, shape = [n_samples, n_features]

Training set.

$y$ : array-like, shape = [n_samples]

Labels for $X$.

**Returns** $z$ : float

**set_params** (**params**)

Set the parameters of the estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The former have parameters of the form <component>__<parameter> so that it’s possible to update each component of a nested object.

**Returns** self :
1.8.14 sklearn.lda: Linear Discriminant Analysis

The sklearn.lda module implements Linear Discriminant Analysis (LDA).

User guide: See the Linear and Quadratic Discriminant Analysis section for further details.

```python
lda.LDA(n_components, priors) Linear Discriminant Analysis (LDA)
```

```
scikit-learn user guide, Release 0.12-git

422 Chapter 1. User Guide

```
<table>
<thead>
<tr>
<th>Method</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>decision_function(X)</code></td>
<td>This function return the decision function values related to each class on an array of test vectors X.</td>
</tr>
<tr>
<td><code>fit(X, y[, store_covariance, tol])</code></td>
<td>Fit the LDA model according to the given training data and parameters.</td>
</tr>
<tr>
<td><code>fit_transform(X[, y])</code></td>
<td>Fit to data, then transform it.</td>
</tr>
<tr>
<td><code>get_params([deep])</code></td>
<td>Get parameters for the estimator.</td>
</tr>
<tr>
<td><code>predict(X)</code></td>
<td>This function does classification on an array of test vectors X.</td>
</tr>
<tr>
<td><code>predict_log_proba(X)</code></td>
<td>This function return posterior log-probabilities of classification</td>
</tr>
<tr>
<td><code>predict_proba(X)</code></td>
<td>This function return posterior probabilities of classification</td>
</tr>
<tr>
<td><code>score(X, y)</code></td>
<td>Returns the mean accuracy on the given test data and labels.</td>
</tr>
<tr>
<td><code>set_params(**params)</code></td>
<td>Set the parameters of the estimator.</td>
</tr>
<tr>
<td><code>transform(X)</code></td>
<td>Project the data so as to maximize class separation (large separation between projected class mean).</td>
</tr>
</tbody>
</table>

__init__ (n_components=None, priors=None)

decision_function (X)
   This function return the decision function values related to each class on an array of test vectors X.
   Parameters X : array-like, shape = [n_samples, n_features]
   Returns C : array, shape = [n_samples, n_classes]

fit (X, y, store_covariance=False, tol=0.0001)
   Fit the LDA model according to the given training data and parameters.
   Parameters X : array-like, shape = [n_samples, n_features]
      Training vector, where n_samples in the number of samples and n_features is the number of features.
   y : array, shape = [n_samples]
      Target values (integers)
   store_covariance : boolean
      If True the covariance matrix (shared by all classes) is computed and stored in self.covariance_ attribute.

fit_transform (X, y=None, **fit_params)
   Fit to data, then transform it
   Fits transformer to X and y with optional parameters fit_params and returns a transformed version of X.
   Parameters X : numpy array of shape [n_samples, n_features]
      Training set.
   y : numpy array of shape [n_samples]
      Target values.
   Returns X_new : numpy array of shape [n_samples, n_features_new]
      Transformed array.

Notes

This method just calls fit and transform consecutively, i.e., it is not an optimized implementation of fit_transform, unlike other transformers such as PCA.

get_params (deep=True)
   Get parameters for the estimator
Parameters deep: boolean, optional:

If True, will return the parameters for this estimator and contained subobjects that are
estimators.

predict (X)

This function does classification on an array of test vectors X.

The predicted class C for each sample in X is returned.

Parameters X : array-like, shape = [n_samples, n_features]

Returns C : array, shape = [n_samples]

predict_log_proba (X)

This function return posterior log-probabilities of classification according to each class on an array of test
vectors X.

Parameters X : array-like, shape = [n_samples, n_features]

Returns C : array, shape = [n_samples, n_classes]

predict_proba (X)

This function return posterior probabilities of classification according to each class on an array of test
vectors X.

Parameters X : array-like, shape = [n_samples, n_features]

Returns C : array, shape = [n_samples, n_classes]

score (X, y)

Returns the mean accuracy on the given test data and labels.

Parameters X : array-like, shape = [n_samples, n_features]

Training set.

y : array-like, shape = [n_samples]

Labels for X.

Returns z : float

set_params (**params)

Set the parameters of the estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The former have
parameters of the form <component>__<parameter> so that it's possible to update each component
of a nested object.

Returns self :

transform (X)

Project the data so as to maximize class separation (large separation between projected class means and
small variance within each class).

Parameters X : array-like, shape = [n_samples, n_features]

Returns X_new : array, shape = [n_samples, n_components]

1.8.15 sklearn.linear_model: Generalized Linear Models

The sklearn.linear_model module implements genelarized linear models. It includes Ridge regression,
Bayesian Regression, Lasso and Elastic Net estimators computed with Least Angle Regression and coordinate de-
scent. It also implements Stochastic Gradient Descent related algorithms.
**User guide:** See the *Generalized Linear Models* section for further details.

### For dense data

<table>
<thead>
<tr>
<th>Class</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>linear_model.LinearRegression</code></td>
<td>Ordinary least squares Linear Regression.</td>
</tr>
<tr>
<td><code>linear_model.Ridge</code></td>
<td>Linear least squares with L2 regularization.</td>
</tr>
<tr>
<td><code>linear_model.RidgeClassifier</code></td>
<td>Classifier using Ridge regression.</td>
</tr>
<tr>
<td><code>linear_model.RidgeClassifierCV</code></td>
<td>Ridge classifier with built-in cross-validation.</td>
</tr>
<tr>
<td><code>linear_model.RidgeCV</code></td>
<td>Ridge regression with built-in cross-validation.</td>
</tr>
<tr>
<td><code>linear_model.Lasso</code></td>
<td>Linear Model trained with L1 prior as regularizer (aka the Lasso)</td>
</tr>
<tr>
<td><code>linear_model.LassoCV</code></td>
<td>Lasso linear model with iterative fitting along a regularization path.</td>
</tr>
<tr>
<td><code>linear_model.ElasticNet</code></td>
<td>Linear Model trained with L1 and L2 prior as regularizer</td>
</tr>
<tr>
<td><code>linear_model.ElasticNetCV</code></td>
<td>Elastic Net model with iterative fitting along a regularization path.</td>
</tr>
<tr>
<td><code>linear_model.Lars</code></td>
<td>Least Angle Regression model a.k.a. LAR</td>
</tr>
<tr>
<td><code>linear_model.LassoLars</code></td>
<td>Lasso model fit with Least Angle Regression a.k.a. LAR</td>
</tr>
<tr>
<td><code>linear_model.LassoLarsCV</code></td>
<td>Cross-validated Least Angle Regression model</td>
</tr>
<tr>
<td><code>linear_model.LassoLarsIC</code></td>
<td>Cross-validated Lasso, using the LARS algorithm</td>
</tr>
<tr>
<td><code>linear_model.LogisticRegression</code></td>
<td>Logistic Regression (aka logit, MaxEnt) classifier.</td>
</tr>
<tr>
<td><code>linear_model.OrthogonalMatchingPursuit</code></td>
<td>Orthogonal Matching Pursuit model (OMP)</td>
</tr>
<tr>
<td><code>linear_model.Perceptron</code></td>
<td>Perceptron</td>
</tr>
<tr>
<td><code>linear_model.SGDClassifier</code></td>
<td>Linear model fitted by minimizing a regularized empirical loss with Stochastic Gradient Descent.</td>
</tr>
<tr>
<td><code>linear_model.SGDRegressor</code></td>
<td>Linear model fitted by minimizing a regularized empirical loss with Stochastic Gradient Descent.</td>
</tr>
<tr>
<td><code>linear_model.BayesianRidge</code></td>
<td>Bayesian ridge regression</td>
</tr>
<tr>
<td><code>linear_model.ARDRidge</code></td>
<td>Bayesian ARD regression.</td>
</tr>
<tr>
<td><code>linear_model.RandomizedLasso</code></td>
<td>Randomized Lasso</td>
</tr>
</tbody>
</table>

**sklearn.linear_model.LinearRegression**

**class** `sklearn.linear_model.LinearRegression` *(fit_intercept=True, normalize=False, copy_X=True)*

Ordinary least squares Linear Regression.

**Parameters**

- `fit_intercept` : boolean, optional

  wether to calculate the intercept for this model. If set to false, no intercept will be used in calculations (e.g. data is expected to be already centered).

- `normalize` : boolean, optional

  If True, the regressors X are normalized

**Notes**

From the implementation point of view, this is just plain Ordinary Least Squares (numpy.linalg.lstsq) wrapped as a predictor object.
Attributes

<table>
<thead>
<tr>
<th>Attribute</th>
<th>Type</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>coef_</td>
<td>array</td>
<td>Estimated coefficients for the linear regression problem.</td>
</tr>
<tr>
<td>intercept_</td>
<td>array</td>
<td>Independent term in the linear model.</td>
</tr>
</tbody>
</table>

Methods

<table>
<thead>
<tr>
<th>Method</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>decision_function(X)</td>
<td>Decision function of the linear model</td>
</tr>
<tr>
<td>fit(X, y[, n_jobs])</td>
<td>Fit linear model</td>
</tr>
<tr>
<td>get_params([deep])</td>
<td>Get parameters for the estimator</td>
</tr>
<tr>
<td>predict(X)</td>
<td>Predict using the linear model</td>
</tr>
<tr>
<td>score(X, y)</td>
<td>Returns the coefficient of determination $R^2$ of the prediction.</td>
</tr>
<tr>
<td>set_params(**params)</td>
<td>Set the parameters of the estimator.</td>
</tr>
</tbody>
</table>

__init__ (fit_intercept=True, normalize=False, copy_X=True)

decision_function(X)

Decision function of the linear model

Parameters:

- **X**: numpy array of shape [n_samples, n_features]

Returns:

- **C**: array, shape = [n_samples]

  Returns predicted values.

fit(X, y, n_jobs=1)

Fit linear model.

Parameters:

- **X**: numpy array or sparse matrix of shape [n_samples, n_features]

  Training data

- **y**: numpy array of shape [n_samples, n_responses]

  Target values

- **n_jobs**: The number of jobs to use for the computation.

  If -1 all CPUs are used. This will only provide speedup for n_response > 1 and sufficient large problems

Returns **self**: returns an instance of self.

get_params (deep=True)

Get parameters for the estimator

Parameters:

- **deep**: boolean, optional

  If True, will return the parameters for this estimator and contained subobjects that are estimators.

predict(X)

Predict using the linear model

Parameters:

- **X**: numpy array of shape [n_samples, n_features]

Returns **C**: array, shape = [n_samples]

  Returns predicted values.
score \((X, y)\)

Returns the coefficient of determination \(R^2\) of the prediction.

The coefficient \(R^2\) is defined as \((1 - u/v)\), where \(u\) is the regression sum of squares \(((y - y_{\text{pred}})^2).\sum()\) and \(v\) is the residual sum of squares \(((y_{\text{true}} - y_{\text{true.mean}})^2).\sum()\). Best possible score is 1.0, lower values are worse.

**Parameters**

\(X\) : array-like, shape \([n_{\text{samples}}, n_{\text{features}}]\)

Training set.

\(y\) : array-like, shape \([n_{\text{samples}}]\)

Returns \(z\) : float

set_params (**params)**

Set the parameters of the estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The former have parameters of the form \(<\text{component}>__.<\text{parameter}>\) so that it's possible to update each component of a nested object.

Returns self :

sklearn.linear_model.Ridge

class sklearn.linear_model.Ridge (\(alpha=1.0, fit\_intercept=True, normalize=False, copy\_X=True, tol=0.001\))

Linear least squares with l2 regularization.

This model solves a regression model where the loss function is the linear least squares function and regularization is given by the l2-norm. Also known as Ridge Regression or Tikhonov regularization. This estimator has built-in support for multi-variate regression (i.e., when y is a 2d-array of shape \([n_{\text{samples}}, n_{\text{responses}}]\)).

**Parameters**

\(alpha\) : float

Small positive values of \(alpha\) improve the conditioning of the problem and reduce the variance of the estimates. \(alpha\) corresponds to \((2*C)^{-1}\) in other linear models such as LogisticRegression or LinearSVC.

\(fit\_intercept\) : boolean

Whether to calculate the intercept for this model. If set to false, no intercept will be used in calculations (e.g. data is expected to be already centered).

\(normalize\) : boolean, optional

If True, the regressors \(X\) are normalized.

\(copy\_X\) : boolean, optional, default True

If True, \(X\) will be copied; else, it may be overwritten.

\(tol\) : float :

Precision of the solution.

See Also:

RidgeClassifier, RidgeCV
Examples

```python
>>> from sklearn.linear_model import Ridge
>>> import numpy as np

>>> n_samples, n_features = 10, 5
>>> np.random.seed(0)
>>> y = np.random.randn(n_samples)
>>> X = np.random.randn(n_samples, n_features)
>>> clf = Ridge(alpha=1.0)
>>> clf.fit(X, y)
Ridge(alpha=1.0, copy_X=True, fit_intercept=True, normalize=False, tol=0.001)
```

Attributes

| coef_ | array, shape = [n_features] or [n_responses, n_features] | Weight vector(s). |

Methods

| decision_function(X) | Decision function of the linear model |
| fit(X, y[, sample_weight, solver]) | Fit Ridge regression model |
| get_params([deep]) | Get parameters for the estimator |
| predict(X) | Predict using the linear model |
| score(X, y) | Returns the coefficient of determination R^2 of the prediction. |
| set_params(**params) | Set the parameters of the estimator. |

__init__(alpha=1.0, fit_intercept=True, normalize=False, copy_X=True, tol=0.001)

decision_function(X)

Decision function of the linear model

Parameters X : numpy array of shape [n_samples, n_features]

Returns C : array, shape = [n_samples]

Returns predicted values.

fit (X, y, sample_weight=1.0, solver='auto')

Fit Ridge regression model

Parameters X : {array-like, sparse matrix}, shape = [n_samples, n_features]

Training data

y : array-like, shape = [n_samples] or [n_samples, n_responses]

Target values

sample_weight : float or numpy array of shape [n_samples]

Individual weights for each sample

solver : {‘auto’, ‘dense_cholesky’, ‘sparse_cg’}

Solver to use in the computational routines. ‘dense_cholesky’ will use the standard scipy.linalg.solve function, ‘sparse_cg’ will use the conjugate gradient solver as found
in scipy.sparse.linalg.cg while ‘auto’ will chose the most appropriate depending on the
matrix X.

**Returns self**: returns an instance of self.

**get_params**(deep=True)
Get parameters for the estimator

**Parameters deep**: boolean, optional:
If True, will return the parameters for this estimator and contained subobjects that are
estimators.

**predict**(X)
Predict using the linear model

**Parameters X**: numpy array of shape [n_samples, n_features]

**Returns C**: array, shape = [n_samples]
Returns predicted values.

**score**(X, y)
Returns the coefficient of determination R^2 of the prediction.
The coefficient R^2 is defined as (1 - u/v), where u is the regression sum of squares ((y - y_pred) **
2).sum() and v is the residual sum of squares ((y_true - y_true.mean()) ** 2).sum(). Best possible score is
1.0, lower values are worse.

**Parameters X**: array-like, shape = [n_samples, n_features]
Training set.

**y**: array-like, shape = [n_samples]

**Returns z**: float

**set_params**(**params)
Set the parameters of the estimator.
The method works on simple estimators as well as on nested objects (such as pipelines). The former have
parameters of the form <component>__<parameter> so that it’s possible to update each component
of a nested object.

**Returns self**: 

**sklearn.linear_model.RidgeClassifier**

class sklearn.linear_model.RidgeClassifier(alpha=1.0, fit_intercept=True, normalize=False,
copy_X=True, tol=0.001, class_weight=None)
Classifier using Ridge regression.

**Parameters alpha**: float
Small positive values of alpha improve the conditioning of the problem and reduce the
variance of the estimates. Alpha corresponds to (2*C)^-1 in other linear models such as
LogisticRegression or LinearSVC.

**fit_intercept**: boolean
Whether to calculate the intercept for this model. If set to false, no intercept will be
used in calculations (e.g. data is expected to be already centered).

**normalize**: boolean, optional
If True, the regressors $X$ are normalized

**copy_X**: boolean, optional, default True

If True, $X$ will be copied; else, it may be overwritten.

**tol**: float

Precision of the solution.

**class_weight**: dict, optional

Weights associated with classes in the form `{class_label : weight}`. If not given, all classes are supposed to have weight one.

**See Also:**

`Ridge`, `RidgeClassifierCV`

**Notes**

For multi-class classification, $n_{\text{class}}$ classifiers are trained in a one-versus-all approach. Concretely, this is implemented by taking advantage of the multi-variate response support in Ridge.

**Attributes**

- **`coef_`**: array, shape = `[n_features]` or `[n_classes, n_features]`  
  Weight vector(s).

**Methods**

```python

def decision_function(X)
    Fit Ridge regression model.

def fit(X, y[, solver])
    Get parameters for the estimator

def predict(X)
    Predict target values according to the fitted model.

def score(X, y)
    Returns the coefficient of determination $R^2$ of the prediction.

def set_params(**params)
    Set the parameters of the estimator.
```

**__init__**(alpha=1.0,  
  fit_intercept=True,  
  normalize=False,  
  copy_X=True,  
  tol=0.001,  
  class_weight=None)

**fit**(X, y, solver='auto')

Fit Ridge regression model.

**Parameters**

- **X**: array-like, sparse matrix, shape = `[n_samples, n_features]`
  Training data
- **y**: array-like, shape = `[n_samples]`
  Target values
- **solver**: `{‘auto’, ‘dense_cholesky’, ‘sparse_cg’}`
  Solver to use in the computational routines. ‘dense_cholesky’ will use the standard scipy.linalg.solve function, ‘sparse_cg’ will use the conjugate gradient solver as found in scipy.sparse.linalg.cg while ‘auto’ will chose the most appropriate depending on the matrix $X$. 

430 Chapter 1. User Guide
**Returns self**: returns an instance of self.

**get_params**(deep=True)

Get parameters for the estimator

**Parameters deep**: boolean, optional:

If True, will return the parameters for this estimator and contained subobjects that are estimators.

**predict**(X)

Predict target values according to the fitted model.

**Parameters X**: array-like, shape = [n_samples, n_features]

**Returns y**: array, shape = [n_samples]

**score**(X, y)

Returns the coefficient of determination $R^2$ of the prediction.

The coefficient $R^2$ is defined as $(1 - u/v)$, where $u$ is the regression sum of squares $((y - y\_pred) ** 2).sum()$ and $v$ is the residual sum of squares $((y\_true - y\_true.mean()) ** 2).sum()$. Best possible score is 1.0, lower values are worse.

**Parameters X**: array-like, shape = [n_samples, n_features]

Training set.

**y**: array-like, shape = [n_samples]

**Returns z**: float

**set_params**(**params)

Set the parameters of the estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The former have parameters of the form `<component>__<parameter>` so that it's possible to update each component of a nested object.

**Returns self**: 

**sklearn.linear_model.RidgeClassifierCV**

**class sklearn.linear_model.RidgeClassifierCV**(alphas=array([0.1, 1., 10.]),

fit_intercept=True,

normalize=False,

score_func=None, loss_func=None,

cv=None, class_weight=None)

Ridge classifier with built-in cross-validation.

By default, it performs Generalized Cross-Validation, which is a form of efficient Leave-One-Out cross-validation. Currently, only the n_features > n_samples case is handled efficiently.

**Parameters alphas**: numpy array of shape [n_alpha]

Array of alpha values to try. Small positive values of alpha improve the conditioning of the problem and reduce the variance of the estimates. Alpha corresponds to $(2*C)^{-1}$ in other linear models such as LogisticRegression or LinearSVC.

**fit_intercept**: boolean

Whether to calculate the intercept for this model. If set to false, no intercept will be used in calculations (e.g. data is expected to be already centered).

**normalize**: boolean, optional
If True, the regressors X are normalized.

score_func: callable, optional:
function that takes 2 arguments and compares them in order to evaluate the performance of prediction (big is good) if None is passed, the score of the estimator is maximized.

loss_func: callable, optional:
function that takes 2 arguments and compares them in order to evaluate the performance of prediction (small is good) if None is passed, the score of the estimator is maximized.

cv: cross-validation generator, optional
If None, Generalized Cross-Validation (efficient Leave-One-Out) will be used.

class_weight: dict, optional
Weights associated with classes in the form {class_label : weight}. If not given, all classes are supposed to have weight one.

See Also:
Ridge: Ridge regression
RidgeClassifier: Ridge classifier
RidgeCV: Ridge regression with built-in cross validation

Notes
For multi-class classification, n_class classifiers are trained in a one-versus-all approach. Concretely, this is implemented by taking advantage of the multi-variate response support in Ridge.

Methods

decision_function(X)

fit(X, y[, sample_weight, class_weight]) Fit the ridge classifier.

get_params([deep]) Get parameters for the estimator.
predict(X) Predict target values according to the fitted model.
score(X, y) Returns the coefficient of determination R^2 of the prediction.

set_params(**params) Set the parameters of the estimator.

__init__(alphas=array([ 0.1, 1. , 10. ]), fit_intercept=True, normalize=False, score_func=None, loss_func=None, cv=None, class_weight=None)

fit (X, y, sample_weight=1.0, class_weight=None) Fit the ridge classifier.

Parameters X : array-like, shape = [n_samples, n_features]
Training vectors, where n_samples is the number of samples and n_features is the number of features.

y : array-like, shape = [n_samples]
Target values.

sample_weight : float or numpy array of shape [n_samples]
Sample weight

**class_weight** : dict, optional

Weights associated with classes in the form {class_label : weight}. If not given, all classes are supposed to have weight one.

**Returns self** : object

Returns self.

**get_params**(deep=True)

Get parameters for the estimator

**Parameters deep** : boolean, optional:

If True, will return the parameters for this estimator and contained subobjects that are estimators.

**predict**(X)

Predict target values according to the fitted model.

**Parameters X** : array-like, shape = [n_samples, n_features]

**Returns y** : array, shape = [n_samples]

**score**(X, y)

Returns the coefficient of determination $R^2$ of the prediction.

The coefficient $R^2$ is defined as $(1 - u/v)$, where $u$ is the regression sum of squares (($y - y\_pred$)**2).sum() and $v$ is the residual sum of squares (($y\_true - y\_true\_mean()$)**2).sum(). Best possible score is 1.0, lower values are worse.

**Parameters X** : array-like, shape = [n_samples, n_features]

Training set.

**y** : array-like, shape = [n_samples]

**Returns z** : float

**set_params**(**params)

Set the parameters of the estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The former have parameters of the form `<component>__<parameter>` so that it’s possible to update each component of a nested object.

**Returns self** :

**sklearn.linear_model.RidgeCV**

**class sklearn.linear_model.RidgeCV** (alphas=array([ 0.1, 1., 10.]), fit_intercept=True, normalize=False, score_func=None, loss_func=None, cv=None, gcv_mode=None)

Ridge regression with built-in cross-validation.

By default, it performs Generalized Cross-Validation, which is a form of efficient Leave-One-Out cross-validation.

**Parameters alphas** : numpy array of shape [n_alpha] :
Array of alpha values to try. Small positive values of alpha improve the conditioning of
the problem and reduce the variance of the estimates. Alpha corresponds to \((2+C)^{-1}\)
in other linear models such as LogisticRegression or LinearSVC.

**fit_intercept** : boolean

Whether to calculate the intercept for this model. If set to false, no intercept will be
used in calculations (e.g. data is expected to be already centered).

**normalize** : boolean, optional

If True, the regressors X are normalized

**score_func** : callable, optional :

function that takes 2 arguments and compares them in order to evaluate the performance
of prediction (big is good) if None is passed, the score of the estimator is maximized

**loss_func** : callable, optional :

function that takes 2 arguments and compares them in order to evaluate the performance
of prediction (small is good) if None is passed, the score of the estimator is maximized

**cv** : cross-validation generator, optional

If None, Generalized Cross-Validation (efficient Leave-One-Out) will be used.

See Also:

- **Ridge**
Ridge regression

- **RidgeClassifier**
Ridge classifier

- **RidgeCV**
Ridge regression with built-in cross validation

Attributes

**coef_**

array, shape = [n_features] or [n_classes, n_features]

Weight vector(s).

**gcv_mode**

\{None, ‘auto’, ‘svd’, ‘eigen’\}, optional

Flag indicating which strategy to use when performing Generalized Cross-Validation. Options are:

- ‘auto’ : use svd if n_samples > n
- ‘svd’ : force computation via singular
- ‘eigen’ : force computation via eigen

The ‘auto’ mode is the default and is intended to pick the cheaper option of the two depending upon the
shape of the training data.

Methods

**decision_function(X)**

Decision function of the linear model

**fit(X, y[, sample_weight])**

Fit Ridge regression model

**get_params([deep])**

Get parameters for the estimator

Continued on next page
**predict**(X) Predict using the linear model

**score**(X, y) Returns the coefficient of determination \( R^2 \) of the prediction.

**set_params**(**params**) Set the parameters of the estimator.

```
__init__(alphas=array([ 0.1, 1., 10. ]), fit_intercept=True, normalize=False, score_func=None, loss_func=None, cv=None, gcv_mode=None)
```

**decision_function**(X) Decision function of the linear model

Parameters X : numpy array of shape [n_samples, n_features]

Returns C : array, shape = [n_samples]

Returns predicted values.

**fit**(X, y, sample_weight=1.0) Fit Ridge regression model

Parameters X : array-like, shape = [n_samples, n_features]

Training data

y : array-like, shape = [n_samples] or [n_samples, n_responses]

Target values

sample_weight : float or array-like of shape [n_samples]

Sample weight

Returns self : Returns self.

**get_params**(deep=True) Get parameters for the estimator

Parameters deep: boolean, optional:

If True, will return the parameters for this estimator and contained subobjects that are
estimators.

**predict**(X) Predict using the linear model

Parameters X : numpy array of shape [n_samples, n_features]

Returns C : array, shape = [n_samples]

Returns predicted values.

**score**(X, y) Returns the coefficient of determination \( R^2 \) of the prediction.

The coefficient \( R^2 \) is defined as \( (1 - u/v) \), where \( u \) is the regression sum of squares \( ((y - y_{\text{pred}})^2).\text{sum()} \) and \( v \) is the residual sum of squares \( ((y_{\text{true}} - y_{\text{true}.\text{mean()}})^2).\text{sum()} \). Best possible score is 1.0, lower values are worse.

Parameters X : array-like, shape = [n_samples, n_features]

Training set.

y : array-like, shape = [n_samples]

Returns z : float
set_params (**params)**

Set the parameters of the estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The former have parameters of the form `<component>__<parameter>` so that it's possible to update each component of a nested object.

Returns self:

```
sklearn.linear_model.Lasso
```

Linear Model trained with L1 prior as regularizer (aka the Lasso)

The optimization objective for Lasso is:

\[
(1 / (2 \times n\_samples)) \times ||y - Xw||^2_2 + alpha \times ||w||_1
\]

Technically the Lasso model is optimizing the same objective function as the Elastic Net with rho=1.0 (no L2 penalty).

Parameters

**alpha** : float, optional

Constant that multiplies the L1 term. Defaults to 1.0

**fit_intercept** : boolean

Whether to calculate the intercept for this model. If set to False, no intercept will be used in calculations (e.g. data is expected to be already centered).

**normalize** : boolean, optional

If True, the regressors X are normalized

**copy_X** : boolean, optional, default True

If True, X will be copied; else, it may be overwritten.

**precompute** : True | False | ‘auto’ | array-like

Whether to use a precomputed Gram matrix to speed up calculations. If set to ‘auto’ let us decide. The Gram matrix can also be passed as argument.

**max_iter** : int, optional

The maximum number of iterations

**tol** : float, optional

The tolerance for the optimization: if the updates are smaller than ‘tol’, the optimization code checks the dual gap for optimality and continues until it is smaller than tol.

**warm_start** : bool, optional

When set to True, reuse the solution of the previous call to fit as initialization, otherwise, just erase the previous solution.

**positive** : bool, optional

When set to True, forces the coefficients to be positive.
See Also:

lars_path, lasso_path, LassoLars, LassoCV, LassoLarsCV, sklearn.decomposition.sparse_encode

Notes

The algorithm used to fit the model is coordinate descent.

To avoid unnecessary memory duplication the X argument of the fit method should be directly passed as a fortran contiguous numpy array.

Examples

```python
>>> from sklearn import linear_model

>>> clf = linear_model.Lasso(alpha=0.1)

>>> clf.fit([[0, 0], [1, 1], [2, 2]], [0, 1, 2])
Lasso(alpha=0.1, copy_X=True, fit_intercept=True, max_iter=1000,
      normalize=False, positive=False, precompute='auto', tol=0.0001,
      warm_start=False)

>>> print(clf.coef_)
[ 0.85 0. ]

>>> print(clf.intercept_)
0.15
```

Attributes

<table>
<thead>
<tr>
<th>coef_</th>
<th>array, shape = [n_features]</th>
<th>parameter vector (w in the formulation formula)</th>
</tr>
</thead>
<tbody>
<tr>
<td>intercept_</td>
<td>float</td>
<td>independent term in decision function.</td>
</tr>
</tbody>
</table>

Methods

<table>
<thead>
<tr>
<th>decision_function(X)</th>
<th>Decision function of the linear model</th>
</tr>
</thead>
<tbody>
<tr>
<td>fit(X, y, Xy, coef_init)</td>
<td>Fit Elastic Net model with coordinate descent</td>
</tr>
<tr>
<td>get_params([deep])</td>
<td>Get parameters for the estimator</td>
</tr>
<tr>
<td>predict(X)</td>
<td>Predict using the linear model</td>
</tr>
<tr>
<td>score(X, y)</td>
<td>Returns the coefficient of determination $R^2$ of the prediction.</td>
</tr>
<tr>
<td>set_params(**params)</td>
<td>Set the parameters of the estimator.</td>
</tr>
</tbody>
</table>

__init__(alpha=1.0, fit_intercept=True, normalize=False, precompute='auto', copy_X=True, max_iter=1000, tol=0.0001, warm_start=False, positive=False)

decision_function(X)

Decision function of the linear model

Parameters

- X : numpy array of shape [n_samples, n_features]

Returns

- C : array, shape = [n_samples]

Returns predicted values.

fit(X, y=None, coef_init=None)

Fit Elastic Net model with coordinate descent
Parameters X: ndarray, (n_samples, n_features):
Data

y: ndarray, (n_samples):
Target

Xy : array-like, optional
Xy = np.dot(X.T, y) that can be precomputed. It is useful only when the Gram matrix is
precomputed.

coef_init: ndarray of shape n_features:
The initial coefficients to warm-start the optimization

Notes

Coordinate descent is an algorithm that considers each column of data at a time hence it will automatically
convert the X input as a fortran contiguous numpy array if necessary.
To avoid memory re-allocation it is advised to allocate the initial data in memory directly using that format.

get_params (deep=True)
Get parameters for the estimator

Parameters deep: boolean, optional:
If True, will return the parameters for this estimator and contained subobjects that are
estimators.

predict (X)
Predict using the linear model

Parameters X : numpy array of shape [n_samples, n_features]

Returns C : array, shape = [n_samples]
Returns predicted values.

score (X, y)
Returns the coefficient of determination R^2 of the prediction.
The coefficient R^2 is defined as (1 - u/v), where u is the regression sum of squares ((y - y_pred) **
2).sum() and v is the residual sum of squares ((y_true - y_true.mean()) ** 2).sum(). Best possible score is
1.0, lower values are worse.

Parameters X : array-like, shape = [n_samples, n_features]

Returns z : float

set_params (**params)
Set the parameters of the estimator.
The method works on simple estimators as well as on nested objects (such as pipelines). The former have
parameters of the form <component>__<parameter> so that it’s possible to update each component of
a nested object.

Returns self:
sklearn.linear_model.LassoCV

class sklearn.linear_model.LassoCV (eps=0.001, n_alphas=100, alphas=None, fit_intercept=True, normalize=False, precompute='auto', max_iter=1000, tol=0.0001, copy_X=True, cv=None, verbose=False)

Lasso linear model with iterative fitting along a regularization path

The best model is selected by cross-validation.

The optimization objective for Lasso is:

\[
\frac{1}{(2 \times n_{\text{samples}})} \times ||y - Xw||^2_2 + \alpha \times ||w||_1
\]

Parameters:

eps : float, optional
    Length of the path. eps=1e-3 means that alpha_min / alpha_max = 1e-3.

n_alphas : int, optional
    Number of alphas along the regularization path

alphas : numpy array, optional
    List of alphas where to compute the models. If None alphas are set automatically

precompute : True | False | ‘auto’ | array-like
    Whether to use a precomputed Gram matrix to speed up calculations. If set to ‘auto’ let us decide. The Gram matrix can also be passed as argument.

max_iter: int, optional:
    The maximum number of iterations

tol: float, optional:
    The tolerance for the optimization: if the updates are smaller than ‘tol’, the optimization code checks the dual gap for optimality and continues until it is smaller than tol.

cv : integer or crossvalidation generator, optional
    If an integer is passed, it is the number of fold (default 3). Specific crossvalidation objects can be passed, see sklearn.cross_validation module for the list of possible objects

verbose : bool or integer
    amount of verbosity

See Also:
lars_path, lasso_path, LassoLars, Lasso, LassoLarsCV

Notes

See examples/linear_model/lasso_path_with_crossvalidation.py for an example.

To avoid unnecessary memory duplication the X argument of the fit method should be directly passed as a fortran contiguous numpy array.

1.8. Reference
Attributes

<table>
<thead>
<tr>
<th>attribute</th>
<th>description</th>
</tr>
</thead>
<tbody>
<tr>
<td>alpha_</td>
<td>float</td>
</tr>
<tr>
<td>coef_</td>
<td>array, shape = [n_features]</td>
</tr>
<tr>
<td>intercept_</td>
<td>float</td>
</tr>
<tr>
<td>mse_path_</td>
<td>array, shape = [n_alphas, n_folds]</td>
</tr>
</tbody>
</table>

Methods

- **decision_function**(*X*) Decision function of the linear model
  - **Parameters**
    - X : numpy array of shape [n_samples, n_features]
  - **Returns**
    - C : array, shape = [n_samples]
      Returns predicted values.

- **fit**(*X*, *y*) Fit linear model with coordinate descent along decreasing alphas using cross-validation
  - **Parameters**
    - X : numpy array of shape [n_samples,n_features]
      Training data. Pass directly as fortran contiguous data to avoid unnecessary memory duplication
    - y : numpy array of shape [n_samples]
      Target values
  - **get_params**(deep=True)
    Get parameters for the estimator
    - **Parameters**
      - deep: boolean, optional
        If True, will return the parameters for this estimator and contained subobjects that are estimators.
  - **static** *path* (*X*, *y*, *eps=0.001*, *n_alphas=100*, *alphas=None*, *precompute='auto'*, *max_iter=1000*, *tol=0.0001*, *copy_X=True*, *cv=None*, *verbose=False*, **params**)
    Compute Lasso path with coordinate descent
    The optimization objective for Lasso is:
(1 / (2 * n_samples)) * ||y - Xw||^2_2 + alpha * ||w||_1

Parameters

X : numpy array of shape [n_samples, n_features]

Training data. Pass directly as fortran contiguous data to avoid unnecessary memory duplication.

y : numpy array of shape [n_samples]

Target values.

eps : float, optional

Length of the path. eps=1e-3 means that alpha_min / alpha_max = 1e-3.

n_alphas : int, optional

Number of alphas along the regularization path.

alphas : numpy array, optional

List of alphas where to compute the models. If None alphas are set automatically.

precompute : True | False | ‘auto’ | array-like

Whether to use a precomputed Gram matrix to speed up calculations. If set to ‘auto’ let us decide. The Gram matrix can also be passed as argument.

Xy : array-like, optional

Xy = np.dot(X.T, y) that can be precomputed. It is useful only when the Gram matrix is precomputed.

fit_intercept : bool

Fit or not an intercept.

normalize : boolean, optional

If True, the regressors X are normalized.

copy_X : boolean, optional, default True

If True, X will be copied; else, it may be overwritten.

verbose : bool or integer

Amount of verbosity.

params : kwargs

Keyword arguments passed to the Lasso objects.

Returns

models : a list of models along the regularization path.

See Also:

lars_path, Lasso, LassoLars, LassoCV, LassoLarsCV, sklearn.decomposition.sparse_encode

Notes

See examples/linear_model/plot_lasso_coordinate_descent_path.py for an example.
To avoid unnecessary memory duplication the \texttt{X} argument of the \texttt{fit} method should be directly passed as a fortran contiguous numpy array.

\texttt{predict} \((X)\)

Predict using the linear model

\textbf{Parameters} \(X\) : numpy array of shape \([n\_samples, n\_features]\)

\textbf{Returns} \(C\) : array, shape = \([n\_samples]\)

Returns predicted values.

\texttt{score} \((X, y)\)

Returns the coefficient of determination \(R^2\) of the prediction.

The coefficient \(R^2\) is defined as \(1 - \frac{u}{v}\), where \(u\) is the regression sum of squares \((y - y\_pred)^2\).sum() and \(v\) is the residual sum of squares \((y\_true - y\_true\_mean)^2\).sum(). Best possible score is 1.0, lower values are worse.

\textbf{Parameters} \(X\) : array-like, shape = \([n\_samples, n\_features]\)

Training set.

\(y\) : array-like, shape = \([n\_samples]\)

\textbf{Returns} \(z\) : float

\texttt{set_params} \((**params)\)

Set the parameters of the estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The former have parameters of the form \(<\text{component}>\_\_\text{<parameter>}\) so that it’s possible to update each component of a nested object.

\textbf{Returns} \texttt{self}:

\texttt{sklearn.linear_model.ElasticNet}

\texttt{class} \texttt{sklearn.linear_model.ElasticNet} (alpha=1.0, rho=0.5, fit_intercept=True, normalize=False, precompute='auto', max_iter=1000, copy_X=True, tol=0.0001, warm_start=False, positive=False)

Linear Model trained with L1 and L2 prior as regularizer

Minimizes the objective function:

\[
\frac{1}{2 \times n\_samples} \times (||y - Xw||^2_2 + \\
+ \alpha \times \rho \times ||w||_1 + 0.5 \times \alpha \times (1 - \rho) \times ||w||^2_2)
\]

If you are interested in controlling the L1 and L2 penalty separately, keep in mind that this is equivalent to:

\(a \times \text{L1} + b \times \text{L2}\)

where:

\(alpha = a + b\) and \(rho = a \div (a + b)\)

The parameter \(rho\) corresponds to \(alpha\) in the glmnet R package while \(alpha\) corresponds to the lambda parameter in glmnet. Specifically, \(rho = 1\) is the lasso penalty. Currently, \(rho <= 0.01\) is not reliable, unless you supply your own sequence of \(alpha\).

\textbf{Parameters} \(alpha\) : float
Constant that multiplies the penalty terms. Defaults to 1.0 See the notes for the exact mathematical meaning of this parameter

**rho** : float

The ElasticNet mixing parameter, with 0 < rho <= 1. For rho = 0 the penalty is an L1 penalty. For rho = 1 it is an L2 penalty. For 0 < rho < 1, the penalty is a combination of L1 and L2

**fit_intercept** : bool

Whether the intercept should be estimated or not. If False, the data is assumed to be already centered.

**normalize** : boolean, optional

If True, the regressors X are normalized

**precompute** : True | False | ‘auto’ | array-like

Whether to use a precomputed Gram matrix to speed up calculations. If set to ‘auto’ let us decide. The Gram matrix can also be passed as argument.

**max_iter** : int, optional

The maximum number of iterations

**copy_X** : boolean, optional, default False

If True, X will be copied; else, it may be overwritten.

**tol** : float, optional

The tolerance for the optimization: if the updates are smaller than ‘tol’, the optimization code checks the dual gap for optimality and continues until it is smaller than tol.

**warm_start** : bool, optional

When set to True, reuse the solution of the previous call to fit as initialization, otherwise, just erase the previous solution.

**positive** : bool, optional

When set to True, forces the coefficients to be positive.

Notes

To avoid unnecessary memory duplication the X argument of the fit method should be directly passed as a fortran contiguous numpy array.

Methods

decision_function(X) Decision function of the linear model

fit(X, y[, Xy, coef_init]) Fit Elastic Net model with coordinate descent

get_params([deep]) Get parameters for the estimator

predict(X) Predict using the linear model

score(X, y) Returns the coefficient of determination R^2 of the prediction.

set_params(**params) Set the parameters of the estimator.
```python
__init__(alpha=1.0, rho=0.5, fit_intercept=True, normalize=False, precompute='auto', max_iter=1000, copy_X=True, tol=0.0001, warm_start=False, positive=False)

decision_function (X)
Decision function of the linear model

Parameters X : numpy array of shape [n_samples, n_features]

Returns C : array, shape = [n_samples]
Returns predicted values.

fit (X, y, Xy=None, coef_init=None)
Fit Elastic Net model with coordinate descent

Parameters X: ndarray, (n_samples, n_features):
Data

y: ndarray, (n_samples):
Target

Xy : array-like, optional
Xy = np.dot(X.T, y) that can be precomputed. It is useful only when the Gram matrix is precomputed.

coeff_init: ndarray of shape n_features:
The initial coefficients to warm-start the optimization

Notes
Coordinate descent is an algorithm that considers each column of data at a time hence it will automatically convert the X input as a fortran contiguous numpy array if necessary.

To avoid memory re-allocation it is advised to allocate the initial data in memory directly using that format.

get_params (deep=True)
Get parameters for the estimator

Parameters deep: boolean, optional:
If True, will return the parameters for this estimator and contained subobjects that are estimators.

predict (X)
Predict using the linear model

Parameters X : numpy array of shape [n_samples, n_features]

Returns C : array, shape = [n_samples]
Returns predicted values.

score (X, y)
Returns the coefficient of determination R^2 of the prediction.
The coefficient R^2 is defined as (1 - u/v), where u is the regression sum of squares ((y - y_pred) ** 2).sum() and v is the residual sum of squares ((y_true - y_true.mean()) ** 2).sum(). Best possible score is 1.0, lower values are worse.

Parameters X : array-like, shape = [n_samples, n_features]
Training set.
```
y : array-like, shape = [n_samples]

Returns z : float

set_params (**params)
Set the parameters of the estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The former have parameters of the form <component>__<parameter> so that it’s possible to update each component of a nested object.

Returns self :

sklearn.linear_model.ElasticNetCV

class sklearn.linear_model.ElasticNetCV(rho=0.5, eps=0.001, n_alphas=100, alphas=None, fit_intercept=True, normalize=False, precompute='auto', max_iter=1000, tol=0.0001, cv=None, copy_X=True, verbose=0, n_jobs=1)

Elastic Net model with iterative fitting along a regularization path

The best model is selected by cross-validation.

Parameters rho : float, optional

float between 0 and 1 passed to ElasticNet (scaling between l1 and l2 penalties). For rho = 0 the penalty is an L1 penalty. For rho = 1 it is an L2 penalty. For 0 < rho < 1, the penalty is a combination of L1 and L2 This parameter can be a list, in which case the different values are tested by cross-validation and the one giving the best prediction score is used. Note that a good choice of list of values for rho is often to put more values close to 1 (i.e. Lasso) and less close to 0 (i.e. Ridge), as in [.1, .5, .7, .9, .95, .99, 1]

eps : float, optional

Length of the path. eps=1e-3 means that alpha_min / alpha_max = 1e-3.

n_alphas : int, optional

Number of alphas along the regularization path

alphas : numpy array, optional

List of alphas where to compute the models. If None alphas are set automatically

precompute : True | False | ‘auto’ | array-like

Whether to use a precomputed Gram matrix to speed up calculations. If set to ‘auto’ let us decide. The Gram matrix can also be passed as argument.

max_iter: int, optional :

The maximum number of iterations

tol: float, optional :

The tolerance for the optimization: if the updates are smaller than ‘tol’, the optimization code checks the dual gap for optimality and continues until it is smaller than tol.

cv : integer or crossvalidation generator, optional

If an integer is passed, it is the number of fold (default 3). Specific crossvalidation objects can be passed, see sklearn.cross_validation module for the list of possible objects

verbose : bool or integer
amount of verbosity

n_jobs : integer, optional

Number of CPUs to use during the cross validation. If ‘-1’, use all the CPUs. Note that
this is used only if multiple values for rho are given.

See Also:
enet_path, ElasticNet

Notes

See examples/linear_model/lasso_path_with_crossvalidation.py for an example.

To avoid unnecessary memory duplication the X argument of the fit method should be directly passed as a fortran
contiguous numpy array.

The parameter rho corresponds to alpha in the glmnet R package while alpha corresponds to the lambda param-
eter in glmnet. More specifically, the optimization objective is:

\[
\frac{1}{2n_{\text{samples}}} ||y - Xw||^2_2 + \\
+ \alpha \cdot \rho \cdot ||w||_1 + 0.5 \cdot \alpha \cdot (1 - \rho) \cdot ||w||^2_2
\]

If you are interested in controlling the L1 and L2 penalty separately, keep in mind that this is equivalent to:

\[
a \cdot L1 + b \cdot L2
\]

for:

\[
\alpha = a + b \quad \text{and} \quad \rho = a / (a + b)
\]

Attributes

<table>
<thead>
<tr>
<th>attribute</th>
<th>description</th>
</tr>
</thead>
<tbody>
<tr>
<td>alpha_</td>
<td>float</td>
</tr>
<tr>
<td>rho_</td>
<td>float</td>
</tr>
<tr>
<td>coef_</td>
<td>array, shape = [n_features]</td>
</tr>
<tr>
<td>intercept_</td>
<td>float</td>
</tr>
<tr>
<td>mse_path_</td>
<td>array, shape = [n_rho, n_alpha, n_folds]</td>
</tr>
</tbody>
</table>

Methods

<table>
<thead>
<tr>
<th>method</th>
<th>description</th>
</tr>
</thead>
<tbody>
<tr>
<td>decision_function(X)</td>
<td>Decision function of the linear model</td>
</tr>
<tr>
<td>fit(X, y)</td>
<td>Fit linear model with coordinate descent along decreasing alphas</td>
</tr>
<tr>
<td>get_params([deep])</td>
<td>Get parameters for the estimator</td>
</tr>
<tr>
<td>path(X, y[, rho, eps, n_alphas, alphas, ...])</td>
<td>Compute Elastic-Net path with coordinate descent</td>
</tr>
<tr>
<td>predict(X)</td>
<td>Predict using the linear model</td>
</tr>
<tr>
<td>score(X, y)</td>
<td>Returns the coefficient of determination R^2 of the prediction.</td>
</tr>
<tr>
<td>set_params(**params)</td>
<td>Set the parameters of the estimator.</td>
</tr>
</tbody>
</table>
__init__ (rho=0.5, eps=0.001, n_alphas=100, alphas=None, fit_intercept=True, normalize=False, precompute='auto', max_iter=1000, tol=0.0001, cv=None, copy_X=True, verbose=0, n_jobs=1)

decision_function (X)
    Decision function of the linear model

    Parameters X : numpy array of shape [n_samples, n_features]

    Returns C : array, shape = [n_samples]
        Returns predicted values.

fit (X, y)
    Fit linear model with coordinate descent along decreasing alphas using cross-validation

    Parameters X : numpy array of shape [n_samples, n_features]
        Training data. Pass directly as fortran contiguous data to avoid unnecessary memory duplication

    y : numpy array of shape [n_samples]
        Target values

get_params (deep=True)
    Get parameters for the estimator

    Parameters deep: boolean, optional :
        If True, will return the parameters for this estimator and contained subobjects that are estimators.

static path (X, y, rho=0.5, eps=0.001, n_alphas=100, alphas=None, precompute='auto', Xy=None, fit_intercept=True, normalize=False, copy_X=True, verbose=False, **params)
    Compute Elastic-Net path with coordinate descent

    The Elastic Net optimization function is:

    1 / (2 * n_samples) * ||y - Xw||^2_2 +
    + alpha * rho * ||w||_1 + 0.5 * alpha * (1 - rho) * ||w||^2_2

    Parameters X : numpy array of shape [n_samples, n_features]
        Training data. Pass directly as fortran contiguous data to avoid unnecessary memory duplication

    y : numpy array of shape [n_samples]
        Target values

    rho : float, optional
        float between 0 and 1 passed to ElasticNet (scaling between l1 and l2 penalties). rho=1 corresponds to the Lasso

    eps : float
        Length of the path. eps=1e-3 means that alpha_min / alpha_max = 1e-3

    n_alphas : int, optional
        Number of alphas along the regularization path

    alphas : numpy array, optional
        List of alphas where to compute the models. If None alphas are set automatically
**precompute** : True | False | ‘auto’ | array-like

Whether to use a precomputed Gram matrix to speed up calculations. If set to ‘auto’ let us decide. The Gram matrix can also be passed as argument.

**Xy** : array-like, optional

Xy = np.dot(X.T, y) that can be precomputed. It is useful only when the Gram matrix is precomputed.

**fit_intercept** : bool

Fit or not an intercept

**normalize** : boolean, optional

If True, the regressors X are normalized

**copy_X** : boolean, optional, default True

If True, X will be copied; else, it may be overwritten.

**verbose** : bool or integer

Amount of verbosity

**params** : kwargs

keyword arguments passed to the Lasso objects

**Returns models** : a list of models along the regularization path

**See Also**:

ElasticNet, ElasticNetCV

**Notes**

See examples/plot_lasso_coordinate_descent_path.py for an example.

**predict**(X)

Predict using the linear model

**Parameters** X : numpy array of shape [n_samples, n_features]

**Returns** C : array, shape = [n_samples]

Returns predicted values.

**score**(X, y)

Returns the coefficient of determination R^2 of the prediction.

The coefficient R^2 is defined as (1 - u/v), where u is the regression sum of squares ((y - y_pred) ** 2).sum() and v is the residual sum of squares ((y_true - y_true.mean()) ** 2).sum(). Best possible score is 1.0, lower values are worse.

**Parameters** X : array-like, shape = [n_samples, n_features]

Training set.

y : array-like, shape = [n_samples]

**Returns** z : float
**set_params(**params**)

Set the parameters of the estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The former have parameters of the form `<component>__<parameter>` so that it’s possible to update each component of a nested object.

Returns self:

```
scikit-learn user guide, Release 0.12-git

set_params(**params)
Set the parameters of the estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The former have parameters of the form `<component>__<parameter>` so that it’s possible to update each component of a nested object.

Returns self:

sklearn.linear_model.Lars

class sklearn.linear_model.Lars (fit_intercept=True, verbose=False, normalize=True, precompute='auto', n_nonzero_coefs=500, eps=2.2204460492503131e-16, copy_X=True)

Least Angle Regression model a.k.a. LAR

Parameters n_nonzero_coefs : int, optional

Target number of non-zero coefficients. Use np.inf for no limit.

fit_intercept : boolean

Whether to calculate the intercept for this model. If set to false, no intercept will be used in calculations (e.g. data is expected to be already centered).

verbose : boolean or integer, optional

Sets the verbosity amount

normalize : boolean, optional

If True, the regressors X are normalized

precompute : True | False | ‘auto’ | array-like

Whether to use a precomputed Gram matrix to speed up calculations. If set to ‘auto’ let us decide. The Gram matrix can also be passed as argument.

copy_X : boolean, optional, default True

If True, X will be copied; else, it may be overwritten.

eps: float, optional:

The machine-precision regularization in the computation of the Cholesky diagonal factors. Increase this for very ill-conditioned systems. Unlike the ‘tol’ parameter in some iterative optimization-based algorithms, this parameter does not control the tolerance of the optimization.

See Also:

lars_path, LarsCV, sklearn.decomposition.sparse_encode

http://en.wikipedia.org/wiki/Least_angle_regression

Examples

```python
>>> from sklearn import linear_model
>>> clf = linear_model.Lars(n_nonzero_coefs=1)
>>> clf.fit([(−1, 1), [0, 0], [1, 1]], [-1.1111, 0, -1.1111])
...Lars(copy_X=True, eps=..., fit_intercept=True, n_nonzero_coefs=1,
```
```python
>>> clf = LinearRegression(normalize=True, precompute='auto', verbose=False)
>>> print(clf.coef_)
[ 0. -1.11...]
```

**Attributes**

<table>
<thead>
<tr>
<th>coef_</th>
<th>array, shape = [n_features]</th>
<th>parameter vector (w in the formulation formula)</th>
</tr>
</thead>
<tbody>
<tr>
<td>intercept_</td>
<td>float</td>
<td>independent term in decision function.</td>
</tr>
</tbody>
</table>

**Methods**

<table>
<thead>
<tr>
<th>decision_function(X)</th>
<th>Decision function of the linear model</th>
</tr>
</thead>
<tbody>
<tr>
<td>fit(X, y)</td>
<td>Fit the model using X, y as training data.</td>
</tr>
<tr>
<td>get_params([deep])</td>
<td>Get parameters for the estimator</td>
</tr>
<tr>
<td>predict(X)</td>
<td>Predict using the linear model</td>
</tr>
<tr>
<td>score(X, y)</td>
<td>Returns the coefficient of determination R^2 of the prediction.</td>
</tr>
<tr>
<td>set_params(**params)</td>
<td>Set the parameters of the estimator.</td>
</tr>
</tbody>
</table>

```python
__init__(fit_intercept=True, verbose=False, normalize=True, precompute='auto', n_nonzero_coefs=500, eps=2.2204460492503131e-16, copy_X=True)
```

**decision_function (X)**

Decision function of the linear model

**Parameters**

- **X**: numpy array of shape [n_samples, n_features]

**Returns**

- **C**: array, shape = [n_samples]
  
  Returns predicted values.

**fit (X, y)**

Fit the model using X, y as training data.

**Parameters**

- **X**: array-like, shape = [n_samples, n_features]
  
  training data.

- **y**: array-like, shape = [n_samples]
  
  target values.

**Returns**

- **self**: object
  
  returns an instance of self.

**get_params (deep=True)**

Get parameters for the estimator

**Parameters**

- **deep**: boolean, optional
  
  If True, will return the parameters for this estimator and contained subobjects that are estimators.

**predict (X)**

Predict using the linear model

**Parameters**

- **X**: numpy array of shape [n_samples, n_features]
Returns C : array, shape = [n_samples]
 Returns predicted values.

score (X, y)
Returns the coefficient of determination $R^2$ of the prediction.

The coefficient $R^2$ is defined as $(1 - u/v)$, where $u$ is the regression sum of squares $(y - y_{\text{pred}})^2$.sum() and $v$ is the residual sum of squares $(y_{\text{true}} - y_{\text{true.mean}})^2$.sum(). Best possible score is 1.0, lower values are worse.

Parameters X : array-like, shape = [n_samples, n_features]
 Training set.

y : array-like, shape = [n_samples]

Returns z : float

set_params (**params)
Set the parameters of the estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The former have parameters of the form <component>__<parameter> so that it’s possible to update each component of a nested object.

Returns self :

sklearn.linear_model.LassoLars

class sklearn.linear_model.LassoLars (alpha=1.0, fit_intercept=True, verbose=False, normalize=True, precompute='auto', max_iter=500, eps=2.220446049250313e-16, copy_X=True)

Lasso model fit with Least Angle Regression a.k.a. Lars

It is a Linear Model trained with an L1 prior as regularizer.

The optimization objective for Lasso is:

$(1 / (2 * n_{\text{samples}})) \sum ||y - Xw||^2 + \alpha \sum |w|$

Parameters fit_intercept : boolean
 whether to calculate the intercept for this model. If set to false, no intercept will be used in calculations (e.g. data is expected to be already centered).

verbose : boolean or integer, optional
 Sets the verbosity amount

normalize : boolean, optional
 If True, the regressors X are normalized

copy_X : boolean, optional, default True
 If True, X will be copied; else, it may be overwritten.

precompute : True | False | ‘auto’ | array-like
 Whether to use a precomputed Gram matrix to speed up calculations. If set to ‘auto’ let us decide. The Gram matrix can also be passed as argument.

max_iter: integer, optional :
Maximum number of iterations to perform.

**eps: float, optional**

The machine-precision regularization in the computation of the Cholesky diagonal factors. Increase this for very ill-conditioned systems. Unlike the ‘tol’ parameter in some iterative optimization-based algorithms, this parameter does not control the tolerance of the optimization.

**See Also:**

lars_path, lasso_path, Lasso, LassoCV, LassoLarsCV, sklearn.decomposition.sparse_encode

**http://en.wikipedia.org/wiki/Least_angle_regression**

**Examples**

```python
>>> from sklearn import linear_model
>>> clf = linear_model.LassoLars(alpha=0.01)
>>> clf.fit([[-1, 1], [0, 0], [1, 1]], [-1, 0, -1])
...
LassoLars(alpha=0.01, copy_X=True, eps=..., fit_intercept=True,
max_iter=500, normalize=True, precompute='auto', verbose=False)
>>> print(clf.coef_)
[ 0. -0.963257...]
```

**Attributes**

<table>
<thead>
<tr>
<th>coef_</th>
<th>array, shape = [n_features]</th>
<th>parameter vector (w in the formulation formula)</th>
</tr>
</thead>
<tbody>
<tr>
<td>intercept_</td>
<td>float</td>
<td>independent term in decision function.</td>
</tr>
</tbody>
</table>

**Methods**

| decision_function(X) | Decision function of the linear model |
| fit(X, y) | Fit the model using X, y as training data. |
| get_params([deep]) | Get parameters for the estimator |
| predict(X) | Predict using the linear model |
| score(X, y) | Returns the coefficient of determination $R^2$ of the prediction. |
| set_params(**params) | Set the parameters of the estimator. |

**__init__(alpha=1.0, fit_intercept=True, verbose=False, normalize=True, precompute='auto', max_iter=500, eps=2.2204460492503131e-16, copy_X=True)**

**decision_function (X)**

Decision function of the linear model

**Parameters**

| X : numpy array of shape [n_samples, n_features] |

**Returns**

| C : array, shape = [n_samples] |

| Returns predicted values. |

**fit (X, y)**

Fit the model using X, y as training data.
Parameters X : array-like, shape = [n_samples, n_features]
   training data.
y : array-like, shape = [n_samples]
   target values.

Returns self : object
   returns an instance of self.

get_params (deep=True)
   Get parameters for the estimator

   Parameters deep: boolean, optional :
      If True, will return the parameters for this estimator and contained subobjects that are
      estimators.

predict (X)
   Predict using the linear model

   Parameters X : numpy array of shape [n_samples, n_features]

   Returns C : array, shape = [n_samples]
      Returns predicted values.

score (X, y)
   Returns the coefficient of determination R^2 of the prediction.

   The coefficient R^2 is defined as (1 - u/v), where u is the regression sum of squares ((y - y_pred) **
   2).sum() and v is the residual sum of squares ((y_true - y_true.mean()) ** 2).sum(). Best possible score is
   1.0, lower values are worse.

   Parameters X : array-like, shape = [n_samples, n_features]

   Returns z : float

set_params (**params)
   Set the parameters of the estimator.

   The method works on simple estimators as well as on nested objects (such as pipelines). The former have
   parameters of the form <component>__<parameter> so that it’s possible to update each component
   of a nested object.

   Returns self :

sklearn.linear_model.LarsCV

class sklearn.linear_model.LarsCV (fit_intercept=True, verbose=False, max_iter=500, normalize=True, precompute='auto', cv=None, max_n_alphas=1000, n_jobs=1, eps=2.2204460492503131e-16, copy_X=True)

Cross-validated Least Angle Regression model

Parameters fit_intercept : boolean
   whether to calculate the intercept for this model. If set to false, no intercept will be used
   in calculations (e.g. data is expected to be already centered).
verbose : boolean or integer, optional
    Sets the verbosity amount
normalize : boolean, optional
    If True, the regressors X are normalized
copy_X : boolean, optional, default True
    If True, X will be copied; else, it may be overwritten.
precompute : True | False | ‘auto’ | array-like
    Whether to use a precomputed Gram matrix to speed up calculations. If set to ‘auto’ let
    us decide. The Gram matrix can also be passed as argument.
max_iter: integer, optional :
    Maximum number of iterations to perform.
cv : crossvalidation generator, optional
    see sklearn.cross_validation module. If None is passed, default to a 5-fold strategy
max_n_alphas : integer, optional
    The maximum number of points on the path used to compute the residuals in the cross-
    validation
n_jobs : integer, optional
    Number of CPUs to use during the cross validation. If ‘-1’, use all the CPUs
eps: float, optional :
    The machine-precision regularization in the computation of the Cholesky diagonal fac-
    tors. Increase this for very ill-conditioned systems.

See Also:
    lars_path, LassoLARS, LassoLarsCV

Attributes

| coef_  | array, shape = [n_features] | parameter vector (w in the formulation formula) |
| intercept_ | float | independent term in decision function. |
| coef_path: array, shape = [n_features, n_alpha] | the varying values of the coefficients along the path |

Methods

- decision_function(X)
  Decision function of the linear model
- fit(X, y)
  Fit the model using X, y as training data.
- get_params([deep])
  Get parameters for the estimator
- predict(X)
  Predict using the linear model
- score(X, y)
  Returns the coefficient of determination R^2 of the prediction.
- set_params(**params)
  Set the parameters of the estimator.
__init__(fit_intercept=True, verbose=False, max_iter=500, normalize=True, precompute='auto', cv=None, max_n_alphas=1000, n_jobs=1, eps=2.2204460492503131e-16, copy_X=True)

decision_function(X)

Decision function of the linear model

Parameters X : numpy array of shape [n_samples, n_features]

Returns C : array, shape = [n_samples]

Returns predicted values.

fit(X, y)

Fit the model using X, y as training data.

Parameters X : array-like, shape = [n_samples, n_features]

Training data.

y : array-like, shape = [n_samples]

Target values.

Returns self : object

returns an instance of self.

get_params(deep=True)

Get parameters for the estimator

Parameters deep: boolean, optional :

If True, will return the parameters for this estimator and contained subobjects that are estimators.

predict(X)

Predict using the linear model

Parameters X : numpy array of shape [n_samples, n_features]

Returns C : array, shape = [n_samples]

Returns predicted values.

score(X, y)

Returns the coefficient of determination R^2 of the prediction.

The coefficient R^2 is defined as (1 - u/v), where u is the regression sum of squares ((y - y_pred) ** 2).sum() and v is the residual sum of squares ((y_true - y_true.mean()) ** 2).sum(). Best possible score is 1.0, lower values are worse.

Parameters X : array-like, shape = [n_samples, n_features]

Training set.

y : array-like, shape = [n_samples]

Returns z : float

set_params(**params)

Set the parameters of the estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The former have parameters of the form <component>__<parameter> so that it’s possible to update each component of a nested object.

Returns self :
Cross-validated Lasso, using the LARS algorithm

The optimization objective for Lasso is:

\[(1 / (2 \times n\_samples)) \times ||y - Xw||^2_2 + \alpha \times ||w||_1\]

**Parameters**

**fit_intercept** : boolean

whether to calculate the intercept for this model. If set to false, no intercept will be used in calculations (e.g. data is expected to be already centered).

**verbose** : boolean or integer, optional

Sets the verbosity amount

**normalize** : boolean, optional

If True, the regressors X are normalized

**precompute** : True | False | ‘auto’ | array-like

Whether to use a precomputed Gram matrix to speed up calculations. If set to ‘auto’ let us decide. The Gram matrix can also be passed as argument.

**max_iter** : integer, optional

Maximum number of iterations to perform.

**cv** : crossvalidation generator, optional

see sklearn.cross_validation module. If None is passed, default to a 5-fold strategy

**max_n_alphas** : integer, optional

The maximum number of points on the path used to compute the residuals in the cross-validation

**n_jobs** : integer, optional

Number of CPUs to use during the cross validation. If ‘-1’, use all the CPUs

**eps** : float, optional

The machine-precision regularization in the computation of the Cholesky diagonal factors. Increase this for very ill-conditioned systems.

**copy_X** : boolean, optional, default True

If True, X will be copied; else, it may be overwritten.

See Also:

lars_path, LassoLars, LarsCV, LassoCV
Notes

The object solves the same problem as the LassoCV object. However, unlike the LassoCV, it find the relevant alphas values by itself. In general, because of this property, it will be more stable. However, it is more fragile to heavily multicollinear datasets.

It is more efficient than the LassoCV if only a small number of features are selected compared to the total number, for instance if there are very few samples compared to the number of features.

Attributes

<table>
<thead>
<tr>
<th>Attribute</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>coef_</code></td>
<td>array, shape = [n_features] parameter vector (w in the formulation formula)</td>
</tr>
<tr>
<td><code>intercept_</code></td>
<td>float independent term in decision function.</td>
</tr>
<tr>
<td><code>coef_path</code></td>
<td>array, shape = [n_features, n_alpha] the varying values of the coefficients along the path</td>
</tr>
<tr>
<td><code>alphas_</code></td>
<td>array, shape = [n_alpha] the different values of alpha along the path</td>
</tr>
<tr>
<td><code>cv_alphas</code></td>
<td>array, shape = [n_cv_alphas] all the values of alpha along the path for the different folds</td>
</tr>
<tr>
<td><code>cv_mse_path_</code></td>
<td>array, shape = [n_folds, n_cv_alphas] the mean square error on left-out for each fold along the path (alpha values given by cv_alphas)</td>
</tr>
</tbody>
</table>

Methods

<table>
<thead>
<tr>
<th>Method</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>decision_function(X)</code></td>
<td>Decision function of the linear model</td>
</tr>
<tr>
<td><code>fit(X, y)</code></td>
<td>Fit the model using X, y as training data.</td>
</tr>
<tr>
<td><code>get_params([deep])</code></td>
<td>Get parameters for the estimator</td>
</tr>
<tr>
<td><code>predict(X)</code></td>
<td>Predict using the linear model</td>
</tr>
<tr>
<td><code>score(X, y)</code></td>
<td>Returns the coefficient of determination R^2 of the prediction.</td>
</tr>
<tr>
<td><code>set_params(**params)</code></td>
<td>Set the parameters of the estimator</td>
</tr>
</tbody>
</table>

`__init__`(fit_intercept=True, verbose=False, max_iter=500, normalize=True, precompute='auto', cv=None, max_n_alphas=1000, n_jobs=1, eps=2.2204460492503131e-16, copy_X=True)

decision_function (X)

Decision function of the linear model

Parameters X : numpy array of shape [n_samples, n_features]

Returns C : array, shape = [n_samples]

Returns predicted values.

fit (X, y)

Fit the model using X, y as training data.

Parameters X : array-like, shape = [n_samples, n_features]

Training data.

y : array-like, shape = [n_samples]

Target values.
Returns self : object
returns an instance of self.

get_params (deep=True)
Get parameters for the estimator

Parameters deep: boolean, optional :
If True, will return the parameters for this estimator and contained subobjects that are estimators.

predict (X)
Predict using the linear model

Parameters X : numpy array of shape [n_samples, n_features]

Returns C : array, shape = [n_samples]
Returns predicted values.

score (X, y)
Returns the coefficient of determination R^2 of the prediction.
The coefficient R^2 is defined as (1 - u/v), where u is the regression sum of squares ((y - y_pred) ** 2).sum() and v is the residual sum of squares ((y_true - y_true.mean()) ** 2).sum(). Best possible score is 1.0, lower values are worse.

Parameters X : array-like, shape = [n_samples, n_features]

    Training set.

    y : array-like, shape = [n_samples]

Returns z : float

set_params (**params)
Set the parameters of the estimator.
The method works on simple estimators as well as on nested objects (such as pipelines). The former have parameters of the form <component>__<parameter> so that it's possible to update each component of a nested object.

Returns self :

sklearn.linear_model.LassoLarsIC

class sklearn.linear_model.LassoLarsIC (criterion='aic', fit_intercept=True, verbose=False, normalize=True, precompute='auto', max_iter=500, eps=2.2204460492503131e-16, copy_X=True)
Lasso model fit with Lars using BIC or AIC for model selection

The optimization objective for Lasso is:

(1 / (2 * n_samples)) * ||y - Xw||^2_2 + alpha * ||w||_1

AIC is the Akaike information criterion and BIC is the Bayes Information criterion. Such criteria are useful to select the value of the regularization parameter by making a trade-off between the goodness of fit and the complexity of the model. A good model should explain well the data while being simple.

Parameters criterion: ‘bic’ | ‘aic’ :
The type of criterion to use.
**fit_intercept** : boolean

whether to calculate the intercept for this model. If set to false, no intercept will be used in calculations (e.g. data is expected to be already centered).

**verbose** : boolean or integer, optional

Sets the verbosity amount

**normalize** : boolean, optional

If True, the regressors X are normalized

**copy_X** : boolean, optional, default True

If True, X will be copied; else, it may be overwritten.

**precompute** : True | False | ‘auto’ | array-like

Whether to use a precomputed Gram matrix to speed up calculations. If set to ‘auto’ let us decide. The Gram matrix can also be passed as argument.

**max_iter** : integer, optional

Maximum number of iterations to perform. Can be used for early stopping.

**eps** : float, optional

The machine-precision regularization in the computation of the Cholesky diagonal factors. Increase this for very ill-conditioned systems. Unlike the ‘tol’ parameter in some iterative optimization-based algorithms, this parameter does not control the tolerance of the optimization.

**See Also:**

lars_path, LassoLars, LassoLarsCV

**Notes**

The estimation of the number of degrees of freedom is given by:


**Examples**

```python
>>> from sklearn import linear_model
>>> clf = linear_model.LassoLarsIC(criterion='bic')
>>> clf.fit([[1, 2], [1, 2], [3, 4], [3, 4]], [-3.3333, -3.3333, 1.6667, 1.6667])
...
LassoLarsIC(copy_X=True, criterion='bic', eps..., fit_intercept=True,
        max_iter=500, normalize=True, precompute='auto',
        verbose=False)
>>> print(clf.coef_)
[ 0.  1.  2.  3.]
```
### Attributes

<table>
<thead>
<tr>
<th>Attribute</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>coef_</td>
<td>array, shape = [n_features]</td>
</tr>
<tr>
<td>intercept_</td>
<td>float</td>
</tr>
<tr>
<td>alpha_</td>
<td>float</td>
</tr>
</tbody>
</table>

### Methods

<table>
<thead>
<tr>
<th>Method</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>decision_function(X)</td>
<td>Decision function of the linear model</td>
</tr>
<tr>
<td>fit(X, y[, copy_X])</td>
<td>Fit the model using X, y as training data.</td>
</tr>
<tr>
<td>get_params([deep])</td>
<td>Get parameters for the estimator</td>
</tr>
<tr>
<td>predict(X)</td>
<td>Predict using the linear model</td>
</tr>
<tr>
<td>score(X, y)</td>
<td>Returns the coefficient of determination R^2 of the prediction.</td>
</tr>
<tr>
<td>set_params(**params)</td>
<td>Set the parameters of the estimator.</td>
</tr>
</tbody>
</table>

__init__ (criterion='aic', fit_intercept=True, verbose=False, normalize=True, precompute='auto', max_iter=500, eps=2.2204460492503131e-16, copy_X=True)

### decision_function (X)

Decision function of the linear model

Parameters X : numpy array of shape [n_samples, n_features]

Returns C : array, shape = [n_samples]

Returns predicted values.

### fit (X, y, copy_X=True)

Fit the model using X, y as training data.

Parameters x : array-like, shape = [n_samples, n_features]

training data.

y : array-like, shape = [n_samples]

target values.

Returns self : object

returns an instance of self.

### get_params (deep=True)

Get parameters for the estimator

Parameters deep: boolean, optional :

If True, will return the parameters for this estimator and contained subobjects that are estimators.

### predict (X)

Predict using the linear model

Parameters X : numpy array of shape [n_samples, n_features]

Returns C : array, shape = [n_samples]

Returns predicted values.
score \((X, y)\)

Returns the coefficient of determination \(R^2\) of the prediction.

The coefficient \(R^2\) is defined as \((1 - u/v)\), where \(u\) is the regression sum of squares \(((y - y_{\text{pred}})^2)\).sum()\) and \(v\) is the residual sum of squares \(((y_{\text{true}} - y_{\text{true.mean}})^2)\).sum(). Best possible score is 1.0, lower values are worse.

**Parameters**

- \(X\) : array-like, shape \([n_{\text{samples}}, n_{\text{features}}]\)
  Training set.

- \(y\) : array-like, shape \([n_{\text{samples}}]\)

**Returns**

- \(z\) : float

set_params (**params**)

Set the parameters of the estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The former have parameters of the form \(<\text{component}>__<\text{parameter}>\) so that it's possible to update each component of a nested object.

**Returns**

- self

**sklearn.linear_model.LogisticRegression**

class sklearn.linear_model.LogisticRegression(\(\text{penalty='l2', dual=False, tol=0.0001, C=1.0, fit_intercept=True, \text{intercept_scaling=1, class_weight=None}}\))

Logistic Regression (aka logit, MaxEnt) classifier.

In the multiclass case, the training algorithm uses a one-vs.-all (OvA) scheme, rather than the “true” multinomial LR.

This class implements L1 and L2 regularized logistic regression using the liblinear library. It can handle both dense and sparse input. Use C-ordered arrays or CSR matrices containing 64-bit floats for optimal performance; any other input format will be converted (and copied).

**Parameters**

- \(\text{penalty}\) : string, ‘l1’ or ‘l2’
  Used to specify the norm used in the penalization

- \(\text{dual}\) : boolean
  Dual or primal formulation. Dual formulation is only implemented for \(l2\) penalty. Prefer \(\text{dual=}\text{False}\) when \(n_{\text{samples}} > n_{\text{features}}\).

- \(C\) : float or None, optional (default=None)
  Specifies the strength of the regularization. The smaller it is the bigger in the regular-
  ization. If None then \(C\) is set to \(n_{\text{samples}}\).

- \(\text{fit_intercept}\) : bool, default: True
  Specifies if a constant (a.k.a. bias or intercept) should be added the decision function

- \(\text{intercept_scaling}\) : float, default: 1
  when \(\text{self.fit_intercept}\) is True, instance vector \(x\) becomes \([x, \text{self.intercept_scaling}]\), i.e. a “synthetic” feature with constant value equals to intercept_scaling is appended to the instance vector. The intercept becomes intercept_scaling * synthetic feature weight

Note! the synthetic feature weight is subject to \(l1/l2\) regularization as all other features.
To lessen the effect of regularization on synthetic feature weight (and therefore on the
intercept) intercept_scaling has to be increased

**tol**: float, optional :
tolerance for stopping criteria

See Also:
LinearSVC

Notes

The underlying C implementation uses a random number generator to select features when fitting the model.
It is thus not uncommon, to have slightly different results for the same input data. If that happens, try with a
smaller tol parameter.

References:
LIBLINEAR – A Library for Large Linear Classification http://www.csie.ntu.edu.tw/~cjlin/liblinear/

Attributes

<table>
<thead>
<tr>
<th>coef_</th>
<th>array, shape = [n_classes-1, n_features]</th>
<th>Coefficient of the features in the decision function. coef_ is readonly property derived from raw_coef_ that follows the internal memory layout of liblinear.</th>
</tr>
</thead>
<tbody>
<tr>
<td>intercept_</td>
<td>array, shape = [n_classes-1]</td>
<td>intercept (a.k.a. bias) added to the decision function. It is available only when parameter intercept is set to True</td>
</tr>
</tbody>
</table>

Methods

<table>
<thead>
<tr>
<th>decision_function(X)</th>
<th>Decision function value for X according to the trained model.</th>
</tr>
</thead>
<tbody>
<tr>
<td>fit(X, y[, class_weight])</td>
<td>Fit the model according to the given training data.</td>
</tr>
<tr>
<td>fit_transform(X[, y])</td>
<td>Fit to data, then transform it</td>
</tr>
<tr>
<td>get_params([deep])</td>
<td>Get parameters for the estimator</td>
</tr>
<tr>
<td>predict(X)</td>
<td>Predict target values of X according to the fitted model.</td>
</tr>
<tr>
<td>predict_log_proba(X)</td>
<td>Log of Probability estimates.</td>
</tr>
<tr>
<td>predict_proba(X)</td>
<td>Probability estimates.</td>
</tr>
<tr>
<td>score(X, y)</td>
<td>Returns the mean accuracy on the given test data and labels.</td>
</tr>
<tr>
<td>set_params(**params)</td>
<td>Set the parameters of the estimator.</td>
</tr>
<tr>
<td>transform(X[, threshold])</td>
<td>Reduce X to its most important features.</td>
</tr>
</tbody>
</table>

**init**(penalty='l2', dual=False, tol=0.0001, C=1.0, fit_intercept=True, intercept_scaling=1, class_weight=None)

**decision_function**(X)

Decision function value for X according to the trained model.

Parameters X : array-like, shape = [n_samples, n_features]
Returns $T$ : array-like, shape = [n_samples, n_class]

Returns the decision function of the sample for each class in the model.

**fit** ($X, y, \text{class\_weight=None}$)

Fit the model according to the given training data.

**Parameters** $X$ : [array-like, sparse matrix], shape = [n_samples, n_features]

Training vector, where n_samples is the number of samples and n_features is the number of features.

$y$ : array-like, shape = [n_samples]

Target vector relative to $X$

**class\_weight** : {dict, ‘auto’}, optional

Weights associated with classes. If not given, all classes are supposed to have weight one.

**Returns self** : object

Returns self.

**fit\_transform** ($X, y=\text{None}, **\text{fit\_params}$)

Fit to data, then transform it

Fits transformer to $X$ and $y$ with optional parameters fit\_params and returns a transformed version of $X$.

**Parameters** $X$ : numpy array of shape [n_samples, n_features]

Training set.

$y$ : numpy array of shape [n_samples]

Target values.

**Returns** $X\_\text{new}$ : numpy array of shape [n_samples, n\_features\_new]

Transformed array.

**Notes**

This method just calls fit and transform consecutively, i.e., it is not an optimized implementation of fit\_transform, unlike other transformers such as PCA.

**get\_params** ($deep=\text{True}$)

Get parameters for the estimator

**Parameters** $deep$: boolean, optional:

If True, will return the parameters for this estimator and contained subobjects that are estimators.

**predict** ($X$)

Predict target values of $X$ according to the fitted model.

**Parameters** $X$ : [array-like, sparse matrix], shape = [n_samples, n_features]

**Returns** $C$ : array, shape = [n_samples]

**predict\_log\_proba** ($X$)

Log of Probability estimates.

The returned estimates for all classes are ordered by the label of classes.
Parameters **X** : array-like, shape = [n_samples, n_features]

Returns **T** : array-like, shape = [n_samples, n_classes]

Returns the log-probabilities of the sample for each class in the model, where classes are ordered by arithmetical order.

**predict_proba** (**X**)

Probability estimates.

The returned estimates for all classes are ordered by the label of classes.

Parameters **X** : array-like, shape = [n_samples, n_features]

Returns **T** : array-like, shape = [n_samples, n_classes]

Returns the probability of the sample for each class in the model, where classes are ordered by arithmetical order.

**score** (**X**, **y**)

Returns the mean accuracy on the given test data and labels.

Parameters **X** : array-like, shape = [n_samples, n_features]

Training set.

**y** : array-like, shape = [n_samples]

Labels for **X**.

Returns **z** : float

**set_params** (****params**)

Set the parameters of the estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The former have parameters of the form `<component>__<parameter>` so that it’s possible to update each component of a nested object.

Returns **self** :

**transform** (**X**, **threshold=None**)

Reduce **X** to its most important features.

Parameters **X** : array or scipy sparse matrix of shape [n_samples, n_features]

The input samples.

**threshold** : string, float or None, optional (default=None)

The threshold value to use for feature selection. Features whose importance is greater or equal are kept while the others are discarded. If “median” (resp. “mean”), then the threshold value is the median (resp. the mean) of the feature importances. A scaling factor (e.g., “1.25*mean”) may also be used. If None and if available, the object attribute **threshold** is used. Otherwise, “mean” is used by default.

Returns **X_r** : array of shape [n_samples, n_selected_features]

The input samples with only the selected features.
Orthogonal Matching Pursuit model (OMP)

**Parameters**

- `n_nonzero_coefs` : int, optional
  - Desired number of non-zero entries in the solution. If None (by default) this value is set to 10% of `n_features`.

- `tol` : float, optional
  - Maximum norm of the residual. If not None, overrides `n_nonzero_coefs`.

- `fit_intercept` : boolean, optional
  - Whether to calculate the intercept for this model. If set to false, no intercept will be used in calculations (e.g. data is expected to be already centered).

- `normalize` : boolean, optional
  - If False, the regressors X are assumed to be already normalized.

- `precompute_gram` : {True, False, ‘auto’},
  - Whether to use a precomputed Gram and Xy matrix to speed up calculations. Improves performance when `n_targets` or `n_samples` is very large. Note that if you already have such matrices, you can pass them directly to the fit method.

- `copy_X` : bool, optional
  - Whether the design matrix X must be copied by the algorithm. A false value is only helpful if X is already Fortran-ordered, otherwise a copy is made anyway.

- `copy_Gram` : bool, optional
  - Whether the gram matrix must be copied by the algorithm. A false value is only helpful if X is already Fortran-ordered, otherwise a copy is made anyway.

- `copy_Xy` : bool, optional
  - Whether the covariance vector Xy must be copied by the algorithm. If False, it may be overwritten.

**See Also:**

- `orthogonal_mp`
- `orthogonal_mp_gram`
- `lars_path`
- `Lars`
- `LassoLars`
- `decomposition.sparse_encode`
- `decomposition.sparse_encode_parallel`

**Notes**


**Attributes**

<table>
<thead>
<tr>
<th>Attributes</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>coef_</code></td>
<td>array, shape = (n_features,) or (n_features, n_targets)</td>
</tr>
<tr>
<td><code>intercept_</code></td>
<td>float or array, shape = (n_targets,)</td>
</tr>
</tbody>
</table>

**Methods**

- `decision_function(X)` Decision function of the linear model
- `fit(X, y[, Gram, Xy])` Fit the model using X, y as training data.
- `get_params([deep])` Get parameters for the estimator
- `predict(X)` Predict using the linear model
- `score(X, y)` Returns the coefficient of determination $R^2$ of the prediction.
- `set_params(**params)` Set the parameters of the estimator.

```python
__init__(copy_X=True, copy_Gram=True, copy_Xy=True, n_nonzero_coefs=None, tol=None, fit_intercept=True, normalize=True, precompute_gram=False)
```

**decision_function**($X$)

- **Parameters**
  - $X$: numpy array of shape [n_samples, n_features]
- **Returns**
  - $C$: array, shape = [n_samples]
  - Returns predicted values.

```python
fit(X, y, Gram=None, Xy=None)
```

- **Parameters**
  - $X$: array-like, shape = (n_samples, n_features)
    - Training data.
  - $y$: array-like, shape = (n_samples,) or (n_samples, n_targets)
    - Target values.
  - `Gram`: array-like, shape = (n_features, n_features) (optional)
    - Gram matrix of the input data: $X.T * X$
  - `Xy`: array-like, shape = (n_features,) or (n_features, n_targets) (optional)
    - Input targets multiplied by $X$: $X.T * y$
- **Returns**
  - `self`: object
    - returns an instance of self.

```python
get_params(deep=True)
```

- **Parameters**
  - `deep`: boolean, optional
    - If True, will return the parameters for this estimator and contained subobjects that are estimators.
**predict** (*X*)

Predict using the linear model

Parameters

- **X**: numpy array of shape [n_samples, n_features]
  - Returns **C**: array, shape = [n_samples]
  - Returns predicted values.

**score** (*X*, *y*)

Returns the coefficient of determination R^2 of the prediction.

The coefficient R^2 is defined as (1 - u/v), where u is the regression sum of squares ((y - y_pred)**2).sum() and v is the residual sum of squares ((y_true - y_true.mean())**2).sum(). Best possible score is 1.0, lower values are worse.

Parameters

- **X**: array-like, shape = [n_samples, n_features]
  - Returns **y**: array-like, shape = [n_samples]
  - Returns **z**: float

**set_params** (**params**)  

Set the parameters of the estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The former have parameters of the form `<component>__<parameter>` so that it's possible to update each component of a nested object.

Returns **self**:

**sklearn.linear_model.Perceptron**

**class** `sklearn.linear_model.Perceptron`  

```
penalty=None, alpha=0.0001, fit_intercept=True,  
n_iter=5, shuffle=False, verbose=0, eta0=1.0, n_jobs=1,  
seed=0, class_weight=None, warm_start=False)
```

Perceptron

Parameters

- **penalty**: None, ‘l2’ or ‘l1’ or ‘elasticnet’
  - The penalty (aka regularization term) to be used. Defaults to None.

- **alpha**: float
  - Constant that multiplies the regularization term if regularization is used. Defaults to 0.0001

- **fit_intercept**: bool
  - Whether the intercept should be estimated or not. If False, the data is assumed to be already centered. Defaults to True.

- **n_iter**: int, optional
  - The number of passes over the training data (aka epochs). Defaults to 5.

- **shuffle**: bool, optional
  - Whether or not the training data should be shuffled after each epoch. Defaults to False.

- **seed**: int, optional
  - The seed of the pseudo random number generator to use when shuffling the data.
verbose: integer, optional:
The verbosity level

n_jobs: integer, optional:
The number of CPUs to use to do the OVA (One Versus All, for multi-class problems) computation. -1 means ‘all CPUs’. Defaults to 1.

eta0: double
Constant by which the updates are multiplied. Defaults to 1.

class_weight: dict, {class_label
Preset for the class_weight fit parameter.
Weights associated with classes. If not given, all classes are supposed to have weight one.
The “auto” mode uses the values of y to automatically adjust weights inversely proportional to class frequencies.

warm_start: bool, optional
When set to True, reuse the solution of the previous call to fit as initialization, otherwise, just erase the previous solution.

See Also:
SGDClassifier

Notes

Perceptron and SGDClassifier share the same underlying implementation. In fact, Perceptron() is equivalent to SGDClassifier(loss="perceptron", eta0=1, learning_rate="constant", penalty=None).

References


Attributes

coefficients_ array, shape = [1, n_features] if n_classes == 2 else [n_classes, n_features]
Weights assigned to the features.

intercept_ array, shape = [1] if n_classes == 2 else [n_classes]
Constants in decision function.

Methods

decision_function(X) Predict signed ‘distance’ to the hyperplane (aka confidence score)
fit(X, y[, coef_init, intercept_init, ...]) Fit linear model with Stochastic Gradient Descent.
fit_transform(X[, y]) Fit to data, then transform it
get_params([deep]) Get parameters for the estimator
partial_fit(X, y[, classes, class_weight, ...]) Fit linear model with Stochastic Gradient Descent.
Table 1.118 – continued from previous page

<table>
<thead>
<tr>
<th>Method</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>predict(X)</td>
<td>Predict using the linear model</td>
</tr>
<tr>
<td>predict_proba(X)</td>
<td>Predict class membership probability</td>
</tr>
<tr>
<td>score(X, y)</td>
<td>Returns the mean accuracy on the given test data and labels.</td>
</tr>
<tr>
<td>set_params(**params)</td>
<td>Set the parameters of the estimator.</td>
</tr>
<tr>
<td>transform(X[, threshold])</td>
<td>Reduce X to its most important features.</td>
</tr>
</tbody>
</table>

__init__ (penalty=None, alpha=0.0001, fit_intercept=True, n_iter=5, shuffle=False, verbose=0, eta0=1.0, n_jobs=1, seed=0, class_weight=None, warm_start=False)

classes
DEPRECATED: to be removed in v0.12; use classes_ instead.

decision_function (X)
Predict signed ‘distance’ to the hyperplane (aka confidence score)

Parameters X : {array-like, sparse matrix}, shape = [n_samples, n_features]

Returns array, shape = [n_samples] if n_classes == 2 else [n_samples,n_classes] :
  The signed ‘distances’ to the hyperplane(s).

fit (X, y, coef_init=None, intercept_init=None, class_weight=None, sample_weight=None)
Fit linear model with Stochastic Gradient Descent.

Parameters X : {array-like, sparse matrix}, shape = [n_samples, n_features]

  Training data

  y : numpy array of shape [n_samples]
  Target values

  coef_init : array, shape = [n_classes,n_features]
  The initial coefficients to warm-start the optimization.

  intercept_init : array, shape = [n_classes]
  The initial intercept to warm-start the optimization.

  sample_weight : array-like, shape = [n_samples], optional
  Weights applied to individual samples. If not provided, uniform weights are assumed.

Returns self : returns an instance of self.

fit_transform (X, y=None, **fit_params)
Fit to data, then transform it

Fits transformer to X and y with optional parameters fit_params and returns a transformed version of X.

Parameters X : numpy array of shape [n_samples, n_features]

  Training set.

  y : numpy array of shape [n_samples]
  Target values.

Returns X_new : numpy array of shape [n_samples, n_features_new]
  Transformed array.
Notes

This method just calls fit and transform consecutively, i.e., it is not an optimized implementation of fit_transform, unlike other transformers such as PCA.

**get_params**(deep=True)

Get parameters for the estimator

**Parameters**

- **deep**: boolean, optional
  
  If True, will return the parameters for this estimator and contained subobjects that are estimators.

**partial_fit**(X, y, classes=None, class_weight=None, sample_weight=None)

Fit linear model with Stochastic Gradient Descent.

**Parameters**

- **X**: {array-like, sparse matrix}, shape = [n_samples, n_features]
  
  Subset of the training data
- **y**: numpy array of shape [n_samples]
  
  Subset of the target values
- **classes**: array, shape = [n_classes]
  
  Classes across all calls to partial_fit. Can be obtained by via np.unique(y_all), where y_all is the target vector of the entire dataset. This argument is required for the first call to partial_fit and can be omitted in the subsequent calls. Note that y doesn’t need to contain all labels in classes.
- **sample_weight**: array-like, shape = [n_samples], optional
  
  Weights applied to individual samples. If not provided, uniform weights are assumed.

**Returns**

- **self**: returns an instance of self.

**predict**(X)

Predict using the linear model

**Parameters**

- **X**: {array-like, sparse matrix}, shape = [n_samples, n_features]

**Returns**

- **array**, shape = [n_samples]
  
  Array containing the predicted class labels.

**predict_proba**(X)

Predict class membership probability

**Parameters**

- **X**: {array-like, sparse matrix}, shape = [n_samples, n_features]

**Returns**

- **array**, shape = [n_samples] if n_classes == 2 else [n_samples, :
  
  n_classes]

  Contains the membership probabilities of the positive class.

**References**

The justification for the formula in the loss="modified_huber" case is in the appendix B in:
http://jmlr.csail.mit.edu/papers/volume2/zhang02c/zhang02c.pdf

**score**(X, y)

Returns the mean accuracy on the given test data and labels.
Parameters $X$: array-like, shape = [n_samples, n_features]

Training set.

$y$: array-like, shape = [n_samples]

Labels for $X$.

Returns $z$: float

$set\_params$ (**params)

Set the parameters of the estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The former have parameters of the form `<component>__<parameter>` so that it’s possible to update each component of a nested object.

Returns $self$:

$transform$ ($X$, threshold=None)

Reduce $X$ to its most important features.

Parameters $X$: array or scipy sparse matrix of shape [n_samples, n_features]

The input samples.

threshold: string, float or None, optional (default=None)

The threshold value to use for feature selection. Features whose importance is greater or equal are kept while the others are discarded. If “median” (resp. “mean”), then the threshold value is the median (resp. the mean) of the feature importances. A scaling factor (e.g., “1.25*mean”) may also be used. If None and if available, the object attribute threshold is used. Otherwise, “mean” is used by default.

Returns $X\_r$: array of shape [n_samples, n_selected_features]

The input samples with only the selected features.

**sklearn.linear_model.SGDClassifier**

```python
class sklearn.linear_model.SGDClassifier (loss='hinge', penalty='l2', alpha=0.0001, rho=0.85, fit_intercept=True, n_iter=5, shuffle=False, verbose=0, epsilon=0.1, n_jobs=1, seed=0, learning_rate='optimal', eta0=0.0, power_t=0.5, class_weight=None, warm_start=False)
```

Linear model fitted by minimizing a regularized empirical loss with SGD.

SGD stands for Stochastic Gradient Descent: the gradient of the loss is estimated each sample at a time and the model is updated along the way with a decreasing strength schedule (aka learning rate).

The regularizer is a penalty added to the loss function that shrinks model parameters towards the zero vector using either the squared euclidean norm $L2$ or the absolute norm $L1$ or a combination of both (Elastic Net). If the parameter update crosses the 0.0 value because of the regularizer, the update is truncated to 0.0 to allow for learning sparse models and achieve online feature selection.

This implementation works with data represented as dense numpy arrays of floating point values for the features.

Parameters $loss$: str, ‘hinge’ or ‘log’ or ‘modified_huber’

The loss function to be used. Defaults to ‘hinge’. The hinge loss is a margin loss used by standard linear SVM models. The ‘log’ loss is the loss of logistic regression models.
and can be used for probability estimation in binary classifiers. ‘modified_huber’ is another smooth loss that brings tolerance to outliers.

**penalty** : str, ‘l2’ or ‘l1’ or ‘elasticnet’

The penalty (aka regularization term) to be used. Defaults to ‘l2’ which is the standard regularizer for linear SVM models. ‘l1’ and ‘elasticnet’ might bring sparsity to the model (feature selection) not achievable with ‘l2’.

**alpha** : float

Constant that multiplies the regularization term. Defaults to 0.0001

**rho** : float

The Elastic Net mixing parameter, with 0 < rho <= 1. Defaults to 0.85.

**fit_intercept** : bool

Whether the intercept should be estimated or not. If False, the data is assumed to be already centered. Defaults to True.

**n_iter** : int, optional

The number of passes over the training data (aka epochs). Defaults to 5.

**shuffle** : bool, optional

Whether or not the training data should be shuffled after each epoch. Defaults to False.

**seed** : int, optional

The seed of the pseudo random number generator to use when shuffling the data.

**verbose** : integer, optional

The verbosity level

**n_jobs** : integer, optional

The number of CPUs to use to do the OVA (One Versus All, for multi-class problems) computation. -1 means ‘all CPUs’. Defaults to 1.

**learning_rate** : string, optional

The learning rate: constant: eta = eta0 optimal: eta = 1.0/(t+t0) [default] invscaling: eta = eta0 / pow(t, power_t)

**eta0** : double

The initial learning rate [default 0.01].

**power_t** : double

The exponent for inverse scaling learning rate [default 0.25].

**class_weight** : dict, {class_label

Preset for the class_weight fit parameter. Weights associated with classes. If not given, all classes are supposed to have weight one.

The “auto” mode uses the values of y to automatically adjust weights inversely proportional to class frequencies.

**warm_start** : bool, optional
When set to True, reuse the solution of the previous call to fit as initialization, otherwise, just erase the previous solution.

See Also:

LinearSVC, LogisticRegression, Perceptron

Examples

```python
>>> import numpy as np
>>> from sklearn import linear_model
>>> X = np.array([[-1, -1], [-2, -1], [1, 1], [2, 1]])
>>> Y = np.array([1, 1, 2, 2])
>>> clf = linear_model.SGDClassifier()
>>> clf.fit(X, Y)
...
SGDClassifier(alpha=0.0001, class_weight=None, epsilon=0.1, eta0=0.0,
              fit_intercept=True, learning_rate='optimal', loss='hinge',
              n_iter=5, n_jobs=1, penalty='l2', power_t=0.5, rho=0.85, seed=0,
              shuffle=False, verbose=0, warm_start=False)
>>> print(clf.predict([[-0.8, -1]]))
[1]
```

Attributes

- `coef_`  
  array, shape = [1, n_features] if n_classes == 2 else [n_classes, n_features]  
  Weights assigned to the features.

- `intercept_`  
  array, shape = [1] if n_classes == 2 else [n_classes]  
  Constants in decision function.

Methods

- `decision_function(X)`  
  Predict signed ‘distance’ to the hyperplane (aka confidence score).

- `fit(X, y[, coef_init, intercept_init, ...])`  
  Fit linear model with Stochastic Gradient Descent.

- `fit_transform(X[, y])`  
  Fit to data, then transform it

- `get_params([deep])`  
  Get parameters for the estimator

- `partial_fit(X, y[, classes, class_weight, ...])`  
  Fit linear model with Stochastic Gradient Descent.

- `predict(X)`  
  Predict using the linear model

- `predict_proba(X)`  
  Predict class membership probability

- `score(X, y)`  
  Returns the mean accuracy on the given test data and labels.

- `set_params(**params)`  
  Set the parameters of the estimator.

- `transform(X[, threshold])`  
  Reduce X to its most important features.

```python
__init__(loss='hinge', penalty='l2', alpha=0.0001, rho=0.85, fit_intercept=True, n_iter=5, shuffle=False, verbose=0, epsilon=0.1, n_jobs=1, seed=0, learning_rate='optimal', eta0=0.0, power_t=0.5, class_weight=None, warm_start=False)
```

- `classes`  
  DEPRECATED: to be removed in v0.12; use `classes_` instead.

- `decision_function(X)`  
  Predict signed ‘distance’ to the hyperplane (aka confidence score)

  Parameters

  - `X` : [array-like, sparse matrix], shape = [n_samples, n_features]
Returns array, shape = [n_samples] if n_classes == 2 else [n_samples,n_classes] :

The signed ‘distances’ to the hyperplane(s).

fit (X, y, coef_init=None, intercept_init=None, class_weight=None, sample_weight=None)
Fit linear model with Stochastic Gradient Descent.

Parameters X : {array-like, sparse matrix}, shape = [n_samples, n_features]
Training data
y : numpy array of shape [n_samples]
Target values
coef_init : array, shape = [n_classes,n_features]
The initial coefficients to warm-start the optimization.
intercept_init : array, shape = [n_classes]
The initial intercept to warm-start the optimization.
sample_weight : array-like, shape = [n_samples], optional
Weights applied to individual samples. If not provided, uniform weights are assumed.

Returns self : returns an instance of self.

fit_transform (X, y=None, **fit_params)
Fit to data, then transform it

Fits transformer to X and y with optional parameters fit_params and returns a transformed version of X.

Parameters X : numpy array of shape [n_samples, n_features]
Training set.
y : numpy array of shape [n_samples]
Target values.

Returns X_new : numpy array of shape [n_samples, n_features_new]
Transformed array.

Notes

This method just calls fit and transform consecutively, i.e., it is not an optimized implementation of
fit_transform, unlike other transformers such as PCA.

get_params (deep=True)
Get parameters for the estimator

Parameters deep: boolean, optional :

If True, will return the parameters for this estimator and contained subobjects that are estimators.

partial_fit (X, y, classes=None, class_weight=None, sample_weight=None)
Fit linear model with Stochastic Gradient Descent.

Parameters X : {array-like, sparse matrix}, shape = [n_samples, n_features]
Subset of the training data
y : numpy array of shape [n_samples]
Subset of the target values

**classes** : array, shape = [n_classes]

Classes across all calls to partial_fit. Can be obtained by via `np.unique(y_all)`, where y_all is the target vector of the entire dataset. This argument is required for the first call to partial_fit and can be omitted in the subsequent calls. Note that y doesn’t need to contain all labels in classes.

**sample_weight** : array-like, shape = [n_samples], optional

Weights applied to individual samples. If not provided, uniform weights are assumed.

**Returns** self : returns an instance of self.

**predict** (X)

Predict using the linear model

**Parameters** X : {array-like, sparse matrix}, shape = [n_samples, n_features]

**Returns** array, shape = [n_samples] :

Array containing the predicted class labels.

**predict_proba** (X)

Predict class membership probability

**Parameters** X : {array-like, sparse matrix}, shape = [n_samples, n_features]

**Returns** array, shape = [n_samples] if n_classes == 2 else [n_samples, :

n_classes] :

Contains the membership probabilities of the positive class.

References

The justification for the formula in the loss="modified_huber" case is in the appendix B in: http://jmlr.csail.mit.edu/papers/volume2/zhang02c/zhang02c.pdf

**score** (X, y)

Returns the mean accuracy on the given test data and labels.

**Parameters** X : array-like, shape = [n_samples, n_features]

Training set.

y : array-like, shape = [n_samples]

Labels for X.

**Returns** z : float

**set_params** (**params)

Set the parameters of the estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The former have parameters of the form <component>__<parameter> so that it’s possible to update each component of a nested object.

**Returns** self :

**transform** (X, threshold=None)

Reduce X to its most important features.
Parameters $X$ : array or scipy sparse matrix of shape [n_samples, n_features]

The input samples.

threshold : string, float or None, optional (default=None)

The threshold value to use for feature selection. Features whose importance is greater or equal are kept while the others are discarded. If “median” (resp. “mean”), then the threshold value is the median (resp. the mean) of the feature importances. A scaling factor (e.g., “1.25*mean”) may also be used. If None and if available, the object attribute threshold is used. Otherwise, “mean” is used by default.

Returns $X_r$ : array of shape [n_samples, n_selected_features]

The input samples with only the selected features.

sklearn.linear_model.SGDRegressor

class sklearn.linear_model.SGDRegressor (loss='squared_loss', penalty='l2', alpha=0.0001, rho=0.85, fit_intercept=True, n_iter=5, shuffle=False, verbose=0, epsilon=0.1, p=None, learning_rate='invscaling', eta0=0.01, power_t=0.25, warm_start=False)

Linear model fitted by minimizing a regularized empirical loss with SGD

SGD stands for Stochastic Gradient Descent: the gradient of the loss is estimated each sample at a time and the model is updated along the way with a decreasing strength schedule (aka learning rate).

The regularizer is a penalty added to the loss function that shrinks model parameters towards the zero vector using either the squared euclidean norm L2 or the absolute norm L1 or a combination of both (Elastic Net). If the parameter update crosses the 0.0 value because of the regularizer, the update is truncated to 0.0 to allow for learning sparse models and achieve online feature selection.

This implementation works with data represented as dense numpy arrays of floating point values for the features.

Parameters loss : str, ‘squared_loss’ or ‘huber’

The loss function to be used. Defaults to ‘squared_loss’ which refers to the ordinary least squares fit. ‘huber’ is an epsilon insensitive loss function for robust regression.

penalty : str, ‘l2’ or ‘l1’ or ‘elasticnet’

The penalty (aka regularization term) to be used. Defaults to ‘l2’ which is the standard regularizer for linear SVM models. ‘l1’ and ‘elasticnet’ migh bring sparsity to the model (feature selection) not achievable with ‘l2’.

alpha : float

Constant that multiplies the regularization term. Defaults to 0.0001

rho : float

The Elastic Net mixing parameter, with 0 < rho <= 1. Defaults to 0.85.

fit_intercept: bool :

Whether the intercept should be estimated or not. If False, the data is assumed to be already centered. Defaults to True.

n_iter: int, optional :

The number of passes over the training data (aka epochs). Defaults to 5.

shuffle: bool, optional :
Whether or not the training data should be shuffled after each epoch. Defaults to False.

**seed**: int, optional

The seed of the pseudo random number generator to use when shuffling the data.

**verbose**: integer, optional

The verbosity level.

**epsilon**: float

Epsilon in the epsilon-insensitive huber loss function; only if **loss**=='huber'.

**learning_rate**: string, optional

The learning rate: constant: eta = eta0 optimal: eta = 1.0/(t+t0) invscaling: eta = eta0 / pow(t, power_t) [default]

**eta0**: double, optional

The initial learning rate [default 0.01].

**power_t**: double, optional

The exponent for inverse scaling learning rate [default 0.25].

**warm_start**: bool, optional

When set to True, reuse the solution of the previous call to fit as initialization, otherwise, just erase the previous solution.

See Also:

Ridge, ElasticNet, Lasso, SVR

Examples

```python
>>> import numpy as np
>>> from sklearn import linear_model
>>> n_samples, n_features = 10, 5
>>> np.random.seed(0)
>>> y = np.random.randn(n_samples)
>>> X = np.random.randn(n_samples, n_features)
>>> clf = linear_model.SGDRegressor()
>>> clf.fit(X, y)
SGDRegressor(alpha=0.0001, epsilon=0.1, eta0=0.01, fit_intercept=True,
             learning_rate='invscaling', loss='squared_loss', n_iter=5, p=None,
             penalty='l2', power_t=0.25, rho=0.85, seed=0, shuffle=False,
             verbose=0, warm_start=False)
```

Attributes

| coef_     | array, shape = [n_features] | Weights assigned to the features. |

Methods
decision_function(X) Predict using the linear model

fit(X, y[, coef_init, intercept_init, ...]) Fit linear model with Stochastic Gradient Descent.

fit_transform(X[, y]) Fit to data, then transform it

get_params([deep]) Get parameters for the estimator

partial_fit(X, y[, sample_weight]) Fit linear model with Stochastic Gradient Descent.

predict(X) Predict using the linear model

score(X, y) Returns the coefficient of determination $R^2$ of the prediction.

set_params(**params) Set the parameters of the estimator.

transform(X[, threshold]) Reduce X to its most important features.

__init__(loss='squared_loss', penalty='l2', alpha=0.0001, rho=0.85, fit_intercept=True, n_iter=5, shuffle=False, verbose=0, epsilon=0.1, p=None, seed=0, learning_rate='invscaling', eta0=0.01, power_t=0.25, warm_start=False)

decision_function (X)

Predict using the linear model

Parameters X : {array-like, sparse matrix}, shape = [n_samples, n_features]

Returns array, shape = [n_samples] :

Predicted target values per element in X.

fit (X, y, coef_init=None, intercept_init=None, sample_weight=None)

Fit linear model with Stochastic Gradient Descent.

Parameters X : {array-like, sparse matrix}, shape = [n_samples, n_features]

Training data

y : numpy array of shape [n_samples]

Target values

coef_init : array, shape = [n_features]

The initial coefficients to warm-start the optimization.

intercept_init : array, shape = [1]

The initial intercept to warm-start the optimization.

sample_weight : array-like, shape = [n_samples], optional

Weights applied to individual samples (1. for unweighted).

Returns self : returns an instance of self.

fit_transform (X, y=None, **fit_params)

Fit to data, then transform it

Fits transformer to X and y with optional parameters fit_params and returns a transformed version of X.

Parameters X : numpy array of shape [n_samples, n_features]

Training set.

y : numpy array of shape [n_samples]

Target values.

Returns X_new : numpy array of shape [n_samples, n_features_new]

Transformed array.
Notes

This method just calls fit and transform consecutively, i.e., it is not an optimized implementation of fit_transform, unlike other transformers such as PCA.

get_params (deep=True)
Get parameters for the estimator

Parameters deep: boolean, optional:
If True, will return the parameters for this estimator and contained subobjects that are estimators.

partial_fit (X, y, sample_weight=None)
Fit linear model with Stochastic Gradient Descent.

Parameters X: [array-like, sparse matrix], shape = [n_samples, n_features]
Subset of training data
y: numpy array of shape [n_samples]
Subset of target values
sample_weight: array-like, shape = [n_samples], optional
Weights applied to individual samples. If not provided, uniform weights are assumed.

Returns self: returns an instance of self.

predict (X)
Predict using the linear model

Parameters X: [array-like, sparse matrix], shape = [n_samples, n_features]

Returns array, shape = [n_samples]:
Predicted target values per element in X.

score (X, y)
Returns the coefficient of determination R^2 of the prediction.

The coefficient R^2 is defined as (1 - u/v), where u is the regression sum of squares ((y - y_pred) ** 2).sum() and v is the residual sum of squares ((y_true - y_true.mean()) ** 2).sum(). Best possible score is 1.0, lower values are worse.

Parameters X: array-like, shape = [n_samples, n_features]
Training set.
y: array-like, shape = [n_samples]

Returns z: float

set_params (**params)
Set the parameters of the estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The former have parameters of the form <component>__<parameter> so that it’s possible to update each component of a nested object.

Returns self:

transform (X, threshold=None)
Reduce X to its most important features.

Parameters X: array or scipy sparse matrix of shape [n_samples, n_features]
The input samples.

**threshold** : string, float or None, optional (default=None)

The threshold value to use for feature selection. Features whose importance is greater or equal are kept while the others are discarded. If “median” (resp. “mean”), then the threshold value is the median (resp. the mean) of the feature importances. A scaling factor (e.g., “1.25*mean”) may also be used. If None and if available, the object attribute `threshold` is used. Otherwise, “mean” is used by default.

**Returns**

- **X_r** : array of shape [n_samples, n_selected_features]

The input samples with only the selected features.

**sklearn.linear_model.BayesianRidge**

class `sklearn.linear_model.BayesianRidge` (n_iter=300, tol=0.001, alpha_1=1e-06, alpha_2=1e-06, lambda_1=1e-06, lambda_2=1e-06, compute_score=False, fit_intercept=True, normalize=False, copy_X=True, verbose=False)

Bayesian ridge regression

Fit a Bayesian ridge model and optimize the regularization parameters lambda (precision of the weights) and alpha (precision of the noise).

**Parameters**

- **X** : array, shape = (n_samples, n_features)
  Training vectors.

- **y** : array, shape = (length)
  Target values for training vectors

- **n_iter** : int, optional
  Maximum number of iterations. Default is 300.

- **tol** : float, optional
  Stop the algorithm if w has converged. Default is 1.e-3.

- **alpha_1** : float, optional
  Hyper-parameter : shape parameter for the Gamma distribution prior over the alpha parameter. Default is 1.e-6

- **alpha_2** : float, optional
  Hyper-parameter : inverse scale parameter (rate parameter) for the Gamma distribution prior over the alpha parameter. Default is 1.e-6.

- **lambda_1** : float, optional
  Hyper-parameter : shape parameter for the Gamma distribution prior over the lambda parameter. Default is 1.e-6.

- **lambda_2** : float, optional
  Hyper-parameter : inverse scale parameter (rate parameter) for the Gamma distribution prior over the lambda parameter. Default is 1.e-6.

- **compute_score** : boolean, optional
  If True, compute the objective function at each step of the model. Default is False
fit_intercept : boolean, optional
wether to calculate the intercept for this model. If set to false, no intercept will be used
in calculations (e.g. data is expected to be already centered). Default is True.

normalize : boolean, optional, default False
If True, the regressors X are normalized

copy_X : boolean, optional, default True
If True, X will be copied; else, it may be overwritten.

verbose : boolean, optional, default False
Verbose mode when fitting the model.

Notes

See examples/linear_model/plot_bayesian_ridge.py for an example.

Examples

```python
>>> from sklearn import linear_model
>>> clf = linear_model.BayesianRidge()
>>> clf.fit([[0,0], [1, 1], [2, 2]], [0, 1, 2])
... BayesianRidge(alpha_1=1e-06, alpha_2=1e-06, compute_score=False,
               copy_X=True, fit_intercept=True, lambda_1=1e-06, lambda_2=1e-06,
               n_iter=300, normalize=False, tol=0.001, verbose=False)
>>> clf.predict([[1, 1]])
array([ 1.])
```

Attributes

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>coef_</td>
<td>array, shape = (n_features)</td>
</tr>
<tr>
<td>alpha_</td>
<td>float</td>
</tr>
<tr>
<td>lambda_</td>
<td>array, shape = (n_features)</td>
</tr>
<tr>
<td>scores_</td>
<td>float</td>
</tr>
</tbody>
</table>

Methods

<table>
<thead>
<tr>
<th>Method</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>decision_function(X)</td>
<td>Decision function of the linear model</td>
</tr>
<tr>
<td>fit(X, y)</td>
<td>Fit the model</td>
</tr>
<tr>
<td>get_params([deep])</td>
<td>Get parameters for the estimator</td>
</tr>
<tr>
<td>predict(X)</td>
<td>Predict using the linear model</td>
</tr>
<tr>
<td>score(X, y)</td>
<td>Returns the coefficient of determination R^2 of the prediction.</td>
</tr>
<tr>
<td>set_params(**params)</td>
<td>Set the parameters of the estimator.</td>
</tr>
</tbody>
</table>

__init__ (n_iter=300, tol=0.001, alpha_1=1e-06, alpha_2=1e-06, lambda_1=1e-06, lambda_2=1e-06, compute_score=False, fit_intercept=True, normalize=False, copy_X=True, verbose=False)
**decision_function** \((X)\)
Decision function of the linear model

**Parameters**
- \(X\) : numpy array of shape \([n\_samples, n\_features]\)

**Returns**
- \(C\) : array, shape \([n\_samples]\)
  Returns predicted values.

**fit** \((X, y)\)
Fit the model

**Parameters**
- \(X\) : numpy array of shape \([n\_samples,n\_features]\)
  Training data
- \(y\) : numpy array of shape \([n\_samples]\)
  Target values

**Returns**
- \(self\) : returns an instance of \(self\).

**get_params** \((deep=True)\)
Get parameters for the estimator

**Parameters**
- **deep**: boolean, optional
  If True, will return the parameters for this estimator and contained subobjects that are estimators.

**predict** \((X)\)
Predict using the linear model

**Parameters**
- \(X\) : numpy array of shape \([n\_samples, n\_features]\)

**Returns**
- \(C\) : array, shape \([n\_samples]\)
  Returns predicted values.

**score** \((X, y)\)
Returns the coefficient of determination \(R^2\) of the prediction.

The coefficient \(R^2\) is defined as \((1 - u/v)\), where \(u\) is the regression sum of squares \(((y - y\_pred) ** 2)\).sum()\) and \(v\) is the residual sum of squares \(((y\_true - y\_true\_mean()) ** 2)\).sum(). Best possible score is 1.0, lower values are worse.

**Parameters**
- \(X\) : array-like, shape \([n\_samples, n\_features]\)
  Training set.
- \(y\) : array-like, shape \([n\_samples]\)

**Returns**
- \(z\) : float

**set_params** \((**params)\)
Set the parameters of the estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The former have parameters of the form \(<component>\_<parameter>\) so that it's possible to update each component of a nested object.

**Returns**
- \(self\) :
Bayesian ARD regression.

Fit the weights of a regression model, using an ARD prior. The weights of the regression model are assumed to be in Gaussian distributions. Also estimate the parameters lambda (precisions of the distributions of the weights) and alpha (precision of the distribution of the noise). The estimation is done by an iterative procedures (Evidence Maximization)

**Parameters**

- **X**: array, shape = (n_samples, n_features)
  - Training vectors.
- **y**: array, shape = (n_samples)
  - Target values for training vectors
- **n_iter**: int, optional
  - Maximum number of iterations. Default is 300
- **tol**: float, optional
  - Stop the algorithm if w has converged. Default is 1.e-3.
- **alpha_1**: float, optional
  - Hyper-parameter : shape parameter for the Gamma distribution prior over the alpha parameter. Default is 1.e-6.
- **alpha_2**: float, optional
  - Hyper-parameter : inverse scale parameter (rate parameter) for the Gamma distribution prior over the alpha parameter. Default is 1.e-6.
- **lambda_1**: float, optional
  - Hyper-parameter : shape parameter for the Gamma distribution prior over the lambda parameter. Default is 1.e-6.
- **lambda_2**: float, optional
  - Hyper-parameter : inverse scale parameter (rate parameter) for the Gamma distribution prior over the lambda parameter. Default is 1.e-6.
- **compute_score**: boolean, optional
  - If True, compute the objective function at each step of the model. Default is False.
- **threshold_lambda**: float, optional
  - threshold for removing (pruning) weights with high precision from the computation. Default is 1.e+4.
- **fit_intercept**: boolean, optional
  - Whether to calculate the intercept for this model. If set to false, no intercept will be used in calculations (e.g. data is expected to be already centered). Default is True.
- **normalize**: boolean, optional
If True, the regressors X are normalized

**copy_X**: boolean, optional, default True.
If True, X will be copied; else, it may be overwritten.

**verbose**: boolean, optional, default False
Verbose mode when fitting the model.

**Notes**
See examples/linear_model/plot_ard.py for an example.

**Examples**

```python
>>> from sklearn import linear_model
>>> clf = linear_model.ARDRegression()
>>> clf.fit([[0,0], [1, 1], [2, 2]], [0, 1, 2])
...  ARDRegression(alpha_1=1e-06, alpha_2=1e-06, compute_score=False,
                  copy_X=True, fit_intercept=True, lambda_1=1e-06, lambda_2=1e-06,
                  n_iter=300, normalize=False, threshold_lambda=10000.0, tol=0.001,
                  verbose=False)
>>> clf.predict([[1, 1]])
array([ 1.])
```

**Attributes**

<table>
<thead>
<tr>
<th>coef_</th>
<th>array, shape = (n_features)</th>
<th>Coefficients of the regression model (mean of distribution)</th>
</tr>
</thead>
<tbody>
<tr>
<td>alpha</td>
<td>float</td>
<td>estimated precision of the noise.</td>
</tr>
<tr>
<td>lambda</td>
<td>array, shape = (n_features)</td>
<td>estimated precisions of the weights.</td>
</tr>
<tr>
<td>sigma_</td>
<td>array, shape = (n_features, n_features)</td>
<td>estimated variance-covariance matrix of the weights</td>
</tr>
<tr>
<td>scores_</td>
<td>float</td>
<td>if computed, value of the objective function (to be maximized)</td>
</tr>
</tbody>
</table>

**Methods**

<table>
<thead>
<tr>
<th>decision_function(X)</th>
<th>Decision function of the linear model</th>
</tr>
</thead>
<tbody>
<tr>
<td>fit(X, y)</td>
<td>Fit the ARDRegression model according to the given training data</td>
</tr>
<tr>
<td>get_params((deep))</td>
<td>Get parameters for the estimator</td>
</tr>
<tr>
<td>predict(X)</td>
<td>Predict using the linear model</td>
</tr>
<tr>
<td>score(X, y)</td>
<td>Returns the coefficient of determination $R^2$ of the prediction.</td>
</tr>
<tr>
<td>set_params(**params)</td>
<td>Set the parameters of the estimator.</td>
</tr>
</tbody>
</table>

```python
__init__(n_iter=300, tol=0.001, alpha_1=1e-06, alpha_2=1e-06, lambda_1=1e-06, lambda_2=1e-06,
         compute_score=False, threshold_lambda=10000.0, fit_intercept=True, normalize=False,
         copy_X=True, verbose=False)
```

decision_function(X)  
Decision function of the linear model

**Parameters**

**X**: numpy array of shape [n_samples, n_features]
Returns $C$: array, shape = [n_samples]

Returns predicted values.

**fit** $(X, y)$

Fit the ARDRegression model according to the given training data and parameters.

Iterative procedure to maximize the evidence

**Parameters** $X$: array-like, shape = [n_samples, n_features]

Training vector, where n_samples is the number of samples and n_features is the number of features.

$y$: array, shape = [n_samples]

Target values (integers)

**Returns self**: returns an instance of self.

**get_params** *(deep=True)*

Get parameters for the estimator

**Parameters** **deep**: boolean, optional:

If True, will return the parameters for this estimator and contained subobjects that are estimators.

**predict** $(X)$

Predict using the linear model

**Parameters** $X$: numpy array of shape [n_samples, n_features]

**Returns** $C$: array, shape = [n_samples]

Returns predicted values.

**score** $(X, y)$

Returns the coefficient of determination $R^2$ of the prediction.

The coefficient $R^2$ is defined as $1 - u/v$, where $u$ is the regression sum of squares ($(y - y_{\text{pred}})^2$).sum() and $v$ is the residual sum of squares ($(y_{\text{true}} - y_{\text{true.mean}})^2$).sum(). Best possible score is 1.0, lower values are worse.

**Parameters** $X$: array-like, shape = [n_samples, n_features]

Training set.

$y$: array-like, shape = [n_samples]

**Returns** $z$: float

**set_params** *(**params)*

Set the parameters of the estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The former have parameters of the form `<component>__<parameter>` so that it’s possible to update each component of a nested object.

**Returns self**: 
Randomized Lasso

Randomized Lasso works by resampling the train data and computing a Lasso on each resampling. In short, the features selected more often are good features. It is also known as stability selection.

**Parameters**

- **alpha** : float, ‘aic’, or ‘bic’
  
  The regularization parameter alpha parameter in the Lasso. Warning: this is not the alpha parameter in the stability selection article which is scaling.

- **scaling** : float
  
  The alpha parameter in the stability selection article used to randomly scale the features. Should be between 0 and 1.

- **sample_fraction** : float
  
  The fraction of samples to be used in each randomized design. Should be between 0 and 1. If 1, all samples are used.

- **fit_intercept** : boolean
  
  Whether to calculate the intercept for this model. If set to false, no intercept will be used in calculations (e.g. data is expected to be already centered).

- **verbose** : boolean or integer, optional
  
  Sets the verbosity amount

- **normalize** : boolean, optional
  
  If True, the regressors X are normalized

- **precompute** : True | False | ‘auto’
  
  Whether to use a precomputed Gram matrix to speed up calculations. If set to ‘auto’ let us decide. The Gram matrix can also be passed as argument.

- **max_iter** : integer, optional
  
  Maximum number of iterations to perform in the Lars algorithm.

- **eps** : float, optional
  
  The machine-precision regularization in the computation of the Cholesky diagonal factors. Increase this for very ill-conditioned systems. Unlike the ‘tol’ parameter in some iterative optimization-based algorithms, this parameter does not control the tolerance of the optimization.

- **n_jobs** : integer, optional
  
  Number of CPUs to use during the resampling. If ‘-1’, use all the CPUs.

- **random_state** : int, RandomState instance or None, optional (default=None)
If int, random_state is the seed used by the random number generator; If RandomState instance, random_state is the random number generator; If None, the random number generator is the RandomState instance used by `np.random`.

**pre_dispatch**: int, or string, optional

Controls the number of jobs that get dispatched during parallel execution. Reducing this number can be useful to avoid an explosion of memory consumption when more jobs get dispatched than CPUs can process. This parameter can be:

- •None, in which case all the jobs are immediately created and spawned. Use this for lightweight and fast-running jobs, to avoid delays due to on-demand spawning of the jobs
- •An int, giving the exact number of total jobs that are spawned
- •A string, giving an expression as a function of n_jobs, as in ‘2*n_jobs’

**memory**: Instance of joblib.Memory or string

Used for internal caching. By default, no caching is done. If a string is given, it is the path to the caching directory.

See Also:

`RandomizedLogisticRegression`, `LogisticRegression`

**Notes**

See examples/linear_model/plot_sparse_recovery.py for an example.

**References**


**Examples**

```python
>>> from sklearn.linear_model import RandomizedLasso
>>> randomized_lasso = RandomizedLasso()
```

**Attributes**

<table>
<thead>
<tr>
<th>scores</th>
<th>array, shape = [n_features]</th>
<th>Feature scores between 0 and 1.</th>
</tr>
</thead>
<tbody>
<tr>
<td>all_scores</td>
<td>array, shape = [n_features, n_reg_parameter]</td>
<td>Feature scores between 0 and 1 for all values of the regularization parameter. The reference article suggests <code>scores_</code> is the max of <code>all_scores_</code>.</td>
</tr>
</tbody>
</table>

**Methods**

```python
fit(X, y)  
Fit the model using X, y as training data.
```

Continued on next page
Table 1.123 – continued from previous page

<table>
<thead>
<tr>
<th>Method</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>fit_transform(X, y)</td>
<td>Fit to data, then transform it</td>
</tr>
<tr>
<td>get_params([deep])</td>
<td>Get parameters for the estimator</td>
</tr>
<tr>
<td>get_support([indices])</td>
<td>Return a mask, or list, of the features/indices selected.</td>
</tr>
<tr>
<td>inverse_transform(X)</td>
<td>Transform a new matrix using the selected features</td>
</tr>
<tr>
<td>set_params(**params)</td>
<td>Set the parameters of the estimator.</td>
</tr>
<tr>
<td>transform(X)</td>
<td>Transform a new matrix using the selected features</td>
</tr>
</tbody>
</table>

```
__init__(alpha='aic', scaling=0.5, sample_fraction=0.75, n_resampling=200, selection_threshold=0.25, fit_intercept=True, verbose=False, normalize=True, precompute='auto', max_iter=500, eps=2.2204460492503131e-16, random_state=None, n_jobs=1, pre_dispatch='3*n_jobs', memory=Memory(cachedir=None))
```

**fit** *(X, y)*

Fit the model using X, y as training data.

**Parameters**

- **X**: array-like, shape = [n_samples, n_features]
  - training data.
- **y**: array-like, shape = [n_samples]
  - target values.

**Returns**

**self**: object

returns an instance of self.

**fit_transform** *(X, y=None, **fit_params)*

Fit to data, then transform it

Fits transformer to X and y with optional parameters fit_params and returns a transformed version of X.

**Parameters**

- **X**: numpy array of shape [n_samples, n_features]
  - Training set.
- **y**: numpy array of shape [n_samples]
  - Target values.

**Returns**

**X_new**: numpy array of shape [n_samples, n_features_new]

Transformed array.

**Notes**

This method just calls fit and transform consecutively, i.e., it is not an optimized implementation of fit_transform, unlike other transformers such as PCA.

**get_params** *(deep=True)*

Get parameters for the estimator

**Parameters**

- **deep**: boolean, optional :
  - If True, will return the parameters for this estimator and contained subobjects that are estimators.

**get_support** *(indices=False)*

Return a mask, or list, of the features/indices selected.
inverse_transform $(X)$  
Transform a new matrix using the selected features

set_params (**params)  
Set the parameters of the estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The former have parameters of the form `<component>__<parameter>` so that it’s possible to update each component of a nested object.

Returns self:

transform $(X)$  
Transform a new matrix using the selected features

sklearn.linear_model.RandomizedLogisticRegression

class sklearn.linear_model.RandomizedLogisticRegression ($C=1$, $scaling=0.5$, $sample_fraction=0.75$, $n_resampling=200$, $selection_threshold=0.25$, $tol=0.001$, $fit_intercept=True$, $verbose=False$, $normalize=True$, $random_state=None$, $n_jobs=1$, $pre_dispatch='3*n_jobs$', $memory=Memory(cachedir=None)$)

Randomized Logistic Regression

Randomized Regression works by resampling the train data and computing a LogisticRegression on each resampling. In short, the features selected more often are good features. It is also known as stability selection.

Parameters

- **C**: float  
The regularization parameter C in the LogisticRegression.

- **scaling**: float  
The alpha parameter in the stability selection article used to randomly scale the features. Should be between 0 and 1.

- **sample_fraction**: float  
The fraction of samples to be used in each randomized design. Should be between 0 and 1. If 1, all samples are used.

- **fit_intercept**: boolean  
Whether to calculate the intercept for this model. If set to false, no intercept will be used in calculations (e.g. data is expected to be already centered).

- **verbose**: boolean or integer, optional  
Sets the verbosity amount

- **normalize**: boolean, optional  
If True, the regressors X are normalized

- **tol**: float, optional  
tolerance for stopping criteria of LogisticRegression
n_jobs : integer, optional

Number of CPUs to use during the resampling. If ‘-1’, use all the CPUs

random_state : int, RandomState instance or None, optional (default=None)

If int, random_state is the seed used by the random number generator; If RandomState instance, random_state is the random number generator; If None, the random number generator is the RandomState instance used by np.random.

pre_dispatch : int, or string, optional

Controls the number of jobs that get dispatched during parallel execution. Reducing this number can be useful to avoid an explosion of memory consumption when more jobs get dispatched than CPUs can process. This parameter can be:

- None, in which case all the jobs are immediately created and spawned. Use this for lightweight and fast-running jobs, to avoid delays due to on-demand spawning of the jobs
- An int, giving the exact number of total jobs that are spawned
- A string, giving an expression as a function of n_jobs, as in ‘2*n_jobs’

memory : Instance of joblib.Memory or string

Used for internal caching. By default, no caching is done. If a string is given, it is the path to the caching directory.

See Also:

RandomizedLasso, Lasso, ElasticNet

Notes

See examples/linear_model/plot_randomized_lasso.py for an example.

References


Examples

```python
>>> from sklearn.linear_model import RandomizedLogisticRegression
>>> randomized_logistic = RandomizedLogisticRegression()
```
Methods

<table>
<thead>
<tr>
<th>Method</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>fit(X, y)</code></td>
<td>Fit the model using X, y as training data.</td>
</tr>
<tr>
<td><code>fit_transform(X[, y])</code></td>
<td>Fit to data, then transform it</td>
</tr>
<tr>
<td><code>get_params([deep])</code></td>
<td>Get parameters for the estimator</td>
</tr>
<tr>
<td><code>get_support([indices])</code></td>
<td>Return a mask, or list, of the features/indices selected.</td>
</tr>
<tr>
<td><code>inverse_transform(X)</code></td>
<td>Transform a new matrix using the selected features</td>
</tr>
<tr>
<td><code>set_params(**params)</code></td>
<td>Set the parameters of the estimator.</td>
</tr>
<tr>
<td><code>transform(X)</code></td>
<td>Transform a new matrix using the selected features</td>
</tr>
</tbody>
</table>

**init**(C=1, scaling=0.5, sample_fraction=0.75, n_resampling=200, selection_threshold=0.25, tol=0.001, fit_intercept=True, verbose=False, normalize=True, random_state=None, n_jobs=1, pre_dispatch='3*n_jobs', memory=Memory(cachedir=None))

`fit`(X, y)
Fit the model using X, y as training data.

**Parameters**

- **X** : array-like, shape = [n_samples, n_features]
  training data.
- **y** : array-like, shape = [n_samples]
  target values.

**Returns**

- **self** : object
  returns an instance of self.

`fit_transform`(X, y=None, **fit_params)
Fit to data, then transform it
Fits transformer to X and y with optional parameters fit_params and returns a transformed version of X.

**Parameters**

- **X** : numpy array of shape [n_samples, n_features]
  Training set.
- **y** : numpy array of shape [n_samples]
  Target values.

**Returns**

- **X_new** : numpy array of shape [n_samples, n_features_new]
  Transformed array.

**Notes**

This method just calls fit and transform consecutively, i.e., it is not an optimized implementation of fit_transform, unlike other transformers such as PCA.

`get_params`(deep=True)
Get parameters for the estimator

**Parameters**

- **deep** : boolean, optional
  If True, will return the parameters for this estimator and contained subobjects that are estimators.

`get_support`(indices=False)
Return a mask, or list, of the features/indices selected.
**inverse_transform** *(X)*
Transform a new matrix using the selected features

**set_params**( **params**)
Set the parameters of the estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The former have parameters of the form `<component>__<parameter>` so that it’s possible to update each component of a nested object.

Returns self:

**transform** *(X)*
Transform a new matrix using the selected features

```
linear_model.lasso_path(X, y[, eps, ...])
Compute Lasso path with coordinate descent

linear_model.lars_path(X, y[, Xy, Gram, ...])
Compute Least Angle Regression and Lasso path

linear_model.orthogonal_mp(X, y[, ...])
Orthogonal Matching Pursuit (OMP)

linear_model.orthogonal_mp_gram(Gram, Xy[, ...])
Gram Orthogonal Matching Pursuit (OMP)

linear_model.lasso_stability_path(X, y[, ...])
Stability path based on randomized Lasso estimates
```

sklearn.linear_model.lasso_path

sklearn.linear_model.lasso_path *(X, y, eps=0.001, n_alphas=100, alphas=None, precompute='auto', Xy=None, fit_intercept=True, normalize=False, copy_X=True, verbose=False, **params)*
Compute Lasso path with coordinate descent

The optimization objective for Lasso is:

\[
\frac{1}{2n_{samples}} \|y - Xw\|_2^2 + \alpha \|w\|_1
\]

Parameters **X** : numpy array of shape `[n_samples, n_features]`
Training data. Pass directly as fortran contiguous data to avoid unnecessary memory duplication

**y** : numpy array of shape `[n_samples]`
Target values

**eps** : float, optional
Length of the path. eps=1e-3 means that alpha_min / alpha_max = 1e-3

**n_alphas** : int, optional
Number of alphas along the regularization path

**alphas** : numpy array, optional
List of alphas where to compute the models. If None alphas are set automatically

**precompute** : True | False | ‘auto’ | array-like
Whether to use a precomputed Gram matrix to speed up calculations. If set to ‘auto’ let us decide. The Gram matrix can also be passed as argument.

**Xy** : array-like, optional

\[
Xy = \text{np.dot}(X.T, y)
\]
that can be precomputed. It is useful only when the Gram matrix is precomputed.
fit_intercept : bool
    Fit or not an intercept

normalize : boolean, optional
    If True, the regressors X are normalized

copy_X : boolean, optional, default True
    If True, X will be copied; else, it may be overwritten.

verbose : bool or integer
    Amount of verbosity

params : kwargs
    keyword arguments passed to the Lasso objects

Returns models : a list of models along the regularization path

See Also:
   lars_path, Lasso, LassoLars, LassoCV, LassoLarsCV, sklearn.decomposition.sparse_encode

Notes

See examples/linear_model/plot_lasso_coordinate_descent_path.py for an example.

To avoid unnecessary memory duplication the X argument of the fit method should be directly passed as a fortran contiguous numpy array.

sklearn.linear_model.lars_path

sklearn.linear_model.lars_path(X, y, Xy=None, Gram=None, max_iter=500, alpha_min=0, method='lar', copy_X=True, eps=2.2204460492503131e-16, copy_Gram=True, verbose=False)

Compute Least Angle Regression and Lasso path

The optimization objective for Lasso is:

\[(1 / (2 * n_samples)) * ||y - Xw||^2_2 + alpha * ||w||_1\]

Parameters X: array, shape: (n_samples, n_features) :
    Input data

y: array, shape: (n_samples) :
    Input targets

max_iter: integer, optional :
    Maximum number of iterations to perform, set to infinity for no limit.

Gram: None, ‘auto’, array, shape: (n_features, n_features), optional :
    Precomputed Gram matrix (X’ * X), if ‘auto’, the Gram matrix is precomputed from the given X, if there are more samples than features

alpha_min: float, optional :
Minimum correlation along the path. It corresponds to the regularization parameter alpha parameter in the Lasso.

**method**: ‘lar’, ‘lasso’:

Specifies the returned model. Select ‘lar’ for Least Angle Regression, ‘lasso’ for the Lasso.

**eps**: float, optional:

The machine-precision regularization in the computation of the Cholesky diagonal factors. Increase this for very ill-conditioned systems.

**copy_X**: bool:

If False, X is overwritten.

**copy_Gram**: bool:

If False, Gram is overwritten.

**Returns**

**alphas**: array, shape: (max_features + 1,):

Maximum of covariances (in absolute value) at each iteration.

**active**: array, shape (max_features,):

Indices of active variables at the end of the path.

**coefs**: array, shape (n_features, max_features + 1):

Coefficients along the path

See Also:

lasso_path, LassoLars, Lars, LassoLarsCV, LarsCV, sklearn.decomposition.sparse_encode

Notes


**sklearn.linear_model.orthogonal_mp**

sklearn.linear_model.orthogonal_mp (X, y, n_nonzero_coefs=None, tol=None, precompute_gram=False, copy_X=True)

Orthogonal Matching Pursuit (OMP)

Solves n_targets Orthogonal Matching Pursuit problems. An instance of the problem has the form:

When parametrized by the number of non-zero coefficients using n_nonzero_coefs: argmin ||y - Xgamma||^2 subject to ||gamma||_0 <= n_{nonzero coefs}

When parametrized by error using the parameter tol: argmin ||gamma||_0 subject to ||y - Xgamma||^2 <= tol

**Parameters**

**X**: array, shape = (n_samples, n_features):

Input data. Columns are assumed to have unit norm.

**y**: array, shape = (n_samples,) or (n_samples, n_targets):

Input targets
n_nonzero_coefs: int :

Desired number of non-zero entries in the solution. If None (by default) this value is set to 10% of n_features.

tol: float :

Maximum norm of the residual. If not None, overrides n_nonzero_coefs.

precompute_gram: {True, False, ‘auto’}, :

Whether to perform precomputations. Improves performance when n_targets or n_samples is very large.

copy_X: bool, optional :

Whether the design matrix X must be copied by the algorithm. A false value is only helpful if X is already Fortran-ordered, otherwise a copy is made anyway.

Returns coef: array, shape = (n_features,) or (n_features, n_targets) :

Coefficients of the OMP solution

See Also:
OrthogonalMatchingPursuit, orthogonal_mp_gram, lars_path,
decomposition.sparse_encode, decomposition.sparse_encode_parallel

Notes


sklearn.linear_model.orthogonal_mp_gram

sklearn.linear_model.orthogonal_mp_gram(Gram, Xy, n_nonzero_coefs=None, tol=None, norms_squared=None, copy_Gram=True, copy_Xy=True)

Gram Orthogonal Matching Pursuit (OMP)

Solves n_targets Orthogonal Matching Pursuit problems using only the Gram matrix X.T * X and the product X.T * y.

Parameters Gram: array, shape = (n_features, n_features) :

Gram matrix of the input data: X.T * X

Xy: array, shape = (n_features,) or (n_features, n_targets) :

Input targets multiplied by X: X.T * y

n_nonzero_coefs: int :

Desired number of non-zero entries in the solution. If None (by default) this value is set to 10% of n_features.

tol: float :
Maximum norm of the residual. If not None, overrides n_nonzero_coefs.

**norms_squared**: array-like, shape = (n_targets,)

Squared L2 norms of the lines of y. Required if tol is not None.

**copy_Gram**: bool, optional

Whether the gram matrix must be copied by the algorithm. A false value is only helpful if it is already Fortran-ordered, otherwise a copy is made anyway.

**copy_Xy**: bool, optional

Whether the covariance vector Xy must be copied by the algorithm. If False, it may be overwritten.

**Returns**

**coef**: array, shape = (n_features,) or (n_features, n_targets)

Coefficients of the OMP solution

**See Also**

OrthogonalMatchingPursuit, orthogonal_mp, lars_path, decomposition.sparse_encode, decomposition.sparse_encode_parallel

**Notes**


**sklearn.linear_model.lasso_stability_path**

**sklearn.linear_model.lasso_stability_path**(X, y, scaling=0.5, random_state=None, n_resampling=200, n_grid=100, sample_fraction=0.75, eps=8.8817841970012523e-16, n_jobs=1, verbose=False)

Stability path based on randomized Lasso estimates

**Parameters**

**X**: array-like, shape = [n_samples, n_features]

training data.

**y**: array-like, shape = [n_samples]

target values.

**scaling**: float

The alpha parameter in the stability selection article used to randomly scale the features. Should be between 0 and 1.

**random_state**: integer or numpy.RandomState, optional

The generator used to randomize the design.

**n_resampling**: int
Number of randomized models.

**n_grid** : int

Number of grid points. The path is linearly reinterpolated on a grid between 0 and 1 before computing the scores.

**sample_fraction** : float

The fraction of samples to be used in each randomized design. Should be between 0 and 1. If 1, all samples are used.

**eps** : float

Smallest value of alpha / alpha_max considered

**n_jobs** : integer, optional

Number of CPUs to use during the resampling. If `-1`, use all the CPUs

**verbose** : boolean or integer, optional

Sets the verbosity amount

Returns **alphas_grid** : array, shape ~ [n_grid]

The grid points between 0 and 1: alpha/alpha_max

**scores_path** : array, shape = [n_features, n_grid]

The scores for each feature along the path.

**Notes**

See examples/linear_model/plot_randomized_lasso.py for an example.

**For sparse data**

The `sklearn.linear_model.sparse` submodule is the sparse counterpart of the `sklearn.linear_model` module.

**User guide:** See the *Generalized Linear Models* section for further details.

<table>
<thead>
<tr>
<th>Function</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>linear_model.sparse.Lasso([alpha, ...])</code></td>
<td>Linear Model trained with L1 prior as regularizer</td>
</tr>
<tr>
<td><code>linear_model.sparse.ElasticNet([alpha, rho, ...])</code></td>
<td>Linear Model trained with L1 and L2 prior as regularizer</td>
</tr>
<tr>
<td><code>linear_model.sparse.SGDClassifier(*args, ...)</code></td>
<td></td>
</tr>
<tr>
<td><code>linear_model.sparse.SGDRegressor(*args, **kwargs)</code></td>
<td></td>
</tr>
<tr>
<td><code>linear_model.LogisticRegression([penalty, ...])</code></td>
<td>Logistic Regression (aka logit, MaxEnt) classifier.</td>
</tr>
</tbody>
</table>

**sklearn.linear_model.sparse.Lasso**

**class** `sklearn.linear_model.sparse.Lasso` *(alpha=1.0, fit_intercept=False, normalize=False, max_iter=1000, tol=0.0001, positive=False)*

Linear Model trained with L1 prior as regularizer

This implementation works on scipy.sparse X and dense `coef_`. Technically this is the same as Elastic Net with the L2 penalty set to zero.

**Parameters alpha** : float
Constant that multiplies the L1 term. Defaults to 1.0

`coef_` : ndarray of shape n_features

The initial coefficients to warm-start the optimization

`fit_intercept` : bool :

Whether the intercept should be estimated or not. If False, the data is assumed to be already centered.

Methods

```python
__init__(alpha=1.0, fit_intercept=False, normalize=False, max_iter=1000, tol=0.0001, positive=False)

decision_function(X)  # Decision function of the linear model
fit(X, y)  # Fit current model with coordinate descent
get_params([deep])  # Get parameters for the estimator
predict(X)  # Predict using the linear model
score(X, y)  # Returns the coefficient of determination R^2 of the prediction.
set_params(**params)  # Set the parameters of the estimator.
```

```python
def decision_function(X):
    # Decision function of the linear model
    Parameters X : scipy.sparse matrix of shape [n_samples, n_features]
    Returns array, shape = [n_samples] with the predicted real values :

def fit(X, y):
    # Fit current model with coordinate descent
    X is expected to be a sparse matrix. For maximum efficiency, use a sparse matrix in CSC format (scipy.sparse.csc_matrix)

def get_params(deep=True):
    # Get parameters for the estimator
    Parameters deep: boolean, optional :
        If True, will return the parameters for this estimator and contained subobjects that are estimators.

def predict(X):
    # Predict using the linear model
    Parameters X : numpy array of shape [n_samples, n_features]
    Returns C : array, shape = [n_samples]
        Returns predicted values.

def score(X, y):
    # Returns the coefficient of determination R^2 of the prediction.
    The coefficient R^2 is defined as (1 - u/v), where u is the regression sum of squares ((y - y_pred) ** 2).sum() and v is the residual sum of squares ((y_true - y_true.mean()) ** 2).sum(). Best possible score is 1.0, lower values are worse.
    Parameters X : array-like, shape = [n_samples, n_features]
```

498 Chapter 1. User Guide
Training set.

\[ y \]: array-like, shape = [n_samples]

Returns \( z \): float

**set_params** (**params**)

Set the parameters of the estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The former have parameters of the form `<component>__<parameter>` so that it’s possible to update each component of a nested object.

Returns self:

```python
sklearn.linear_model.sparse.ElasticNet
```

**Parameters**

- **alpha**: float
  
  Constant that multiplies the L1 term. Defaults to 1.0

- **rho**: float
  
  The ElasticNet mixing parameter, with \( 0 < \rho \leq 1 \).

- **fit_intercept**: bool
  
  Whether the intercept should be estimated or not. If False, the data is assumed to be already centered.

  TODO: fit_intercept=True is not yet implemented

**Notes**

The parameter \( \rho \) corresponds to alpha in the glmnet R package while alpha corresponds to the lambda parameter in glmnet.

**Methods**

- **decision_function**(X)  
  
  Decision function of the linear model

- **fit**(X, y)  
  
  Fit current model with coordinate descent

- **get_params**(deep)
  
  Get parameters for the estimator

- **predict**(X)
  
  Predict using the linear model

- **score**(X, y)
  
  Returns the coefficient of determination \( R^2 \) of the prediction.

- **set_params**(**params**)
  
  Set the parameters of the estimator.

```python
__init__(alpha=1.0, rho=0.5, fit_intercept=False, normalize=False, max_iter=1000, tol=0.0001, positive=False)
```
**decision_function** *(X)*

Decision function of the linear model

**Parameters**

- **X**: scipy.sparse matrix of shape [n_samples, n_features]

**Returns**

- array, shape = [n_samples] with the predicted real values:

**fit** *(X, y)*

Fit current model with coordinate descent

- X is expected to be a sparse matrix. For maximum efficiency, use a sparse matrix in CSC format (scipy.sparse.csc_matrix)

**get_params**(deep=True)

Get parameters for the estimator

**Parameters**

- **deep**: boolean, optional
  
  If True, will return the parameters for this estimator and contained subobjects that are estimators.

**predict** *(X)*

Predict using the linear model

**Parameters**

- **X**: numpy array of shape [n_samples, n_features]

**Returns**

- **C**: array, shape = [n_samples]
  
  Returns predicted values.

**score** *(X, y)*

Returns the coefficient of determination $R^2$ of the prediction.

- The coefficient $R^2$ is defined as $(1 - \frac{u}{v})$, where $u$ is the regression sum of squares $\sum(y - y_{\text{pred}})^2$ and $v$ is the residual sum of squares $\sum(y_{\text{true}} - y_{\text{true.mean}})^2$. Best possible score is 1.0, lower values are worse.

**Parameters**

- **X**: array-like, shape = [n_samples, n_features]
  
  Training set.

- **y**: array-like, shape = [n_samples]

**Returns**

- **z**: float

**set_params**(**params**)

Set the parameters of the estimator.

- The method works on simple estimators as well as on nested objects (such as pipelines). The former have parameters of the form `<component>__<parameter>` so that it's possible to update each component of a nested object.

**Returns**

- **self**:

**sklearn.linear_model.sparse.SGDClassifier**

**class**

**sklearn.linear_model.sparse.SGDClassifier**(*args, **kwargs*)

**Methods**

**decision_function** *(X)*

Predict signed ’distance’ to the hyperplane (aka confidence score)

Continued on next page
**Table 1.129 – continued from previous page**

<table>
<thead>
<tr>
<th>Method</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>fit(X, y[, coef_init, intercept_init, ...])</code></td>
<td>Fit linear model with Stochastic Gradient Descent.</td>
</tr>
<tr>
<td><code>fit_transform(X[, y])</code></td>
<td>Fit to data, then transform it</td>
</tr>
<tr>
<td><code>get_params([deep])</code></td>
<td>Get parameters for the estimator</td>
</tr>
<tr>
<td><code>partial_fit(X, y[, classes, class_weight, ...])</code></td>
<td>Fit linear model with Stochastic Gradient Descent.</td>
</tr>
<tr>
<td><code>predict(X)</code></td>
<td>Predict using the linear model</td>
</tr>
<tr>
<td><code>predict_proba(X)</code></td>
<td>Predict class membership probability</td>
</tr>
<tr>
<td><code>score(X, y)</code></td>
<td>Returns the mean accuracy on the given test data and labels.</td>
</tr>
<tr>
<td><code>set_params(**params)</code></td>
<td>Set the parameters of the estimator</td>
</tr>
<tr>
<td><code>transform(X[, threshold])</code></td>
<td>Reduce X to its most important features.</td>
</tr>
</tbody>
</table>

**__init__(*args, **kwargs)**

DEPRECATED: to be removed in v0.12; use sklearn.linear_model.SGDClassifier directly

**classes**

DEPRECATED: to be removed in v0.12; use `classes_` instead.

**decision_function (X)**

Predict signed ‘distance’ to the hyperplane (aka confidence score)

Parameters

- `X`: array-like, sparse matrix, shape = [n_samples, n_features]

Returns

- `array`, shape = [n_samples] if `n_classes` == 2 else [n_samples, n_classes]:
  - The signed ‘distances’ to the hyperplane(s).

**fit (X, y, coef_init=None, intercept_init=None, class_weight=None, sample_weight=None)**

Fit linear model with Stochastic Gradient Descent.

Parameters

- `X`: array-like, sparse matrix, shape = [n_samples, n_features]
  - Training data

- `y`: numpy array of shape [n_samples]
  - Target values

- `coef_init`: array, shape = [n_classes, n_features]
  - The initial coefficients to warm-start the optimization.

- `intercept_init`: array, shape = [n_classes]
  - The initial intercept to warm-start the optimization.

- `sample_weight`: array-like, shape = [n_samples], optional
  - Weights applied to individual samples. If not provided, uniform weights are assumed.

Returns

- `self`: returns an instance of self.

**fit_transform (X, y=None, **fit_params)**

Fit to data, then transform it

Fits transformer to X and y with optional parameters `fit_params` and returns a transformed version of X.

Parameters

- `X`: numpy array of shape [n_samples, n_features]
  - Training set.

- `y`: numpy array of shape [n_samples]
  - Target values.

Returns

- `X_new`: numpy array of shape [n_samples, n_features_new]
Transformed array.

Notes

This method just calls fit and transform consecutively, i.e., it is not an optimized implementation of fit_transform, unlike other transformers such as PCA.

get_params (deep=True)
Get parameters for the estimator

Parameters deep: boolean, optional:
If True, will return the parameters for this estimator and contained subobjects that are estimators.

partial_fit (X, y, classes=None, class_weight=None, sample_weight=None)
Fit linear model with Stochastic Gradient Descent.

Parameters X : {array-like, sparse matrix}, shape = [n_samples, n_features]
Subset of the training data

y : numpy array of shape [n_samples]
Subset of the target values

classes : array, shape = [n_classes]
Classes across all calls to partial_fit. Can be obtained by via np.unique(y_all), where y_all is the target vector of the entire dataset. This argument is required for the first call to partial_fit and can be omitted in the subsequent calls. Note that y doesn’t need to contain all labels in classes.

sample_weight : array-like, shape = [n_samples], optional
Weights applied to individual samples. If not provided, uniform weights are assumed.

Returns self : returns an instance of self.

predict (X)
Predict using the linear model

Parameters X : {array-like, sparse matrix}, shape = [n_samples, n_features]

Returns array, shape = [n_samples] :
Array containing the predicted class labels.

predict_proba (X)
Predict class membership probability

Parameters X : {array-like, sparse matrix}, shape = [n_samples, n_features]

Returns array, shape = [n_samples] if n_classes == 2 else [n_samples, :

n_classes] :
Contains the membership probabilities of the positive class.

References

The justification for the formula in the loss="modified_huber" case is in the appendix B in:
http://jmlr.csail.mit.edu/papers/volume2/zhang02c/zhang02c.pdf
score \((X, y)\)

Returns the mean accuracy on the given test data and labels.

**Parameters**

- **X**: array-like, shape = [n_samples, n_features]
  - Training set.
- **y**: array-like, shape = [n_samples]
  - Labels for X.

**Returns**

- **z**: float

set_params (**params)**

Set the parameters of the estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The former have parameters of the form `<component>__<parameter>` so that it’s possible to update each component of a nested object.

**Returns**

- **self**

transform \((X, threshold=None)\)

Reduce X to its most important features.

**Parameters**

- **X**: array or scipy sparse matrix of shape [n_samples, n_features]
  - The input samples.
- **threshold**: string, float or None, optional (default=None)
  - The threshold value to use for feature selection. Features whose importance is greater or equal are kept while the others are discarded. If “median” (resp. “mean”), then the threshold value is the median (resp. the mean) of the feature importances. A scaling factor (e.g., “1.25*mean”) may also be used. If None and if available, the object attribute threshold is used. Otherwise, “mean” is used by default.

**Returns**

- **X_r**: array of shape [n_samples, n_selected_features]
  - The input samples with only the selected features.

### sklearn.linear_model.sparse.SGDRegressor

class sklearn.linear_model.sparse.SGDRegressor(*args, **kwargs)

**Methods**

<table>
<thead>
<tr>
<th>Method</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>decision_function(X)</td>
<td>Predict using the linear model</td>
</tr>
<tr>
<td>fit(X, y[, coef_init, intercept_init, ...])</td>
<td>Fit linear model with Stochastic Gradient Descent.</td>
</tr>
<tr>
<td>fit_transform(X[, y])</td>
<td>Fit to data, then transform it</td>
</tr>
<tr>
<td>get_params([deep])</td>
<td>Get parameters for the estimator</td>
</tr>
<tr>
<td>partial_fit(X[, y[, sample_weight]])</td>
<td>Fit linear model with Stochastic Gradient Descent.</td>
</tr>
<tr>
<td>predict(X)</td>
<td>Predict using the linear model</td>
</tr>
<tr>
<td>score(X, y)</td>
<td>Returns the coefficient of determination R^2 of the prediction.</td>
</tr>
<tr>
<td>set_params(**params)</td>
<td>Set the parameters of the estimator.</td>
</tr>
<tr>
<td>transform(X[, threshold])</td>
<td>Reduce X to its most important features.</td>
</tr>
</tbody>
</table>
__init__(\*args, **kwargs)
DEPRECATED: to be removed in v0.12; use sklearn.linear_model.SGDRegressor directly

decision_function(X)
Predict using the linear model

Parameters X : {array-like, sparse matrix}, shape = [n_samples, n_features]

Returns array, shape = [n_samples] :
Predicted target values per element in X.

fit(X, y, coef_init=None, intercept_init=None, sample_weight=None)
Fit linear model with Stochastic Gradient Descent.

Parameters X : {array-like, sparse matrix}, shape = [n_samples, n_features]

Training data
y : numpy array of shape [n_samples]
Target values
coef_init : array, shape = [n_features]
The initial coefficients to warm-start the optimization.
intercept_init : array, shape = [1]
The initial intercept to warm-start the optimization.
sample_weight : array-like, shape = [n_samples], optional
Weights applied to individual samples (1. for unweighted).

Returns self : returns an instance of self.

fit_transform(X, y=None, **fit_params)
Fit to data, then transform it
Fits transformer to X and y with optional parameters fit_params and returns a transformed version of X.

Parameters X : numpy array of shape [n_samples, n_features]

Training set.
y : numpy array of shape [n_samples]
Target values.

Returns X_new : numpy array of shape [n_samples, n_features_new]
Transformed array.

Notes
This method just calls fit and transform consecutively, i.e., it is not an optimized implementation of fit_transform, unlike other transformers such as PCA.

get_params(deep=True)
Get parameters for the estimator

Parameters deep: boolean, optional :
If True, will return the parameters for this estimator and contained subobjects that are estimators.
partial_fit (X, y, sample_weight=None)

Fit linear model with Stochastic Gradient Descent.

**Parameters**
- `X` : array-like, sparse matrix, shape = [n_samples, n_features]
  - Subset of training data
- `y` : numpy array of shape [n_samples]
  - Subset of target values
- `sample_weight` : array-like, shape = [n_samples], optional
  - Weights applied to individual samples. If not provided, uniform weights are assumed.

**Returns**
- `self` : returns an instance of self.

predict (X)

Predict using the linear model

**Parameters**
- `X` : array-like, sparse matrix, shape = [n_samples, n_features]

**Returns**
- `array`, shape = [n_samples]
  - Predicted target values per element in X.

score (X, y)

Returns the coefficient of determination $R^2$ of the prediction.

The coefficient $R^2$ is defined as $(1 - \frac{u}{v})$, where $u$ is the regression sum of squares $(\sum (y - y_{\text{pred}})^2)$ and $v$ is the residual sum of squares $(\sum (y_{\text{true}} - y_{\text{true,mean}})^2)$. Best possible score is 1.0, lower values are worse.

**Parameters**
- `X` : array-like, shape = [n_samples, n_features]
  - Training set.
- `y` : array-like, shape = [n_samples]

**Returns**
- `z` : float

set_params (**params)

Set the parameters of the estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The former have parameters of the form `<component>__<parameter>` so that it’s possible to update each component of a nested object.

**Returns**
- `self`

transform (X, threshold=None)

Reduce X to its most important features.

**Parameters**
- `X` : array or scipy sparse matrix of shape [n_samples, n_features]
  - The input samples.
- `threshold` : string, float or None, optional (default=None)
  - The threshold value to use for feature selection. Features whose importance is greater or equal are kept while the others are discarded. If “median” (resp. “mean”), then the threshold value is the median (resp. the mean) of the feature importances. A scaling factor (e.g., “1.25*mean”) may also be used. If None and if available, the object attribute `threshold` is used. Otherwise, “mean” is used by default.

**Returns**
- `X_r` : array of shape [n_samples, n_selected_features]
The input samples with only the selected features.

**sklearn.linear_model.LogisticRegression**

class sklearn.linear_model.LogisticRegression(penalty='l2', dual=False, tol=0.0001, C=1.0, fit_intercept=True, intercept_scaling=1, class_weight=None)

Logistic Regression (aka logit, MaxEnt) classifier.

In the multiclass case, the training algorithm uses a one-vs.-all (OvA) scheme, rather than the “true” multinomial LR.

This class implements L1 and L2 regularized logistic regression using the liblinear library. It can handle both dense and sparse input. Use C-ordered arrays or CSR matrices containing 64-bit floats for optimal performance; any other input format will be converted (and copied).

**Parameters**

- **penalty**: string, ‘l1’ or ‘l2’
  Used to specify the norm used in the penalization

- **dual**: boolean
  Dual or primal formulation. Dual formulation is only implemented for l2 penalty. Prefer dual=False when n_samples > n_features.

- **C**: float or None, optional (default=None)
  Specifies the strength of the regularization. The smaller it is the bigger in the regularization. If None then C is set to n_samples.

- **fit_intercept**: bool, default: True
  Specifies if a constant (a.k.a. bias or intercept) should be added the decision function

- **intercept_scaling**: float, default: 1
  When self.fit_intercept is True, instance vector x becomes [x, self.intercept_scaling], i.e. a “synthetic” feature with constant value equals to intercept_scaling is appended to the instance vector. The intercept becomes intercept_scaling * synthetic feature weight

  Note! the synthetic feature weight is subject to l1/l2 regularization as all other features.

- **tol**: float, optional
  Tolerance for stopping criteria

**See Also:**

LinearSVC

**Notes**

The underlying C implementation uses a random number generator to select features when fitting the model. It is thus not uncommon, to have slightly different results for the same input data. If that happens, try with a smaller tol parameter.

**References:**

LIBLINEAR – A Library for Large Linear Classification [http://www.csie.ntu.edu.tw/~cjlin/liblinear/](http://www.csie.ntu.edu.tw/~cjlin/liblinear/)

Attributes

<table>
<thead>
<tr>
<th>Attribute</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>coef_</td>
<td>Coefficient of the features in the decision function. coef_ is readonly property derived from raw_coef_ that follows the internal memory layout of liblinear.</td>
</tr>
<tr>
<td>intercept_</td>
<td>Intercept (a.k.a. bias) added to the decision function. It is available only when parameter intercept is set to True.</td>
</tr>
</tbody>
</table>

Methods

<table>
<thead>
<tr>
<th>Method</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>decision_function(X)</td>
<td>Decision function value for X according to the trained model.</td>
</tr>
<tr>
<td>fit(X, y[, class_weight])</td>
<td>Fit the model according to the given training data.</td>
</tr>
<tr>
<td>fit_transform(X[, y])</td>
<td>Fit to data, then transform it.</td>
</tr>
<tr>
<td>get_params([deep])</td>
<td>Get parameters for the estimator</td>
</tr>
<tr>
<td>predict(X)</td>
<td>Predict target values of X according to the fitted model.</td>
</tr>
<tr>
<td>predict_log_proba(X)</td>
<td>Log of Probability estimates.</td>
</tr>
<tr>
<td>predict_proba(X)</td>
<td>Probability estimates.</td>
</tr>
<tr>
<td>score(X, y)</td>
<td>Returns the mean accuracy on the given test data and labels.</td>
</tr>
<tr>
<td>set_params(**params)</td>
<td>Set the parameters of the estimator.</td>
</tr>
<tr>
<td>transform(X[, threshold])</td>
<td>Reduce X to its most important features.</td>
</tr>
</tbody>
</table>

__init__(penalty='l2', dual=False, tol=0.0001, C=1.0, fit_intercept=True, intercept_scaling=1, class_weight=None)

decision_function(X)

Decision function value for X according to the trained model.

Parameters X : array-like, shape = [n_samples, n_features]

Returns T : array-like, shape = [n_samples, n_class]

Returns the decision function of the sample for each class in the model.

fit(X, y, class_weight=None)

Fit the model according to the given training data.

Parameters X : {array-like, sparse matrix}, shape = [n_samples, n_features]

Training vector, where n_samples is the number of samples and n_features is the number of features.

y : array-like, shape = [n_samples]

Target vector relative to X

class_weight : {dict, ‘auto’}, optional

Weights associated with classes. If not given, all classes are supposed to have weight one.

Returns self : object
Returns self.

**fit_transform** *(X, y=None, **fit_params)*
Fit to data, then transform it

Fits transformer to X and y with optional parameters fit_params and returns a transformed version of X.

**Parameters**
- **X**: numpy array of shape `[n_samples, n_features]`
  - Training set.
- **y**: numpy array of shape `[n_samples]`
  - Target values.

**Returns**
- **X_new**: numpy array of shape `[n_samples, n_features_new]`
  - Transformed array.

**Notes**
This method just calls fit and transform consecutively, i.e., it is not an optimized implementation of fit_transform, unlike other transformers such as PCA.

**get_params** *(deep=True)*
Get parameters for the estimator

**Parameters**
- **deep** (default: `True`): boolean, optional
  - If True, will return the parameters for this estimator and contained subobjects that are estimators.

**predict** *(X)*
Predict target values of X according to the fitted model.

**Parameters**
- **X**: {array-like, sparse matrix}, shape = `[n_samples, n_features]`

**Returns**
- **C**: array, shape = `[n_samples]`

**predict_log_proba** *(X)*
Log of Probability estimates.

The returned estimates for all classes are ordered by the label of classes.

**Parameters**
- **X**: array-like, shape = `[n_samples, n_features]`

**Returns**
- **T**: array-like, shape = `[n_samples, n_classes]`
  - Returns the log-probabilities of the sample for each class in the model, where classes are ordered by arithmetical order.

**predict_proba** *(X)*
Probability estimates.

The returned estimates for all classes are ordered by the label of classes.

**Parameters**
- **X**: array-like, shape = `[n_samples, n_features]`

**Returns**
- **T**: array-like, shape = `[n_samples, n_classes]`
  - Returns the probability of the sample for each class in the model, where classes are ordered by arithmetical order.

**score** *(X, y)*
Returns the mean accuracy on the given test data and labels.
**Parameters**

**X**: array-like, shape = [n_samples, n_features]

Training set.

**y**: array-like, shape = [n_samples]

Labels for X.

**Returns**

**z**: float

set_params (**params**)

Set the parameters of the estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The former have parameters of the form `<component>__<parameter>` so that it’s possible to update each component of a nested object.

**Returns**

**self**:

transform (**X**, **threshold**=None)

Reduce X to its most important features.

**Parameters**

**X**: array or scipy sparse matrix of shape [n_samples, n_features]

The input samples.

**threshold**: string, float or None, optional (default=None)

The threshold value to use for feature selection. Features whose importance is greater or equal are kept while the others are discarded. If “median” (resp. “mean”), then the threshold value is the median (resp. the mean) of the feature importances. A scaling factor (e.g., “1.25*mean”) may also be used. If None and if available, the object attribute threshold is used. Otherwise, “mean” is used by default.

**Returns**

**X_r**: array of shape [n_samples, n_selected_features]

The input samples with only the selected features.

---

### 1.8.16 sklearn.manifold: Manifold Learning

The sklearn.manifold module implements data embedding techniques.

**User guide:** See the Manifold learning section for further details.

<table>
<thead>
<tr>
<th>Class</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>manifold.LocallyLinearEmbedding(...)</td>
<td>Locally Linear Embedding</td>
</tr>
<tr>
<td>manifold.Isomap(..., n_neighbors, n_components...)</td>
<td>Isomap Embedding</td>
</tr>
<tr>
<td>manifold.MDS(..., n_components, metric, n_init...)</td>
<td>Multidimensional scaling</td>
</tr>
</tbody>
</table>

**sklearn.manifold.LocallyLinearEmbedding**

**class** sklearn.manifold.LocallyLinearEmbedding (**n_neighbors**=5, **n_components**=2, **reg**=0.001, **eigen_solver**=’auto’, **tol**=1e-06, **max_iter**=100, **method**=’standard’, **hessian_tol**=0.0001, **modified_tol**=1e-12, **neighbors_algorithm**=’auto’, **random_state**=None, **out_dim**=None)

Locally Linear Embedding

**Parameters**

**n_neighbors**: integer

number of neighbors to consider for each point.
**n_components**: integer

number of coordinates for the manifold

**reg**: float

regularization constant, multiplies the trace of the local covariance matrix of the distances.

**eigen_solver**: string, {‘auto’, ‘arpack’, ‘dense’}

- **auto**: algorithm will attempt to choose the best method for input data
- **arpack**: use arnoldi iteration in shift-invert mode. For this method, M may be a dense matrix, sparse matrix, or general linear operator.
- **dense**: use standard dense matrix operations for the eigenvalue decomposition. For this method, M must be an array or matrix type. This method should be avoided for large problems.

**tol**: float, optional

Tolerance for ‘arpack’ method Not used if eigen_solver==’dense’.

**max_iter**: integer

maximum number of iterations for the arpack solver. Not used if eigen_solver==’dense’.

**method**: string [‘standard’ | ‘hessian’ | ‘modified’]

- **standard**: use the standard locally linear embedding algorithm. see reference [1]
- **hessian**: use the Hessian eigenmap method. This method requires n_neighbors > n_components * (1 + (n_components + 1) / 2. see reference [2]
- **modified**: use the modified locally linear embedding algorithm. see reference [3]
- **ltsa**: use local tangent space alignment algorithm see reference [4]

**hessian_tol**: float, optional

Tolerance for Hessian eigenmapping method. Only used if method == ‘hessian’

**modified_tol**: float, optional

Tolerance for modified LLE method. Only used if method == ‘modified’

**neighbors_algorithm**: string [‘auto’|’brute’|’kd_tree’|’ball_tree’]

algorithm to use for nearest neighbors search, passed to neighbors.NearestNeighbors instance

**random_state**: numpy.RandomState, optional

The generator used to initialize the centers. Defaults to numpy.random. Used to determine the starting vector for arpack iterations

**References**

[R63], [R64], [R65], [R66]
Attributes

<table>
<thead>
<tr>
<th>Attribute</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>embedding_vectors_</td>
<td>array-like, shape [n_components, n_samples] Stores the embedding vectors</td>
</tr>
<tr>
<td>reconstruction_error_</td>
<td>float Reconstruction error associated with embedding_vectors_</td>
</tr>
<tr>
<td>nbrs_</td>
<td>NearestNeighbors object Stores nearest neighbors instance, including BallTree or KDtree if applicable.</td>
</tr>
</tbody>
</table>

Methods

<table>
<thead>
<tr>
<th>Method</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>fit(X[, y])</td>
<td>Compute the embedding vectors for data X</td>
</tr>
<tr>
<td>fit_transform(X[, y])</td>
<td>Compute the embedding vectors for data X and transform X.</td>
</tr>
<tr>
<td>get_params([deep])</td>
<td>Get parameters for the estimator</td>
</tr>
<tr>
<td>set_params(**params)</td>
<td>Set the parameters of the estimator.</td>
</tr>
<tr>
<td>transform(X)</td>
<td>Transform new points into embedding space.</td>
</tr>
</tbody>
</table>

**__init__** (n_neighbors=5, n_components=2, reg=0.001, eigen_solver='auto', tol=1e-06, max_iter=100, method='standard', hessian_tol=0.0001, modified_tol=1e-12, neighbors_algorithm='auto', random_state=None, out_dim=None)

**fit** (X, y=None)
Compute the embedding vectors for data X


Returns self : returns an instance of self.

**fit_transform** (X, y=None)
Compute the embedding vectors for data X and transform X.


Returns X_new : array-like, shape (n_samples, n_components) :

**get_params** (deep=True)
Get parameters for the estimator

Parameters deep: boolean, optional : If True, will return the parameters for this estimator and contained subobjects that are estimators.

**set_params** (**params)
Set the parameters of the estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The former have parameters of the form <component>__<parameter> so that it's possible to update each component of a nested object.

Returns self :

**transform** (X)
Transform new points into embedding space.
**Parameters** X : array-like, shape = [n_samples, n_features]

**Returns** X_new : array, shape = [n_samples, n_components]

**Notes**

Because of scaling performed by this method, it is discouraged to use it together with methods that are not scale-invariant (like SVMs)

**sklearn.manifold.Isomap**

class sklearn.manifold.Isomap(n_neighbors=5, n_components=2, eigen_solver='auto', tol=0, max_iter=None, path_method='auto', neighbors_algorithm='auto', out_dim=None)

Isomap Embedding

Non-linear dimensionality reduction through Isometric Mapping

**Parameters**

n_neighbors : integer
    number of neighbors to consider for each point.

n_components : integer
    number of coordinates for the manifold

eigen_solver : ['auto'|'arpack'|'dense']
    'auto'[attempt to choose the most efficient solver] for the given problem.
    'arpack'[use Arnoldi decomposition to find the eigenvalues] and eigenvectors. Note
    that arpack can handle both dense and sparse data efficiently
    'dense'[use a direct solver (i.e. LAPACK)] for the eigenvalue decomposition.

tol : float
    convergence tolerance passed to arpack or lobpcg. not used if eigen_solver == 'dense'

max_iter : integer
    maximum number of iterations for the arpack solver. not used if eigen_solver == 'dense'

path_method : string ['auto'|'FW'|'D']
    method to use in finding shortest path. ‘auto’ : attempt to choose the best algorithm
    automatically ‘FW’ : Floyd-Warshall algorithm ‘D’ : Dijkstra algorithm with Fibonacci
    Heaps

neighbors_algorithm : string ['auto'|'brute'|'kd_tree'|'ball_tree']
    algorithm to use for nearest neighbors search, passed to neighbors.NearestNeighbors
    instance

**References**

Attributes

<table>
<thead>
<tr>
<th>Attribute</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>embedding_</code></td>
<td>array-like, shape (n_samples, n_components) Stores the embedding vectors</td>
</tr>
<tr>
<td><code>kernel_pca_</code></td>
<td>KernelPCA object used to implement the embedding</td>
</tr>
<tr>
<td><code>training_data_</code></td>
<td>array-like, shape (n_samples, n_features) Stores the training data</td>
</tr>
<tr>
<td><code>nbrs_</code></td>
<td>sklearn.neighbors.NearestNeighbors instance Stores nearest neighbors instance, including BallTree or KDtree if applicable.</td>
</tr>
<tr>
<td><code>dist_matrix_</code></td>
<td>array-like, shape (n_samples, n_samples) Stores the geodesic distance matrix of training data</td>
</tr>
</tbody>
</table>

Methods

<table>
<thead>
<tr>
<th>Method</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>fit(X[, y])</code></td>
<td>Compute the embedding vectors for data X</td>
</tr>
<tr>
<td><code>fit_transform(X[, y])</code></td>
<td>Fit the model from data in X and transform X.</td>
</tr>
<tr>
<td><code>get_params([deep])</code></td>
<td>Get parameters for the estimator</td>
</tr>
<tr>
<td><code>reconstruction_error()</code></td>
<td>Compute the reconstruction error for the embedding.</td>
</tr>
<tr>
<td><code>set_params(**params)</code></td>
<td>Set the parameters of the estimator.</td>
</tr>
<tr>
<td><code>transform(X)</code></td>
<td>Transform X.</td>
</tr>
</tbody>
</table>

```
__init__(n_neighbors=5, n_components=2, eigen_solver='auto', tol=0, max_iter=None, path_method='auto', neighbors_algorithm='auto', out_dim=None)

fit (X, y=None)
Compute the embedding vectors for data X

Parameters X : {array-like, sparse matrix, BallTree, cKDTree, NearestNeighbors}
Sample data, shape = (n_samples, n_features), in the form of a numpy array, sparse array, precomputed tree, or NearestNeighbors object.

Returns self : returns an instance of self.
```

```
fit_transform (X, y=None)
Fit the model from data in X and transform X.

Parameters X: {array-like, sparse matrix, BallTree, cKDTree} :
Training vector, where n_samples in the number of samples and n_features is the number of features.

Returns X_new: array-like, shape (n_samples, n_components) :
```

```
get_params (deep=True)
Get parameters for the estimator

Parameters deep: boolean, optional :
If True, will return the parameters for this estimator and contained subobjects that are estimators.
```

```
reconstruction_error ()
Compute the reconstruction error for the embedding.

Returns reconstruction_error : float
```
Notes

The cost function of an isomap embedding is
\[
E = \text{frobenius_norm}[K(D) - K(D_{\text{fit}})] / n_{\text{samples}}
\]
Where \( D \) is the matrix of distances for the input data \( X \), \( D_{\text{fit}} \) is the matrix of distances for the output embedding \( X_{\text{fit}} \), and \( K \) is the isomap kernel:
\[
K(D) = -0.5 * (I - 1/n_{\text{samples}}) * D^2 * (I - 1/n_{\text{samples}})
\]

**set_params(****params**)**
Set the parameters of the estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The former have parameters of the form \(<\text{component}>__<\text{parameter}>\) so that it's possible to update each component of a nested object.

**Returns self :**

**transform(X)**
Transform \( X \).

This is implemented by linking the points \( X \) into the graph of geodesic distances of the training data. First the \( n_{\text{neighbors}} \) nearest neighbors of \( X \) are found in the training data, and from these the shortest geodesic distances from each point in \( X \) to each point in the training data are computed in order to construct the kernel. The embedding of \( X \) is the projection of this kernel onto the embedding vectors of the training set.

**Parameters X: array-like, shape (n_samples, n_features) :**

**Returns X_new: array-like, shape (n_samples, n_components) :**

**sklearn.manifold.MDS**

*class* sklearn.manifold.MDS (n_components=2, metric=True, n_init=4, max_iter=300, verbose=0, eps=0.001, n_jobs=1, random_state=None)

Multidimensional scaling

**Parameters metric :** boolean, optional, default: True

- compute metric or nonmetric SMACOF (Scaling by Majorizing a Complicated Function) algorithm

**n_components :** int, optional, default: 2

- number of dimension in which to immerse the similarities overridden if initial array is provided.

**n_init :** int, optional, default: 4

- Number of time the smacof algorithm will be run with different initialisation. The final results will be the best output of the n_init consecutive runs in terms of stress.

**max_iter :** int, optional, default: 300

- Maximum number of iterations of the SMACOF algorithm for a single run

**verbose :** int, optional, default: 0

- level of verbosity

**eps :** float, optional, default: 1e-6

- relative tolerance w.r.t stress to declare converge
**n_jobs**: int, optional, default: 1

The number of jobs to use for the computation. This works by breaking down the pairwise matrix into n_jobs even slices and computing them in parallel.

If -1 all CPUs are used. If 1 is given, no parallel computing code is used at all, which is useful for debugging. For n_jobs below -1, (n_cpus + 1 - n_jobs) are used. Thus for n_jobs = -2, all CPUs but one are used.

**random_state**: integer or numpy.RandomState, optional

The generator used to initialize the centers. If an integer is given, it fixes the seed. Defaults to the global numpy random number generator.

**Notes**


“Nonmetric multidimensional scaling: a numerical method” Kruskal, J. Psychometrika, 29 (1964)

“Multidimensional scaling by optimizing goodness of fit to a nonmetric hypothesis” Kruskal, J. Psychometrika, 29, (1964)

**Attributes**

| embedding | array-like, shape | Stores the position of the dataset in the embedding space |
| stress_ | float | The final value of the stress (sum of squared distance of the disparities and the distances for all constrained points) |

**Methods**

| fit(X[, init, y]) | Computes the position of the points in the embedding space |
| fit_transform(X[, init, y]) | Fit the data from X, and returns the embedded coordinates |
| get_params([deep]) | Get parameters for the estimator |
| set_params(**params) | Set the parameters of the estimator. |

**__init__**(n_components=2, metric=True, n_init=4, max_iter=300, verbose=0, eps=0.001, n_jobs=1, random_state=None)

**fit**(X, init=None, y=None)

Computes the position of the points in the embedding space

**Parameters** X: array, shape=[n_samples, n_samples], symmetric :

Proximity matrix

**init**: {None or ndarray, shape (n_samples,)}

if None, randomly chooses the initial configuration if ndarray, initialize the SMACOF algorithm with this array

**fit_transform**(X, init=None, y=None)

Fit the data from X, and returns the embedded coordinates
Parameters X: array, shape=[n_samples, n_samples], symetric:

Proximity matrice

init: {None or ndarray, shape (n_samples,)}

if None, randomly chooses the initial configuration if ndarray, initialize the SMACOF algorithm with this array

get_params (deep=True)

Get parameters for the estimator

Parameters deep: boolean, optional:

If True, will return the parameters for this estimator and contained subobjects that are estimators.

set_params (**params)

Set the parameters of the estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The former have parameters of the form <component>_<parameter> so that it’s possible to update each component of a nested object.

Returns self:

manifold.locally_linear_embedding(X, ...[, ...]) Perform a Locally Linear Embedding analysis on the data.

sklearn.manifold.locally_linear_embedding

sklearn.manifold.locally_linear_embedding (X, n_neighbors, n_components, reg=0.001, eigen_solver='auto', tol=1e-06, max_iter=100, method='standard', hessian_tol=0.0001, modified_tol=1e-12, random_state=None, out_dim=None)

Perform a Locally Linear Embedding analysis on the data.

Parameters X: {array-like, sparse matrix, BallTree, cKDTree, NearestNeighbors}

Sample data, shape = (n_samples, n_features), in the form of a numpy array, sparse array, precomputed tree, or NearestNeighbors object.

n_neighbors : integer

number of neighbors to consider for each point.

n_components : integer

number of coordinates for the manifold.

reg : float

regularization constant, multiplies the trace of the local covariance matrix of the distances.

eigen_solver : string, {'auto', 'arpack', 'dense'}

auto : algorithm will attempt to choose the best method for input data

arpack [use arnoldi iteration in shift-invert mode.] For this method, M may be a dense matrix, sparse matrix, or general linear operator.
**dense** [use standard dense matrix operations for the eigenvalue decomposition. For this method, M must be an array or matrix type. This method should be avoided for large problems.]

**tol** : float, optional
Tolerance for ‘arpack’ method. Not used if eigen_solver=='dense'.

**max_iter** : integer
maximum number of iterations for the arpack solver.

- **standard** [use the standard locally linear embedding algorithm. see reference [R67]]
- **hessian** [use the Hessian eigenmap method. This method requires n_neighbors > n_components * (1 + (n_components + 1) / 2. see reference [R68]]
- **modified** [use the modified locally linear embedding algorithm. see reference [R69]]
- **ltsa** [use local tangent space alignment algorithm] see reference [R70]

**hessian_tol** : float, optional
Tolerance for Hessian eigenmapping method. Only used if method == ‘hessian’

**modified_tol** : float, optional
Tolerance for modified LLE method. Only used if method == ‘modified’

**random_state** : numpy.RandomState, optional
The generator used to initialize the centers. Defaults to numpy.random.

**Returns**
- **Y** : array-like, shape [n_samples, n_components]
  Embedding vectors.
- **squared_error** : float
  Reconstruction error for the embedding vectors. Equivalent to \( \| Y - W Y, 'fro' \|^2 \), where W are the reconstruction weights.

**References**

[R67], [R68], [R69], [R70]

### 1.8.17 sklearn.metrics: Metrics

The sklearn.metrics module includes score functions, performance metrics and pairwise metrics and distance computations.

**Classification metrics**

- **metrics.confusion_matrix(y_true, y_pred[, ...])** Compute confusion matrix to evaluate the accuracy of a classification
- **metrics.roc_curve(y_true, y_score)** compute Receiver operating characteristic (ROC)
- **metrics.auc(x, y)** Compute Area Under the Curve (AUC) using the trapezoidal rule
- **metrics.precision_score(y_true, y_pred[, ...])** Compute the precision
Table 1.137 – continued from previous page

<table>
<thead>
<tr>
<th>Function</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>metrics.recall_score(y_true, y_pred[, ...])</code></td>
<td>Compute the recall</td>
</tr>
<tr>
<td><code>metrics.fbeta_score(y_true, y_pred, beta[, ...])</code></td>
<td>Compute fbeta score</td>
</tr>
<tr>
<td><code>metrics.f1_score(y_true, y_pred[, labels, ...])</code></td>
<td>Compute f1 score</td>
</tr>
<tr>
<td><code>metrics.precision_recall_fscore_support(...)</code></td>
<td>Compute precisions, recalls, f-measures and support for each class</td>
</tr>
<tr>
<td><code>metrics.classification_report(y_true, y_pred)</code></td>
<td>Build a text report showing the main classification metrics</td>
</tr>
<tr>
<td><code>metrics.precision_recall_curve(y_true, ...)</code></td>
<td>Compute precision-recall pairs for different probability thresholds</td>
</tr>
<tr>
<td><code>metrics.zero_one_score(y_true, y_pred)</code></td>
<td>Zero-one classification score (accuracy)</td>
</tr>
<tr>
<td><code>metrics.zero_one(y_true, y_pred)</code></td>
<td>Zero-One classification loss</td>
</tr>
<tr>
<td><code>metrics.hinge_loss(y_true, pred_decision[, ...])</code></td>
<td>Cumulated hinge loss (non-regularized).</td>
</tr>
</tbody>
</table>

**sklearn.metrics.confusion_matrix**

`sklearn.metrics.confusion_matrix(y_true, y_pred, labels=None)`

Compute confusion matrix to evaluate the accuracy of a classification.

By definition a confusion matrix cm is such that cm[i, j] is equal to the number of observations known to be in group i but predicted to be in group j.

**Parameters**

- `y_true` : array, shape = [n_samples]
  true targets
- `y_pred` : array, shape = [n_samples]
  estimated targets
- `labels` : array, shape = [n_classes]
  lists all labels occuring in the dataset. If none is given, those that appear at least once in y_true or y_pred are used.

**Returns**

- `CM` : array, shape = [n_classes, n_classes]
  confusion matrix

**References**


**sklearn.metrics.roc_curve**

`sklearn.metrics.roc_curve(y_true, y_score)`

compute Receiver operating characteristic (ROC)

Note: this implementation is restricted to the binary classification task.

**Parameters**

- `y_true` : array, shape = [n_samples]
  true binary labels
- `y_score` : array, shape = [n_samples]
  target scores, can either be probability estimates of the positive class, confidence values, or binary decisions.

**Returns**

- `fpr` : array, shape = [>2]
  False Positive Rates
**tpr**: array, shape = [>2]

True Positive Rates

**thresholds**: array, shape = [>2]

Thresholds on y_score used to compute fpr and tpr

References

http://en.wikipedia.org/wiki/Receiver_operating_characteristic

Examples

```python
>>> import numpy as np
>>> from sklearn import metrics

>>> y = np.array([1, 1, 2, 2])
>>> scores = np.array([0.1, 0.4, 0.35, 0.8])
>>> fpr, tpr, thresholds = metrics.roc_curve(y, scores)
>>> fpr
array([ 0. , 0.5, 0.5, 1. ])
```

**sklearn.metrics.auc**

`sklearn.metrics.auc(x, y)`

Compute Area Under the Curve (AUC) using the trapezoidal rule

**Parameters**

- **x**: array, shape = [n]
  - x coordinates
- **y**: array, shape = [n]
  - y coordinates

**Returns**

- **auc**: float

Examples

```python
>>> import numpy as np
>>> from sklearn import metrics

>>> y = np.array([1, 1, 2, 2])
>>> pred = np.array([0.1, 0.4, 0.35, 0.8])
>>> fpr, tpr, thresholds = metrics.roc_curve(y, pred)
>>> metrics.auc(fpr, tpr)
0.75
```

**sklearn.metrics.precision_score**

`sklearn.metrics.precision_score(y_true, y_pred, labels=None, pos_label=1, average='weighted')`

Compute the precision
The precision is the ratio \( \frac{tp}{tp + fp} \) where \( tp \) is the number of true positives and \( fp \) the number of false positives. The precision is intuitively the ability of the classifier not to label as positive a sample that is negative. The best value is 1 and the worst value is 0.

**Parameters**

- `y_true`: array, shape=[n_samples]
  True targets
- `y_pred`: array, shape=[n_samples]
  Predicted targets
- `labels`: array
  Integer array of labels
- `pos_label`: int
  In the binary classification case, give the label of the positive class (default is 1). Everything else but `pos_label` is considered to belong to the negative class. Set to None in the case of multiclass classification.
- `average`: string, [None, 'micro', 'macro', 'weighted'](default)
  In the multiclass classification case, this determines the type of averaging performed on the data.
  - `macro`: Average over classes (does not take imbalance into account).
  - `micro`: Average over instances (takes imbalance into account). This implies that \( \text{precision} = \text{recall} = \text{f1} \)
  - `weighted`: Average weighted by support (takes imbalance into account). Can result in f1 score that is not between precision and recall.

**Returns**

- `precision`: float
  Precision of the positive class in binary classification or weighted average of the precision of each class for the multiclass task.

### sklearn.metrics.recall_score

**sklearn.metrics.recall_score** *(y_true, y_pred, labels=None, pos_label=1, average='weighted')*

Compute the recall

The recall is the ratio \( \frac{tp}{tp + fn} \) where \( tp \) is the number of true positives and \( fn \) the number of false negatives. The recall is intuitively the ability of the classifier to find all the positive samples.

The best value is 1 and the worst value is 0.

**Parameters**

- `y_true`: array, shape=[n_samples]
  True targets
- `y_pred`: array, shape=[n_samples]
  Predicted targets
- `labels`: array
  Integer array of labels
- `pos_label`: int
In the binary classification case, give the label of the positive class (default is 1). Everything else but ‘pos_label’ is considered to belong to the negative class. Set to None in the case of multiclass classification.

**average** : string, [None, ‘micro’, ‘macro’, ‘weighted’(default)]

In the multiclass classification case, this determines the type of averaging performed on the data.

- **macro**: Average over classes (does not take imbalance into account).
- **micro**: Average over instances (takes imbalance into account). This implies that precision == recall == f1
- **weighted**: Average weighted by support (takes imbalance into account). Can result in f1 score that is not between precision and recall.

**Returns recall** : float

Recall of the positive class in binary classification or weighted average of the recall of each class for the multiclass task.

**sklearn.metrics.fbeta_score**

`sklearn.metrics.fbeta_score(y_true, y_pred, beta, labels=None, pos_label=1, average='weighted')`

Compute fbeta score

The F_beta score is the weighted harmonic mean of precision and recall, reaching its optimal value at 1 and its worst value at 0.

The beta parameter determines the weight of precision in the combined score. beta < 1 lends more weight to precision, while beta > 1 favors precision (beta == 0 considers only precision, beta == inf only recall).

**Parameters y_true** : array, shape = [n_samples]

True targets

- **y_pred** : array, shape = [n_samples]

Predicted targets

- **beta**: float :

  Weight of precision in harmonic mean.

- **labels** : array

  Integer array of labels

- **pos_label** : int

  In the binary classification case, give the label of the positive class (default is 1). Everything else but ‘pos_label’ is considered to belong to the negative class. Set to None in the case of multiclass classification.

- **average** : string, [None, ‘micro’, ‘macro’, ‘weighted’(default)]

  In the multiclass classification case, this determines the type of averaging performed on the data.

- **macro**: Average over classes (does not take imbalance into account).
**f1_score**

The F1 score can be interpreted as a weighted average of the precision and recall, where an F1 score reaches its best value at 1 and worst score at 0. The relative contribution of precision and recall to the F1 score are equal. The formula for the F_1 score is:

\[
F_1 = \frac{2 \times \text{precision} \times \text{recall}}{\text{precision} + \text{recall}}
\]

See: [http://en.wikipedia.org/wiki/F1_score](http://en.wikipedia.org/wiki/F1_score)  

In the multi-class case, this is the weighted average of the f1-score of each class.

**Parameters**

- **y_true**: array, shape = [n_samples]  
  True targets

- **y_pred**: array, shape = [n_samples]  
  Predicted targets

- **labels**: array  
  Integer array of labels

- **pos_label**: int  
  In the binary classification case, give the label of the positive class (default is 1). Everything else but ‘pos_label’ is considered to belong to the negative class. Set to None in the case of multiclass classification.

- **average**: string, [None, ‘micro’, ‘macro’, ‘weighted’ (default)]  
  In the multiclass classification case, this determines the type of averaging performed on the data.

  - **macro**: Average over classes (does not take imbalance into account).
  - **micro**: Average over instances (takes imbalance into account). This implies that \(\text{precision} == \text{recall} == \text{f1}\)
  - **weighted**: Average weighted by support (takes imbalance into account). Can result in f1 score that is not between precision and recall.
Returns \texttt{f1_score} : float

\texttt{f1_score} of the positive class in binary classification or weighted average of the \texttt{f1_scores} of each class for the multiclass task

References

http://en.wikipedia.org/wiki/F1_score

\texttt{sklearn.metrics.precision_recall_fscore_support}

\texttt{sklearn.metrics.precision_recall_fscore_support}(\texttt{y_true}, \texttt{y_pred}, beta=1.0, labels=None, pos_label=1, average=None)

Compute precisions, recalls, f-measures and support for each class

The precision is the ratio \( tp/(tp + fp) \), where \( tp \) is the number of true positives and \( fp \) the number of false positives. The precision is intuitively the ability of the classifier not to label as positive a sample that is negative.

The recall is the ratio \( tp/(tp + fn) \), where \( tp \) is the number of true positives and \( fn \) the number of false negatives. The recall is intuitively the ability of the classifier to find all the positive samples.

The \( F_\beta \) score can be interpreted as a weighted harmonic mean of the precision and recall, where an \( F_\beta \) score reaches its best value at 1 and worst score at 0.

The \( F_\beta \) score weights recall \( \beta \) as much as precision. \( \beta = 1.0 \) means recall and precision are equally important.

The support is the number of occurrences of each class in \texttt{y_true}.

If \texttt{pos_label} is None, this function returns the average precision, recall and f-measure if \texttt{average} is one of ‘micro’, ‘macro’, ‘weighted’.

Parameters \texttt{y_true} : array, shape = [n_samples]

True targets

\texttt{y_pred} : array, shape = [n_samples]

Predicted targets

\texttt{beta} : float, 1.0 by default

The strength of recall versus precision in the f-score.

\texttt{labels} : array

Integer array of labels

\texttt{pos_label} : int

In the binary classification case, give the label of the positive class (default is 1). Everything else but ‘pos_label’ is considered to belong to the negative class. Set to None in the case of multiclass classification.

\texttt{average} : string, [None, ‘micro’, ‘macro’, ‘weighted’(default)]

In the multiclass classification case, this determines the type of averaging performed on the data.

\texttt{macro}: Average over classes (does not take imbalance into account).
**micro**: Average over instances (takes imbalance into account). This implies that precision == recall == f1

**weighted**: Average weighted by support (takes imbalance into account). Can result in f1 score that is not between precision and recall.

Returns:

- precision: array, shape = [n_unique_labels], dtype = np.double
- recall: array, shape = [n_unique_labels], dtype = np.double
- f1_score: array, shape = [n_unique_labels], dtype = np.double
- support: array, shape = [n_unique_labels], dtype = np.long

**References**

http://en.wikipedia.org/wiki/Precision_and_recall

```
from sklearn.metrics import classification_report

classification_report(y_true, y_pred, labels=None, target_names=None)
```

Build a text report showing the main classification metrics

**Parameters**

- `y_true`: array, shape = [n_samples]
  
  True targets

- `y_pred`: array, shape = [n_samples]
  
  Estimated targets

- `labels`: array, shape = [n_labels]
  
  Optional list of label indices to include in the report

- `target_names`: list of strings
  
  Optional display names matching the labels (same order)

**Returns**

- `report`: string
  
  Text summary of the precision, recall, f1-score for each class

```
from sklearn.metrics import precision_recall_curve

precision_recall_curve(y_true, probas_pred)
```

Compute precision-recall curves for different probability thresholds.

**Note**: this implementation is restricted to the binary classification task.

The precision is the ratio $tp/(tp + fp)$ where tp is the number of true positives and fp the number of false positives. The precision is intuitively the ability of the classifier not to label as positive a sample that is negative.

The recall is the ratio $tp/(tp + fn)$ where tp is the number of true positives and fn the number of false negatives. The recall is intuitively the ability of the classifier to find all the positive samples.

The last precision and recall values are 1. and 0. respectively and do not have a corresponding threshold. This ensures that the graph starts on the x axis.

**Parameters**

- `y_true`: array, shape = [n_samples]
  
  True targets of binary classification in range {-1, 1} or {0, 1}
**probas_pred** : array, shape = [n_samples]
Estimated probabilities

**Returns precision** : array, shape = [n + 1]
Precision values

**recall** : array, shape = [n + 1]
Recall values

**thresholds** : array, shape = [n]
Thresholds on y_score used to compute precision and recall

### sklearn.metrics.zero_one_score

```python
sklearn.metrics.zero_one_score(y_true, y_pred)
```

Zero-one classification score (accuracy)

Positive integer (number of good classifications). The best performance is 1.

Return the fraction of correct predictions in y_pred.

**Parameters**

- **y_true** : array-like, shape = n_samples
  Gold standard labels.

- **y_pred** : array-like, shape = n_samples
  Predicted labels, as returned by a classifier.

**Returns**

- **score** : float

### sklearn.metrics.zero_one

```python
sklearn.metrics.zero_one(y_true, y_pred)
```

Zero-One classification loss

Positive integer (number of misclassifications). The best performance is 0.

Return the number of errors

**Parameters**

- **y_true** : array-like

- **y_pred** : array-like

**Returns**

- **loss** : float

### sklearn.metrics.hinge_loss

```python
sklearn.metrics.hinge_loss(y_true, pred_decision, pos_label=1, neg_label=-1)
```

Cumulated hinge loss (non-regularized).

Assuming labels in y_true are encoded with +1 and -1, when a prediction mistake is made, margin = y_true * pred_decision is always negative (since the signs disagree), therefore 1 - margin is always greater than 1. The cumulated hinge loss therefore upperbounds the number of mistakes made by the classifier.

**Parameters**

- **y_true** : array, shape = [n_samples]
  True target (integers)
pred_decision : array, shape = [n_samples] or [n_samples, n_classes]

Predicted decisions, as output by decision_function (floats)

Regression metrics

<table>
<thead>
<tr>
<th>Function</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>metrics.r2_score(y_true, y_pred)</td>
<td>R^2 (coefficient of determination) regression score function</td>
</tr>
<tr>
<td>metrics.mean_squared_error(y_true, y_pred)</td>
<td>Mean squared error regression loss</td>
</tr>
</tbody>
</table>

sklearn.metrics.r2_score

sklearn.metrics.r2_score(y_true, y_pred)

R^2 (coefficient of determination) regression score function

- Best possible score is 1.0, lower values are worse.

**Parameters**

- y_true : array-like
- y_pred : array-like

**Returns**

- z : float
  - The R^2 score

**Notes**

This is not a symmetric function.

**References**


sklearn.metrics.mean_squared_error

sklearn.metrics.mean_squared_error(y_true, y_pred)

Mean squared error regression loss

- Return a a positive floating point value (the best value is 0.0).

**Parameters**

- y_true : array-like
- y_pred : array-like

**Returns**

- loss : float

Clustering metrics

See the Clustering section of the user guide for further details. The sklearn.metrics.cluster submodule contains evaluation metrics for cluster analysis results. There are two forms of evaluation:

- supervised, which uses a ground truth class values for each sample.
- unsupervised, which does not and measures the ‘quality’ of the model itself.
<table>
<thead>
<tr>
<th>Function</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>metrics.adjusted_mutual_info_score</code></td>
<td>Adjusted Mutual Information between two clusterings</td>
</tr>
<tr>
<td><code>metrics.adjusted_rand_score</code></td>
<td>Rand index adjusted for chance</td>
</tr>
<tr>
<td><code>metrics.completeness_score</code></td>
<td>Completeness metric of a cluster labeling given a ground truth</td>
</tr>
<tr>
<td><code>metrics.homogeneity_completeness_v_measure</code></td>
<td>Compute the homogeneity and completeness and V-measure scores</td>
</tr>
<tr>
<td><code>metrics.homogeneity_score</code></td>
<td>Homogeneity metric of a cluster labeling given a ground truth</td>
</tr>
<tr>
<td><code>metrics.mutual_info_score</code></td>
<td>Mutual Information between two clusterings</td>
</tr>
<tr>
<td><code>metrics.normalized_mutual_info_score</code></td>
<td>Normalized Mutual Information between two clusterings</td>
</tr>
<tr>
<td><code>metrics.silhouette_score</code></td>
<td>Compute the mean Silhouette Coefficient of all samples.</td>
</tr>
<tr>
<td><code>metrics.v_measure_score</code></td>
<td>V-Measure cluster labeling given a ground truth.</td>
</tr>
</tbody>
</table>

**sklearn.metrics.adjusted_mutual_info_score**

`sklearn.metrics.adjusted_mutual_info_score(labels_true, labels_pred)`

Adjusted Mutual Information (AMI) is an adjustement of the Mutual Information (MI) score to account for chance. It accounts for the fact that the MI is generally higher for two clusterings with a larger number of clusters, regardless of whether there is actually more information shared. For two clusterings U and V, the AMI is given as:

\[
AMI(U, V) = \frac{MI(U, V) - E(MI(U, V))}{\max(H(U), H(V)) - E(MI(U, V))}
\]

This metric is independent of the absolute values of the labels: a permutation of the class or cluster label values won’t change the score value in any way.

This metric is furthermore symmetric: switching `label_true` with `label_pred` will return the same score value. This can be useful to measure the agreement of two independent label assignments strategies on the same dataset when the real ground truth is not known.

Be mindful that this function is an order of magnitude slower than other metrics, such as the Adjusted Rand Index.

**Parameters**

- `labels_true` : int array, shape = [n_samples]
  A clustering of the data into disjoint subsets.

- `labels_pred` : array, shape = [n_samples]
  A clustering of the data into disjoint subsets.

**Returns**

- `ami` : float
  score between 0.0 and 1.0. 1.0 stands for perfectly complete labeling

**See Also**

- `adjusted_rand_score`: Adjusted Rand Index
- `mutual_information_score`: Mutual Information (not adjusted for chance)

**References**

[R41], [R42]
Examples

Perfect labelings are both homogeneous and complete, hence have score 1.0:

```python
>>> from sklearn.metrics.cluster import adjusted_mutual_info_score
>>> adjusted_mutual_info_score([0, 0, 1, 1], [0, 0, 1, 1])
1.0
>>> adjusted_mutual_info_score([0, 0, 1, 1], [1, 1, 0, 0])
1.0
```

If classes members are completely split across different clusters, the assignment is totally in-complete, hence the AMI is null:

```python
>>> adjusted_mutual_info_score([0, 0, 0, 0], [0, 1, 2, 3])
0.0
```

sklearn.metrics.adjusted_rand_score

sklearn.metrics.adjusted_rand_score(labels_true, labels_pred)

Rand index adjusted for chance

The Rand Index computes a similarity measure between two clusterings by considering all pairs of samples and counting pairs that are assigned in the same or different clusters in the predicted and true clusterings.

The raw RI score is then “adjusted for chance” into the ARI score using the following scheme:

\[
ARI = \frac{RI - Expected\_RI}{max(RI) - Expected\_RI}
\]

The adjusted Rand index is thus ensured to have a value close to 0.0 for random labeling independently of the number of clusters and samples and exactly 1.0 when the clusterings are identical (up to a permutation).

ARI is a symmetric measure:

```python
adjusted_rand_score(a, b) == adjusted_rand_score(b, a)
```

Parameters

- **labels_true**: int array, shape = [n_samples]
  
  Ground truth class labels to be used as a reference

- **labels_pred**: array, shape = [n_samples]
  
  Cluster labels to evaluate

Returns

- **ari**: float
  
  Similarity score between -1.0 and 1.0. Random labelings have an ARI close to 0.0. 1.0 stands for perfect match.

See Also:

- adjusted_mutual_info_score: Adjusted Mutual Information

References

[Hubert1985], [wk]
Examples

Perfectly matching labelings have a score of 1 even

```python
>>> from sklearn.metrics.cluster import adjusted_rand_score
>>> adjusted_rand_score([0, 0, 1, 1], [0, 0, 1, 1])
1.0
>>> adjusted_rand_score([0, 0, 1, 1], [1, 1, 0, 0])
1.0
```

Labelings that assign all classes members to the same clusters are complete but not always pure, hence penalized:

```python
>>> adjusted_rand_score([0, 0, 1, 2], [0, 0, 1, 1])
0.57...
```

ARI is symmetric, so labelings that have pure clusters with members coming from the same classes but unnecessary splits are penalized:

```python
>>> adjusted_rand_score([0, 0, 1, 1], [0, 0, 1, 2])
0.57...
```

If classes members are completely split across different clusters, the assignment is totally incomplete, hence the ARI is very low:

```python
>>> adjusted_rand_score([0, 0, 0, 0], [0, 1, 2, 3])
0.0
```

**sklearn.metrics.completeness_score**

`sklearn.metrics.completeness_score(labels_true, labels_pred)`

Completeness metric of a cluster labeling given a ground truth

A clustering result satisfies completeness if all the data points that are members of a given class are elements of the same cluster.

This metric is independent of the absolute values of the labels: a permutation of the class or cluster label values won’t change the score value in any way.

This metric is not symmetric: switching `label_true` with `label_pred` will return the homogeneity_score which will be different in general.

**Parameters**

- `labels_true` : int array, shape = [n_samples]
  ground truth class labels to be used as a reference

- `labels_pred` : array, shape = [n_samples]
  cluster labels to evaluate

**Returns**

- `completeness` : float
  score between 0.0 and 1.0. 1.0 stands for perfectly complete labeling

**See Also**

- `homogeneity_score`, `v_measure_score`

**References**

Examples

Perfect labelings are complete:

```python
>>> from sklearn.metrics.cluster import completeness_score
>>> completeness_score([0, 0, 1, 1], [1, 1, 0, 0])
1.0
```

Non-perfect labelings that assign all classes members to the same clusters are still complete:

```python
>>> completeness_score([0, 0, 1, 1], [0, 0, 0, 0])
1.0
>>> completeness_score([0, 1, 2, 3], [0, 0, 1, 1])
1.0
```

If classes members are splitted across different clusters, the assignment cannot be complete:

```python
>>> completeness_score([0, 0, 1, 1], [0, 1, 0, 1])
0.0
>>> completeness_score([0, 0, 0, 0], [0, 1, 2, 3])
0.0
```

**sklearn.metrics.homogeneity_completeness_v_measure**

`sklearn.metrics.homogeneity_completeness_v_measure(labels_true, labels_pred)`

Compute the homogeneity and completeness and V-measure scores at once.

Those metrics are based on normalized conditional entropy measures of the clustering labeling to evaluate given the knowledge of a Ground Truth class labels of the same samples.

A clustering result satisfies homogeneity if all of its clusters contain only data points which are members of a single class.

A clustering result satisfies completeness if all the data points that are members of a given class are elements of the same cluster.

Both scores have positive values between 0.0 and 1.0, larger values being desirable.

Those 3 metrics are independent of the absolute values of the labels: a permutation of the class or cluster label values won’t change the score values in any way.

V-Measure is furthermore symmetric: swapping `labels_true` and `labels_pred` will give the same score. This does not hold for homogeneity and completeness.

Parameters

- `labels_true` : int array, shape = [n_samples]
  
  ground truth class labels to be used as a reference

- `labels_pred` : array, shape = [n_samples]
  
  cluster labels to evaluate

Returns

- `homogeneity` : float :
  
  score between 0.0 and 1.0. 1.0 stands for perfectly homogeneous labeling

- `completeness` : float :
  
  score between 0.0 and 1.0. 1.0 stands for perfectly complete labeling

- `v_measure` : float :
  
  harmonic mean of the first two
See Also:

`homogeneity_score`, `completeness_score`, `v_measure_score`

`sklearn.metrics.homogeneity_score`

`sklearn.metrics.homogeneity_score(labels_true, labels_pred)`

Homogeneity metric of a cluster labeling given a ground truth

A clustering result satisfies homogeneity if all of its clusters contain only data points which are members of a single class.

This metric is independent of the absolute values of the labels: a permutation of the class or cluster label values won't change the score value in any way.

This metric is not symmetric: switching `label_true` with `label_pred` will return the `completeness_score` which will be different in general.

**Parameters**

- **labels_true**: int array, shape = [n_samples]
  ground truth class labels to be used as a reference
- **labels_pred**: array, shape = [n_samples]
  cluster labels to evaluate

**Returns**

- **homogeneity**: float
  score between 0.0 and 1.0. 1.0 stands for perfectly homogeneous labeling

See Also:

`completeness_score`, `v_measure_score`

References


Examples

Perfect labelings are homegenous:

```python
>>> from sklearn.metrics.cluster import homogeneity_score
>>> homogeneity_score([0, 0, 1, 1], [1, 1, 0, 0])
1.0
```

Non-pefect labelings that further split classes into more clusters can be perfectly homogeneous:

```python
>>> homogeneity_score([0, 0, 1, 1], [0, 0, 1, 2])
1.0
```

```python
>>> homogeneity_score([0, 0, 1, 1], [0, 1, 2, 3])
1.0
```

Clusters that include samples from different classes do not make for an homogeneous labeling:

```python
>>> homogeneity_score([0, 0, 1, 1], [0, 1, 0, 1])
0.0
```

```python
>>> homogeneity_score([0, 0, 1, 1], [0, 0, 0, 0])
0.0
```
sklearn.metrics.mutual_info_score

sklearn.metrics.mutual_info_score(labels_true, labels_pred, contingency=None)

Mutual Information between two clusterings

The Mutual Information is a measure of the similarity between two labels of the same data. Where \( P(i) \) is the probability of a random sample occurring in cluster \( U_i \) and \( P'(j) \) is the probability of a random sample occurring in cluster \( V_j \), the Mutual information between clusterings \( U \) and \( V \) is given as:

\[
MI(U,V) = \sum_{i=1}^{R} \sum_{j=1}^{C} P(i,j) \log \frac{P(i,j)}{P(i)P'(j)}
\]

This is equal to the Kullback-Leibler divergence of the joint distribution with the product distribution of the marginals.

This metric is independent of the absolute values of the labels: a permutation of the class or cluster label values won’t change the score value in any way.

This metric is furthermore symmetric: switching \( label_true \) with \( label_pred \) will return the same score value. This can be useful to measure the agreement of two independent label assignments strategies on the same dataset when the real ground truth is not known.

Parameters

- **labels_true**: int array, shape = [n_samples]
  A clustering of the data into disjoint subsets.

- **labels_pred**: array, shape = [n_samples]
  A clustering of the data into disjoint subsets.

- **contingency**: None or array, shape = [n_classes_true, n_classes_pred]
  A contingency matrix given by the contingency_matrix function. If value is None, it will be computed, otherwise the given value is used, with labels_true and labels_pred ignored.

Returns **mi**: float

- Mutual information, a non-negative value

See Also:

- adjusted_mutual_info_score: Adjusted against chance Mutual Information
- normalized_mutual_info_score: Normalized Mutual Information

sklearn.metrics.normalized_mutual_info_score

sklearn.metrics.normalized_mutual_info_score(labels_true, labels_pred)

Normalized Mutual Information between two clusterings

Normalized Mutual Information (NMI) is an normalization of the Mutual Information (MI) score to scale the results between 0 (no mutual information) and 1 (perfect correlation).

This measure is not adjusted for chance. Therefore adjusted_mutual_info_score might be preferred.

This metric is independent of the absolute values of the labels: a permutation of the class or cluster label values won’t change the score value in any way.

This metric is furthermore symmetric: switching \( label_true \) with \( label_pred \) will return the same score value. This can be useful to measure the agreement of two independent label assignments strategies on the same dataset when the real ground truth is not known.
Parameters **labels_true**: int array, shape = [n_samples]
A clustering of the data into disjoint subsets.

**labels_pred**: array, shape = [n_samples]
A clustering of the data into disjoint subsets.

Returns nmi: float:
score between 0.0 and 1.0. 1.0 stands for perfectly complete labeling

See Also:

*adjusted_rand_score*Adjusted Rand Index
*adjusted_mutual_information_score*Adjusted Mutual Information (adjusted against chance)

Examples

Perfect labelings are both homogeneous and complete, hence have score 1.0:

```python
>>> from sklearn.metrics.cluster import normalized_mutual_info_score
>>> normalized_mutual_info_score([0, 0, 1, 1], [0, 0, 1, 1])
1.0
>>> normalized_mutual_info_score([0, 0, 1, 1], [1, 1, 0, 0])
0.0
```

If classes members are completly splitted across different clusters, the assignment is totally in-complete, hence the NMI is null:

```python
>>> normalized_mutual_info_score([0, 0, 0, 0], [0, 1, 2, 3])
0.0
```

**sklearn.metrics.silhouette_score**

*sklearn.metrics.silhouette_score(X, labels, metric='euclidean', sample_size=None, random_state=None, **kwds)*

Compute the mean Silhouette Coefficient of all samples.

The Silhouette Coefficient is calculated using the mean intra-cluster distance (a) and the mean nearest-cluster distance (b) for each sample. The Silhouette Coefficient for a sample is \((b - a) / \max(a, b)\). To clarify, b is the distance between a sample and the nearest cluster that b is not a part of.

This function returns the mean Silhouette Coefficient over all samples. To obtain the values for each sample, use silhouette_samples

The best value is 1 and the worst value is -1. Values near 0 indicate overlapping clusters. Negative values generally indicate that a sample has been assigned to the wrong cluster, as a different cluster is more similar.

Parameters **X** : array [n_samples_a, n_samples_a] if metric == “precomputed”, or [n_samples_a, n_features] otherwise

Array of pairwise distances between samples, or a feature array.

**labels** : array, shape = [n_samples]
label values for each sample

**metric** : string, or callable
The metric to use when calculating distance between instances in a feature array. If metric is a string, it must be one of the options allowed by metrics.pairwise.pairwise_distances. If X is the distance array itself, use “precomputed” as the metric.

**sample_size** : int or None

The size of the sample to use when computing the Silhouette Coefficient. If sample_size is None, no sampling is used.

**random_state** : integer or numpy.RandomState, optional

The generator used to initialize the centers. If an integer is given, it fixes the seed. Defaults to the global numpy random number generator.

**kwds** : optional keyword parameters

Any further parameters are passed directly to the distance function. If using a scipy.spatial.distance metric, the parameters are still metric dependent. See the scipy docs for usage examples.

**Returns**

**silhouette** : float

Mean Silhouette Coefficient for all samples.

**References**


**sklearn.metrics.v_measure_score**

**sklearn.metrics.v_measure_score(labels_true, labels_pred)**

This score is identical to normalized_mutual_info_score.

The V-Measure is the harmonic mean between homogeneity and completeness:

\[
V = \frac{2 \times \text{homogeneity} \times \text{completeness}}{\text{homogeneity} + \text{completeness}}
\]

This metric is independent of the absolute values of the labels: a permutation of the class or cluster label values won’t change the score value in any way.

This metric is furthermore symmetric: switching label_true with label_pred will return the same score value. This can be useful to measure the agreement of two independent label assignments strategies on the same dataset when the real ground truth is not known.

**Parameters**

labels_true : int array, shape = [n_samples]

ground truth class labels to be used as a reference

labels_pred : array, shape = [n_samples]

cluster labels to evaluate

**Returns**

completeness : float

score between 0.0 and 1.0. 1.0 stands for perfectly complete labeling
Perfect labelings are both homogeneous and complete, hence have score 1.0:

```
>>> from sklearn.metrics.cluster import v_measure_score
>>> v_measure_score([0, 0, 1, 1], [0, 0, 1, 1])
1.0
>>> v_measure_score([0, 0, 1, 1], [1, 1, 0, 0])
1.0
```

Labelings that assign all classes members to the same clusters are complete but not homogeneous, hence penalized:

```
>>> v_measure_score([0, 0, 1, 2], [0, 0, 1, 1])
0.8...
>>> v_measure_score([0, 1, 2, 3], [0, 0, 1, 1])
0.66...
```

Labelings that have pure clusters with members coming from the same classes are homogeneous but unnecessary splits harms completeness and thus penalize V-measure as well:

```
>>> v_measure_score([0, 0, 1, 1], [0, 0, 1, 2])
0.8...
>>> v_measure_score([0, 0, 1, 1], [0, 1, 2, 3])
0.66...
```

If classes members are completely splitted across different clusters, the assignment is totally incomplete, hence the v-measure is null:

```
>>> v_measure_score([0, 0, 0, 0], [0, 1, 2, 3])
0.0
```

Clusters that include samples from totally different classes totally destroy the homogeneity of the labeling, hence:

```
>>> v_measure_score([0, 0, 1, 1], [0, 0, 0, 0])
0.0
```

**Pairwise metrics**

The **sklearn.metrics.pairwise** submodule implements utilities to evaluate pairwise distances or affinity of sets of samples.

This module contains both distance metrics and kernels. A brief summary is given on the two here.

Distance metrics are a function \( d(a, b) \) such that \( d(a, b) < d(a, c) \) if objects \( a \) and \( b \) are considered “more similar” to objects \( a \) and \( c \). Two objects exactly alike would have a distance of zero. One of the most popular examples is Euclidean distance. To be a ‘true’ metric, it must obey the following four conditions:
1. \(d(a, b) \geq 0\), for all \(a\) and \(b\)
2. \(d(a, b) = 0\), if and only if \(a = b\), positive definiteness
3. \(d(a, b) = d(b, a)\), symmetry
4. \(d(a, c) \leq d(a, b) + d(b, c)\), the triangle inequality

Kernels are measures of similarity, i.e. \(s(a, b) > s(a, c)\) if objects \(a\) and \(b\) are considered “more similar” to objects \(a\) and \(c\). A kernel must also be positive semi-definite.

There are a number of ways to convert between a distance metric and a similarity measure, such as a kernel. Let \(D\) be the distance, and \(S\) be the kernel:
1. \(S = \exp(-D \times \text{gamma})\), where one heuristic for choosing \(\text{gamma}\) is \(1 / \text{num}\_\text{features}\)
2. \(S = 1 / (D / \text{np}\_\text{max}(D))\)

scikit-learn user guide, Release 0.12-git

sklearn.metrics.pairwise.euclidean_distances

Considering the rows of \(X\) (and \(Y=\text{X}\)) as vectors, compute the distance matrix between each pair of vectors. For efficiency reasons, the euclidean distance between a pair of row vector \(x\) and \(y\) is computed as:
\[
\text{dist}(x, y) = \sqrt{\text{dot}(x, x) - 2 \times \text{dot}(x, y) + \text{dot}(y, y)}
\]

This formulation has two main advantages. First, it is computationally efficient when dealing with sparse data. Second, if \(x\) varies but \(y\) remains unchanged, then the right-most dot-product \(\text{dot}(y, y)\) can be pre-computed.

**Parameters**

- **\(X\)** : array-like, sparse matrix, shape = [n_samples_1, n_features]
- **\(Y\)** : array-like, sparse matrix, shape = [n_samples_2, n_features]
- **\(Y\_\text{norm}\_\text{sq}uared\)** : array-like, shape = [n_samples_2], optional
  - Pre-computed dot-products of vectors in \(Y\) (e.g., \((Y**2)\).sum(axis=1))
- **\(\text{squared}\)** : boolean, optional
  - Return squared Euclidean distances.

**Returns**

- **\(\text{distances}\)** : array, sparse matrix, shape = [n_samples_1, n_samples_2]

**Examples**

```python
>>> from sklearn.metrics.pairwise import euclidean_distances
>>> X = [[0, 1], [1, 1]]
>>> # distance between rows of X
```
euclidean_distances(X, X)
array([[ 0.,  1.],
       [ 1.,  0.]])

# get distance to origin
euclidean_distances(X, [[0, 0]])
array([[ 1.],
       [1.41421356]])

sklearn.metrics.pairwise.manhattan_distances

sklearn.metrics.pairwise.manhattan_distances(X, Y=None, sum_over_features=True)
Compute the L1 distances between the vectors in X and Y.

With sum_over_features equal to False it returns the componentwise distances.

Parameters:
X : array_like
    An array with shape (n_samples_X, n_features).
Y : array_like, optional
    An array with shape (n_samples_Y, n_features).
sum_over_features : bool, default=True
    If True the function returns the pairwise distance matrix else it returns the componentwise L1 pairwise-distances.

Returns:
D : array
    If sum_over_features is False shape is (n_samples_X * n_samples_Y, n_features) and D contains the componentwise L1 pairwise-distances (ie. absolute difference), else shape is (n_samples_X, n_samples_Y) and D contains the pairwise L1 distances.

Examples

from sklearn.metrics.pairwise import manhattan_distances
manhattan_distances(3, 3)
array([[ 0.]])
manhattan_distances(3, 2)
array([[ 1.]])
manhattan_distances(2, 3)
array([[1.]])
manhattan_distances([[1, 2], [3, 4]], [[1, 2], [0, 3]])
array([[ 0.,  2.],
       [ 4.,  4.]])
import numpy as np
X = np.ones((1, 2))
y = 2 * np.ones((2, 2))
manhattan_distances(X, y, sum_over_features=False)
array([[1., 1.],
       [1., 1.]])

sklearn.metrics.pairwise.linear_kernel

sklearn.metrics.pairwise.linear_kernel(X, Y=None)
Compute the linear kernel between X and Y.

1.8. Reference
**Parameters** X : array of shape (n_samples_1, n_features)

Y : array of shape (n_samples_2, n_features)

**Returns** Gram matrix : array of shape (n_samples_1, n_samples_2)

sklearn.metrics.pairwise.polynomial_kernel

sklearn.metrics.pairwise.polynomial_kernel(X, Y=None, degree=3, gamma=0, coef0=1)

Compute the polynomial kernel between X and Y:

\[ K(X, Y) = (\text{gamma} \langle X, Y \rangle + \text{coef0})^{\text{degree}} \]

**Parameters** X : array of shape (n_samples_1, n_features)

Y : array of shape (n_samples_2, n_features)

degree : int

**Returns** Gram matrix : array of shape (n_samples_1, n_samples_2)

sklearn.metrics.pairwise.rbf_kernel

sklearn.metrics.pairwise.rbf_kernel(X, Y=None, gamma=0)

Compute the rbf (gaussian) kernel between X and Y:

\[ K(X, Y) = \exp(-\text{gamma} \|X-Y\|^2) \]

**Parameters** X : array of shape (n_samples_1, n_features)

Y : array of shape (n_samples_2, n_features)

gamma : float

**Returns** Gram matrix : array of shape (n_samples_1, n_samples_2)

sklearn.metrics.pairwise.distance_metrics

sklearn.metrics.pairwise.distance_metrics()

Valid metrics for pairwise_distances

This function simply returns the valid pairwise distance metrics. It exists, however, to allow for a verbose description of the mapping for each of the valid strings.

The valid distance metrics, and the function they map to, are:

<table>
<thead>
<tr>
<th>metric</th>
<th>Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>’cityblock’</td>
<td>sklearn.pairwise.manhattan_distances</td>
</tr>
<tr>
<td>’euclidean’</td>
<td>sklearn.pairwise.euclidean_distances</td>
</tr>
<tr>
<td>’l1’</td>
<td>sklearn.pairwise.manhattan_distances</td>
</tr>
<tr>
<td>’l2’</td>
<td>sklearn.pairwise.euclidean_distances</td>
</tr>
<tr>
<td>’manhattan’</td>
<td>sklearn.pairwise.manhattan_distances</td>
</tr>
</tbody>
</table>
sklearn.metrics.pairwise.pairwise_distances

**sklearn.metrics.pairwise.pairwise_distances** (*X*, *Y=None*, *metric='euclidean'*, *n_jobs=1*, **kwds**)

Compute the distance matrix from a vector array *X* and optional *Y*.

This method takes either a vector array or a distance matrix, and returns a distance matrix. If the input is a vector array, the distances are computed. If the input is a distances matrix, it is returned instead.

This method provides a safe way to take a distance matrix as input, while preserving compatibility with many other algorithms that take a vector array.

If *Y* is given (default is None), then the returned matrix is the pairwise distance between the arrays from both *X* and *Y*.

Please note that support for sparse matrices is currently limited to those metrics listed in pairwise.pairwise_distance_functions.

Valid values for metric are:


See the documentation for scipy.spatial.distance for details on these metrics.

Note in the case of ‘euclidean’ and ‘cityblock’ (which are valid scipy.spatial.distance metrics), the values will use the scikit-learn implementation, which is faster and has support for sparse matrices. For a verbose description of the metrics from scikit-learn, see the __doc__ of the sklearn.pairwise.distance_metrics function.

**Parameters**  

**X** : array[n_samples_a, n_samples_a] if metric == “precomputed”, or, [n_samples_a, n_features] otherwise

Array of pairwise distances between samples, or a feature array.

**Y** : array[n_samples_b, n_features]

A second feature array only if *X* has shape [n_samples_a, n_features].

**metric** : string, or callable

The metric to use when calculating distance between instances in a feature array. If metric is a string, it must be one of the options allowed by scipy.spatial.distance.pdist for its metric parameter, or a metric listed in pairwise.pairwise_distance_functions. If metric is “precomputed”, *X* is assumed to be a distance matrix. Alternatively, if metric is a callable function, it is called on each pair of instances (rows) and the resulting value recorded. The callable should take two arrays from *X* as input and return a value indicating the distance between them.

**n_jobs** : int

The number of jobs to use for the computation. This works by breaking down the pairwise matrix into n_jobs even slices and computing them in parallel.

If -1 all CPUs are used. If 1 is given, no parallel computing code is used at all, which is useful for debugging. For n_jobs below -1, (n_cpus + 1 - n_jobs) are used. Thus for n_jobs = -2, all CPUs but one are used.

**kwds** : optional keyword parameters

### 1.8. Reference
Any further parameters are passed directly to the distance function. If using a scipy.spatial.distance metric, the parameters are still metric dependent. See the scipy docs for usage examples.

**Returns** 

D : array [n_samples_a, n_samples_a] or [n_samples_a, n_samples_b]  
A distance matrix D such that D_[i, j] is the distance between the ith and jth vectors of the given matrix X, if Y is None. If Y is not None, then D_[i, j] is the distance between the ith array from X and the jth array from Y.

### sklearn.metrics.pairwise.pairwise_kernels

**sklearn.metrics.pairwise.pairwise_kernels**()  
Valid metrics for pairwise_kernels  
This function simply returns the valid pairwise distance metrics. It exists, however, to allow for a verbose description of the mapping for each of the valid strings.

<table>
<thead>
<tr>
<th>metric</th>
<th>Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>'linear'</td>
<td>sklearn.pairwise.linear_kernel</td>
</tr>
<tr>
<td>'poly'</td>
<td>sklearn.pairwise.polynomial_kernel</td>
</tr>
<tr>
<td>'polynomial'</td>
<td>sklearn.pairwise.polynomial_kernel</td>
</tr>
<tr>
<td>'rbf'</td>
<td>sklearn.pairwise.rbf_kernel</td>
</tr>
<tr>
<td>'sigmoid'</td>
<td>sklearn.pairwise.sigmoid_kernel</td>
</tr>
</tbody>
</table>

### sklearn.metrics.pairwise.pairwise_kernels

**sklearn.metrics.pairwise.pairwise_kernels**(X, Y=None, metric='linear', filter_params=False, n_jobs=1, **kwds)  
Compute the kernel between arrays X and optional array Y.  
This method takes either a vector array or a kernel matrix, and returns a kernel matrix. If the input is a vector array, the kernels are computed. If the input is a kernel matrix, it is returned instead.  
This method provides a safe way to take a kernel matrix as input, while preserving compatability with many other algorithms that take a vector array.  
If Y is given (default is None), then the returned matrix is the pairwise kernel between the arrays from both X and Y.  
**Valid values for metric are:** ['rbf', 'sigmoid', 'polynomial', 'poly', 'linear']

**Parameters**  

X : array [n_samples_a, n_samples_a] if metric == "precomputed", or, [n_samples_a, n_features] otherwise  
Array of pairwise kernels between samples, or a feature array.  
Y : array [n_samples_b, n_features]  
A second feature array only if X has shape [n_samples_a, n_features].  
metric : string, or callable  
The metric to use when calculating kernel between instances in a feature array. If metric is a string, it must be one of the metrics in pairwise.pairwise_kernel_functions. If metric is "precomputed", X is assumed to be a kernel matrix. Alternatively, if metric is a callable function, it is called on each pair of instances (rows) and the resulting value recorded. The callable should take two arrays from X as input and return a value indicating the distance between them.
n_jobs : int

The number of jobs to use for the computation. This works by breaking down the pairwise matrix into n_jobs even slices and computing them in parallel.

If -1 all CPUs are used. If 1 is given, no parallel computing code is used at all, which is useful for debugging. For n_jobs below -1, (n_cpus + 1 - n_jobs) are used. Thus for n_jobs = -2, all CPUs but one are used.

filter_params : boolean

Whether to filter invalid parameters or not.

**kwds** : optional keyword parameters

Any further parameters are passed directly to the kernel function.

Returns K : array [n_samples_a, n_samples_a] or [n_samples_a, n_samples_b]

A kernel matrix K such that K_{i, j} is the kernel between the ith and jth vectors of the given matrix X, if Y is None. If Y is not None, then K_{i, j} is the kernel between the ith array from X and the jth array from Y.

1.8.18 sklearn.mixture: Gaussian Mixture Models

The sklearn.mixture module implements mixture modeling algorithms.

User guide: See the Gaussian mixture models section for further details.

<table>
<thead>
<tr>
<th>sklearn.mixture.GMM( [n_components, covariance_type, ...])</th>
<th>Gaussian Mixture Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>mixture.DPGMM( [n_components, ...])</td>
<td>Variational Inference for the Infinite Gaussian Mixture Model.</td>
</tr>
<tr>
<td>mixture.VBGMM( [n_components, ...])</td>
<td>Variational Inference for the Gaussian Mixture Model</td>
</tr>
</tbody>
</table>

**sklearn.mixture.GMM**

class sklearn.mixture.GMM( n_components=1, covariance_type=’diag’, random_state=None, thresh=0.01, min_covar=0.001, n_iter=100, n_init=1, params=’wmc’, init_params=’wmc’)

Gaussian Mixture Model

Representation of a Gaussian mixture model probability distribution. This class allows for easy evaluation of, sampling from, and maximum-likelihood estimation of the parameters of a GMM distribution.

Initializes parameters such that every mixture component has zero mean and identity covariance.

Parameters n_components : int, optional

Number of mixture components. Defaults to 1.

covariance_type : string, optional

String describing the type of covariance parameters to use. Must be one of ‘spherical’, ‘tied’, ‘diag’, ‘full’. Defaults to ‘diag’.

random_state: RandomState or an int seed (0 by default) :

A random number generator instance

min_covar : float, optional

Floor on the diagonal of the covariance matrix to prevent overfitting. Defaults to 1e-3.

1.8. Reference
thresh : float, optional
    Convergence threshold.

n_iter : int, optional
    Number of EM iterations to perform.

n_init : int, optional
    Number of initializations to perform. the best results is kept

params : string, optional
    Controls which parameters are updated in the training process. Can contain any combina-
    tion of ‘w’ for weights, ‘m’ for means, and ‘c’ for covars. Defaults to ‘wmc’.

init_params : string, optional
    Controls which parameters are updated in the initialization process. Can contain any
    combination of ‘w’ for weights, ‘m’ for means, and ‘c’ for covars. Defaults to ‘wmc’.

See Also:

DPGMM: Infinite gaussian mixture model, using the dirichlet process, fit with a variational algorithm

VBGMM: Finite gaussian mixture model fit with a variational algorithm, better for situations where there might be
    too little data to get a good estimate of the covariance matrix.

Examples

>>> import numpy as np
>>> from sklearn import mixture
>>> np.random.seed(1)
>>> g = mixture.GMM(n_components=2)
>>> # Generate random observations with two modes centered on 0
>>> # and 10 to use for training.
>>> obs = np.concatenate((np.random.randn(100, 1),
...    10 + np.random.randn(300, 1)))
>>> g.fit(obs)
GMM(covariance_type=None, init_params='wmc', min_covar=0.001,
    n_components=2, n_init=1, n_iter=100, params='wmc',
    random_state=None, thresh=0.01)
>>> np.round(g.weights_, 2)
array([0.75, 0.25])
>>> np.round(g.means_, 2)
array([[10.05],
    [ 0.06]])
>>> np.round(g.covars_, 2)
array([[1.02],
    [0.96]])
>>> g.predict([[0], [2], [9], [10]])
array([1, 1, 0, 0])
>>> np.round(g.score([[0], [2], [9], [10]]), 2)
array([-2.19, -4.58, -1.75, -1.21])
>>> # Refit the model on new data (initial parameters remain the
>>> # same), this time with an even split between the two modes.
>>> g.fit(20 * [[0]] + 20 * [[10]])
GMM(covariance_type=None, init_params='wmc', min_covar=0.001,
    n_components=2, n_init=1, n_iter=100, params='wmc',
    random_state=None, thresh=0.01)
```python
>>> np.round(g.weights_, 2)
array([ 0.5, 0.5])
```

### Attributes

<table>
<thead>
<tr>
<th>Attribute</th>
<th>Type</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>weights_</code></td>
<td>array, shape (n_components,)</td>
<td>This attribute stores the mixing weights for each mixture component.</td>
</tr>
<tr>
<td><code>means_</code></td>
<td>array, shape (n_components, n_features)</td>
<td>Mean parameters for each mixture component.</td>
</tr>
</tbody>
</table>
| `covars_` | array | Covariance parameters for each mixture component. The shape depends on `covariance_type`:
| | | (n_components,)
| | | (n_features, n_features)
| | | (n_components, n_features)
| | | (n_components, n_features, n_features) |
| `converged_` | bool | True when convergence was reached in `fit()`, False otherwise. |

### Methods

<table>
<thead>
<tr>
<th>Method</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>aic(X)</code></td>
<td>Akaike information criterion for the current model fit and the proposed data</td>
</tr>
<tr>
<td><code>bic(X)</code></td>
<td>Bayesian information criterion for the current model fit and the proposed data</td>
</tr>
<tr>
<td><code>decode(*args, **kwargs)</code></td>
<td>DEPRECATED: will be removed in v0.12;</td>
</tr>
<tr>
<td><code>eval(X)</code></td>
<td>Evaluate the model on data</td>
</tr>
<tr>
<td><code>fit(X, **kwargs)</code></td>
<td>Estimate model parameters with the expectation-maximization algorithm.</td>
</tr>
<tr>
<td><code>get_params([deep])</code></td>
<td>Get parameters for the estimator</td>
</tr>
<tr>
<td><code>predict(X)</code></td>
<td>Predict label for data.</td>
</tr>
<tr>
<td><code>predict_proba(X)</code></td>
<td>Predict posterior probability of data under each Gaussian</td>
</tr>
<tr>
<td><code>rvs(*args, **kwargs)</code></td>
<td>DEPRECATED: will be removed in v0.12;</td>
</tr>
<tr>
<td><code>sample([n_samples, random_state])</code></td>
<td>Generate random samples from the model.</td>
</tr>
<tr>
<td><code>score(X)</code></td>
<td>Compute the log probability under the model.</td>
</tr>
<tr>
<td><code>set_params(**params)</code></td>
<td>Set the parameters of the estimator.</td>
</tr>
</tbody>
</table>

__init__ (n_components=1, covariance_type='diag', random_state=None, thresh=0.01, min_covar=0.001, n_iter=100, n_init=1, params='wmc', init_params='wmc')

### References

1.8. Reference 543
decode (*args, **kwargs)

DEPRECATED: will be removed in v0.12; use the score or predict method instead, depending on the question.

Find most likely mixture components for each point in X.

DEPRECATED IN VERSION 0.10; WILL BE REMOVED IN VERSION 0.12 use the score or predict method instead, depending on the question.

Parameters X : array_like, shape (n, n_features)
List of n_features-dimensional data points. Each row corresponds to a single data point.

Returns logprobs : array_like, shape (n_samples,)
Log probability of each point in obs under the model.

components[array_like, shape (n_samples,)] Index of the most likely hood mixture components for each observation.

eval (X)
Evaluate the model on data

Compute the log probability of X under the model and return the posterior distribution (responsibilities) of each mixture component for each element of X.

Parameters X: array_like, shape (n_samples, n_features):
List of n_features-dimensional data points. Each row corresponds to a single data point.

Returns logprob: array_like, shape (n_samples,):
Log probabilities of each data point in X

responsibilities: array_like, shape (n_samples, n_components):
Posterior probabilities of each mixture component for each observation

fit (X, **kwargs)
Estimate model parameters with the expectation-maximization algorithm.

A initialization step is performed before entering the em algorithm. If you want to avoid this step, set the keyword argument init_params to the empty string '' when creating the GMM object. Likewise, if you would like just to do an initialization, set n_iter=0.

Parameters X : array_like, shape (n, n_features)
List of n_features-dimensional data points. Each row corresponds to a single data point.

get_params (deep=True)
Get parameters for the estimator

Parameters deep: boolean, optional:
If True, will return the parameters for this estimator and contained subobjects that are estimators.

predict (X)
Predict label for data.

Parameters X : array-like, shape = [n_samples, n_features]

Returns C : array, shape = (n_samples,)
**predict_proba** *(X)*
Predict posterior probability of data under each Gaussian in the model.

**Parameters**

* X : array-like, shape = [n_samples, n_features]

**Returns**

* responsibilities : array-like, shape = (n_samples, n_components)

Returns the probability of the sample for each Gaussian (state) in the model.

**rvs** *(args, **kwargs)*
DEPRECATED: will be removed in v0.12; use the score or predict method instead, depending on the question
Generate random samples from the model.

**Parameters**

* n_samples : int, optional
  Number of samples to generate. Defaults to 1.

**Returns**

* X : array-like, shape (n_samples, n_features)
  List of samples

**score** *(X)*
Compute the log probability under the model.

**Parameters**

* X : array_like, shape (n_samples, n_features)

**Returns**

* logprob : array_like, shape (n_samples,)
  Log probabilities of each data point in X

**set_params** *(**params)*
Set the parameters of the estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The former have parameters of the form `<component>__<parameter>` so that it’s possible to update each component of a nested object.

**Returns**

* self :

**sklearn.mixture.DPGMM**

**class**

* **sklearn.mixture.DPGMM** *(n_components=1, covariance_type='diag', alpha=1.0, random_state=None, thresh=0.01, verbose=False, min_covar=None, n_iter=10, params='wmc', init_params='wmc')*

Variational Inference for the Infinite Gaussian Mixture Model.

DPGMM stands for Dirichlet Process Gaussian Mixture Model, and it is an infinite mixture model with the Dirichlet Process as a prior distribution on the number of clusters. In practice the approximate inference algorithm uses a truncated distribution with a fixed maximum number of components, but almost always the number of components actually used depends on the data.

Stick-breaking Representation of a Gaussian mixture model probability distribution. This class allows for easy and efficient inference of an approximate posterior distribution over the parameters of a Gaussian mixture model with a variable number of components (smaller than the truncation parameter n_components).
Initialization is with normally-distributed means and identity covariance, for proper convergence.

**Parameters**

- **n_components**: `int, optional`:
  - Number of mixture components. Defaults to 1.

- **covariance_type**: `string, optional`:
  - String describing the type of covariance parameters to use. Must be one of ‘spherical’, ‘tied’, ‘diag’, ‘full’. Defaults to ‘diag’.

- **alpha**: `float, optional`:
  - Real number representing the concentration parameter of the dirichlet process. Intuitively, the Dirichlet Process is as likely to start a new cluster for a point as it is to add that point to a cluster with alpha elements. A higher alpha means more clusters, as the expected number of clusters is $\alpha \log(N)$. Defaults to 1.

- **thresh**: `float, optional`:
  - Convergence threshold.

- **n_iter**: `int, optional`:
  - Maximum number of iterations to perform before convergence.

- **params**: `string, optional`:
  - Controls which parameters are updated in the training process. Can contain any combination of ‘w’ for weights, ‘m’ for means, and ‘c’ for covars. Defaults to ‘wmc’.

- **init_params**: `string, optional`:
  - Controls which parameters are updated in the initialization process. Can contain any combination of ‘w’ for weights, ‘m’ for means, and ‘c’ for covars. Defaults to ‘wmc’.

**See Also:**

- **GMM**: Finite Gaussian mixture model fit with EM
- **VBGMM**: Finite Gaussian mixture model fit with a variational algorithm, better for situations where there might be too little data to get a good estimate of the covariance matrix.
Attributes

<table>
<thead>
<tr>
<th>Attribute</th>
<th>Type</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>covariance_type</td>
<td>string</td>
<td>String describing the type of covariance parameters used by the DP-GMM. Must be one of ‘spherical’, ‘tied’, ‘diag’, ‘full’.</td>
</tr>
<tr>
<td>n_components</td>
<td>int</td>
<td>Number of mixture components.</td>
</tr>
<tr>
<td>weights_</td>
<td>array, shape (n_components,)</td>
<td>Mixing weights for each mixture component.</td>
</tr>
<tr>
<td>means_</td>
<td>array, shape (n_components, n_features)</td>
<td>Mean parameters for each mixture component.</td>
</tr>
<tr>
<td>precisions_</td>
<td>array</td>
<td>Precision (inverse covariance) parameters for each mixture component. The shape depends on covariance_type: ('n_components', 'n_features') ('n_features', 'n_features') ('n_components', 'n_features') ('n_components', 'n_features', 'n_features')</td>
</tr>
<tr>
<td>converged_</td>
<td>bool</td>
<td>True when convergence was reached in fit(), False otherwise.</td>
</tr>
</tbody>
</table>

Methods

- **aic**(X) Akaike information criterion for the current model fit
- **bic**(X) Bayesian information criterion for the current model fit
- **decode**(args, **kwargs) DEPRECATED: will be removed in v0.12;
- **eval**(X) Evaluate the model on data
- **fit**(X, **kwargs) Estimate model parameters with the variational algorithm.
- **get_params**(deep) Get parameters for the estimator
- **lower_bound**(X, z) returns a lower bound on model evidence based on X and membership
- **predict**(X) Predict label for data.
- **predict_proba**(X) Predict posterior probability of data under each Gaussian
- **rvs**(args, **kwargs) DEPRECATED: will be removed in v0.12;
- **sample**(n_samples, random_state) Generate random samples from the model.
- **score**(X) Compute the log probability under the model.
- **set_params**(**params) Set the parameters of the estimator.

**__init__**(n_components=1, covariance_type='diag', alpha=1.0, random_state=None, thresh=0.01, verbose=False, min_covar=None, n_iter=10, params='wmc', init_params='wmc')

**aic**(X) Akaike information criterion for the current model fit and the proposed data

Parameters X : array of shape(n_samples, n_dimensions)

Returns aic: float (the lower the better) :

**bic**(X) Bayesian information criterion for the current model fit and the proposed data

Parameters X : array of shape(n_samples, n_dimensions)
Returns bic: float (the lower the better)

**decode** (*args, **kwargs)

DEPRECATED: will be removed in v0.12; use the score or predict method instead, depending on the question

Find most likely mixture components for each point in X.

DEPRECATED IN VERSION 0.10; WILL BE REMOVED IN VERSION 0.12 use the score or predict method instead, depending on the question.

**Parameters**

- **X**: array_like, shape (n, n_features)
  List of n_features-dimensional data points. Each row corresponds to a single data point.

**Returns**

- **logprobs**: array_like, shape (n_samples,)
  Log probability of each point in obs under the model.

- **components**: array_like, shape (n_samples,)
  Index of the most likely mixture components for each observation.

**eval** (X)

Evaluate the model on data

Compute the bound on log probability of X under the model and return the posterior distribution (responsibilities) of each mixture component for each element of X.

This is done by computing the parameters for the mean-field of z for each observation.

**Parameters**

- **X**: array_like, shape (n_samples, n_features)
  List of n_features-dimensional data points. Each row corresponds to a single data point.

**Returns**

- **logprob**: array_like, shape (n_samples,)
  Log probabilities of each data point in X

- **responsibilities**: array_like, shape (n_samples, n_components)
  Posterior probabilities of each mixture component for each observation.

**fit** (X, **kwargs)

Estimate model parameters with the variational algorithm.

For a full derivation and description of the algorithm see doc/dp-derivation/dp-derivation.tex

A initialization step is performed before entering the em algorithm. If you want to avoid this step, set the keyword argument init_params to the empty string ‘’ when when creating the object. Likewise, if you would like just to do an initialization, set n_iter=0.

**Parameters**

- **X**: array_like, shape (n, n_features)
  List of n_features-dimensional data points. Each row corresponds to a single data point.

**get_params** (deep=True)

Get parameters for the estimator

**Parameters**

- **deep**: boolean, optional
  If True, will return the parameters for this estimator and contained subobjects that are estimators.

**lower_bound** (X, z)

returns a lower bound on model evidence based on X and membership
predict \((X)\)
Predict label for data.

Parameters \(X\) : array-like, shape = [n_samples, n_features]
Returns \(C\) : array, shape = (n_samples,)

predict_proba \((X)\)
Predict posterior probability of data under each Gaussian in the model.

Parameters \(X\) : array-like, shape = [n_samples, n_features]
Returns responsibilities : array-like, shape = (n_samples, n_components)
              Returns the probability of the sample for each Gaussian (state) in the model.

rvs (**args, **kwargs)
DEPRECATED: will be removed in v0.12; use the score or predict method instead, depending on the question
Generate random samples from the model.

DEPRECATED IN VERSION 0.11; WILL BE REMOVED IN VERSION 0.12 use sample instead

sample \((n\_samples=1, \text{random\_state}=\text{None})\)
Generate random samples from the model.

Parameters \(n\_samples\) : int, optional
Number of samples to generate. Defaults to 1.
Returns \(X\) : array_like, shape (n_samples, n_features)
List of samples

score \((X)\)
Compute the log probability under the model.

Parameters \(X\) : array_like, shape (n_samples, n_features)
List of n_features-dimensional data points. Each row corresponds to a single data point.
Returns logprob : array_like, shape (n_samples,)
Log probabilities of each data point in X

set_params (**params)
Set the parameters of the estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The former have parameters of the form \(<\text{component}>__<\text{parameter}>\) so that it’s possible to update each component of a nested object.

Returns self:

sklearn.mixture.VBGMM

class sklearn.mixture.VBGMM \((n\_components=1, \text{covariance\_type}=\text{diag'}, \alpha=1.0, \text{random\_state}=\text{None}, \text{thresh}=0.01, \text{verbose}=\text{False}, \text{min\_covar}=\text{None}, \text{n\_iter}=10, \text{params}=\text{wmc'}, \text{init\_params}=\text{wmc'})\)
Variational Inference for the Gaussian Mixture Model
Variational inference for a Gaussian mixture model probability distribution. This class allows for easy and efficient inference of an approximate posterior distribution over the parameters of a Gaussian mixture model with a fixed number of components.

Initialization is with normally-distributed means and identity covariance, for proper convergence.

**Parameters**

- **n_components**: int, optional
  Number of mixture components. Defaults to 1.

- **covariance_type**: string, optional
  String describing the type of covariance parameters to use. Must be one of ‘spherical’, ‘tied’, ‘diag’, ‘full’. Defaults to ‘diag’.

- **alpha**: float, optional
  Real number representing the concentration parameter of the dirichlet distribution. Intuitively, the higher the value of alpha the more likely the variational mixture of Gaussians model will use all components it can. Defaults to 1.

**See Also:**

- **GMM**: Finite Gaussian mixture model fit with EM
- **DPGMM**: Infinite Gaussian mixture model, using the dirichlet process, fit with a variational algorithm

**Attributes**

<table>
<thead>
<tr>
<th>Attribute</th>
<th>Type</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>covariance_type</td>
<td>string</td>
<td>String describing the type of covariance parameters used by the DP-GMM. Must be one of ‘spherical’, ‘tied’, ‘diag’, ‘full’.</td>
</tr>
<tr>
<td>n_features</td>
<td>int</td>
<td>Dimensionality of the Gaussians.</td>
</tr>
<tr>
<td>n_components</td>
<td>int (read-only)</td>
<td>Number of mixture components.</td>
</tr>
<tr>
<td>weights_</td>
<td>array, shape (n_components,)</td>
<td>Mixing weights for each mixture component.</td>
</tr>
<tr>
<td>means_</td>
<td>array, shape (n_components, n_features)</td>
<td>Mean parameters for each mixture component.</td>
</tr>
<tr>
<td>precisions_</td>
<td>array</td>
<td>Precision (inverse covariance) parameters for each mixture component. The shape depends on covariance_type:</td>
</tr>
<tr>
<td>converged_</td>
<td>bool</td>
<td>True when convergence was reached in fit(). False otherwise.</td>
</tr>
</tbody>
</table>

**Methods**

- **aic(X)**: Akaike information criterion for the current model fit
### Table 1.144 – continued from previous page

<table>
<thead>
<tr>
<th>Method</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>bic(X)</code></td>
<td>Bayesian information criterion for the current model fit</td>
</tr>
<tr>
<td><code>decode(*args, **kwargs)</code></td>
<td>DEPRECATED: will be removed in v0.12; use the score or predict method instead, depending on the question</td>
</tr>
<tr>
<td><code>eval(X)</code></td>
<td>Evaluate the model on data</td>
</tr>
<tr>
<td><code>fit(X, **kwargs)</code></td>
<td>Estimate model parameters with the variational algorithm.</td>
</tr>
<tr>
<td><code>get_params([deep])</code></td>
<td>Get parameters for the estimator</td>
</tr>
<tr>
<td><code>lower_bound(X, z)</code></td>
<td>returns a lower bound on model evidence based on X and membership</td>
</tr>
<tr>
<td><code>predict(X)</code></td>
<td>Predict label for data</td>
</tr>
<tr>
<td><code>predict_proba(X)</code></td>
<td>Predict posterior probability of data under each Gaussian</td>
</tr>
<tr>
<td><code>rvs(*args, **kwargs)</code></td>
<td>DEPRECATED: will be removed in v0.12; use the score or predict method instead, depending on the question</td>
</tr>
<tr>
<td><code>sample([n_samples, random_state])</code></td>
<td>Generate random samples from the model.</td>
</tr>
<tr>
<td><code>score(X)</code></td>
<td>Compute the log probability under the model.</td>
</tr>
<tr>
<td><code>set_params(**params)</code></td>
<td>Set the parameters of the estimator</td>
</tr>
</tbody>
</table>

```python
__init__ (n_components=1, covariance_type='diag', alpha=1.0, random_state=None, thresh=0.01, verbose=False, min_covar=None, n_iter=10, params='wmc', init_params='wmc')
```

- **aic(X)**
  - Akaike information criterion for the current model fit and the proposed data
  - **Parameters X**: array of shape(n_samples, n_dimensions)
  - **Returns aic**: float (the lower the better):

- **bic(X)**
  - Bayesian information criterion for the current model fit and the proposed data
  - **Parameters X**: array of shape(n_samples, n_dimensions)
  - **Returns bic**: float (the lower the better):

- **decode(*args, **kwargs)**
  - DEPRECATED: will be removed in v0.12; use the score or predict method instead, depending on the question
  - Find most likely mixture components for each point in X.
  - DEPRECATED IN VERSION 0.10; WILL BE REMOVED IN VERSION 0.12 use the score or predict method instead, depending on the question.
  - **Parameters X**: array_like, shape (n, n_features)
  - List of n_features-dimensional data points. Each row corresponds to a single data point.
  - **Returns logprobs**: array_like, shape (n_samples,)
  - Log probability of each point in obs under the model.

- **components[array_like, shape (n_samples,)]**
  - Index of the most likely mixture components for each observation

- **eval(X)**
  - Evaluate the model on data
  - Compute the bound on log probability of X under the model and return the posterior distribution (responsibilities) of each mixture component for each element of X.
  - This is done by computing the parameters for the mean-field of z for each observation.
  - **Parameters X**: array_like, shape (n_samples, n_features)
List of n_features-dimensional data points. Each row corresponds to a single data point.

**Returns logprob** : array_like, shape (n_samples,)

Log probabilities of each data point in X

**responsibilities** : array_like, shape (n_samples, n_components)

Posterior probabilities of each mixture component for each observation

**fit** (X, **kwargs)

Estimate model parameters with the variational algorithm.

For a full derivation and description of the algorithm see doc/dp-derivation/dp-derivation.tex

A initialization step is performed before entering the em algorithm. If you want to avoid this step, set the keyword argument init_params to the empty string ‘’ when when creating the object. Likewise, if you would like just to do an initialization, set n_iter=0.

**Parameters X** : array_like, shape (n, n_features)

List of n_features-dimensional data points. Each row corresponds to a single data point.

**get_params** (deep=True)

Get parameters for the estimator

**Parameters deep** : boolean, optional

If True, will return the parameters for this estimator and contained subobjects that are estimators.

**lower_bound** (X, z)

returns a lower bound on model evidence based on X and membership

**predict** (X)

Predict label for data.

**Parameters X** : array-like, shape = [n_samples, n_features]

**Returns C** : array, shape = (n_samples,)

**predict_proba** (X)

Predict posterior probability of data under each Gaussian in the model.

**Parameters X** : array-like, shape = [n_samples, n_features]

**Returns responsibilities** : array-like, shape = (n_samples, n_components)

Returns the probability of the sample for each Gaussian (state) in the model.

**rvs** (*args, **kwargs)

DEPRECATED: will be removed in v0.12; use the score or predict method instead, depending on the question

Generate random samples from the model.

DEPRECATED IN VERSION 0.11; WILL BE REMOVED IN VERSION 0.12 use sample instead

**sample** (n_samples=1, random_state=None)

Generate random samples from the model.

**Parameters n_samples** : int, optional

Number of samples to generate. Defaults to 1.

**Returns X** : array_like, shape (n_samples, n_features)
List of samples

**score** (*X*)
Compute the log probability under the model.

**Parameters**
- **X**: array_like, shape (n_samples, n_features)
  List of n_features-dimensional data points. Each row corresponds to a single data point.

**Returns**
- **logprob**: array_like, shape (n_samples,)
  Log probabilities of each data point in X

**set_params** (**params**)
Set the parameters of the estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The former have parameters of the form `<component>__<parameter>` so that it’s possible to update each component of a nested object.

**Returns**
- **self** :

### 1.8.19 sklearn.multiclass: Multiclass and multilabel classification

**Multiclass and multilabel classification strategies**

This module implements multiclass learning algorithms:

- one-vs-the-rest / one-vs-all
- one-vs-one
- error correcting output codes

The estimators provided in this module are meta-estimators: they require a base estimator to be provided in their constructor. For example, it is possible to use these estimators to turn a binary classifier or a regressor into a multiclass classifier. It is also possible to use these estimators with multiclass estimators in the hope that their accuracy or runtime performance improves.

**User guide**: See the *Multiclass and multilabel algorithms* section for further details.

<table>
<thead>
<tr>
<th>Class</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>sklearn.multiclass.OneVsRestClassifier(estimator)</td>
<td>One-vs-the-rest (OvR) multiclass/multilabel strategy</td>
</tr>
<tr>
<td>sklearn.multiclass.OneVsOneClassifier(estimator)</td>
<td>One-vs-one multiclass strategy</td>
</tr>
<tr>
<td>sklearn.multiclass.OutputCodeClassifier(estimator[, ...])</td>
<td>(Error-Correcting) Output-Code multiclass strategy</td>
</tr>
</tbody>
</table>

**sklearn.multiclass.OneVsRestClassifier**

Also known as one-vs-all, this strategy consists in fitting one classifier per class. For each classifier, the class is fitted against all the other classes. In addition to its computational efficiency (only n_classes classifiers are needed), one advantage of this approach is its interpretability. Since each class is represented by one and one classifier only, it is possible to gain knowledge about the class by inspecting its corresponding classifier. This is the most commonly used strategy for multiclass classification and is a fair default choice.

This strategy can also be used for multilabel learning, where a classifier is used to predict multiple labels for instance, by fitting on a sequence of sequences of labels (e.g., a list of tuples) rather than a single target vector. For multilabel learning, the number of classes must be at least three, since otherwise OvR reduces to binary
classification.

**Parameters estimator** : estimator object

An estimator object implementing `fit` and one of `decision_function` or `predict_proba`.

**Attributes**

<table>
<thead>
<tr>
<th>Attribute</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>estimators_</td>
<td>list of <code>n_classes</code> estimators</td>
</tr>
<tr>
<td>label_binarizer_</td>
<td>LabelBinarizer object</td>
</tr>
<tr>
<td>multilabel_</td>
<td>boolean</td>
</tr>
</tbody>
</table>

**Methods**

- `fit(X, y)` Fit underlying estimators.
- `get_params([deep])` Get parameters for the estimator
- `predict(X)` Predict multi-class targets using underlying estimators.
- `score(X, y)`
- `set_params(**params)` Set the parameters of the estimator.

**__init__(estimator)**

- `fit (X, y)` Fit underlying estimators.

  **Parameters**

  - `X` : {array-like, sparse matrix}, shape = [n_samples, n_features]
    - Data.
  - `y` : array-like, shape = [n_samples]
    - or sequence of sequences, `len = n_samples` Multi-class targets. A sequence of sequences turns on multilabel classification.

  **Returns**

  - `self`:

- `get_params(deep=True)` Get parameters for the estimator

  **Parameters**

  - `deep` : boolean, optional
    - If True, will return the parameters for this estimator and contained subobjects that are estimators.

- `multilabel_` Whether this is a multilabel classifier

- `predict (X)` Predict multi-class targets using underlying estimators.

  **Parameters**

  - `X` : {array-like, sparse matrix}, shape = [n_samples, n_features]
    - Data.

  **Returns**

  - `y` : array-like, shape = [n_samples]
    - Predicted multi-class targets.
**set_params** (**params**)

Set the parameters of the estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The former have parameters of the form `<component>__<parameter>` so that it’s possible to update each component of a nested object.

**Returns** self :

**sklearn.multiclass.OneVsOneClassifier**

class sklearn.multiclass.OneVsOneClassifier (**estimator**)

One-vs-one multiclass strategy

This strategy consists in fitting one classifier per class pair. At prediction time, the class which received the most votes is selected. Since it requires to fit \( n_{\text{classes}} \times (n_{\text{classes}} - 1) / 2 \) classifiers, this method is usually slower than one-vs-the-rest, due to its \( O(n_{\text{classes}}^2) \) complexity. However, this method may be advantageous for algorithms such as kernel algorithms which don’t scale well with \( n_{\text{samples}} \). This is because each individual learning problem only involves a small subset of the data whereas, with one-vs-the-rest, the complete dataset is used \( n_{\text{classes}} \) times.

**Parameters**

- **estimator** : estimator object

  An estimator object implementing fit and predict.

**Attributes**

- **estimators_** : list of \( n_{\text{classes}} \times (n_{\text{classes}} - 1) / 2 \) estimators

  Estimators used for predictions.

- **classes_** : numpy array of shape \([n_{\text{classes}}]\)

  Array containing labels.

**Methods**

- **fit**(X, y)

  Fit underlying estimators.

- **get_params**(deep=True)

  Get parameters for the estimator

- **predict**(X)

  Predict multi-class targets using underlying estimators.

- **score**(X, y)

  Returns the mean accuracy on the given test data and labels.

- **set_params**(**params**)

  Set the parameters of the estimator.

**__init__**(**estimator**)

**fit**(X, y)

Fit underlying estimators.

**Parameters**

- **X** : array-like, sparse matrix, shape = [n_samples, n_features]

  Data.

- **y** : numpy array of shape [n_samples]

  Multi-class targets.

**Returns**

self :

**get_params**(deep=True)

Get parameters for the estimator
Parameters deep: boolean, optional:
If True, will return the parameters for this estimator and contained subobjects that are estimators.

predict(X)
Predict multi-class targets using underlying estimators.

Parameters X : array-like, sparse matrix, shape = [n_samples, n_features]
Data.

Returns y : numpy array of shape [n_samples]
Predicted multi-class targets.

score(X, y)
Returns the mean accuracy on the given test data and labels.

Parameters X : array-like, shape = [n_samples, n_features]
Training set.
y : array-like, shape = [n_samples]
Labels for X.

Returns z : float

set_params(**params)
Set the parameters of the estimator.
The method works on simple estimators as well as on nested objects (such as pipelines). The former have parameters of the form <component>__<parameter> so that it’s possible to update each component of a nested object.

Returns self:

sklearn.multiclass.OutputCodeClassifier

class sklearn.multiclass.OutputCodeClassifier(estimator, code_size=1.5, random_state=None)
(Error-Correcting) Output-Code multiclass strategy

Output-code based strategies consist in representing each class with a binary code (an array of 0s and 1s). At fitting time, one binary classifier per bit in the code book is fitted. At prediction time, the classifiers are used to project new points in the class space and the class closest to the points is chosen. The main advantage of these strategies is that the number of classifiers used can be controlled by the user, either for compressing the model (0 < code_size < 1) or for making the model more robust to errors (code_size > 1). See the documentation for more details.

Parameters estimator : estimator object
An estimator object implementing fit and one of decision_function or predict_proba.

code_size : float
Percentage of the number of classes to be used to create the code book. A number between 0 and 1 will require fewer classifiers than one-vs-the-rest. A number greater than 1 will require more classifiers than one-vs-the-rest.

random_state : numpy.RandomState, optional
The generator used to initialize the codebook. Defaults to numpy.random.
References

[R73], [R74], [R75]

Attributes

<table>
<thead>
<tr>
<th><code>estimators_</code></th>
<th>list of int(n_classes * code_size) estimators</th>
<th>Estimators used for predictions.</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>classes_</code></td>
<td>numpy array of shape [n_classes]</td>
<td>Array containing labels.</td>
</tr>
<tr>
<td><code>code_book_</code></td>
<td>nump array of shape [n_classes, code_size]</td>
<td>Binary array containing the code of each class.</td>
</tr>
</tbody>
</table>

Methods

```python
fit(X, y)  Fit underlying estimators.
get_params([deep])  Get parameters for the estimator
predict(X)  Predict multi-class targets using underlying estimators.
score(X, y)  Returns the mean accuracy on the given test data and labels.
set_params(**params)  Set the parameters of the estimator.
```

```
__init__(estimator, code_size=1.5, random_state=None)
fit(X, y)
    Fit underlying estimators.
    Parameters:
    X: {array-like, sparse matrix}, shape = [n_samples, n_features] :
        Data.
    y: nump array of shape [n_samples]
        Multi-class targets.
    Returns:
    self:
get_params (deep=True)
    Get parameters for the estimator
    Parameters:
    deep: boolean, optional :
        If True, will return the parameters for this estimator and contained subobjects that are estimators.
predict (X)
    Predict multi-class targets using underlying estimators.
    Parameters:
    X: {array-like, sparse matrix}, shape = [n_samples, n_features] :
        Data.
    Returns:
    y: nump array of shape [n_samples]
        Predicted multi-class targets.
score (X, y)
    Returns the mean accuracy on the given test data and labels.
    Parameters:
    X: array-like, shape = [n_samples, n_features]
        Training set.
```

1.8. Reference
y : array-like, shape = [n_samples]
   Labels for X.

Returns z : float

set_params (**params)
   Set the parameters of the estimator.

   The method works on simple estimators as well as on nested objects (such as pipelines). The former have
   parameters of the form <component>_<parameter> so that it’s possible to update each component
   of a nested object.

   Returns self :

   multiclass.fit_ovr(estimator, X, y)  Fit a one-vs-the-rest strategy.
   multiclass.predict_ovr(estimators, ...) Make predictions using the one-vs-the-rest strategy.
   multiclass.fit_ovo(estimator, X, y)  Fit a one-vs-one strategy.
   multiclass.predict_ovo(estimators, classes, X) Make predictions using the one-vs-one strategy.
   multiclass.fit_ecoc(estimator, X, y[, ...])  Fit an error-correcting output-code strategy.
   multiclass.predict_ecoc(estimators, classes, ...) Make predictions using the error-correcting output-code strategy.

sklearn.multiclass.fit_ovr

   multiclass.fit_ovr (estimator, X, y)
      Fit a one-vs-the-rest strategy.

sklearn.multiclass.predict_ovr

   multiclass.predict_ovr (estimators, label_binarizer, X)
      Make predictions using the one-vs-the-rest strategy.

sklearn.multiclass.fit_ovo

   multiclass.fit_ovo (estimator, X, y)
      Fit a one-vs-one strategy.

sklearn.multiclass.predict_ovo

   multiclass.predict_ovo (estimators, classes, X)
      Make predictions using the one-vs-one strategy.

sklearn.multiclass.fit_ecoc

   multiclass.fit_ecoc (estimator, X, y, code_size=1.5, random_state=None)
      Fit an error-correcting output-code strategy.

   Parameters estimator : estimator object
      An estimator object implementing fit and one of decision_function or predict_proba.

   code_size : float, optional :
      Percentage of the number of classes to be used to create the code book.
random_state: numpy.RandomState, optional:

The generator used to initialize the codebook. Defaults to numpy.random.

Returns:
estimators : list of int(n_classes * code_size) estimators

Estimators used for predictions.

classes : numpy array of shape [n_classes]

Array containing labels.

‘code_book_‘: numpy array of shape [n_classes, code_size] :

Binary array containing the code of each class.

sklearn.multiclass.predict_ecoc

sklearn.multiclass.predict_ecoc(estimators, classes, code_book, X)

Make predictions using the error-correcting output-code strategy.

1.8.20 sklearn.naive_bayes: Naive Bayes

The sklearn.naive_bayes module implements Naive Bayes algorithms. These are supervised learning methods based on applying Bayes’ theorem with strong (naive) feature independence assumptions.

User guide: See the Naive Bayes section for further details.

<table>
<thead>
<tr>
<th>naive_bayes.GaussianNB</th>
<th>Gaussian Naive Bayes (GaussianNB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>naive_bayes.MultinomialNB([alpha, fit_prior])</td>
<td>Naive Bayes classifier for multinomial models</td>
</tr>
<tr>
<td>naive_bayes.BernoulliNB([alpha, binarize, ...])</td>
<td>Naive Bayes classifier for multivariate Bernoulli models</td>
</tr>
</tbody>
</table>

sklearn.naive_bayes.GaussianNB

class sklearn.naive_bayes.GaussianNB

Gaussian Naive Bayes (GaussianNB)

Parameters:

X : array-like, shape = [n_samples, n_features]

Training vector, where n_samples in the number of samples and n_features is the number of features.

y : array, shape = [n_samples]

Target vector relative to X

Examples

```python
>>> import numpy as np
>>> X = np.array([[[-1, -1], [-2, -1], [-3, -2], [1, 1], [2, 1], [3, 2]]])
>>> Y = np.array([1, 1, 1, 2, 2, 2])
>>> from sklearn.naive_bayes import GaussianNB
>>> clf = GaussianNB()
>>> clf.fit(X, Y)
GaussianNB()
>>> print(clf.predict([[-0.8, -1]]))
[1]
```
Attributes

<table>
<thead>
<tr>
<th>Name</th>
<th>Type</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>class_prior_</td>
<td>array, shape = [n_classes]</td>
<td>probability of each class.</td>
</tr>
<tr>
<td>theta_</td>
<td>array, shape = [n_classes, n_features]</td>
<td>mean of each feature per class</td>
</tr>
<tr>
<td>sigma_</td>
<td>array, shape = [n_classes, n_features]</td>
<td>variance of each feature per class</td>
</tr>
</tbody>
</table>

Methods

- **fit**(X, y) Fit Gaussian Naive Bayes according to X, y
  - **Parameters**
    - X : array-like, shape = [n_samples, n_features]
      - Training vectors, where n_samples is the number of samples and n_features is the number of features.
    - y : array-like, shape = [n_samples]
      - Target values.
  - **Returns** self : object

- **get_params**(deep=True) Get parameters for the estimator
  - **Parameters**
    - deep : boolean, optional
      - If True, will return the parameters for this estimator and contained subobjects that are estimators.

- **predict**(X) Perform classification on an array of test vectors X.
  - **Parameters**
    - X : array-like, shape = [n_samples, n_features]
  - **Returns** C : array, shape = [n_samples]
    - Predicted target values for X

- **predict_log_proba**(X) Return log-probability estimates for the test vector X.

- **predict_proba**(X) Return probability estimates for the test vector X.

- **score**(X, y) Returns the mean accuracy on the given test data and labels.

- **set_params**(**params**) Set the parameters of the estimator.

__init__() x.__init__(...) initializes x; see help(type(x)) for signature

class prior

DEPRECATED: GaussianNB.class_prior is deprecated and will be removed in version 0.12. Please use GaussianNB.class_prior_ instead.

**fit**(X, y) Fit Gaussian Naive Bayes according to X, y

**Parameters**

- X : array-like, shape = [n_samples, n_features]
  - Training vectors, where n_samples is the number of samples and n_features is the number of features.
- y : array-like, shape = [n_samples]
  - Target values.

**Returns** self : object

**get_params**(deep=True)

Get parameters for the estimator

**Parameters**

- deep : boolean, optional
  - If True, will return the parameters for this estimator and contained subobjects that are estimators.

**predict**(X)

Perform classification on an array of test vectors X.

**Parameters**

- X : array-like, shape = [n_samples, n_features]

**Returns** C : array, shape = [n_samples]

- Predicted target values for X

**predict_log_proba**(X)

Return log-probability estimates for the test vector X.
Parameters $X$: array-like, shape = [n_samples, n_features]

Returns $C$: array-like, shape = [n_samples, n_classes]

Returns the log-probability of the sample for each class in the model, where classes are ordered arithmetically.

**predict_proba** ($X$)
Return probability estimates for the test vector $X$.

Parameters $X$: array-like, shape = [n_samples, n_features]

Returns $C$: array-like, shape = [n_samples, n_classes]

Returns the probability of the sample for each class in the model, where classes are ordered arithmetically.

score ($X, y$)
Returns the mean accuracy on the given test data and labels.

Parameters $X$: array-like, shape = [n_samples, n_features]

Training set.

$y$: array-like, shape = [n_samples]
Labels for $X$.

Returns $z$: float

set_params (**params)
Set the parameters of the estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The former have parameters of the form `<component>__<parameter>` so that it’s possible to update each component of a nested object.

Returns self:

**sigma**
DEPRECATED: GaussianNB.sigma is deprecated and will be removed in version 0.12. Please use GaussianNB.sigma_ instead.

**theta**
DEPRECATED: GaussianNB.theta is deprecated and will be removed in version 0.12. Please use GaussianNB.theta_ instead.

**sklearn.naive_bayes.MultinomialNB**

class **sklearn.naive_bayes.MultinomialNB** (alpha=1.0, fit_prior=True)

Naive Bayes classifier for multinomial models

The multinomial Naive Bayes classifier is suitable for classification with discrete features (e.g., word counts for text classification). The multinomial distribution normally requires integer feature counts. However, in practice, fractional counts such as tf-idf may also work.

Parameters alpha: float, optional (default=1.0) :

Additive (Laplace/Lidstone) smoothing parameter (0 for no smoothing).

fit_prior: boolean :

Whether to learn class prior probabilities or not. If false, a uniform prior will be used.
Notes

For the rationale behind the names `coef_` and `intercept_`, i.e. naive Bayes as a linear classifier, see J. Rennie et al. (2003), Tackling the poor assumptions of naive Bayes text classifiers, ICML.

Examples

```python
>>> import numpy as np
>>> X = np.random.randint(5, size=(6, 100))
>>> Y = np.array([1, 2, 3, 4, 5, 6])
>>> from sklearn.naive_bayes import MultinomialNB
>>> clf = MultinomialNB()
>>> clf.fit(X, Y)
MultinomialNB(alpha=1.0, fit_prior=True)
>>> print(clf.predict(X[2]))
[3]
```

Attributes

<table>
<thead>
<tr>
<th>Name</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>intercept_</code></td>
<td>Smoothened empirical log probability for each class.</td>
</tr>
<tr>
<td><code>class_log_prior_</code></td>
<td>Array, shape = [n_classes] Smoothed empirical log probability for each class.</td>
</tr>
<tr>
<td><code>feature_log_prob_</code></td>
<td>Array, shape = [n_classes, n_features] Empirical log probability of features given a class, P(x_i</td>
</tr>
<tr>
<td><code>coef_</code></td>
<td><code>intercept_</code> and <code>coef_</code> are properties referring to <code>class_log_prior_</code> and <code>feature_log_prob_</code>, respectively.</td>
</tr>
</tbody>
</table>

Methods

- `fit(X, y[, sample_weight, class_prior])` Fit Naive Bayes classifier according to X, y
- `get_params([deep])` Get parameters for the estimator
- `predict(X)` Perform classification on an array of test vectors X.
- `predict_log_proba(X)` Return log-probability estimates for the test vector X.
- `predict_proba(X)` Return probability estimates for the test vector X.
- `score(X, y)` Returns the mean accuracy on the given test data and labels.
- `set_params(**params)` Set the mean accuracy on the given test data and labels.

`__init__(alpha=1.0, fit_prior=True)`

**Parameters**

- `X`: array-like, sparse matrix, shape = [n_samples, n_features]
  Training vectors, where n_samples is the number of samples and n_features is the number of features.
- `y`: array-like, shape = [n_samples]
  Target values.
- `sample_weight`: array-like, shape = [n_samples], optional
  Weights applied to individual samples (1. for unweighted).
class_prior : array, shape [n_classes]
Custom prior probability per class. Overrides the fit_prior parameter.

Returns self : object

Returns self.

get_params (deep=True)
Get parameters for the estimator

Parameters deep: boolean, optional :
If True, will return the parameters for this estimator and contained subobjects that are
estimators.

predict (X)
Perform classification on an array of test vectors X.

Parameters X : array-like, shape = [n_samples, n_features]

Returns C : array, shape = [n_samples]
Predicted target values for X

predict_log_proba (X)
Return log-probability estimates for the test vector X.

Parameters X : array-like, shape = [n_samples, n_features]

Returns C : array-like, shape = [n_samples, n_classes]
Returns the log-probability of the sample for each class in the model, where classes are
ordered arithmetically.

predict_proba (X)
Return probability estimates for the test vector X.

Parameters X : array-like, shape = [n_samples, n_features]

Returns C : array-like, shape = [n_samples, n_classes]
Returns the probability of the sample for each class in the model, where classes are
ordered arithmetically.

score (X, y)
Returns the mean accuracy on the given test data and labels.

Parameters X : array-like, shape = [n_samples, n_features]
     Training set.
     y : array-like, shape = [n_samples]
     Labels for X.

Returns z : float

set_params (**params)
Set the parameters of the estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The former have
parameters of the form <component>__<parameter> so that it’s possible to update each component
of a nested object.

Returns self :
sklearn.naive_bayes.BernoulliNB

class sklearn.naive_bayes.BernoulliNB (alpha=1.0, binarize=0.0, fit_prior=True)

Naive Bayes classifier for multivariate Bernoulli models.

Like MultinomialNB, this classifier is suitable for discrete data. The difference is that while MultinomialNB works with occurrence counts, BernoulliNB is designed for binary/boolean features.

Parameters

alpha: float, optional (default=1.0) :
    Additive (Laplace/Lidstone) smoothing parameter (0 for no smoothing).

binarize: float or None, optional :
    Threshold for binarizing (mapping to booleans) of sample features. If None, input is presumed to already consist of binary vectors.

fit_prior: boolean :
    Whether to learn class prior probabilities or not. If false, a uniform prior will be used.

References


Examples

```python
>>> import numpy as np
>>> X = np.random.randint(2, size=(6, 100))
>>> Y = np.array([1, 2, 3, 4, 4, 5])
>>> from sklearn.naive_bayes import BernoulliNB
>>> clf = BernoulliNB()
>>> clf.fit(X, Y)
BernoulliNB(alpha=1.0, binarize=0.0, fit_prior=True)
>>> print(clf.predict(X[2]))
[3]
```

Attributes

- `class_log_prior_` array, shape = [n_classes] Log probability of each class (smoothed).
- `feature_log_prob_` array, shape = [n_classes, n_features] Empirical log probability of features given a class, P(x_i|y).

Methods

- `fit(X, y[, sample_weight, class_prior])` Fit Naive Bayes classifier according to X, y

Continued on next page
Table 1.153 – continued from previous page

<table>
<thead>
<tr>
<th>Method</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>get_params([deep])</code></td>
<td>Get parameters for the estimator</td>
</tr>
<tr>
<td><code>predict(X)</code></td>
<td>Perform classification on an array of test vectors X.</td>
</tr>
<tr>
<td><code>predict_log_proba(X)</code></td>
<td>Return log-probability estimates for the test vector X.</td>
</tr>
<tr>
<td><code>predict_proba(X)</code></td>
<td>Return probability estimates for the test vector X.</td>
</tr>
<tr>
<td><code>score(X, y)</code></td>
<td>Returns the mean accuracy on the given test data and labels.</td>
</tr>
<tr>
<td><code>set_params(**params)</code></td>
<td>Set the parameters of the estimator.</td>
</tr>
</tbody>
</table>

`__init__ (alpha=1.0, binarize=0.0, fit_prior=True)`

`fit (X, y, sample_weight=None, class_prior=None)`

Fit Naive Bayes classifier according to X, y

**Parameters**

- **X**: {array-like, sparse matrix}, shape = [n_samples, n_features]
  
  Training vectors, where n_samples is the number of samples and n_features is the number of features.

- **y**: array-like, shape = [n_samples]
  
  Target values.

- **sample_weight**: array-like, shape = [n_samples], optional
  
  Weights applied to individual samples (1. for unweighted).

- **class_prior**: array, shape [n_classes]
  
  Custom prior probability per class. Overrides the fit_prior parameter.

**Returns**

- **self**: object
  
  Returns self.

`get_params (deep=True)`

Get parameters for the estimator

**Parameters**

- **deep**: boolean, optional
  
  If True, will return the parameters for this estimator and contained subobjects that are estimators.

`predict (X)`

Perform classification on an array of test vectors X.

**Parameters**

- **X**: array-like, shape = [n_samples, n_features]

**Returns**

- **C**: array, shape = [n_samples]
  
  Predicted target values for X

`predict_log_proba (X)`

Return log-probability estimates for the test vector X.

**Parameters**

- **X**: array-like, shape = [n_samples, n_features]

**Returns**

- **C**: array-like, shape = [n_samples, n_classes]
  
  Returns the log-probability of the sample for each class in the model, where classes are ordered arithmetically.

`predict_proba (X)`

Return probability estimates for the test vector X.

**Parameters**

- **X**: array-like, shape = [n_samples, n_features]
**Returns C**: array-like, shape = \([n\_samples, n\_classes]\)

Returns the probability of the sample for each class in the model, where classes are ordered arithmetically.

**score** (\(X, y\))

Returns the mean accuracy on the given test data and labels.

**Parameters** \(X\) : array-like, shape = \([n\_samples, n\_features]\)

Training set.

\(y\) : array-like, shape = \([n\_samples]\)

Labels for \(X\).

**Returns** \(z\) : float

**set_params** (**params**)

Set the parameters of the estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The former have parameters of the form \(<component>__<parameter>\) so that it’s possible to update each component of a nested object.

**Returns** self:

### 1.8.21 `sklearn.neighbors`: Nearest Neighbors

The `sklearn.neighbors` module implements the k-nearest neighbors algorithm.

**User guide**: See the `Nearest Neighbors` section for further details.

| `neighbors.NearestNeighbors([n_neighbors, ...])` | Unsupervised learner for implementing neighbor searches. |
| `neighbors.KNeighborsClassifier([...])` | Classifier implementing the k-nearest neighbors vote. |
| `neighbors.RadiusNeighborsClassifier([...])` | Classifier implementing a vote among neighbors within a given radius. |
| `neighbors.KNeighborsRegressor([n_neighbors, ...])` | Regression based on k-nearest neighbors. |
| `neighbors.RadiusNeighborsRegressor([radius, ...])` | Regression based on neighbors within a fixed radius. |
| `neighbors.BallTree` | Ball Tree for fast nearest-neighbor searches. |
| `neighbors.NearestCentroid([metric, ...])` | Nearest centroid classifier. |

#### sklearn.neighbors.NearestNeighbors

**class** `sklearn.neighbors.NearestNeighbors` (\(n\_neighbors=5\), \(radius=1.0\), \(algorithm='auto'\), \(leaf\_size=30\), \(warn\_on\_equidistant=True\), \(p=2\))

Unsupervised learner for implementing neighbor searches.

**Parameters** \(n\_neighbors\) : int, optional (default = 5)

Number of neighbors to use by default for \(k\_neighbors\) queries.

\(radius\) : float, optional (default = 1.0)

Range of parameter space to use by default for :meth:`radius_neighbors` queries.

\(algorithm\) : [‘auto’, ‘ball_tree’, ‘kd_tree’, ‘brute’], optional

Algorithm used to compute the nearest neighbors:

• ‘ball_tree’ will use BallTree
• ‘kd_tree’ will use `scipy.spatial.cKDtree`
• ‘brute’ will use a brute-force search.
• ‘auto’ will attempt to decide the most appropriate algorithm based on the values passed to `fit` method.

Note: fitting on sparse input will override the setting of this parameter, using brute force.

**leaf_size**: int, optional (default = 30)

Leaf size passed to BallTree or cKDTree. This can affect the speed of the construction and query, as well as the memory required to store the tree. The optimal value depends on the nature of the problem.

**warn_on_equidistant**: boolean, optional. Defaults to True.

Generate a warning if equidistant neighbors are discarded. For classification or regression based on k-neighbors, if neighbor k and neighbor k+1 have identical distances but different labels, then the result will be dependent on the ordering of the training data. If the fit method is ‘kd_tree’, no warnings will be generated.

**p**: integer, optional (default = 2):

Parameter for the Minkowski metric from `sklearn.metrics.pairwise.pairwise_distances`. When p = 1, this is equivalent to using manhattan_distance (l1), and euclidean_distance (l2) for p = 2. For arbitrary p, minkowski_distance (l_p) is used.

See Also:

`KNeighborsClassifier`, `RadiusNeighborsClassifier`, `KNeighborsRegressor`, `RadiusNeighborsRegressor`, `BallTree`

Notes

See *Nearest Neighbors* in the online documentation for a discussion of the choice of algorithm and leaf_size.


Examples

```python
>>> from sklearn.neighbors import NearestNeighbors
>>> samples = [[0, 0, 2], [1, 0, 0], [0, 0, 1]]

>>> neigh = NearestNeighbors(2, 0.4)
>>> neigh.fit(samples)
NearestNeighbors(...)

>>> neigh.kneighbors([[0, 0, 1.3]], 2, return_distance=False)
array([[2, 0]])

>>> neigh.radius_neighbors([[0, 0, 1.3]], 0.4, return_distance=False)
array([[[2]]])
```

Methods
<table>
<thead>
<tr>
<th>Method</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>fit(X[, y])</code></td>
<td>Fit the model using X as training data</td>
</tr>
<tr>
<td><code>get_params([deep])</code></td>
<td>Get parameters for the estimator</td>
</tr>
<tr>
<td><code>kneighbors(X[, n_neighbors, return_distance])</code></td>
<td>Finds the K-neighbors of a point.</td>
</tr>
<tr>
<td><code>kneighbors_graph(X[, n_neighbors, mode])</code></td>
<td>Computes the (weighted) graph of k-Neighbors for points in X</td>
</tr>
<tr>
<td><code>radius_neighbors(X[, radius, return_distance])</code></td>
<td>Finds the neighbors of a point within a given radius.</td>
</tr>
<tr>
<td><code>radius_neighbors_graph(X[, radius, mode])</code></td>
<td>Computes the (weighted) graph of Neighbors for points in X</td>
</tr>
<tr>
<td><code>set_params(**params)</code></td>
<td>Set the parameters of the estimator</td>
</tr>
</tbody>
</table>

**__init__**(n_neighbors=5, radius=1.0, algorithm='auto', leaf_size=30, warn_on_equidistant=True, p=2)

**fit**(X, y=None)
Fit the model using X as training data

**Parameters**
X : {array-like, sparse matrix, BallTree, cKDTree}
Training data. If array or matrix, shape = [n_samples, n_features]

**get_params**(deep=True)
Get parameters for the estimator

**Parameters**
deep : boolean, optional
If True, will return the parameters for this estimator and contained subobjects that are estimators.

**kneighbors**(X, n_neighbors=None, return_distance=True)
Finds the K-neighbors of a point.

**Returns**

dist : array
Array representing the lengths to point, only present if return_distance=True

ind : array
Indices of the nearest points in the population matrix.

**Examples**

In the following example, we construct a NeighborsClassifier class from an array representing our data set and ask who’s the closest point to [1,1,1]

```python
>>> samples = [[0., 0., 0.], [0., .5, 0.], [1., 1., .5]]
>>> from sklearn.neighbors import NearestNeighbors
>>> neigh = NearestNeighbors(n_neighbors=1)
>>> neigh.fit(samples)
NearestNeighbors(algorithm='auto', leaf_size=30, ...)
```
As you can see, it returns [[0.5]], and [[2]], which means that the element is at distance 0.5 and is the third element of samples (indexes start at 0). You can also query for multiple points:

```python
>>> X = [[0., 1., 0.], [1., 0., 1.]]
>>> neigh.kneighbors(X, return_distance=False)
array([[1],
[2]])
```

**kneighbors_graph** *(X, n_neighbors=None, mode='connectivity')*
Computes the (weighted) graph of k-Neighbors for points in X

**Parameters**

- **X**: array-like, shape = [n_samples, n_features]
  Sample data

- **n_neighbors**: int
  Number of neighbors for each sample. (default is value passed to the constructor).

- **mode**: {'connectivity', 'distance'}, optional
  Type of returned matrix: ‘connectivity’ will return the connectivity matrix with ones and zeros, in ‘distance’ the edges are Euclidean distance between points.

**Returns**

- **A**: sparse matrix in CSR format, shape = [n_samples, n_samples_fit]
  n_samples_fit is the number of samples in the fitted data A[i, j] is assigned the weight of edge that connects i to j.

**See Also:**

- NearestNeighbors.radius_neighbors_graph

**Examples**

```python
>>> X = [[0], [3], [1]]
>>> from sklearn.neighbors import NearestNeighbors
>>> neigh = NearestNeighbors(n_neighbors=2)
>>> neigh.fit(X)
NearestNeighbors(algorithm='auto', leaf_size=30, ...)
>>> A = neigh.kneighbors_graph(X)
>>> A.todense()
matrix([[ 1.,  0.,  1.],
        [ 0.,  1.,  1.],
        [ 1.,  0.,  1.]])
```

**radius_neighbors** *(X, radius=None, return_distance=True)*
Finds the neighbors of a point within a given radius.

**Parameters**

- **X**: array-like, last dimension same as that of fit data
  The new point.

- **radius**: float
  Limiting distance of neighbors to return. (default is the value passed to the constructor).

- **return_distance**: boolean, optional. Defaults to True.
If False, distances will not be returned

**Returns**

- **dist**: array
  
  Array representing the lengths to point, only present if return_distance=True

- **ind**: array
  
  Indices of the nearest points in the population matrix.

**Examples**

In the following example, we construct a NeighborsClassifier class from an array representing our data set and ask who’s the closest point to [1,1,1]

```python
>>> samples = [[0., 0., 0.], [0., .5, 0.], [1., 1., .5]]
>>> from sklearn.neighbors import NearestNeighbors
>>> neigh = NearestNeighbors(radius=1.6)
>>> neigh.fit(samples)
NearestNeighbors(algorithm='auto', leaf_size=30, ...)
>>> print(neigh.radius_neighbors([1., 1., 1.]))
(array([[ 1.5, 0.5]]...), array([1, 2]]...)
```

The first array returned contains the distances to all points which are closer than 1.6, while the second array returned contains their indices. In general, multiple points can be queried at the same time. Because the number of neighbors of each point is not necessarily equal, radius_neighbors returns an array of objects, where each object is a 1D array of indices.

**radius_neighbors_graph**

*(X, radius=None, mode='connectivity')*

Computes the (weighted) graph of Neighbors for points in X

Neighborhoods are restricted the points at a distance lower than radius.

**Parameters**

- **X**: array-like, shape = [n_samples, n_features]
  
  Sample data

- **radius**: float
  
  Radius of neighborhoods. (default is the value passed to the constructor).

- **mode**: {'connectivity', 'distance'}, optional
  
  Type of returned matrix: ‘connectivity’ will return the connectivity matrix with ones and zeros, in ‘distance’ the edges are Euclidean distance between points.

**Returns**

- **A**: sparse matrix in CSR format, shape = [n_samples, n_samples]
  
  A[i, j] is assigned the weight of edge that connects i to j.

**See Also:**

kneighbors_graph

**Examples**

```python
>>> X = [[0], [3], [1]]
>>> from sklearn.neighbors import NearestNeighbors
>>> neigh = NearestNeighbors(radius=1.5)
>>> neigh.fit(X)
NearestNeighbors(algorithm='auto', leaf_size=30, ...)
```
A = neigh.radius_neighbors_graph(X)
A.todense()

def set_params(**params):
    """Set the parameters of the estimator."
    The method works on simple estimators as well as on nested objects (such as pipelines). The former have
    parameters of the form `<component>__<parameter>` so that it’s possible to update each component
    of a nested object.

    Returns self:

```
# Example
>>> A = neigh.radius_neighbors_graph(X)
>>> A.todense()
matrix([[ 1., 0., 1.],
        [ 0., 1., 0.],
        [ 1., 0., 1.]])
```

**sklearn.neighbors.KNeighborsClassifier**

Class implementing the k-nearest neighbors vote.

**Parameters**

- **n_neighbors**: int, optional (default = 5)
  Number of neighbors to use by default for k_neighbors queries.

- **weights**: str or callable
  weight function used in prediction. Possible values:
  *
  - ‘uniform’ : uniform weights. All points in each neighborhood are weighted equally.
  *
  - ‘distance’ : weight points by the inverse of their distance. in this case, closer neigh-
    bors of a query point will have a greater influence than neighbors which are further
    away.
  *
  - [callable] : a user-defined function which accepts an array of distances, and returns
    an array of the same shape containing the weights.

  Uniform weights are used by default.

- **algorithm**: {'auto', 'ball_tree', 'kd_tree', 'brute'}, optional
  Algorithm used to compute the nearest neighbors:
  *
  - ‘ball_tree’ will use BallTree
  *
  - ‘kd_tree’ will use scipy.spatial.cKDtree
  *
  - ‘brute’ will use a brute-force search.

  *‘auto’ will attempt to decide the most appropriate algorithm based on the values
    passed to fit method.

  Note: fitting on sparse input will override the setting of this parameter, using brute force.

- **leaf_size**: int, optional (default = 30)
  Leaf size passed to BallTree or cKDTree. This can affect the speed of the construction
  and query, as well as the memory required to store the tree. The optimal value depends
  on the nature of the problem.

- **warn_on_equidistant**: boolean, optional. Defaults to True.
Generate a warning if equidistant neighbors are discarded. For classification or regression based on k-neighbors, if neighbor k and neighbor k+1 have identical distances but different labels, then the result will be dependent on the ordering of the training data. If the fit method is ‘kd_tree’, no warnings will be generated.

p: integer, optional (default = 2):
Parameter for the Minkowski metric from sklearn.metrics.pairwise.pairwise_distances. When p = 1, this is equivalent to using manhattan_distance (l1), and euclidean_distance (l2) for p = 2. For arbitrary p, minkowski_distance (l_p) is used.

See Also:
RadiusNeighborsClassifier, KNeighborsRegressor, RadiusNeighborsRegressor, NearestNeighbors

Notes
See Nearest Neighbors in the online documentation for a discussion of the choice of algorithm and leaf_size.

Examples

```python
>>> X = [[0], [1], [2], [3]]
>>> y = [0, 0, 1, 1]
>>> from sklearn.neighbors import KNeighborsClassifier
>>> neigh = KNeighborsClassifier(n_neighbors=2)
>>> neigh.fit(X, y)
>>> print (neigh.predict([[1.5]]))
[0]
```

Methods

<table>
<thead>
<tr>
<th>Method</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>fit(X, y)</td>
<td>Fit the model using X as training data and y as target values</td>
</tr>
<tr>
<td>get_params([deep])</td>
<td>Get parameters for the estimator</td>
</tr>
<tr>
<td>kneighbors(X[, n_neighbors, return_distance])</td>
<td>Finds the K-neighbors of a point.</td>
</tr>
<tr>
<td>kneighbors_graph(X[, n_neighbors, mode])</td>
<td>Computes the (weighted) graph of k-Neighbors for points in X</td>
</tr>
<tr>
<td>predict(X)</td>
<td>Predict the class labels for the provided data</td>
</tr>
<tr>
<td>score(X, y)</td>
<td>Returns the mean accuracy on the given test data and labels.</td>
</tr>
<tr>
<td>set_params(**params)</td>
<td>Set the parameters of the estimator</td>
</tr>
</tbody>
</table>

```
__init__(n_neighbors=5, weights='uniform', algorithm='auto', leaf_size=30, warn_on_equidistant=True, p=2)
```

```python
fit (X, y)
Fit the model using X as training data and y as target values

Parameters X : {array-like, sparse matrix, BallTree, cKDTree}
Training data. If array or matrix, then the shape is [n_samples, n_features]

y : {array-like, sparse matrix}, shape = [n_samples]```
Target values, array of integer values.

**get_params** *(deep=True)*

Get parameters for the estimator

**Parameters**

**deep** : boolean, optional

If True, will return the parameters for this estimator and contained subobjects that are estimators.

**kneighbors** *(X, n_neighbors=None, return_distance=True)*

Finds the K-neighbors of a point.

Returns distance

**Parameters**

**X** : array-like, last dimension same as that of fit data

The new point.

**n_neighbors** : int

Number of neighbors to get (default is the value passed to the constructor).

**return_distance** : boolean, optional. Defaults to True.

If False, distances will not be returned

**Returns**

**dist** : array

Array representing the lengths to point, only present if return_distance=True

**ind** : array

Indices of the nearest points in the population matrix.

**Examples**

In the following example, we construct a NeighborsClassifier class from an array representing our data set and ask who’s the closest point to [1,1,1]

```python
>>> samples = [[0., 0., 0.], [0., 0.5, 0.], [1., 1., 0.5]]
>>> from sklearn.neighbors import NearestNeighbors
>>> neigh = NearestNeighbors(n_neighbors=1)
>>> neigh.fit(samples)
NearestNeighbors(algorithm='auto', leaf_size=30, ...)
>>> print(neigh.kneighbors([1., 1., 1.]))
(array([[ 0.5]]), array([[2]]...))
```

As you can see, it returns [[0.5]], and [[2]], which means that the element is at distance 0.5 and is the third element of samples (indexes start at 0). You can also query for multiple points:

```python
>>> X = [[0., 1., 0.], [1., 0., 1.]]
>>> neigh.kneighbors(X, return_distance=False)
array([[1],
       [2]]...)
```

**kneighbors_graph** *(X, n_neighbors=None, mode='connectivity')*

Computes the (weighted) graph of k-Neighbors for points in X

**Parameters**

**X** : array-like, shape = [n_samples, n_features]

Sample data

**n_neighbors** : int
Number of neighbors for each sample. (default is value passed to the constructor).

**mode** : {'connectivity', 'distance'}, optional

Type of returned matrix: ‘connectivity’ will return the connectivity matrix with ones and zeros, in ‘distance’ the edges are Euclidean distance between points.

**Returns A** : sparse matrix in CSR format, shape = [n_samples, n_samples_fit]

n_samples_fit is the number of samples in the fitted data A[i, j] is assigned the weight of edge that connects i to j.

**See Also:**

NearestNeighbors.radius_neighbors_graph

**Examples**

```python
>>> X = [[0], [3], [1]]
>>> from sklearn.neighbors import NearestNeighbors
>>> neigh = NearestNeighbors(n_neighbors=2)
>>> neigh.fit(X)
NearestNeighbors(algorithm='auto', leaf_size=30, ...)
>>> A = neigh.kneighbors_graph(X)
>>> A.todense()
matrix([[ 1., 0., 1.],
        [ 0., 1., 1.],
        [ 1., 0., 1.]])
```

**predict (X)**

Predict the class labels for the provided data

**Parameters X** : array

A 2-D array representing the test points.

**Returns labels** : array

List of class labels (one for each data sample).

**score (X, y)**

Returns the mean accuracy on the given test data and labels.

**Parameters X** : array-like, shape = [n_samples, n_features]

Training set.

y : array-like, shape = [n_samples]

Labels for X.

**Returns z** : float

**set_params (**params**)

Set the parameters of the estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The former have parameters of the form `<component>__<parameter>` so that it’s possible to update each component of a nested object.

**Returns self** :
**sklearn.neighbors.RadiusNeighborsClassifier**

class `sklearn.neighbors.RadiusNeighborsClassifier` *(radius=1.0, weights='uniform', algorithm='auto', leaf_size=30, p=2, outlier_label=None)*

Classifier implementing a vote among neighbors within a given radius

**Parameters**

- **radius** : float, optional (default = 1.0)
  
  Range of parameter space to use by default for `radius_neighbors` queries.

- **weights** : str or callable
  
  weight function used in prediction. Possible values:
  
  - `'uniform'` : uniform weights. All points in each neighborhood are weighted equally.
  - `'distance'` : weight points by the inverse of their distance. in this case, closer neighbors of a query point will have a greater influence than neighbors which are further away.
  - `[callable]` : a user-defined function which accepts an array of distances, and returns an array of the same shape containing the weights.

  Uniform weights are used by default.

- **algorithm** : {'auto', 'ball_tree', 'kd_tree', 'brute'}, optional
  
  Algorithm used to compute the nearest neighbors:
  
  - `'ball_tree'` will use BallTree
  - `'kd_tree'` will use scipy.spatial.cKDtree
  - `'brute'` will use a brute-force search.
  - `'auto'` will attempt to decide the most appropriate algorithm based on the values passed to `fit` method.

  Note: fitting on sparse input will override the setting of this parameter, using brute force.

- **leaf_size** : int, optional (default = 30)
  
  Leaf size passed to BallTree or cKDTree. This can affect the speed of the construction and query, as well as the memory required to store the tree. The optimal value depends on the nature of the problem.

- **p** : integer, optional (default = 2)
  
  Parameter for the Minkowski metric from sklearn.metrics.pairwise.pairwise_distances. When p = 1, this is equivalent to using manhattan_distance (l1), and euclidean_distance (l2) for p = 2. For arbitrary p, minkowski_distance (l_p) is used.

- **outlier_label** : int, optional (default = None)
  
  Label, which is given for outlier samples (samples with no neighbors on given radius). If set to None, ValueError is raised, when outlier is detected.

**See Also:**

KNeighborsClassifier, RadiusNeighborsRegressor, KNeighborsRegressor, NearestNeighbors
Notes

See Nearest Neighbors in the online documentation for a discussion of the choice of algorithm and leaf_size.


Examples

```python
>>> X = [[0], [1], [2], [3]]
>>> y = [0, 0, 1, 1]
>>> from sklearn.neighbors import RadiusNeighborsClassifier
>>> neigh = RadiusNeighborsClassifier(radius=1.0)
>>> neigh.fit(X, y)
RadiusNeighborsClassifier(...)
>>> print(neigh.predict([[1.5]]))
[0]
```

Methods

<table>
<thead>
<tr>
<th>Function</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>fit(X, y)</td>
<td>Fit the model using X as training data and y as target values</td>
</tr>
<tr>
<td>get_params([deep])</td>
<td>Get parameters for the estimator</td>
</tr>
<tr>
<td>predict(X)</td>
<td>Predict the class labels for the provided data</td>
</tr>
<tr>
<td>radius_neighbors(X[, radius, return_distance])</td>
<td>Finds the neighbors of a point within a given radius.</td>
</tr>
<tr>
<td>radius_neighbors_graph(X[, radius, mode])</td>
<td>Computes the (weighted) graph of Neighbors for points in X</td>
</tr>
<tr>
<td>score(X, y)</td>
<td>Returns the mean accuracy on the given test data and labels.</td>
</tr>
<tr>
<td>set_params(**params)</td>
<td>Set the parameters of the estimator.</td>
</tr>
</tbody>
</table>

__init__(radius=1.0, weights=’uniform’, algorithm=’auto’, leaf_size=30, p=2, outlier_label=None)

fit (X, y)

Fit the model using X as training data and y as target values

Parameters X : {array-like, sparse matrix, BallTree, cKDTree}

Training data. If array or matrix, then the shape is [n_samples, n_features]

y : {array-like, sparse matrix}, shape = [n_samples]

Target values, array of integer values.

get_params (deep=True)

Get parameters for the estimator

Parameters deep: boolean, optional:

If True, will return the parameters for this estimator and contained subobjects that are estimators.

predict (X)

Predict the class labels for the provided data

Parameters X: array:

A 2-D array representing the test points.

Returns labels: array:
List of class labels (one for each data sample).

**radius_neighbors** \((X, \text{radius}=\text{None}, \text{return_distance}=\text{True})\)

Finds the neighbors of a point within a given radius.

Returns distance

**Parameters**

- **X** : array-like, last dimension same as that of fit data
  - The new point.

- **radius** : float
  - Limiting distance of neighbors to return. (default is the value passed to the constructor).

- **return_distance** : boolean, optional. Defaults to True.
  - If False, distances will not be returned

**Returns**

- **dist** : array
  - Array representing the lengths to point, only present if return_distance=True

- **ind** : array
  - Indices of the nearest points in the population matrix.

**Examples**

In the following example, we construct a NeighborsClassifier class from an array representing our data set and ask who’s the closest point to \([1,1,1]\)

```python
>>> samples = [[0., 0., 0.], [0., .5, 0.], [1., 1., .5]]
>>> from sklearn.neighbors import NearestNeighbors
>>> neigh = NearestNeighbors(radius=1.6)
>>> neigh.fit(samples)
NearestNeighbors(algorithm='auto', leaf_size=30, ...)
>>> print(neigh.radius_neighbors([1., 1., 1.]))
(array([[ 1.5, 0.5]]), array([[1, 2]]))
```

The first array returned contains the distances to all points which are closer than 1.6, while the second array returned contains their indices. In general, multiple points can be queried at the same time. Because the number of neighbors of each point is not necessarily equal, *radius_neighbors* returns an array of objects, where each object is a 1D array of indices.

**radius_neighbors_graph** \((X, \text{radius}=\text{None}, \text{mode}='\text{connectivity}')\)

Computes the (weighted) graph of Neighbors for points in \(X\)

Neighborhoods are restricted the points at a distance lower than radius.

**Parameters**

- **X** : array-like, shape = [n_samples, n_features]
  - Sample data

- **radius** : float
  - Radius of neighborhoods. (default is the value passed to the constructor).

- **mode** : {'connectivity', 'distance'}, optional
  - Type of returned matrix: ‘connectivity’ will return the connectivity matrix with ones and zeros, in ‘distance’ the edges are Euclidean distance between points.

**Returns**

- **A** : sparse matrix in CSR format, shape = [n_samples, n_samples]
A[i, j] is assigned the weight of edge that connects i to j.

**See Also:**

kneighbors_graph

**Examples**

```python
>>> X = [[0], [3], [1]]
>>> from sklearn.neighbors import NearestNeighbors
>>> neigh = NearestNeighbors(radius=1.5)
>>> neigh.fit(X)
NearestNeighbors(algorithm='auto', leaf_size=30, ...)
>>> A = neigh.radius_neighbors_graph(X)
>>> A.todense()
matrix([[ 1., 0., 1.],
        [ 0., 1., 0.],
        [ 1., 0., 1.]])
```

**score** *(X, y)*

Returns the mean accuracy on the given test data and labels.

**Parameters**

- **X**: array-like, shape = [n_samples, n_features]
  
  Training set.

- **y**: array-like, shape = [n_samples]
  
  Labels for X.

**Returns**

- **z**: float

**set_params** (**params**)

Set the parameters of the estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The former have parameters of the form `<component>__<parameter>` so that it’s possible to update each component of a nested object.

**Returns**

- **self**:

### sklearn.neighbors.KNeighborsRegressor

**class** sklearn.neighbors.KNeighborsRegressor *(n_neighbors=5, weights='uniform', algorithm='auto', leaf_size=30, warn_on_equidistant=True, p=2)*

Regression based on k-nearest neighbors.

The target is predicted by local interpolation of the targets associated of the nearest neighbors in the training set.

**Parameters**

- **n_neighbors**: int, optional (default = 5)
  
  Number of neighbors to use by default for k_neighbors queries.

- **weights**: str or callable
  
  weight function used in prediction. Possible values:

  - ‘uniform’: uniform weights. All points in each neighborhood are weighted equally.
• ‘distance’ : weight points by the inverse of their distance. In this case, closer neighbors of a query point will have a greater influence than neighbors which are further away.

• [callable] : a user-defined function which accepts an array of distances, and returns an array of the same shape containing the weights.

Uniform weights are used by default.

algorithm : {'auto', 'ball_tree', 'kd_tree', 'brute'}, optional

Algorithm used to compute the nearest neighbors:

• ‘ball_tree’ will use BallTree
• ‘kd_tree’ will use scipy.spatial.cKDTree
• ‘brute’ will use a brute-force search.
• ‘auto’ will attempt to decide the most appropriate algorithm based on the values passed to fit method.

Note: fitting on sparse input will override the setting of this parameter, using brute force.

leaf_size : int, optional (default = 30)

Leaf size passed to BallTree or cKDTree. This can affect the speed of the construction and query, as well as the memory required to store the tree. The optimal value depends on the nature of the problem.

warn_on_equidistant : boolean, optional. Defaults to True.

Generate a warning if equidistant neighbors are discarded. For classification or regression based on k-neighbors, if neighbor k and neighbor k+1 have identical distances but different labels, then the result will be dependent on the ordering of the training data. If the fit method is ‘kd_tree’, no warnings will be generated.

p : integer, optional (default = 2) :

Parameter for the Minkowski metric from sklearn.metrics.pairwise.pairwise_distances. When p = 1, this is equivalent to using manhattan_distance (l1), and euclidean_distance (l2) for p = 2. For arbitrary p, minkowski_distance (l_p) is used.

See Also:

NearestNeighbors, RadiusNeighborsRegressor, KNeighborsClassifier, RadiusNeighborsClassifier

Notes

See Nearest Neighbors in the online documentation for a discussion of the choice of algorithm and leaf_size.


Examples

>>> X = [[0], [1], [2], [3]]
>>> y = [0, 0, 1, 1]
>>> from sklearn.neighbors import KNeighborsRegressor
>>> neigh = KNeighborsRegressor(n_neighbors=2)
>>> neigh.fit(X, y)
KNeighborsRegressor(...)

>>> print(neigh.predict([[1.5]]))
[ 0.5]

**Methods**

<table>
<thead>
<tr>
<th>Method</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>fit(X, y)</td>
<td>Fit the model using X as training data and y as target values</td>
</tr>
<tr>
<td>get_params([deep])</td>
<td>Get parameters for the estimator</td>
</tr>
<tr>
<td>kneighbors(X[, n_neighbors, return_distance])</td>
<td>Finds the K-neighbors of a point.</td>
</tr>
<tr>
<td>kneighbors_graph(X[, n_neighbors, mode])</td>
<td>Computes the (weighted) graph of k-Neighbors for points in X</td>
</tr>
<tr>
<td>predict(X)</td>
<td>Predict the target for the provided data</td>
</tr>
<tr>
<td>score(X, y)</td>
<td>Returns the coefficient of determination $R^2$ of the prediction.</td>
</tr>
<tr>
<td>set_params(**params)</td>
<td>Set the parameters of the estimator.</td>
</tr>
</tbody>
</table>

**init**(n_neighbors=5, weights='uniform', algorithm='auto', leaf_size=30, warn_on_equidistant=True, p=2)

fit(X, y)
Fit the model using X as training data and y as target values

Parameters

- **X**: array-like, sparse matrix, BallTree, cKDTree
  - Training data. If array or matrix, then the shape is [n_samples, n_features]
- **y**: array-like, sparse matrix, shape = [n_samples]
  - Target values, array of float values.

get_params(deep=True)
Get parameters for the estimator

Parameters

- **deep**: boolean, optional
  - If True, will return the parameters for this estimator and contained subobjects that are estimators.

kneighbors(X, n_neighbors=None, return_distance=True)
Finds the K-neighbors of a point.

Parameters

- **X**: array-like, last dimension same as that of fit data
  - The new point.
- **n_neighbors**: int
  - Number of neighbors to get (default is the value passed to the constructor).
- **return_distance**: boolean, optional. Defaults to True.
  - If False, distances will not be returned

Returns

- **dist**: array
  - Array representing the lengths to point, only present if return_distance=True
- **ind**: array
  - Indices of the nearest points in the population matrix.
Examples

In the following example, we construct a NeighborsClassifier class from an array representing our data set and ask who’s the closest point to [1,1,1]

```python
>>> samples = [[0., 0., 0.], [0., .5, 0.], [1., 1., .5]]
>>> from sklearn.neighbors import NearestNeighbors
>>> neigh = NearestNeighbors(n_neighbors=1)
>>> neigh.fit(samples)
NearestNeighbors(algorithm='auto', leaf_size=30, ...)
>>> print(neigh.kneighbors([1., 1., 1.]))
(array([[ 0.5]]), array([[2]]))
```

As you can see, it returns [[0.5]], and [2]], which means that the element is at distance 0.5 and is the third element of samples (indexes start at 0). You can also query for multiple points:

```python
>>> X = [[0., 1., 0.], [1., 0., 1.]]
>>> neigh.kneighbors(X, return_distance=False)
array([[1],
       [2]])
```

**kneighbors_graph** *(X, n_neighbors=None, mode='connectivity')*

Computes the (weighted) graph of k-Neighbors for points in X

Parameters:
- **X**: array-like, shape = [n_samples, n_features]
  Sample data
- **n_neighbors**: int
  Number of neighbors for each sample. (default is value passed to the constructor).
- **mode**: {'connectivity', 'distance'}, optional
  Type of returned matrix: 'connectivity' will return the connectivity matrix with ones and zeros, in 'distance' the edges are Euclidean distance between points.

Returns:
- **A**: sparse matrix in CSR format, shape = [n_samples, n_samples_fit]
  n_samples_fit is the number of samples in the fitted data A[i, j] is assigned the weight of edge that connects i to j.

See Also:
- NearestNeighbors.radius_neighbors_graph

Examples

```python
>>> X = [[0], [3], [1]]
>>> from sklearn.neighbors import NearestNeighbors
>>> neigh = NearestNeighbors(n_neighbors=2)
>>> neigh.fit(X)
NearestNeighbors(algorithm='auto', leaf_size=30, ...)
>>> A = neigh.kneighbors_graph(X)
>>> A.todense()
matrix([[ 1., 0., 1.],
        [ 0., 1., 1.],
        [ 1., 0., 1.]]
```

**predict**(X)

Predict the target for the provided data
Parameters $X$ : array
A 2-D array representing the test data.

Returns $y$: array :
List of target values (one for each data sample).

`score` ($X$, $y$)
Returns the coefficient of determination $R^2$ of the prediction.

The coefficient $R^2$ is defined as $(1 - u/v)$, where $u$ is the regression sum of squares $(y - y_{pred})^2$.sum() and $v$ is the residual sum of squares $(y_{true} - y_{true}.mean())^2$.sum(). Best possible score is 1.0, lower values are worse.

Parameters $X$ : array-like, shape = [n_samples, n_features]
Training set.

$y$ : array-like, shape = [n_samples]

Returns $z$ : float

`set_params` (**params)
Set the parameters of the estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The former have parameters of the form `<component>__<parameter>` so that it’s possible to update each component of a nested object.

Returns self :

**sklearn.neighbors.RadiusNeighborsRegressor**

```python
class sklearn.neighbors.RadiusNeighborsRegressor(radius=1.0, weights='uniform', algorithm='auto', leaf_size=30, p=2)
```
Regression based on neighbors within a fixed radius.

The target is predicted by local interpolation of the targets associated of the nearest neighbors in the training set.

Parameters `radius` : float, optional (default = 1.0)
Range of parameter space to use by default for :meth:`radius_neighbors` queries.

`weights` : str or callable
weight function used in prediction. Possible values:

- ‘uniform’: uniform weights. All points in each neighborhood are weighted equally.
- ‘distance’: weight points by the inverse of their distance. in this case, closer neighbors of a query point will have a greater influence than neighbors which are further away.
- [callable]: a user-defined function which accepts an array of distances, and returns an array of the same shape containing the weights.

Uniform weights are used by default.

Algorithm used to compute the nearest neighbors:

- ‘ball_tree’ will use `BallTree`
- ‘kd_tree’ will use `scipy.spatial.cKDtree`
• 'brute' will use a brute-force search.
• 'auto' will attempt to decide the most appropriate algorithm based on the values passed to \texttt{fit} method.

Note: fitting on sparse input will override the setting of this parameter, using brute force.

\textbf{leaf size} : int, optional (default = 30)

Leaf size passed to BallTree or cKDTree. This can affect the speed of the construction and query, as well as the memory required to store the tree. The optimal value depends on the nature of the problem.

\textbf{p} : integer, optional (default = 2) :

Parameter for the Minkowski metric from sklearn.metrics.pairwise.pairwise_distances. When p = 1, this is equivalent to using manhattan_distance (l1), and euclidean_distance (l2) for p = 2. For arbitrary p, minkowski_distance (l_p) is used.

\textbf{Notes}

See \textit{Nearest Neighbors} in the online documentation for a discussion of the choice of \texttt{algorithm} and \texttt{leaf_size}.


\textbf{Examples}

```python
>>> X = [[0], [1], [2], [3]]
>>> y = [0, 0, 1, 1]
>>> from sklearn.neighbors import RadiusNeighborsRegressor
>>> neigh = RadiusNeighborsRegressor(radius=1.0)
>>> neigh.fit(X, y)
RadiusNeighborsRegressor(...)
>>> print(neigh.predict([[1.5]]))
[ 0.5]
```

\textbf{Methods}

<table>
<thead>
<tr>
<th>Method</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>\texttt{fit(X, y)}</td>
<td>Fit the model using X as training data and y as target values</td>
</tr>
<tr>
<td>\texttt{get_params([deep])}</td>
<td>Get parameters for the estimator</td>
</tr>
<tr>
<td>\texttt{predict(X)}</td>
<td>Predict the target for the provided data</td>
</tr>
<tr>
<td>\texttt{radius_neighbors(X[, radius, return_distance])}</td>
<td>Finds the neighbors of a point within a given radius.</td>
</tr>
<tr>
<td>\texttt{radius_neighbors_graph(X[, radius, mode])}</td>
<td>Computes the (weighted) graph of Neighbors for points in X</td>
</tr>
<tr>
<td>\texttt{score(X, y)}</td>
<td>Returns the coefficient of determination R^2 of the prediction.</td>
</tr>
<tr>
<td>\texttt{set_params(**params)}</td>
<td>Set the parameters of the estimator.</td>
</tr>
</tbody>
</table>

\texttt{__init__}(radius=1.0, weights=’uniform’, algorithm=’auto’, leaf_size=30, p=2)

\texttt{fit(X, y)}
Fit the model using X as training data and y as target values

**Parameters**

- **X**: [array-like, sparse matrix, BallTree, cKDTree]
  - Training data. If array or matrix, then the shape is [n_samples, n_features]
- **y**: [array-like, sparse matrix], shape = [n_samples]
  - Target values, array of float values.

**get_params** *(deep=True)*

- Get parameters for the estimator
  - **Parameters deep**: boolean, optional
    - If True, will return the parameters for this estimator and contained subobjects that are estimators.

**predict** *(X)*

- Predict the target for the provided data
  - **Parameters X**: array
    - A 2-D array representing the test data.
  - **Returns y**: array
    - List of target values (one for each data sample).

**radius_neighbors** *(X, radius=None, return_distance=True)*

- Finds the neighbors of a point within a given radius.
  - **Returns**
    - **dist**: array
      - Array representing the lengths to point, only present if return_distance=True
    - **ind**: array
      - Indices of the nearest points in the population matrix.

**Examples**

In the following example, we construct a NeighborsClassifier class from an array representing our data set and ask who’s the closest point to [1,1,1]

```python
>>> samples = [[0., 0., 0.], [0., .5, 0.], [1., 1., .5]]
>>> from sklearn.neighbors import NearestNeighbors
>>> neigh = NearestNeighbors(radius=1.6)
>>> neigh.fit(samples)
NearestNeighbors(algorithm='auto', leaf_size=30, ...)
>>> print(neigh.radius_neighbors([1., 1., 1.]))
(array([[ 1.5, 0.5]]...), array([[1, 2]]...)
```
The first array returned contains the distances to all points which are closer than 1.6, while the second array returned contains their indices. In general, multiple points can be queried at the same time. Because the number of neighbors of each point is not necessarily equal, `radius_neighbors` returns an array of objects, where each object is a 1D array of indices.

`radius_neighbors_graph(X, radius=None, mode='connectivity')`

Computes the (weighted) graph of Neighbors for points in X.

**Parameters**

- **X**: array-like, shape = [n_samples, n_features]
  Sample data
- **radius**: float
  Radius of neighborhoods. (default is the value passed to the constructor).
- **mode**: {'connectivity', 'distance'}, optional
  Type of returned matrix: ‘connectivity’ will return the connectivity matrix with ones and zeros, in ‘distance’ the edges are Euclidean distance between points.

**Returns**

- **A**: sparse matrix in CSR format, shape = [n_samples, n_samples]
  A[i, j] is assigned the weight of edge that connects i to j.

See Also:

- `kneighbors_graph`

**Examples**

```python
>>> X = [[0], [3], [1]]
>>> from sklearn.neighbors import NearestNeighbors
>>> neigh = NearestNeighbors(radius=1.5)
>>> neigh.fit(X)
NearestNeighbors(algorithm='auto', leaf_size=30, ...)
>>> A = neigh.radius_neighbors_graph(X)
>>> A.todense()
matrix([[ 1., 0., 1.],
        [ 0., 1., 0.],
        [ 1., 0., 1.]])
```

`score(X, y)`

Returns the coefficient of determination $R^2$ of the prediction.

The coefficient $R^2$ is defined as $(1 - u/v)$, where $u$ is the regression sum of squares $(y - y_{pred})^2$.sum() and $v$ is the residual sum of squares $(y_{true} - y_{true}.mean())^2$.sum(). Best possible score is 1.0, lower values are worse.

**Parameters**

- **X**: array-like, shape = [n_samples, n_features]
  Training set.
- **y**: array-like, shape = [n_samples]

**Returns**

- **z**: float

`set_params(**params)`

Set the parameters of the estimator.
The method works on simple estimators as well as on nested objects (such as pipelines). The former have parameters of the form `<component>__<parameter>` so that it’s possible to update each component of a nested object.

**Returns self**:

**sklearn.neighbors.BallTree**

**class sklearn.neighbors.BallTree**

Ball Tree for fast nearest-neighbor searches:

BallTree(X, leaf_size=20, p=2.0)

**Parameters**

- **X**: array-like, shape = [n_samples, n_features]
  
  n_samples is the number of points in the data set, and n_features is the dimension of the parameter space. Note: if X is a C-contiguous array of doubles then data will not be copied. Otherwise, an internal copy will be made.

- **leaf_size**: positive integer (default = 20)
  
  Number of points at which to switch to brute-force. Changing leaf_size will not affect the results of a query, but can significantly impact the speed of a query and the memory required to store the built ball tree. The amount of memory needed to store the tree scales as $2^{(1 + \text{floor}(\log_2((n_{\text{samples}} - 1) / \text{leaf_size})) - 1$ For a specified leaf_size, a leaf node is guaranteed to satisfy $\text{leaf_size} \leq \text{n_points} \leq 2 * \text{leaf_size}$, except in the case that $n_{\text{samples}} < \text{leaf_size}$.

- **p**: distance metric for the BallTree. p encodes the Minkowski p-distance:
  
  \[ D = \sum((X[i] - X[j]) ** p) ** (1. / p) \]

  p must be greater than or equal to 1, so that the triangle inequality will hold. If $p == \text{np.inf}$, then the distance is equivalent to:

  \[ D = \max(X[i] - X[j]) \]

**Examples**

Query for k-nearest neighbors

```
>>> import numpy as np
>>> np.random.seed(0)
>>> X = np.random.random((10,3))  # 10 points in 3 dimensions
>>> ball_tree = BallTree(X, leaf_size=2)
>>> dist, ind = ball_tree.query(X[0], n_neighbors=3)
>>> print ind  # indices of 3 closest neighbors
[0 3 1]
>>> print dist  # distances to 3 closest neighbors
[ 0.19662693 0.29473397]
```

Pickle and Unpickle a ball tree (using protocol = 2). Note that the state of the tree is saved in the pickle operation: the tree is not rebuilt on un-pickling
```python
>>> import numpy as np
>>> import pickle

>>> np.random.seed(0)

>>> X = np.random.random((10,3))  # 10 points in 3 dimensions
>>> ball_tree = BallTree(X, leaf_size=2)
>>> s = pickle.dumps(ball_tree, protocol=2)
>>> ball_tree_copy = pickle.loads(s)

>>> dist, ind = ball_tree_copy.query(X[0], k=3)

>>> print(ind)  # indices of 3 closest neighbors
[0 3 1]
>>> print(dist)  # distances to 3 closest neighbors
[ 0. 0.19662693 0.29473397]
```

**Attributes**

- `data`
- `warning_flag`

**Methods**

- `query(X[, k, return_distance])` query the Ball Tree for the k nearest neighbors

```
Parameters

X : array-like, last dimension self.dim
    An array of points to query

k : integer (default = 1)
    The number of nearest neighbors to return

return_distance : boolean (default = True)
    if True, return a tuple (d,i) if False, return array i

Returns

i : if return_distance == False
    (d,i) : if return_distance == True

    d : array of doubles - shape: x.shape[:-1] + (k,)
        each entry gives the list of distances to the neighbors of the corresponding point (note
        that distances are not sorted)

    i : array of integers - shape: x.shape[:-1] + (k,)
        each entry gives the list of indices of neighbors of the corresponding point (note that
        neighbors are not sorted)
```
Examples

Query for k-nearest neighbors

```python
>>> import numpy as np
>>> np.random.seed(0)
>>> X = np.random.random((10,3))  # 10 points in 3 dimensions
>>> ball_tree = BallTree(X, leaf_size=2)
>>> dist, ind = ball_tree.query(X[0], k=3)
>>> print ind  # indices of 3 closest neighbors
[0 3 1]
>>> print dist  # distances to 3 closest neighbors
[ 0.  0.19662693  0.29473397]
```

`query_radius`

`query_radius(self, X, r, count_only=False)`:

query the Ball Tree for neighbors within a ball of size `r`.

**Parameters**

- **X**: array-like, last dimension `self.dim`
  
  An array of points to query

- **r**: distance within which neighbors are returned
  
  `r` can be a single value, or an array of values of shape `X.shape[:-1]` if different radii are desired for each point.

- **return_distance**: boolean (default = False)
  
  if True, return distances to neighbors of each point if False, return only neighbors. Note that unlike `BallTree.query()`, setting `return_distance=True` adds to the computation time. Not all distances need to be calculated explicitly for `return_distance=False`. Results are not sorted by default: see `sort_results` keyword.

- **count_only**: boolean (default = False)
  
  if True, return only the count of points within distance `r` if False, return the indices of all points within distance `r`. If `return_distance==True`, setting `count_only=True` will result in an error.

- **sort_results**: boolean (default = False)
  
  if True, the distances and indices will be sorted before being returned. If False, the results will not be sorted. If `return_distance == False`, setting `sort_results = True` will result in an error.

**Returns**

- **count**: if `count_only == True`

- **ind**: if `count_only == False` and `return_distance == False`

- **(ind, dist)**: if `count_only == False` and `return_distance == True`

- **count**: array of integers, shape = `X.shape[:-1]`
  
  each entry gives the number of neighbors within a distance `r` of the corresponding point.

- **ind**: array of objects, shape = `X.shape[:-1]`
  
  each element is a numpy integer array listing the indices of neighbors of the corresponding point. Note that unlike the results of `BallTree.query()`, the returned neighbors are not sorted by distance.

- **dist**: array of objects, shape = `X.shape[:-1]`
each element is a numpy double array listing the distances corresponding to indices in i.

Examples

Query for neighbors in a given radius

```python
>>> import numpy as np
>>> np.random.seed(0)
>>> X = np.random.random((10,3))  # 10 points in 3 dimensions
>>> ball_tree = BallTree(X, leaf_size=2)
>>> print ball_tree.query_radius(X[0], r=0.3, count_only=True)
3
>>> ind = ball_tree.query_radius(X[0], r=0.3)
>>> print ind  # indices of neighbors within distance 0.3
[3 0 1]
```

sklearn.neighbors.NearestCentroid

class sklearn.neighbors.NearestCentroid(metric='euclidean', shrink_threshold=None)

Nearest centroid classifier.

Each class is represented by its centroid, with test samples classified to the class with the nearest centroid.

Parameters

metric: string, or callable

The metric to use when calculating distance between instances in a feature array. If metric is a string or callable, it must be one of the options allowed by metrics.pairwise.pairwise_distances for its metric parameter.

shrink_threshold : float, optional

Threshold for shrinking centroids to remove features.

See Also:

sklearn.neighbors.KNeighborsClassifier nearest neighbors classifier

Notes

When used for text classification with tf–idf vectors, this classifier is also known as the Rocchio classifier.

References


Examples
```python
>>> from sklearn.neighbors.nearest_centroid import NearestCentroid
>>> import numpy as np
>>> X = np.array([[-1, -1], [-2, -1], [-3, -2], [1, 1], [2, 1], [3, 2]])
>>> y = np.array([1, 1, 1, 2, 2, 2])
>>> clf = NearestCentroid()
>>> clf.fit(X, y)
NearestCentroid(metric='euclidean', shrink_threshold=None)
>>> print(clf.predict([[-0.8, -1]]))
[1]
```

**Attributes**

- `centroids_` : array-like, shape = [n_classes, n_features] — Centroid of each class

**Methods**

- `fit(X, y)` — Fit the NearestCentroid model according to the given training data.
- `get_params([deep])` — Get parameters for the estimator
- `predict(X)` — Perform classification on an array of test vectors X.
- `score(X, y)` — Returns the mean accuracy on the given test data and labels.
- `set_params(**params)` — Set the parameters of the estimator.

```
__init__(metric='euclidean', shrink_threshold=None)
```

```
fit (X, y)
Fit the NearestCentroid model according to the given training data.

Parameters

- **X** : {array-like, sparse matrix}, shape = [n_samples, n_features]
  Training vector, where n_samples in the number of samples and n_features is the number of features. Note that centroid shrinking cannot be used with sparse matrices.
- **y** : array, shape = [n_samples]
  Target values (integers)

get_params (deep=True)
Get parameters for the estimator

Parameters

- **deep** : boolean, optional
  If True, will return the parameters for this estimator and contained subobjects that are estimators.

predict (X)
Perform classification on an array of test vectors X.

The predicted class C for each sample in X is returned.

Parameters

- **X** : array-like, shape = [n_samples, n_features]

Returns

- **C** : array, shape = [n_samples]
```
Notes

If the metric constructor parameter is “precomputed”, X is assumed to be the distance matrix between the
data to be predicted and self.centroids_.

score(X, y)
Returns the mean accuracy on the given test data and labels.

Parameters:
- **X**: array-like, shape = [n_samples, n_features]
  Training set.
- **y**: array-like, shape = [n_samples]
  Labels for X.

Returns:
- **z**: float

set_params(**params)
Set the parameters of the estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The former have parameters of the form <component>__<parameter> so that it’s possible to update each component of a nested object.

Returns self:

neighbors.kneighbors_graph(X, n_neighbors[, ...])
Computes the (weighted) graph of k-Neighbors for points in X

Parameters:
- **X**: array-like or BallTree, shape = [n_samples, n_features]
  Sample data, in the form of a numpy array or a precomputed BallTree.
- **n_neighbors**: int
  Number of neighbors for each sample.
- **mode**: {'connectivity', 'distance'}, optional
  Type of returned matrix: ‘connectivity’ will return the connectivity matrix with ones and zeros, in ‘distance’ the edges are Euclidean distance between points.

Returns:
- **A**: sparse matrix in CSR format, shape = [n_samples, n_samples]
  A[i, j] is assigned the weight of edge that connects i to j.

See Also:
- radius_neighbors_graph

Examples

```python
>>> X = [[0], [3], [1]]
>>> from sklearn.neighbors import kneighbors_graph
>>> A = kneighbors_graph(X, 2)
```
A.todense()
matrix([[ 1., 0., 1.],
[ 0., 1., 1.],
[ 1., 0., 1.]])

sklearn.neighbors.radius_neighbors_graph

sklearn.neighbors.radius_neighbors_graph(X, radius, mode='connectivity')
Computes the (weighted) graph of Neighbors for points in X.
Neighborhoods are restricted to the points at a distance lower than radius.

Parameters:
X : array-like or BallTree, shape = [n_samples, n_features]
Sample data, in the form of a numpy array or a precomputed BallTree.

radius : float
Radius of neighborhoods.

mode : {'connectivity', 'distance'}, optional
Type of returned matrix: ‘connectivity’ will return the connectivity matrix with ones and zeros, in ‘distance’ the edges are Euclidean distance between points.

Returns:
A : sparse matrix in CSR format, shape = [n_samples, n_samples]
A[i, j] is assigned the weight of edge that connects i to j.

See Also:
kneighbors_graph

Examples:

>>> X = [[0], [3], [1]]
>>> from sklearn.neighbors import radius_neighbors_graph
>>> A = radius_neighbors_graph(X, 1.5)
>>> A.todense()
matrix([[ 1., 0., 1.],
[ 0., 1., 0.],
[ 1., 0., 1.]])

1.8.22 sklearn.pls: Partial Least Squares

The sklearn.pls module implements Partial Least Squares (PLS).

User guide: See the Partial Least Squares section for further details.

<table>
<thead>
<tr>
<th>Function</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>pls.PLSRegression</td>
<td>PLS regression</td>
</tr>
<tr>
<td>pls.PLSCanonical</td>
<td>PLSCanonical implements the 2 blocks canonical PLS of the original Wold</td>
</tr>
<tr>
<td>pls.CCA</td>
<td>CCA Canonical Correlation Analysis. CCA inherits from PLS with</td>
</tr>
<tr>
<td>pls.PLSSVD</td>
<td>Partial Least Square SVD</td>
</tr>
</tbody>
</table>
sklearn.pls.PLSRegression

class sklearn.pls.PLSRegression(n_components=2, scale=True, max_iter=500, tol=1e-06, copy=True)

PLS regression

PLSRegression implements the PLS 2 blocks regression known as PLS2 or PLS1 in case of one dimensional response. This class inherits from _PLS with mode="A", deflation_mode="regression", norm_y_weights=False and algorithm="nipals".

Parameters

X : array-like of predictors, shape = [n_samples, p]
Training vectors, where n_samples is the number of samples and p is the number of predictors.

Y : array-like of response, shape = [n_samples, q]
Training vectors, where n_samples is the number of samples and q is the number of response variables.

n_components : int, (default 2)
Number of components to keep.

scale : boolean, (default True)
whether to scale the data

max_iter : an integer, (default 500)
the maximum number of iterations of the NIPALS inner loop (used only if algorithm="nipals")

tol : non-negative real
Tolerance used in the iterative algorithm default 1e-06.

copy : boolean, default True
Whether the deflation should be done on a copy. Let the default value to True unless you don’t care about side effect

Notes

For each component k, find weights u, v that optimizes: max corr(Xk u, Yk v) * var(Xk u)
var(Yk u), such that |u| = 1

Note that it maximizes both the correlations between the scores and the intra-block variances.

The residual matrix of X (Xk+1) block is obtained by the deflation on the current X score: x_score.

The residual matrix of Y (Yk+1) block is obtained by deflation on the current X score. This performs the PLS regression known as PLS2. This mode is prediction oriented.

This implementation provides the same results that 3 PLS packages provided in the R language (R-project):

• "mixOmics" with function pls(X, Y, mode = "regression")
• "plspm" with function plsreg2(X, Y)
• "pls" with function oscorespls.fit(X, Y)
References


Examples

```python
>>> from sklearn.pls import PLSCanonical, PLSRegression, CCA

>>> X = [[0., 0., 1.], [1.,0.,0.], [2.,2.,2.], [2.,5.,4.]]

>>> Y = [[0.1, -0.2], [0.9, 1.1], [6.2, 5.9], [11.9, 12.3]]

>>> pls2 = PLSRegression(n_components=2)

>>> pls2.fit(X, Y)

...  # Fit the model

>>> Y_pred = pls2.predict(X)

```

Attributes

- `x_weights_`: array, [p, n_components]
  - X block weights vectors.
- `y_weights_`: array, [q, n_components]
  - Y block weights vectors.
- `x_loadings_`: array, [p, n_components]
  - X block loadings vectors.
- `y_loadings_`: array, [q, n_components]
  - Y block loadings vectors.
- `x_scores_`: array, [n_samples, n_components]
  - X scores.
- `y_scores_`: array, [n_samples, n_components]
  - Y scores.
- `x_rotations_`: array, [p, n_components]
  - X block to latents rotations.
- `y_rotations_`: array, [q, n_components]
  - Y block to latents rotations.
- `coefs_`: array, [p, q]
  - The coefficients of the linear model: Y = X coefs + Err

Methods

- `fit(X, Y)`
- `get_params([deep])` Get parameters for the estimator.
- `predict(X[, copy])` Apply the dimension reduction learned on the train data.
- `set_params(**params)` Set the parameters of the estimator.
- `transform(X[, Y, copy])` Apply the dimension reduction learned on the train data.

```python
__init__(n_components=2, scale=True, max_iter=500, tol=1e-06, copy=True)
```

- `get_params`(deep=True)
  - Get parameters for the estimator.

Parameters deep: boolean, optional:
  - If True, will return the parameters for this estimator and contained subobjects that are estimators.

- `predict`(X, copy=True)
  - Apply the dimension reduction learned on the train data.
**Parameters**  

- **X**: array-like of predictors, shape = [n_samples, p]
  
  Training vectors, where n_samples is the number of samples and p is the number of predictors.

- **Y**: array-like of response, shape = [n_samples, q], optional
  
  Training vectors, where n_samples is the number of samples and q is the number of response variables.

- **copy**: boolean
  
  Whether to copy X and Y, or perform in-place normalization.

**Notes**

This call require the estimation of a p x q matrix, which may be an issue in high dimensional space.

**set_params** (**params**)

Set the parameters of the estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The former have parameters of the form `<component>__<parameter>` so that it's possible to update each component of a nested object.

**Returns self**:

**transform** (**X**, **Y**=None, **copy**=True)

Apply the dimension reduction learned on the train data.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>X</strong></td>
<td>array-like of predictors, shape = [n_samples, p]</td>
</tr>
<tr>
<td><strong>Y</strong></td>
<td>array-like of response, shape = [n_samples, q], optional</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>copy</th>
<th>boolean</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Whether to copy X and Y, or perform in-place normalization.</td>
</tr>
</tbody>
</table>

**Returns**

- **x_scores** if **Y** is not given, (**x_scores**, **y_scores**) otherwise.

---

**sklearn.pls.PLSCanonical**

**class** `sklearn.pls.PLSCanonical` (n_components=2, scale=True, algorithm='nipals', max_iter=500, tol=1e-06, copy=True)

PLSCanonical implements the 2 blocks canonical PLS of the original Wold algorithm [Tenenhaus 1998] p.204, refered as PLS-C2A in [Wegelin 2000].

This class inherits from PLS with mode="A" and deflation_mode="canonical", norm_y_weights=True and algorithm="nipals", but svd should provide similar results up to numerical errors.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>X</strong></td>
<td>array-like of predictors, shape = [n_samples, p]</td>
</tr>
<tr>
<td><strong>Y</strong></td>
<td>array-like of response, shape = [n_samples, q]</td>
</tr>
</tbody>
</table>

Training vectors, where n_samples is the number of samples and q is the number of response variables.
n_components : int, number of components to keep. (default 2).

scale : boolean, scale data? (default True)

algorithm : string, “nipals” or “svd”

   The algorithm used to estimate the weights. It will be called n_components times, i.e.
   once for each iteration of the outer loop.

max_iter : an integer, (default 500)

   the maximum number of iterations of the NIPALS inner loop (used only if algo-
   rithm=”nipals”)

tol : non-negative real, default 1e-06

   the tolerance used in the iterative algorithm

copy : boolean, default True

   Whether the deflation should be done on a copy. Let the default value to True unless
   you don’t care about side effect

See Also:

CCA, PLSSVD

Notes

For each component k, find weights u, v that optimize:: max corr(Xk u, Yk v) * var(Xk u) var(Yk u), such that
|u| = |v| = 1

Note that it maximizes both the correlations between the scores and the intra-block variances.

The residual matrix of X (Xk+1) block is obtained by the deflation on the current X score: x_score.

The residual matrix of Y (Yk+1) block is obtained by deflation on the current Y score. This performs a canonical
symetric version of the PLS regression. But slightly different than the CCA. This is mode mostly used for
modeling.

This implementation provides the same results that the “plspm” package provided in the R language (R-
project), using the function plsca(X, Y). Results are equal or colinear with the function
pls(..., mode = "canonical") of the “mixOmics” package. The difference relies in the fact that mixOmics implmentation
does not exactly implement the Wold algorithm since it does not normalize y_weights to one.

References

Jacob A. Wegelin. A survey of Partial Least Squares (PLS) methods, with emphasis on the two-block case.


Examples

```python
>>> from sklearn.pls import PLSCanonical, PLSRegression, CCA
>>> X = [[0., 0., 1.], [1., 0., 0.], [2., 2., 2.], [2., 5., 4.]]
>>> Y = [[0.3, -0.2], [0.5, 1.1], [6.2, 5.9], [11.9, 12.3]]
>>> plsca = PLSCanonical(n_components=2)
>>> plsca.fit(X, Y)
```
PLSCanonical(algorithm='nipals', copy=True, max_iter=500, n_components=2, scale=True, tol=1e-06)

>>> X_c, Y_c = plsca.transform(X, Y)

Attributes

<table>
<thead>
<tr>
<th>Attribute</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>x_weights_</td>
<td>array, shape = [p, n_components] X block weights vectors.</td>
</tr>
<tr>
<td>y_weights_</td>
<td>array, shape = [q, n_components] Y block weights vectors.</td>
</tr>
<tr>
<td>x_loadings_</td>
<td>array, shape = [p, n_components] X block loadings vectors.</td>
</tr>
<tr>
<td>y_loadings_</td>
<td>array, shape = [q, n_components] Y block loadings vectors.</td>
</tr>
<tr>
<td>x_scores_</td>
<td>array, shape = [n_samples, n_components] X scores.</td>
</tr>
<tr>
<td>y_scores_</td>
<td>array, shape = [n_samples, n_components] Y scores.</td>
</tr>
<tr>
<td>x_rotations_</td>
<td>array, shape = [p, n_components] X block to latents rotations.</td>
</tr>
<tr>
<td>y_rotations_</td>
<td>array, shape = [q, n_components] Y block to latents rotations.</td>
</tr>
</tbody>
</table>

Methods

<table>
<thead>
<tr>
<th>Method</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>fit(X, Y)</td>
<td>Get parameters for the estimator</td>
</tr>
<tr>
<td>get_params([deep])</td>
<td>Get parameters for the estimator</td>
</tr>
<tr>
<td>predict(X, copy)</td>
<td>Apply the dimension reduction learned on the train data.</td>
</tr>
<tr>
<td>set_params(**params)</td>
<td>Set the parameters of the estimator.</td>
</tr>
<tr>
<td>transform(X, Y, copy)</td>
<td>Apply the dimension reduction learned on the train data.</td>
</tr>
</tbody>
</table>

**__init__**(n_components=2, scale=True, algorithm='nipals', max_iter=500, tol=1e-06, copy=True)

**get_params**(deep=True)

Get parameters for the estimator

Parameters deep: boolean, optional:

If True, will return the parameters for this estimator and contained subobjects that are estimators.

**predict**(X, copy=True)

Apply the dimension reduction learned on the train data.

Parameters X: array-like of predictors, shape = [n_samples, p]

Training vectors, where n_samples in the number of samples and p is the number of predictors.

copy: boolean

Whether to copy X and Y, or perform in-place normalization.

Notes

This call require the estimation of a p x q matrix, which may be an issue in high dimensional space.

**set_params**(**params)

Set the parameters of the estimator.
The method works on simple estimators as well as on nested objects (such as pipelines). The former have parameters of the form `<component>__<parameter>` so that it’s possible to update each component of a nested object.

**Returns self**

`transform(X, Y=None, copy=True)`  
Apply the dimension reduction learned on the train data.

**Parameters**  
`X` : array-like of predictors, shape = [n_samples, p]  
Training vectors, where n_samples is the number of samples and p is the number of predictors.

`Y` : array-like of response, shape = [n_samples, q], optional  
Training vectors, where n_samples is the number of samples and q is the number of response variables.

`copy` : boolean  
Whether to copy X and Y, or perform in-place normalization.

**Returns**  
`x_scores` if Y is not given, (x_scores, y_scores) otherwise.

```python
sklearn.pls.CCA
```

**class sklearn.pls.CCA (n_components=2, scale=True, max_iter=500, tol=1e-06, copy=True)**  
CCA Canonical Correlation Analysis. CCA inherits from PLS with mode="B" and deflation_mode="canonical".

**Parameters**  
`X` : array-like of predictors, shape = [n_samples, p]  
Training vectors, where n_samples is the number of samples and p is the number of predictors.

`Y` : array-like of response, shape = [n_samples, q]  
Training vectors, where n_samples is the number of samples and q is the number of response variables.

`n_components` : int, (default 2).  
number of components to keep.

`scale` : boolean, (default True)  
whether to scale the data?

`max_iter` : an integer, (default 500)  
the maximum number of iterations of the NIPALS inner loop (used only if algorithm="nipals")

`tol` : non-negative real, default 1e-06.  
the tolerance used in the iterative algorithm

`copy` : boolean  
Whether the deflation be done on a copy. Let the default value to True unless you don’t care about side effects

**See Also:**  
PLSCanonical, PLSSVD
Notes

For each component k, find the weights u, v that maximizes max corr(Xk u, Yk v), such that |u| = |v| = 1

Note that it maximizes only the correlations between the scores.
The residual matrix of X (Xk+1) block is obtained by the deflation on the current X score: x_score.
The residual matrix of Y (Yk+1) block is obtained by deflation on the current Y score.

References


Examples

```python
>>> from sklearn.pls import PLSCanonical, PLSRegression, CCA

>>> X = [[0., 0., 1.], [1., 0., 0.], [2., 2., 2.], [3., 5., 4.]]

>>> Y = [[0.1, -0.2], [0.9, 1.1], [6.2, 5.9], [11.9, 12.3]]

>>> cca = CCA(n_components=1)

>>> cca.fit(X, Y)
...
CCA(copy=True, max_iter=500, n_components=1, scale=True, tol=1e-06)

>>> X_c, Y_c = cca.transform(X, Y)
```

Attributes

- `x_weights_` (array, [p, n_components]) X block weights vectors.
- `y_weights_` (array, [q, n_components]) Y block weights vectors.
- `x_loadings_` (array, [p, n_components]) X block loadings vectors.
- `y_loadings_` (array, [q, n_components]) Y block loadings vectors.
- `x_scores_` (array, [n_samples, n_components]) X scores.
- `y_scores_` (array, [n_samples, n_components]) Y scores.
- `x_rotations_` (array, [p, n_components]) X block to latents rotations.
- `y_rotations_` (array, [q, n_components]) Y block to latents rotations.

Methods

- `fit(X, Y)`
- `get_params([deep])` Get parameters for the estimator
- `predict(X[, copy])` Apply the dimension reduction learned on the train data.
- `set_params(**params)` Set the parameters of the estimator.
- `transform(X[, Y, copy])` Apply the dimension reduction learned on the train data.

```python
__init__ (n_components=2, scale=True, max_iter=500, tol=1e-06, copy=True)
```
scikit-learn user guide, Release 0.12-git

get_params (deep=True)
Get parameters for the estimator

Parameters deep: boolean, optional:
If True, will return the parameters for this estimator and contained subobjects that are estimators.

predict (X, copy=True)
Apply the dimension reduction learned on the train data.

Parameters X : array-like of predictors, shape = [n_samples, p]
Training vectors, where n_samples in the number of samples and p is the number of predictors.

copy : boolean
Whether to copy X and Y, or perform in-place normalization.

Notes
This call require the estimation of a \( p \times q \) matrix, which may be an issue in high dimensional space.

set_params (**params)
Set the parameters of the estimator.
The method works on simple estimators as well as on nested objects (such as pipelines). The former have parameters of the form \(<component>__<parameter>\) so that it’s possible to update each component of a nested object.

Returns self:

transform (X, Y=None, copy=True)
Apply the dimension reduction learned on the train data.

Parameters X : array-like of predictors, shape = [n_samples, p]
Training vectors, where n_samples in the number of samples and p is the number of predictors.

Y : array-like of response, shape = [n_samples, q], optional
Training vectors, where n_samples in the number of samples and q is the number of response variables.

copy : boolean
Whether to copy X and Y, or perform in-place normalization.

Returns x_scores if Y is not given, (x_scores, y_scores) otherwise.

sklearn.pls.PLSSVD

class sklearn.pls.PLSSVD (n_components=2, scale=True, copy=True)
Partial Least Square SVD
Simply perform a svd on the crosscovariance matrix: \( X'Y \) The are no iterative deflation here.

Parameters X : array-like of predictors, shape = [n_samples, p]
Training vector, where n_samples in the number of samples and p is the number of predictors. X will be centered before any analysis.
Y: array-like of response, shape = [n_samples, q]

Training vector, where n_samples in the number of samples and q is the number of
response variables. X will be centered before any analysis.

n_components: int, (default 2).

number of components to keep.

scale: boolean, (default True)

scale X and Y

See Also:

PLSCanonical, CCA

Attributes

<table>
<thead>
<tr>
<th>Attribute</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>x_weights_</td>
<td>array, [p, n_components] X block weights vectors.</td>
</tr>
<tr>
<td>y_weights_</td>
<td>array, [q, n_components] Y block weights vectors.</td>
</tr>
<tr>
<td>x_scores_</td>
<td>array, [n_samples, n_components] X scores.</td>
</tr>
<tr>
<td>y_scores_</td>
<td>array, [n_samples, n_components] Y scores.</td>
</tr>
</tbody>
</table>

Methods

- fit(X, Y)
- get_params([deep]) Get parameters for the estimator
- set_params(**params) Set the parameters of the estimator.
- transform(X[, Y]) Apply the dimension reduction learned on the train data.

__init__ (n_components=2, scale=True, copy=True)

get_params (deep=True)

Get parameters for the estimator

Parameters deep: boolean, optional:

If True, will return the parameters for this estimator and contained subobjects that are
estimators.

set_params (**params)

Set the parameters of the estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The former have
parameters of the form <component>__<parameter> so that it’s possible to update each component
of a nested object.

Returns self:

transform (X, Y=None)

Apply the dimension reduction learned on the train data.

1.8.23 sklearn.pipeline: Pipeline

The sklearn.pipeline module implements utilities to build a composite estimator, as a chain of transforms and estimators.
**pipeline.Pipeline(steps)**  
Pipeline of transforms with a final estimator.

**sklearn.pipeline.Pipeline**

**class sklearn.pipeline.Pipeline(steps)**  
Pipeline of transforms with a final estimator.

Sequentially apply a list of transforms and a final estimator. Intermediate steps of the pipeline must be ‘transforms’, that is, they must implements fit and transform methods. The final estimator needs only implements fit.

The purpose of the pipeline is to assemble several steps that can be cross-validated together while setting different parameters. For this, it enables setting parameters of the various steps using their names and the parameter name separated by a ‘__’, as in the example below.

**Parameters steps: list :**  
List of (name, transform) tuples (implementing fit/transform) that are chained, in the order in which they are chained, with the last object an estimator.

**Examples**

```python  
>>> from sklearn import svm  
>>> from sklearn.datasets import samples_generator  
>>> from sklearn.feature_selection import SelectKBest  
>>> from sklearn.feature_selection import f_regression  
>>> from sklearn.pipeline import Pipeline  

>>> # generate some data to play with  
>>> X, y = samples_generator.make_classification(  
...     n_informative=5, n_redundant=0, random_state=42)  

>>> # ANOVA SVM-C  
>>> anova_filter = SelectKBest(f_regression, k=5)  
>>> clf = svm.SVC(kernel='linear')  
>>> anova_svm = Pipeline([('anova', anova_filter), ('svc', clf)])  

>>> # You can set the parameters using the names issued  
>>> # For instance, fit using a k of 10 in the SelectKBest  
>>> # and a parameter ‘C’ of the svn  
>>> anova_svm.set_params(anova__k=10, svc__C=.1).fit(X, y)  

>>> prediction = anova_svm.predict(X)  
>>> anova_svm.score(X, y)  
0.75  
```

**Attributes**

| steps | list of (name, object) | List of the named object that compose the pipeline, in the order that they are applied on the data. |
Methods

<table>
<thead>
<tr>
<th>Method</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>decision_function</td>
<td>Applies transforms to the data, and the decision_function method of the final estimator. Valid only if the final estimator implements decision_function.</td>
</tr>
<tr>
<td>fit(X[, y])</td>
<td>Fit all the transforms one after the other and transform the data, then fit the transformed data using the final estimator.</td>
</tr>
<tr>
<td>fit_transform(X[, y])</td>
<td>Fit all the transforms one after the other and transform the data, then use fit_transform on transformed data using the final estimator. Valid only if the final estimator implements fit_transform.</td>
</tr>
<tr>
<td>get_params([deep])</td>
<td></td>
</tr>
<tr>
<td>inverse_transform</td>
<td></td>
</tr>
<tr>
<td>predict(X)</td>
<td>Applies transforms to the data, and the predict method of the final estimator. Valid only if the final estimator implements predict.</td>
</tr>
<tr>
<td>predict_log_proba</td>
<td></td>
</tr>
<tr>
<td>predict_proba(X)</td>
<td>Applies transforms to the data, and the predict_proba method of the final estimator. Valid only if the final estimator implements predict_proba.</td>
</tr>
<tr>
<td>score(X[, y])</td>
<td>Applies transforms to the data, and the score method of the final estimator. Valid only if the final estimator implements score.</td>
</tr>
<tr>
<td>set_params(**params)</td>
<td>Set the parameters of the estimator.</td>
</tr>
<tr>
<td>transform(X)</td>
<td>Applies transforms to the data, and the transform method of the final estimator. Valid only if the final estimator implements transform.</td>
</tr>
</tbody>
</table>

__init__(steps)

- decision_function (X)
  Applies transforms to the data, and the decision_function method of the final estimator. Valid only if the final estimator implements decision_function.

- fit (X, y=None, **fit_params)
  Fit all the transforms one after the other and transform the data, then fit the transformed data using the final estimator.

- fit_transform (X, y=None, **fit_params)
  Fit all the transforms one after the other and transform the data, then use fit_transform on transformed data using the final estimator. Valid only if the final estimator implements fit_transform.

- predict (X)
  Applies transforms to the data, and the predict method of the final estimator. Valid only if the final estimator implements predict.

- predict_proba (X)
  Applies transforms to the data, and the predict_proba method of the final estimator. Valid only if the final estimator implements predict_proba.

- score (X, y=None)
  Applies transforms to the data, and the score method of the final estimator. Valid only if the final estimator implements score.

- set_params (**params)
  Set the parameters of the estimator.

  The method works on simple estimators as well as on nested objects (such as pipelines). The former have parameters of the form `<component>__<parameter>` so that it’s possible to update each component of a nested object.

  Returns self:

- transform (X)
  Applies transforms to the data, and the transform method of the final estimator. Valid only if the final estimator implements transform.

1.8.24 sklearn.preprocessing: Preprocessing and Normalization

User guide: See the Preprocessing data section for further details.
**preprocessing.Scaler**

Standardize features by removing the mean and scaling to unit variance

```
preprocessing.Scaler(copy, with_mean, with_std)
```

Centering and scaling happen independently on each feature by computing the relevant statistics on the samples in the training set. Mean and standard deviation are then stored to be used on later data using the `transform` method.

Standardization of a dataset is a common requirement for many machine learning estimators: they might behave badly if the individual feature do not more or less look like standard normally distributed data (e.g. Gaussian with 0 mean and unit variance).

For instance many elements used in the objective function of a learning algorithm (such as the RBF kernel of Support Vector Machines or the L1 and L2 regularizers of linear models) assume that all features are centered around 0 and have variance in the same order. If a feature has a variance that is orders of magnitude larger that others, it might dominate the objective function and make the estimator unable to learn from other features correctly as expected.

**Parameters**

- `with_mean`: boolean, True by default
  - If True, center the data before scaling.

- `with_std`: boolean, True by default
  - If True, scale the data to unit variance (or equivalently, unit standard deviation).

- `copy`: boolean, optional, default is True
  - set to False to perform inplace row normalization and avoid a copy (if the input is already a numpy array or a scipy.sparse CSR matrix and if axis is 1).

**See Also:**

- `sklearn.preprocessing.scale`, `scaling`, `sklearn.decomposition.RandomizedPCA`

**Attributes**

- `mean_`: array of floats with shape [n_features]
  - The mean value for each feature in the training set.

- `std_`: array of floats with shape [n_features]
  - The standard deviation for each feature in the training set.

**Methods**

- `fit(X[, y])`: Compute the mean and std to be used for later scaling
- `fit_transform(X[, y])`: Fit to data, then transform it
- `get_params([deep])`: Get parameters for the estimator
- `inverse_transform(X[, copy])`: Scale back the data to the original representation
Table 1.172 – continued from previous page

<table>
<thead>
<tr>
<th>Method</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>set_params(**params)</td>
<td>Set the parameters of the estimator.</td>
</tr>
<tr>
<td>transform(X[, y, copy])</td>
<td>Perform standardization by centering and scaling</td>
</tr>
</tbody>
</table>

__init__ (copy=True, with_mean=True, with_std=True)

fit (X, y=None)
Compute the mean and std to be used for later scaling

Parameters X : array-like or CSR matrix with shape [n_samples, n_features]
The data used to compute the mean and standard deviation used for later scaling along
the features axis.

fit_transform (X, y=None, **fit_params)
Fit to data, then transform it
Fits transformer to X and y with optional parameters fit_params and returns a transformed version of X.

Parameters X : numpy array of shape [n_samples, n_features]
Training set.
y : numpy array of shape [n_samples]
Target values.

Returns X_new : numpy array of shape [n_samples, n_features_new]
Transformed array.

Notes
This method just calls fit and transform consecutively, i.e., it is not an optimized implementation of
fit_transform, unlike other transformers such as PCA.

get_params (deep=True)
Get parameters for the estimator

Parameters deep: boolean, optional :
If True, will return the parameters for this estimator and contained subobjects that are estimators.

inverse_transform (X, copy=None)
Scale back the data to the original representation

Parameters X : array-like with shape [n_samples, n_features]
The data used to scale along the features axis.

set_params (**params)
Set the parameters of the estimator.
The method works on simple estimators as well as on nested objects (such as pipelines). The former have
parameters of the form <component>__<parameter> so that it’s possible to update each component of
a nested object.

Returns self:

transform (X, y=None, copy=None)
Perform standardization by centering and scaling
Parameters $X$ : array-like with shape [n_samples, n_features]

The data used to scale along the features axis.

**sklearn.preprocessing.Normalizer**

class `sklearn.preprocessing.Normalizer (norm='l2', copy=True)`

Normalize samples individually to unit norm

Each sample (i.e. each row of the data matrix) with at least one non zero component is rescaled independently of other samples so that its norm (l1 or l2) equals one.

This transformer is able to work both with dense numpy arrays and scipy.sparse matrix (use CSR format if you want to avoid the burden of a copy / conversion).

Scaling inputs to unit norms is a common operation for text classification or clustering for instance. For instance the dot product of two l2-normalized TF-IDF vectors is the cosine similarity of the vectors and is the base similarity metric for the Vector Space Model commonly used by the Information Retrieval community.

Parameters **norm** : ‘l1’ or ‘l2’, optional (‘l2’ by default)

The norm to use to normalize each non zero sample.

copy : boolean, optional, default is True

set to False to perform inplace row normalization and avoid a copy (if the input is already a numpy array or a scipy.sparse CSR matrix).

See Also:

`sklearn.preprocessing.normalize`, without

Notes

This estimator is stateless (besides constructor parameters), the fit method does nothing but is useful when used in a pipeline.

Methods

```python
fit(X[, y])
Do nothing and return the estimator unchanged

fit_transform(X[, y])
Fit to data, then transform it

get_params([deep])
Get parameters for the estimator

set_params(**params)
Set the parameters of the estimator.

transform(X[, y, copy])
Scale each non zero row of X to unit norm
```

```python
__init__ (norm='l2', copy=True)

fit (X, y=None)
Do nothing and return the estimator unchanged

This method is just there to implement the usual API and hence work in pipelines.

fit_transform (X, y=None, **fit_params)
Fit to data, then transform it

Fits transformer to X and y with optional parameters fit_params and returns a transformed version of X.
```
Parameters X : numpy array of shape [n_samples, n_features]
    Training set.
y : numpy array of shape [n_samples]
    Target values.
Returns X_new : numpy array of shape [n_samples, n_features_new]
    Transformed array.

Notes

This method just calls fit and transform consecutively, i.e., it is not an optimized implementation of fit_transform, unlike other transformers such as PCA.

get_params (deep=True)
    Get parameters for the estimator

    Parameters deep: boolean, optional :
        If True, will return the parameters for this estimator and contained subobjects that are estimators.

set_params (**params)
    Set the parameters of the estimator.

    The method works on simple estimators as well as on nested objects (such as pipelines). The former have parameters of the form <component>__<parameter> so that it’s possible to update each component of a nested object.

    Returns self :

transform (X, y=None, copy=None)
    Scale each non zero row of X to unit norm

    Parameters X : array or scipy.sparse matrix with shape [n_samples, n_features]
        The data to normalize, row by row. scipy.sparse matrices should be in CSR format to avoid an un-necessary copy.

sklearn.preprocessing.Binarizer

class sklearn.preprocessing.Binarizer (threshold=0.0, copy=True)
    Binarize data (set feature values to 0 or 1) according to a threshold

    The default threshold is 0.0 so that any non-zero values are set to 1.0 and zeros are left untouched.

    Binarization is a common operation on text count data where the analyst can decide to only consider the presence or absence of a feature rather than a quantified number of occurrences for instance.

    It can also be used as a pre-processing step for estimators that consider boolean random variables (e.g. modeled using the Bernoulli distribution in a Bayesian setting).

    Parameters threshold : float, optional (0.0 by default)
        The lower bound that triggers feature values to be replaced by 1.0.

    copy : boolean, optional, default is True
        set to False to perform inplace binarization and avoid a copy (if the input is already a numpy array or a scipy.sparse CSR matrix).
Notes

If the input is a sparse matrix, only the non-zero values are subject to update by the Binarizer class.

This estimator is stateless (besides constructor parameters), the fit method does nothing but is useful when used in a pipeline.

Methods

<table>
<thead>
<tr>
<th>Method</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>fit(X[, y])</td>
<td>Do nothing and return the estimator unchanged</td>
</tr>
<tr>
<td>fit_transform(X[, y])</td>
<td>Fit to data, then transform it</td>
</tr>
<tr>
<td>get_params([deep])</td>
<td>Get parameters for the estimator.</td>
</tr>
<tr>
<td>set_params(**params)</td>
<td>Set the parameters of the estimator.</td>
</tr>
<tr>
<td>transform(X[, y, copy])</td>
<td>Binarize each element of X</td>
</tr>
</tbody>
</table>

__init__(threshold=0.0, copy=True)

fit (X, y=None)
Do nothing and return the estimator unchanged

This method is just there to implement the usual API and hence work in pipelines.

fit_transform (X, y=None, **fit_params)
Fit to data, then transform it

Fits transformer to X and y with optional parameters fit_params and returns a transformed version of X.

Parameters X : numpy array of shape [n_samples, n_features]
Training set.

y : numpy array of shape [n_samples]
Target values.

Returns X_new : numpy array of shape [n_samples, n_features_new]
Transformed array.

Notes

This method just calls fit and transform consecutively, i.e., it is not an optimized implementation of fit_transform, unlike other transformers such as PCA.

get_params (deep=True)
Get parameters for the estimator

Parameters deep : boolean, optional :
If True, will return the parameters for this estimator and contained subobjects that are estimators.

set_params (**params)
Set the parameters of the estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The former have parameters of the form `<component>__<parameter>` so that it’s possible to update each component of a nested object.
Returns `self`:

```
transform(X, y=None, copy=None)
```

Binarize each element of `X`

**Parameters**

- **X**: array or scipy.sparse matrix with shape `[n_samples, n_features]`
  
  The data to binarize, element by element. scipy.sparse matrices should be in CSR format to avoid an un-necessary copy.

### sklearn.preprocessing.LabelBinarizer

**class** `sklearn.preprocessing.LabelBinarizer(neg_label=0, pos_label=1)`

Binarize labels in a one-vs-all fashion.

Several regression and binary classification algorithms are available in the scikit. A simple way to extend these algorithms to the multi-class classification case is to use the so-called one-vs-all scheme.

At learning time, this simply consists in learning one regressor or binary classifier per class. In doing so, one needs to convert multi-class labels to binary labels (belong or does not belong to the class). LabelBinarizer makes this process easy with the transform method.

At prediction time, one assigns the class for which the corresponding model gave the greatest confidence. LabelBinarizer makes this easy with the inverse_transform method.

**Parameters**

- **neg_label**: int (default: 0)
  
  Value with which negative labels must be encoded.

- **pos_label**: int (default: 1)
  
  Value with which positive labels must be encoded.

### Examples

```python
>>> from sklearn import preprocessing
>>> lb = preprocessing.LabelBinarizer()
>>> lb.fit([1, 2, 6, 4, 2])
LabelBinarizer(neg_label=0, pos_label=1)
>>> lb.classes_
array([1, 2, 4, 6])
>>> lb.transform([1, 6])
array([[ 1., 0., 0., 0.],
       [ 0., 0., 0., 1.]])
```

```python
>>> lb.fit_transform([(1, 2), (3,)])
array([[ 1., 1., 0.],
       [ 0., 0., 1.]])
```

```python
>>> lb.classes_
array([1, 2, 3])
```

### Attributes

- **classes_**: array of shape `[n_class]`  
  
  Holds the label for each class.
Methods
### fit(y)
Fit label binarizer

**Parameters**

- **y**: numpy array of shape [n_samples] or sequence of sequences
  
  Target values. In the multilabel case the nested sequences can have variable lengths.

**Returns**

- **self**: returns an instance of self.

### fit_transform(X, y=None, **fit_params)
Fit to data, then transform it

Fits transformer to X and y with optional parameters fit_params and returns a transformed version of X.

**Parameters**

- **X**: numpy array of shape [n_samples, n_features]
  
  Training set.

- **y**: numpy array of shape [n_samples]
  
  Target values.

**Returns**

- **X_new**: numpy array of shape [n_samples, n_features_new]
  
  Transformed array.

**Notes**

This method just calls fit and transform consecutively, i.e., it is not an optimized implementation of fit_transform, unlike other transformers such as PCA.

### get_params(deep=True)
Get parameters for the estimator

**Parameters**

- **deep**: boolean, optional
  
  If True, will return the parameters for this estimator and contained subobjects that are estimators.

### inverse_transform(Y, threshold=None)
Transform binary labels back to multi-class labels

**Parameters**

- **Y**: numpy array of shape [n_samples, n_classes]
  
  Target values.

- **threshold**: float or None
  
  Threshold used in the binary and multi-label cases.

**Use 0 when:**

- Y contains the output of decision_function (classifier)
Use 0.5 when:

- Y contains the output of predict_proba

If None, the threshold is assumed to be half way between neg_label and pos_label.

Returns y : numpy array of shape [n_samples] or sequence of sequences

Target values. In the multilabel case the nested sequences can have variable lengths.

Notes

In the case when the binary labels are fractional (probabilistic), inverse_transform chooses the class with the greatest value. Typically, this allows to use the output of a linear model’s decision_function method directly as the input of inverse_transform.

set_params (**params)

Set the parameters of the estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The former have parameters of the form <component>__<parameter> so that it’s possible to update each component of a nested object.

Returns self :

transform (y)

Transform multi-class labels to binary labels

The output of transform is sometimes referred to by some authors as the 1-of-K coding scheme.

Parameters y : numpy array of shape [n_samples] or sequence of sequences

Target values. In the multilabel case the nested sequences can have variable lengths.

Returns Y : numpy array of shape [n_samples, n_classes]

sklearn.preprocessing.KernelCenterer

class sklearn.preprocessing.KernelCenterer

Center a kernel matrix

This is equivalent to centering phi(X) with sklearn.preprocessing.Scaler(with_std=False).

Methods

<table>
<thead>
<tr>
<th>Method</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>fit(K)</td>
<td>Fit KernelCenterer</td>
</tr>
<tr>
<td>fit_transform(X, y)</td>
<td>Fit to data, then transform it</td>
</tr>
<tr>
<td>get_params([deep])</td>
<td>Get parameters for the estimator.</td>
</tr>
<tr>
<td>set_params(**params)</td>
<td>Set the parameters of the estimator.</td>
</tr>
<tr>
<td>transform(K, copy)</td>
<td>Center kernel</td>
</tr>
</tbody>
</table>

__init__()

x.__init__(...) initializes x; see help(type(x)) for signature

fit (K)

Fit KernelCenterer
Parameters $K$ : numpy array of shape $[n_{\text{samples}}, n_{\text{samples}}]$

Kernel matrix.

Returns self : returns an instance of self.

**fit_transform** $(X, y=\text{None}, **\text{fit_params})$

Fit to data, then transform it

Fits transformer to $X$ and $y$ with optional parameters fit_params and returns a transformed version of $X$.

Parameters $X$ : numpy array of shape $[n_{\text{samples}}, n_{\text{features}}]$

Training set.

$y$ : numpy array of shape $[n_{\text{samples}}]$

Target values.

Returns $X_{\text{new}}$ : numpy array of shape $[n_{\text{samples}}, n_{\text{features}_{\text{new}}}]$

Transformed array.

Notes

This method just calls fit and transform consecutively, i.e., it is not an optimized implementation of fit_transform, unlike other transformers such as PCA.

**get_params** $(\text{deep}=\text{True})$

Get parameters for the estimator

Parameters $\text{deep}$: boolean, optional :

If True, will return the parameters for this estimator and contained subobjects that are estimators.

**set_params** $(**\text{params})$

Set the parameters of the estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The former have parameters of the form $<\text{component}>__<\text{parameter}>$ so that it’s possible to update each component of a nested object.

Returns self :

**transform** $(K, \text{copy}=\text{True})$

Center kernel

Parameters $K$ : numpy array of shape $[n_{\text{samples}_{1}}, n_{\text{samples}_{2}}]$

Kernel matrix.

Returns $K_{\text{new}}$ : numpy array of shape $[n_{\text{samples}_{1}}, n_{\text{samples}_{2}}]$

<table>
<thead>
<tr>
<th>Function</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>preprocessing.scale($X[, \text{axis, with_mean, ...}]$)</td>
<td>Standardize a dataset along any axis</td>
</tr>
<tr>
<td>preprocessing.normalize($X[, \text{norm, axis, copy}]$)</td>
<td>Normalize a dataset along any axis</td>
</tr>
<tr>
<td>preprocessing.binarize($X[, \text{threshold, copy}]$)</td>
<td>Boolean thresholding of array-like or scipy.sparse matrix</td>
</tr>
</tbody>
</table>

**sklearn.preprocessing.scale**

**sklearn.preprocessing.scale** $(X, \text{axis}=0, \text{with\_mean}=\text{True}, \text{with\_std}=\text{True}, \text{copy}=\text{True})$

Standardize a dataset along any axis
Center to the mean and component wise scale to unit variance.

**Parameters**

- **X**: array-like or CSR matrix.
  - The data to center and scale.
- **axis**: int (0 by default)
  - axis used to compute the means and standard deviations along. If 0, independently standardize each feature, otherwise (if 1) standardize each sample.
- **with_mean**: boolean, True by default
  - If True, center the data before scaling.
- **with_std**: boolean, True by default
  - If True, scale the data to unit variance (or equivalently, unit standard deviation).
- **copy**: boolean, optional, default is True
  - set to False to perform inplace row normalization and avoid a copy (if the input is already a numpy array or a scipy.sparse CSR matrix and if axis is 1).

**See Also:**

- `sklearn.preprocessing.Scaler`, `scaling`, `sklearn.pipeline.Pipeline`

**Notes**

This implementation will refuse to center scipy.sparse matrices since it would make them non-sparse and would potentially crash the program with memory exhaustion problems.

Instead the caller is expected to either set explicitly `with_mean=False` (in that case, only variance scaling will be performed on the features of the CSR matrix) or to call `X.toarray()` if he/she expects the materialized dense array to fit in memory.

To avoid memory copy the caller should pass a CSR matrix.

**sklearn.preprocessing.normalize**

```python
sklearn.preprocessing.normalize(X, norm='l2', axis=1, copy=True)
```

Normalize a dataset along any axis

- **X**: array or scipy.sparse matrix with shape [n_samples, n_features]
  - The data to normalize, element by element. scipy.sparse matrices should be in CSR format to avoid an un-necessary copy.
- **norm**: ‘l1’ or ‘l2’, optional (‘l2’ by default)
  - The norm to use to normalize each non zero sample (or each non-zero feature if axis is 0).
- **axis**: 0 or 1, optional (1 by default)
  - axis used to normalize the data along. If 1, independently normalize each sample, otherwise (if 0) normalize each feature.
- **copy**: boolean, optional, default is True
  - set to False to perform inplace row normalization and avoid a copy (if the input is already a numpy array or a scipy.sparse CSR matrix and if axis is 1).
sklearn.preprocessing.binarize

sklearn.preprocessing.binarize(X, threshold=0.0, copy=True)

Boolean thresholding of array-like or scipy.sparse matrix

Parameters

- **X**: array or scipy.sparse matrix with shape [n_samples, n_features]
  - The data to binarize, element by element. scipy.sparse matrices should be in CSR format to avoid an un-necessary copy.
- **threshold**: float, optional (0.0 by default)
  - The lower bound that triggers feature values to be replaced by 1.0.
- **copy**: boolean, optional, default is True
  - Set to False to perform inplace binarization and avoid a copy (if the input is already a numpy array or a scipy.sparse CSR matrix and if axis is 1).

See Also:

- sklearn.preprocessing.Binarizer, using, sklearn.pipeline.Pipeline

1.8.25 sklearn.qda: Quadratic Discriminant Analysis

Quadratic Discriminant Analysis

User guide: See the Linear and Quadratic Discriminant Analysis section for further details.

---

**qda.QDA([priors])**

Quadratic Discriminant Analysis (QDA)

**class sklearn.qda.QDA (priors=None)**

Quadratic Discriminant Analysis (QDA)

A classifier with a quadratic decision boundary, generated by fitting class conditional densities to the data and using Bayes’ rule.

The model fits a Gaussian density to each class.

Parameters

- **priors**: array, optional, shape = [n_classes]
  - Priors on classes

See Also:

- sklearn.lda.LDA
- sklearn.qda.QDA

Examples

```python
>>> from sklearn.qda import QDA
>>> import numpy as np
>>> X = np.array([[-1, -1], [-2, -1], [-3, -2], [1, 1], [2, 1], [3, 2]])
```
```python
>>> y = np.array([1, 1, 1, 2, 2, 2])
>>> clf = QDA()
>>> clf.fit(X, y)
QDA(priors=None)
>>> print(clf.predict([[-0.8, -1]]))
[1]
```

### Attributes

<table>
<thead>
<tr>
<th>means_</th>
<th>array-like, shape = [n_classes, n_features]</th>
<th>Class means</th>
</tr>
</thead>
<tbody>
<tr>
<td>priors_</td>
<td>array-like, shape = [n_classes]</td>
<td>Class priors (sum to 1)</td>
</tr>
<tr>
<td>covariances_</td>
<td>list of array-like, shape = [n_features, n_features]</td>
<td>Covariance matrices of each class</td>
</tr>
</tbody>
</table>

### Methods

- **decision_function**(X)
  - Apply decision function to an array of samples.

- **fit**(X, y[, store_covariances, tol])
  - Fit the QDA model according to the given training data and parameters.

- **get_params**(deep)
  - Get parameters for the estimator

- **predict**(X)
  - Perform classification on an array of test vectors X.

- **predict_log_proba**(X)
  - Return posterior probabilities of classification.

- **predict_proba**(X)
  - Return posterior probabilities of classification.

- **score**(X, y)
  - Returns the mean accuracy on the given test data and labels.

- **set_params**(**params)
  - Set the parameters of the estimator.

---

```python
__init__(priors=None)
```

```python
def decision_function(X)
    Apply decision function to an array of samples.

    Parameters X : array-like, shape = [n_samples, n_features]
        Array of samples (test vectors).

    Returns C : array, shape = [n_samples, n_classes]
        Decision function values related to each class, per sample.
```

```python
def fit(X, y, store_covariances=False, tol=0.0001)
    Fit the QDA model according to the given training data and parameters.

    Parameters X : array-like, shape = [n_samples, n_features]
        Training vector, where n_samples in the number of samples and n_features is the number of features.

    y : array, shape = [n_samples]
        Target values (integers)

    store_covariances : boolean
        If True the covariance matrices are computed and stored in the self.covariances_ attribute.
```

```python
def get_params(deep=True)
    Get parameters for the estimator
```
Parameters deep: boolean, optional

If True, will return the parameters for this estimator and contained subobjects that are estimators.

predict (X)

Perform classification on an array of test vectors X.

The predicted class C for each sample in X is returned.

Parameters X : array-like, shape = [n_samples, n_features]

Returns C : array, shape = [n_samples]

predict_log_proba (X)

Return posterior probabilities of classification.

Parameters X : array-like, shape = [n_samples, n_features]

Array of samples/test vectors.

Returns C : array, shape = [n_samples, n_classes]

Posterior log-probabilities of classification per class.

predict_proba (X)

Return posterior probabilities of classification.

Parameters X : array-like, shape = [n_samples, n_features]

Array of samples/test vectors.

Returns C : array, shape = [n_samples, n_classes]

Posterior probabilities of classification per class.

score (X, y)

Returns the mean accuracy on the given test data and labels.

Parameters X : array-like, shape = [n_samples, n_features]

Training set.

y : array-like, shape = [n_samples]

Labels for X.

Returns z : float

set_params (**params)

Set the parameters of the estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The former have parameters of the form <component>__<parameter> so that it’s possible to update each component of a nested object.

Returns self :

1.8.26 sklearn.svm: Support Vector Machines

The sklearn.svm module includes Support Vector Machine algorithms.

User guide: See the Support Vector Machines section for further details.
**Estimators**

<table>
<thead>
<tr>
<th>Class</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>svm.SVC</code>([C, kernel, degree, gamma, coef0, ...])</td>
<td>C-Support Vector Classification.</td>
</tr>
<tr>
<td><code>svm.LinearSVC</code>([penalty, loss, dual, tol, C, ...])</td>
<td>Linear Support Vector Classification.</td>
</tr>
<tr>
<td><code>svm.NuSVC</code>([nu, kernel, degree, gamma, ...])</td>
<td>Nu-Support Vector Classification.</td>
</tr>
<tr>
<td><code>svm.SVR</code>([kernel, degree, gamma, coef0, tol, ...])</td>
<td>epsilon-Support Vector Regression.</td>
</tr>
<tr>
<td><code>svm.NuSVR</code>([nu, C, kernel, degree, gamma, ...])</td>
<td>Nu Support Vector Regression.</td>
</tr>
<tr>
<td><code>svm.OneClassSVM</code>([kernel, degree, gamma, ...])</td>
<td>Unsupervised Outliers Detection.</td>
</tr>
</tbody>
</table>

```python
# sklearn.svm.SVC

class sklearn.svm.SVC(C=1.0, kernel='rbf', degree=3, gamma=0.0, coef0=0.0, shrinking=True, probability=False, tol=0.001, cache_size=200, class_weight=None, verbose=False)

C-Support Vector Classification.
```

The implementations is based on libsvm. The fit time complexity is more than quadratic with the number of samples which makes it hard to scale to dataset with more than a couple of 10000 samples.

The multiclass support is handled according to a one-vs-one scheme.

For details on the precise mathematical formulation of the provided kernel functions and how `gamma`, `coef0` and `degree` affect each, see the corresponding section in the narrative documentation: [Kernel functions](#).

**Parameters**

- **C**: float or None, optional (default=None)
  Penalty parameter C of the error term. If None then C is set to n_samples.

- **kernel**: string, optional (default='rbf')
  Specifies the kernel type to be used in the algorithm. It must be one of 'linear', 'poly', 'rbf', 'sigmoid', 'precomputed'. If none is given, 'rbf' will be used.

- **degree**: int, optional (default=3)
  Degree of kernel function. It is significant only in 'poly' and 'sigmoid'.

- **gamma**: float, optional (default=0.0)
  Kernel coefficient for 'rbf' and 'poly'. If gamma is 0.0 then 1/n_features will be used instead.

- **coef0**: float, optional (default=0.0)
  Independent term in kernel function. It is only significant in 'poly' and 'sigmoid'.

- **probability**: boolean, optional (default=False)
  Whether to enable probability estimates. This must be enabled prior to calling predict_proba.

- **shrinking**: boolean, optional (default=True)
  Whether to use the shrinking heuristic.

- **tol**: float, optional (default=1e-3)
  Tolerance for stopping criterion.

- **cache_size**: float, optional
  Specify the size of the kernel cache (in MB)

- **class_weight**: {dict, ‘auto’}, optional
Set the parameter C of class i to class_weight[i]*C for SVC. If not given, all classes are supposed to have weight one. The 'auto' mode uses the values of y to automatically adjust weights inversely proportional to class frequencies.

**verbose** : bool, default: False

Enable verbose output. Note that this setting takes advantage of a per-process runtime setting in libsvm that, if enabled, may not work properly in a multithreaded context.

**See Also:**

*SVR* Support Vector Machine for Regression implemented using libsvm.

*LinearSVC* Scalable Linear Support Vector Machine for classification implemented using liblinear. Check the See also section of LinearSVC for more comparison element.

**Examples**

```python
>>> import numpy as np
>>> X = np.array([[-1, -1], [-2, -1], [1, 1], [2, 1]])
>>> y = np.array([1, 1, 2, 2])
>>> from sklearn.svm import SVC
>>> clf = SVC()
>>> clf.fit(X, y)
SVC(C=1.0, cache_size=200, class_weight=None, coef0=0.0, degree=3,
    gamma=0.5, kernel='rbf', probability=False, shrinking=True,
    tol=0.001, verbose=False)
>>> print(clf.predict([[-0.8, -1]]))
[ 1.]
```

**Attributes**

<table>
<thead>
<tr>
<th>Name</th>
<th>Type</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>support_</td>
<td>array-like, shape = [n_SV]</td>
<td>Index of support vectors.</td>
</tr>
<tr>
<td>support_vectors</td>
<td>array-like, shape [n_SV, n_features]</td>
<td>Support vectors.</td>
</tr>
<tr>
<td>n_support</td>
<td>array-like, dtype=int32, shape = [n_class]</td>
<td>number of support vector for each class.</td>
</tr>
<tr>
<td>dual_coef</td>
<td>array, shape = [n_class-1, n_SV]</td>
<td>Coefficients of the support vector in the decision function. For multiclass, coefficient for all 1-vs-1 classifiers. The layout of the coefficients in the multiclass case is somewhat non-trivial. See the section about multi-class classification in the SVM section of the User Guide for details.</td>
</tr>
<tr>
<td>coef_</td>
<td>array, shape = [n_class-1, n_features]</td>
<td>Weights assigned to the features (coefficients in the primal problem). This is only available in the case of linear kernel. coef_ is readonly property derived from dual_coef_ and support_vectors_.</td>
</tr>
<tr>
<td>intercept_</td>
<td>array, shape = [n_class * (n_class-1) / 2]</td>
<td>Constants in decision function.</td>
</tr>
</tbody>
</table>

**Methods**
**decision_function**(X) Distance of the samples X to the separating hyperplane.

**fit**(X, y[, class_weight, sample_weight]) Fit the SVM model according to the given training data.

**get_params**(deep) Get parameters for the estimator

**predict**(X) Perform classification or regression samples in X.

**predict_log_proba**(X) Compute the log likelihoods each possible outcomes of samples in X.

**predict_proba**(X) Compute the likelihoods each possible outcomes of samples in T.

**score**(X, y) Returns the mean accuracy on the given test data and labels.

**set_params**(**params) Set the parameters of the estimator.

**__init__** (C=1.0, kernel='rbf', degree=3, gamma=0.0, coef0=0.0, shrinking=True, probability=False, tol=0.001, cache_size=200, class_weight=None, verbose=False)

**decision_function**(X)

  Distance of the samples X to the separating hyperplane.

  Parameters X : array-like, shape = [n_samples, n_features]
  
  Returns X : array-like, shape = [n_samples, n_class * (n_class-1) / 2]
  
  Returns the decision function of the sample for each class in the model.

**fit**(X, y, class_weight=None, sample_weight=None)

  Fit the SVM model according to the given training data.

  Parameters X : {array-like, sparse matrix}, shape = [n_samples, n_features]
  
  Training vectors, where n_samples is the number of samples and n_features is the number of features.

  y : array-like, shape = [n_samples]
  
  Target values (integers in classification, real numbers in regression)

  sample_weight : array-like, shape = [n_samples], optional
  
  Weights applied to individual samples (1. for unweighted).

  Returns self : object
  
  Returns self.

**Notes**

If X and y are not C-ordered and contiguous arrays of np.float64 and X is not a scipy.sparse.csr_matrix, X and/or y may be copied.

If X is a dense array, then the other methods will not support sparse matrices as input.

**get_params**(deep=True)

  Get parameters for the estimator

  Parameters deep : boolean, optional
  
  If True, will return the parameters for this estimator and contained subobjects that are estimators.

**predict**(X)

  Perform classification or regression samples in X.

  For a classification model, the predicted class for each sample in X is returned. For a regression model, the function value of X calculated is returned.
For an one-class model, +1 or -1 is returned.

Parameters:

- **X**: array-like, sparse matrix, shape = [n_samples, n_features]
- **C**: array, shape = [n_samples]

**predict_log_proba** *(X)*

Compute the log likelihoods each possible outcomes of samples in **X**.

The model need to have probability information computed at training time: fit with attribute `probability` set to True.

Parameters:

- **X**: array-like, shape = [n_samples, n_features]
- **X**: array-like, shape = [n_samples, n_classes]

Returns the log-probabilities of the sample for each class in the model, where classes are ordered by arithmetical order.

**Notes**

The probability model is created using cross validation, so the results can be slightly different than those obtained by predict. Also, it will meaningless results on very small datasets.

**predict_proba** *(X)*

Compute the likelihoods each possible outcomes of samples in **T**.

The model need to have probability information computed at training time: fit with attribute `probability` set to True.

Parameters:

- **X**: array-like, shape = [n_samples, n_features]
- **X**: array-like, shape = [n_samples, n_classes]

Returns the probability of the sample for each class in the model, where classes are ordered by arithmetical order.

**Notes**

The probability model is created using cross validation, so the results can be slightly different than those obtained by predict. Also, it will produce meaningless results on very small datasets.

**score** *(X, y)*

Returns the mean accuracy on the given test data and labels.

Parameters:

- **X**: array-like, shape = [n_samples, n_features]
  
  Training set.

- **y**: array-like, shape = [n_samples]
  
  Labels for **X**.

Returns **z**: float

set_params **(** **params** **)**

Set the parameters of the estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The former have parameters of the form `<component>__<parameter>` so that it's possible to update each component of a nested object.

Returns **self**: 
**sklearn.svm.LinearSVC**

```python
class sklearn.svm.LinearSVC(penalty='l2', loss='l2', dual=True, tol=0.0001, C=1.0, multi_class='ovr', fit_intercept=True, intercept_scaling=1, class_weight=None, verbose=0)
```

Linear Support Vector Classification.

Similar to SVC with parameter kernel='linear', but implemented in terms of liblinear rather than libsvm, so it has more flexibility in the choice of penalties and loss functions and should scale better (to large numbers of samples).

This class supports both dense and sparse input and the multiclass support is handled according to a one-vs-the-rest scheme.

**Parameters**

- **C** : float or None, optional (default=None)
  Penalty parameter C of the error term. If None then C is set to n_samples.

- **loss** : string, ‘l1’ or ‘l2’ (default=’l2’)
  Specifies the loss function. ‘l1’ is the hinge loss (standard SVM) while ‘l2’ is the squared hinge loss.

- **penalty** : string, ‘l1’ or ‘l2’ (default=’l2’)
  Specifies the norm used in the penalization. The ‘l2’ penalty is the standard used in SVC. The ‘l1’ leads to coef_vectors that are sparse.

- **dual** : bool, (default=True)
  Select the algorithm to either solve the dual or primal optimization problem. Prefer dual=False when n_samples > n_features.

- **tol** : float, optional (default=1e-4)
  Tolerance for stopping criteria

- **multi_class** : string, ‘ovr’ or ‘crammer_singer’ (default=’ovr’)
  Determines the multi-class strategy if y contains more than two classes. ovr trains n_classes one-vs-rest classifiers, while crammer_singer optimizes a joint objective over all classes. While crammer_singer is interesting from an theoretical perspective as it is consistent it is seldom used in practice and rarely leads to better accuracy and is more expensive to compute. If crammer_singer is choosen, the options loss, penalty and dual will be ignored.

- **fit_intercept** : boolean, optional (default=True)
  Whether to calculate the intercept for this model. If set to false, no intercept will be used in calculations (e.g. data is expected to be already centered).

- **intercept_scaling** : float, optional (default=1)
  when self.fit_intercept is True, instance vector x becomes [x, self.intercept_scaling], i.e. a “synthetic” feature with constant value equals to intercept_scaling is appended to the instance vector. The intercept becomes intercept_scaling * synthetic feature weight
  Note! the synthetic feature weight is subject to l1/l2 regularization as all other features. To lessen the effect of regularization on synthetic feature weight (and therefore on the intercept) intercept_scaling has to be increased

- **class_weight** : {dict, ‘auto’}, optional
Set the parameter C of class i to class_weight[i]*C for SVC. If not given, all classes are supposed to have weight one. The ‘auto’ mode uses the values of y to automatically adjust weights inversely proportional to class frequencies.

verbose : int, default: 0

Enable verbose output. Note that this setting takes advantage of a per-process runtime setting in liblinear that, if enabled, may not work properly in a multithreaded context.

See Also:

SVC
Implementation of Support Vector Machine classifier using libsvm: the kernel can be non-linear but its SMO algorithm does not scale to large number of samples as LinearSVC does. Furthermore SVC multi-class mode is implemented using one vs one scheme while LinearSVC uses one vs the rest. It is possible to implement one vs the rest with SVC by using the sklearn.multiclass.OneVsRestClassifier wrapper. Finally SVC can fit dense data without memory copy if the input is C-contiguous. Sparse data will still incur memory copy though.

sklearn.linear_model.SGDClassifier
SGDClassifier can optimize the same cost function as LinearSVC by adjusting the penalty and loss parameters. Furthermore SGDClassifier is scalable to large number of samples as it uses a Stochastic Gradient Descent optimizer. Finally SGDClassifier can fit both dense and sparse data without memory copy if the input is C-contiguous or CSR.

Notes

The underlying C implementation uses a random number generator to select features when fitting the model. It is thus not uncommon, to have slightly different results for the same input data. If that happens, try with a smaller tol parameter.

The underlying implementation (liblinear) uses a sparse internal representation for the data that will incur a memory copy.

References: LIBLINEAR: A Library for Large Linear Classification

Attributes

| coef_ | array, shape = [n_features] if n_classes == 2 else [n_classes, n_features] | Weights assigned to the features (coefficients in the primal problem). This is only available in the case of linear kernel. coef_ is readonly property derived from raw_coef_ that follows the internal memory layout of liblinear. |

Methods

| decision_function(X) | Decision function value for X according to the trained model. |
| fit(X, y[, class_weight]) | Fit the model according to the given training data. |
| fit_transform(X[, y]) | Fit to data, then transform it |
| get_params([deep]) | Get parameters for the estimator |
| predict(X) | Predict target values of X according to the fitted model. |
| score(X, y) | Returns the mean accuracy on the given test data and labels. |

Continued on next page
**set_params**(**params**)  
Set the parameters of the estimator.

**transform**(**X**, [**threshold**])  
Reduce X to its most important features.

---

__init__ (penalty='l2', loss='l2', dual=True, tol=0.0001, C=1.0, multi_class='ovr', fit_intercept=True, intercept_scaling=1, class_weight=None, verbose=0)

decision_function (X)
Decision function value for X according to the trained model.

Parameters X : array-like, shape = [n_samples, n_features]

Returns T : array-like, shape = [n_samples, n_class]

Returns the decision function of the sample for each class in the model.

**fit** (**X**, **y**, **class_weight**=None)
Fit the model according to the given training data.

Parameters X : {array-like, sparse matrix}, shape = [n_samples, n_features]

Training vector, where n_samples in the number of samples and n_features is the number of features.

y : array-like, shape = [n_samples]

Target vector relative to X

class_weight : {dict, ‘auto’}, optional

Weights associated with classes. If not given, all classes are supposed to have weight one.

Returns self : object

Returns self.

**fit_transform** (**X**, **y**=None, **fit_params**)  
Fit to data, then transform it

Fits transformer to X and y with optional parameters fit_params and returns a transformed version of X.

Parameters X : numpy array of shape [n_samples, n_features]

Training set.

y : numpy array of shape [n_samples]

Target values.

Returns X_new : numpy array of shape [n_samples, n_features_new]

Transformed array.

Notes

This method just calls fit and transform consecutively, i.e., it is not an optimized implementation of fit_transform, unlike other transformers such as PCA.

**get_params** (**deep**=True)
Get parameters for the estimator

Parameters deep: boolean, optional:
If True, will return the parameters for this estimator and contained subobjects that are estimators.

**predict (X)**

Predict target values of X according to the fitted model.

- **Parameters X**: [array-like, sparse matrix], shape = [n_samples, n_features]
- **Returns C**: array, shape = [n_samples]

**score (X, y)**

Returns the mean accuracy on the given test data and labels.

- **Parameters X**: array-like, shape = [n_samples, n_features]
  - Training set.
- **y**: array-like, shape = [n_samples]
  - Labels for X.
- **Returns z**: float

**set_params (** **kwargs **)**

Set the parameters of the estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The former have parameters of the form `<component>__<parameter>` so that it’s possible to update each component of a nested object.

**Returns self**:

**transform (X, threshold=None)**

Reduce X to its most important features.

- **Parameters X**: array or scipy sparse matrix of shape [n_samples, n_features]
  - The input samples.
- **threshold**: string, float or None, optional (default=None)
  - The threshold value to use for feature selection. Features whose importance is greater or equal are kept while the others are discarded. If “median” (resp. “mean”), then the threshold value is the median (resp. the mean) of the feature importances. A scaling factor (e.g., “1.25*mean”) may also be used. If None and if available, the object attribute `threshold` is used. Otherwise, “mean” is used by default.
- **Returns X_r**: array of shape [n_samples, n_selected_features]
  - The input samples with only the selected features.

**sklearn.svm.NuSVC**

**class sklearn.svm.NuSVC (nu=0.5, kernel='rbf', degree=3, gamma=0.0, coef0=0.0, shrinking=True, probability=False, tol=0.001, cache_size=200, verbose=False)**

Nu-Support Vector Classification.

Similar to SVC but uses a parameter to control the number of support vectors.

The implementation is based on libsvm.

**Parameters nu**: float, optional (default=0.5)

An upper bound on the fraction of training errors and a lower bound of the fraction of support vectors. Should be in the interval (0, 1).
kernel : string, optional (default='rbf')
    Specifies the kernel type to be used in the algorithm. one of ‘linear’, ‘poly’, ‘rbf’, ‘sigmoid’, ‘precomputed’. If none is given ‘rbf’ will be used.

degree : int, optional (default=3)
    degree of kernel function is significant only in poly, rbf, sigmoid

gamma : float, optional (default=0.0)
    kernel coefficient for rbf and poly, if gamma is 0.0 then 1/n_features will be taken.

coeff0 : float, optional (default=0.0)
    independent term in kernel function. It is only significant in poly/sigmoid.

probability: boolean, optional (default=False)
    Whether to enable probability estimates. This must be enabled prior to calling predict_proba.

shrinking: boolean, optional (default=True)
    Whether to use the shrinking heuristic.

tol: float, optional (default=1e-3)
    Tolerance for stopping criterion.

cache_size: float, optional
    Specify the size of the kernel cache (in MB)

class_weight : {dict, ‘auto’}, optional
    Set the parameter C of class i to class_weight[i]*C for SVC. If not given, all classes are supposed to have weight one. The ‘auto’ mode uses the values of y to automatically adjust weights inversely proportional to class frequencies.

verbose : bool, default: False
    Enable verbose output. Note that this setting takes advantage of a per-process runtime setting in libsvm that, if enabled, may not work properly in a multithreaded context.

See Also:

SVC Support Vector Machine for classification using libsvm.
LinearSVC Scalable linear Support Vector Machine for classification using liblinear.

Examples

```python
>>> import numpy as np
>>> X = np.array([[-1, -1], [-2, -1], [1, 1], [2, 1]])
>>> y = np.array([1, 1, 2, 2])
>>> from sklearn.svm import NuSVC
>>> clf = NuSVC()
>>> clf.fit(X, y)
NuSVC(cache_size=200, coef0=0.0, degree=3, gamma=0.5, kernel='rbf', nu=0.5, probability=False, shrinking=True, tol=0.001, verbose=False)
>>> print(clf.predict([[-0.8, -1]]))
[1]
```
### Attributes

<table>
<thead>
<tr>
<th>Attribute</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>support_</code></td>
<td>Index of support vectors.</td>
</tr>
<tr>
<td><code>support_vectors_</code></td>
<td>Support vectors.</td>
</tr>
<tr>
<td><code>n_support_</code></td>
<td>Number of support vector for each class.</td>
</tr>
<tr>
<td><code>dual_coef_</code></td>
<td>Coefficients of the support vector in the decision function. For multiclass, coefficient for all 1-vs-1 classifiers. The layout of the coefficients in the multiclass case is somewhat non-trivial. See the section about multi-class classification in the SVM section of the User Guide for details.</td>
</tr>
<tr>
<td><code>coef_</code></td>
<td>Weights assigned to the features (coefficients in the primal problem). This is only available in the case of linear kernel. coef_ is readonly property derived from dual_coef_ and support_vectors_.</td>
</tr>
<tr>
<td><code>intercept_</code></td>
<td>Constants in decision function.</td>
</tr>
</tbody>
</table>

### Methods

- **decision_function(X)**
  Distance of the samples X to the separating hyperplane.
  
  **Parameters**
  - `X`: array-like, shape = [n_samples, n_features]
  
  **Returns**
  - `X`: array-like, shape = [n_samples, n_class * (n_class-1) / 2]

- **fit(X, y, class_weight=None, sample_weight=None)**
  Fit the SVM model according to the given training data.
  
  **Parameters**
  - `X`: {array-like, sparse matrix}, shape = [n_samples, n_features]
  - `y`: array-like, shape = [n_samples]

  **Returns**
  - `X`: array-like, shape = [n_samples]

- **get_params([deep])**
  Get parameters for the estimator.

- **predict(X)**
  Perform classification or regression samples in X.

- **predict_log_proba(X)**
  Compute the log likelihoods each possible outcomes of samples in X.

- **predict_proba(X)**
  Compute the likelihoods each possible outcomes of samples in T.

- **score(X, y)**
  Returns the mean accuracy on the given test data and labels.

- **set_params(**params)**
  Set the parameters of the estimator.

**__init__(nu=0.5, kernel='rbf', degree=3, gamma=0.0, coef0=0.0, shrinking=True, probability=False, tol=0.001, cache_size=200, verbose=False)**

- **decision_function(X)**
  Distance of the samples X to the separating hyperplane.

  **Parameters**
  - `X`: array-like, shape = [n_samples, n_features]

  **Returns**
  - `X`: array-like, shape = [n_samples, n_class * (n_class-1) / 2]

  Returns the decision function of the sample for each class in the model.

- **fit(X, y, class_weight=None, sample_weight=None)**
  Fit the SVM model according to the given training data.

  **Parameters**
  - `X`: [array-like, sparse matrix], shape = [n_samples, n_features]

  Training vectors, where n_samples is the number of samples and n_features is the number of features.

  - `y`: array-like, shape = [n_samples]

  Target values (integers in classification, real numbers in regression)
sample_weight : array-like, shape = [n_samples], optional
    Weights applied to individual samples (1. for unweighted).

Returns self : object
    Returns self.

Notes

If X and y are not C-ordered and contiguous arrays of np.float64 and X is not a scipy.sparse.csr_matrix, X and/or y may be copied.

If X is a dense array, then the other methods will not support sparse matrices as input.

get_params (deep=True)
    Get parameters for the estimator

    Parameters deep: boolean, optional :
        If True, will return the parameters for this estimator and contained subobjects that are estimators.

predict (X)
    Perform classification or regression samples in X.

    For a classification model, the predicted class for each sample in X is returned. For a regression model, the function value of X calculated is returned.

    For an one-class model, +1 or -1 is returned.

    Parameters X : {array-like, sparse matrix}, shape = [n_samples, n_features]
    Returns C : array, shape = [n_samples]

predict_log_proba (X)
    Compute the log likelihoods each possible outcomes of samples in X.

    The model need to have probability information computed at training time: fit with attribute probability set to True.

    Parameters X : array-like, shape = [n_samples, n_features]
    Returns X : array-like, shape = [n_samples, n_classes]
        Returns the log-probabilities of the sample for each class in the model, where classes are ordered by arithmetical order.

Notes

The probability model is created using cross validation, so the results can be slightly different than those obtained by predict. Also, it will meaningless results on very small datasets.

predict_proba (X)
    Compute the likelihoods each possible outcomes of samples in T.

    The model need to have probability information computed at training time: fit with attribute probability set to True.

    Parameters X : array-like, shape = [n_samples, n_features]
    Returns X : array-like, shape = [n_samples, n_classes]
Returns the probability of the sample for each class in the model, where classes are ordered by arithmetical order.

Notes

The probability model is created using cross validation, so the results can be slightly different than those obtained by predict. Also, it will produce meaningless results on very small datasets.

**score** *(X, y)*

Returns the mean accuracy on the given test data and labels.

**Parameters**

- **X**: array-like, shape = [n_samples, n_features]
  
  Training set.

- **y**: array-like, shape = [n_samples]
  
  Labels for X.

**Returns**

- **z**: float

**set_params** (**params**)

Set the parameters of the estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The former have parameters of the form `<component>__<parameter>` so that it’s possible to update each component of a nested object.

**Returns**

- **self**: 

---

**sklearn.svm.SVR**

**class** **sklearn.svm.SVR** *(kernel='rbf', degree=3, gamma=0.0, coef0=0.0, tol=0.001, C=1.0, epsilon=0.1, shrinking=True, probability=False, cache_size=200, verbose=False)*

epsilon-Support Vector Regression.

The free parameters in the model are C and epsilon.

The implementations is a based on libsvm.

**Parameters**

- **C**: float or None, optional (default=None)
  
  penalty parameter C of the error term. If None then C is set to n_samples.

- **epsilon**: float, optional (default=0.1)
  
  epsilon in the epsilon-SVR model. It specifies the epsilon-tube within which no penalty is associated in the training loss function with points predicted within a distance epsilon from the actual value.

- **kernel**: string, optional (default='rbf')
  
  Specifies the kernel type to be used in the algorithm. one of ‘linear’, ‘poly’, ‘rbf’, ‘sigmoid’, ‘precomputed’. If none is given ‘rbf’ will be used.

- **degree**: int, optional (default=3)
  
  degree of kernel function is significant only in poly, rbf, sigmoid

- **gamma**: float, optional (default=0.0)
  
  kernel coefficient for rbf and poly, if gamma is 0.0 then 1/n_features will be taken.
**coef0** : float, optional (default=0.0)
  independent term in kernel function. It is only significant in poly/sigmoid.

**probability** : boolean, optional (default=False):
  Whether to enable probability estimates. This must be enabled prior to calling predict_proba.

**shrinking** : boolean, optional (default=True):
  Whether to use the shrinking heuristic.

**tol** : float, optional (default=1e-3):
  Tolerance for stopping criterion.

**cache size** : float, optional:
  Specify the size of the kernel cache (in MB)

**verbose** : bool, default: False
  Enable verbose output. Note that this setting takes advantage of a per-process runtime setting in libsvm that, if enabled, may not work properly in a multithreaded context.

**See Also:**

**NuSVR**
Support Vector Machine for regression implemented using libsvm using a parameter to control the number of support vectors.

**Examples**

```python
>>> from sklearn.svm import SVR
>>> import numpy as np
>>> n_samples, n_features = 10, 5
>>> np.random.seed(0)
>>> X = np.random.randn(n_samples, n_features)
>>> y = np.random.randn(n_samples)
>>> clf = SVR(C=1.0, epsilon=0.2)
>>> clf.fit(X, y)
SVR(C=1.0, cache_size=200, coef0=0.0, degree=3, epsilon=0.2, gamma=0.2,
  kernel='rbf', probability=False, shrinking=True, tol=0.001,
  verbose=False)
```
Attributes

<table>
<thead>
<tr>
<th>Attribute</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>support_</td>
<td>Index of support vectors.</td>
</tr>
<tr>
<td>support_vectors_</td>
<td>Support vectors.</td>
</tr>
<tr>
<td>dual_coef_</td>
<td>Coefficients of the support vector in the decision function.</td>
</tr>
<tr>
<td>coef_</td>
<td>Weights assigned to the features (coefficients in the primal problem).</td>
</tr>
<tr>
<td>intercept_</td>
<td>Constants in decision function.</td>
</tr>
</tbody>
</table>

Methods

- **decision_function**(*X*)
  - Distance of the samples *X* to the separating hyperplane.
  - **Parameters** *X* : array-like, shape = [n_samples, n_features]
  - **Returns** *X* : array-like, shape = [n_samples, n_class * (n_class-1) / 2]

- **fit**(*X*, *y*, *class_weight*=None, *sample_weight*=None)
  - Fit the SVM model according to the given training data.
  - **Parameters** *X* : {array-like, sparse matrix}, shape = [n_samples, n_features]
  - *y* : array-like, shape = [n_samples]
  - **Returns** *X* : object

__init__(*kernel*='rbf', *degree*=3, *gamma*=0.0, *coef0*=0.0, *tol*=0.001, *C*=1.0, *epsilon*=0.1, *shrinking*=True, *probability*=False, *cache_size*=200, *verbose*=False)

- **decision_function**(*X*)
  - Distance of the samples *X* to the separating hyperplane.

- **fit**(*X*, *y*, *class_weight*=None, *sample_weight*=None)
  - Fit the SVM model according to the given training data.
Returns self.

Notes

If X and y are not C-ordered and contiguous arrays of np.float64 and X is not a scipy.sparse.csr_matrix, X and/or y may be copied.

If X is a dense array, then the other methods will not support sparse matrices as input.

get_params (deep=True)
Get parameters for the estimator

Parameters deep: boolean, optional:
If True, will return the parameters for this estimator and contained subobjects that are estimators.

predict (X)
Perform classification or regression samples in X.

For a classification model, the predicted class for each sample in X is returned. For a regression model, the function value of X calculated is returned.

For an one-class model, +1 or -1 is returned.

Parameters X: array-like, sparse matrix], shape = [n_samples, n_features]

Returns C: array, shape = [n_samples]

predict_log_proba (X)
Compute the log likehoods each possible outcomes of samples in X.

The model need to have probability information computed at training time: fit with attribute probability set to True.

Parameters X: array-like, shape = [n_samples, n_features]

Returns X: array-like, shape = [n_samples, n_classes]

Returns the log-probabilities of the sample for each class in the model, where classes are ordered by arithmetical order.

Notes

The probability model is created using cross validation, so the results can be slightly different than those obtained by predict. Also, it will meaningless results on very small datasets.

predict_proba (X)
Compute the likehoods each possible outcomes of samples in T.

The model need to have probability information computed at training time: fit with attribute probability set to True.

Parameters X: array-like, shape = [n_samples, n_features]

Returns X: array-like, shape = [n_samples, n_classes]

Returns the probability of the sample for each class in the model, where classes are ordered by arithmetical order.
Notes

The probability model is created using cross validation, so the results can be slightly different than those obtained by predict. Also, it will produce meaningless results on very small datasets.

score \((X, y)\)
Returns the coefficient of determination \(R^2\) of the prediction.

The coefficient \(R^2\) is defined as \((1 - u/v)\), where \(u\) is the regression sum of squares \((y - y_{\text{pred}}) ** 2).sum()\) and \(v\) is the residual sum of squares \((y_{\text{true}} - y_{\text{true.mean()}}) ** 2).sum()\). Best possible score is 1.0, lower values are worse.

**Parameters**

\(X\) : array-like, shape = [n_samples, n_features]
Training set.

\(y\) : array-like, shape = [n_samples]

**Returns**

\(z\) : float

set_params (**params)
Set the parameters of the estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The former have parameters of the form \(<component>__<parameter>\) so that it’s possible to update each component of a nested object.

**Returns**

\(self\) :

sklearn.svm.NuSVR

class sklearn.svm.NuSVR (\(nu=0.5,\ C=1.0,\ kernel='rbf',\ degree=3,\ gamma=0.0,\ coef0=0.0,\ shrinking=True,\ probability=False,\ tol=0.001,\ cache_size=200,\ verbose=False\))

Nu Support Vector Regression.

Similar to NuSVC, for regression, uses a parameter \(nu\) to control the number of support vectors. However, unlike NuSVC, where \(nu\) replaces \(C\), here \(nu\) replaces with the parameter \(epsilon\) of SVR.

The implementations is a based on libsvm.

**Parameters**

\(C\) : float or None, optional (default=None)
penalty parameter \(C\) of the error term. If None then \(C\) is set to \(n_samples\).

\(nu\) : float, optional
An upper bound on the fraction of training errors and a lower bound of the fraction of support vectors. Should be in the interval \((0, 1]\). By default 0.5 will be taken. Only available if impl='nu_svc'.

\(kernel\) : string, optional (default='rbf')
Specifies the kernel type to be used in the algorithm. one of ‘linear’, ‘poly’, ‘rbf’, ‘sigmoid’, ‘precomputed’. If none is given ‘rbf’ will be used.

\(degree\) : int, optional (default=3)
degree of kernel function is significant only in poly, rbf, sigmoid

\(gamma\) : float, optional (default=0.0)
kernal coefficient for rbf and poly, if gamma is 0.0 then \(1/n_features\) will be taken.

\(coef0\) : float, optional (default=0.0)

1.8. Reference
independent term in kernel function. It is only significant in poly/sigmoid.

**probability**: boolean, optional (default=False):

Whether to enable probability estimates. This must be enabled prior to calling predict_proba.

**shrinking**: boolean, optional (default=True):

Whether to use the shrinking heuristic.

**tol**: float, optional (default=1e-3):

Tolerance for stopping criterion.

**cache_size**: float, optional:

Specify the size of the kernel cache (in MB)

**verbose**: bool, default: False

Enable verbose output. Note that this setting takes advantage of a per-process runtime setting in libsvm that, if enabled, may not work properly in a multithreaded context.

See Also:

NuSVC Support Vector Machine for classification implemented with libsvm with a parameter to control the number of support vectors.

SVR Epsilon Support Vector Machine for regression implemented with libsvm.

Examples

```python
>>> from sklearn.svm import NuSVR
>>> import numpy as np
>>> n_samples, n_features = 10, 5
>>> np.random.seed(0)
>>> y = np.random.randn(n_samples)
>>> X = np.random.randn(n_samples, n_features)
>>> clf = NuSVR(C=1.0, nu=0.1)
>>> clf.fit(X, y)
NuSVR(C=1.0, cache_size=200, coef0=0.0, degree=3, gamma=0.2, kernel='rbf',
        nu=0.1, probability=False, shrinking=True, tol=0.001, verbose=False)
```
Attributes

<table>
<thead>
<tr>
<th></th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>support_</strong></td>
<td>array-like, shape = [n_SV]</td>
</tr>
<tr>
<td></td>
<td>Index of support vectors.</td>
</tr>
<tr>
<td><strong>support_vectors_</strong></td>
<td>array-like, shape = [nSV, n_features]</td>
</tr>
<tr>
<td></td>
<td>Support vectors.</td>
</tr>
<tr>
<td><strong>dual_coef_</strong></td>
<td>array, shape = [n_classes-1, n_SV]</td>
</tr>
<tr>
<td></td>
<td>Coefficients of the support vector in the decision function.</td>
</tr>
<tr>
<td><strong>coef_</strong></td>
<td>array, shape = [n_classes-1, n_features]</td>
</tr>
<tr>
<td></td>
<td>Weights assigned to the features (coefficients in the primal problem). This is</td>
</tr>
<tr>
<td></td>
<td>only available in the case of linear kernel.</td>
</tr>
<tr>
<td></td>
<td><code>coef_</code> is readonly property derived from <code>dual_coef_</code> and <code>support_vectors_</code></td>
</tr>
<tr>
<td><strong>intercept_</strong></td>
<td>array, shape = [n_class * (n_class-1) / 2]</td>
</tr>
<tr>
<td></td>
<td>Constants in decision function.</td>
</tr>
</tbody>
</table>

Methods

- **decision_function**(X)  
  Distance of the samples X to the separating hyperplane.

- **fit**(X, y[, class_weight, sample_weight])  
  Fit the SVM model according to the given training data.

- **get_params**(deep)  
  Get parameters for the estimator

- **predict**(X)  
  Perform classification or regression samples in X.

- **predict_log_proba**(X)  
  Compute the log likelihoods each possible outcomes of samples in X.

- **predict_proba**(X)  
  Compute the likelihoods each possible outcomes of samples in T.

- **score**(X, y)  
  Returns the coefficient of determination $R^2$ of the prediction.

- **set_params**(**params)  
  Set the parameters of the estimator.

__init__(nu=0.5, C=1.0, kernel='rbf', degree=3, gamma=0.0, coef0=0.0, shrinking=True, probability=False, tol=0.001, cache_size=200, verbose=False)

**decision_function**(X)  
Distance of the samples X to the separating hyperplane.

**Parameters** X : array-like, shape = [n_samples, n_features]

**Returns** X : array-like, shape = [n_samples, n_class * (n_class-1) / 2]

Returns the decision function of the sample for each class in the model.

**fit**(X, y[, class_weight=None, sample_weight=None])  
Fit the SVM model according to the given training data.

**Parameters** X : {array-like, sparse matrix}, shape = [n_samples, n_features]

Training vectors, where n_samples is the number of samples and n_features is the number of features.

y : array-like, shape = [n_samples]

Target values (integers in classification, real numbers in regression)

**sample_weight** : array-like, shape = [n_samples], optional

Weights applied to individual samples (1. for unweighted).

**Returns** self : object
Returns self.

Notes

If X and y are not C-ordered and contiguous arrays of np.float64 and X is not a scipy.sparse.csr_matrix, X and/or y may be copied.

If X is a dense array, then the other methods will not support sparse matrices as input.

get_params (deep=True)
Get parameters for the estimator

Parameters deep: boolean, optional :
If True, will return the parameters for this estimator and contained subobjects that are estimators.

predict (X)
Perform classification or regression samples in X.

For a classification model, the predicted class for each sample in X is returned. For a regression model, 
the function value of X calculated is returned.

For an one-class model, +1 or -1 is returned.

Parameters X : {array-like, sparse matrix}, shape = [n_samples, n_features]

Returns C : array, shape = [n_samples]

predict_log_proba (X)
Compute the log likehoods each possible outcomes of samples in X.

The model need to have probability information computed at training time: fit with attribute probability set to True.

Parameters X : array-like, shape = [n_samples, n_features]

Returns X : array-like, shape = [n_samples, n_classes]

Returns the log-probabilities of the sample for each class in the model, where classes are ordered by arithmetical order.

Notes

The probability model is created using cross validation, so the results can be slightly different than those obtained by predict. Also, it will meaningless results on very small datasets.

predict_proba (X)
Compute the likehoods each possible outcomes of samples in T.

The model need to have probability information computed at training time: fit with attribute probability set to True.

Parameters X : array-like, shape = [n_samples, n_features]

Returns X : array-like, shape = [n_samples, n_classes]

Returns the probability of the sample for each class in the model, where classes are ordered by arithmetical order.
Notes

The probability model is created using cross validation, so the results can be slightly different than those obtained by predict. Also, it will produce meaningless results on very small datasets.

**score** \((X, y)\)

Returns the coefficient of determination \(R^2\) of the prediction.

The coefficient \(R^2\) is defined as \((1 - u/v)\), where \(u\) is the regression sum of squares \(((y - \text{y\_pred})^2).\text{sum()}\) and \(v\) is the residual sum of squares \(((y\_true - y\_true.\text{mean()})^2).\text{sum()}\). Best possible score is 1.0, lower values are worse.

**Parameters**

- \(X\) : array-like, shape \([n\_samples, n\_features]\)
  - Training set.
- \(y\) : array-like, shape \([n\_samples]\)

**Returns**

- \(z\) : float

**set_params** (**params**)

Set the parameters of the estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The former have parameters of the form `<component>__<parameter>` so that it’s possible to update each component of a nested object.

**Returns**

- `self` :

---

**sklearn.svm.OneClassSVM**

**class** `sklearn.svm.OneClassSVM` (**kernel=’rbf’, degree=3, gamma=0.0, coef0=0.0, tol=0.001, nu=0.5, shrinking=True, cache_size=200, verbose=False)``

Unsupervised Outliers Detection.

Estimate the support of a high-dimensional distribution.

The implementation is based on libsvm.

**Parameters**

- **kernel** : string, optional
  - Specifies the kernel type to be used in the algorithm. Can be one of ‘linear’, ‘poly’, ‘rbf’, ‘sigmoid’, ‘precomputed’. If none is given ‘rbf’ will be used.
- **nu** : float, optional
  - An upper bound on the fraction of training errors and a lower bound of the fraction of support vectors. Should be in the interval \((0, 1]\). By default 0.5 will be taken.
- **degree** : int, optional
  - Degree of kernel function. Significant only in poly, rbf, sigmoid.
- **gamma** : float, optional (default=0.0)
  - kernel coefficient for rbf and poly, if gamma is 0.0 then 1/n_features will be taken.
- **coef0** : float, optional
  - Independent term in kernel function. It is only significant in poly/sigmoid.
- **tol** : float, optional :
  - Tolerance for stopping criterion.
shrinking: boolean, optional:
Whether to use the shrinking heuristic.

cache_size: float, optional:
Specify the size of the kernel cache (in MB)

verbose: bool, default: False
Enable verbose output. Note that this setting takes advantage of a per-process runtime
setting in libsvm that, if enabled, may not work properly in a multithreaded context.

Attributes

<table>
<thead>
<tr>
<th>Attribute</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>support_</td>
<td>array-like, shape = [n_SV] Index of support vectors.</td>
</tr>
<tr>
<td>support_vectors_</td>
<td>array-like, shape = [nSV, n_features] Support vectors.</td>
</tr>
<tr>
<td>dual_coef</td>
<td>array, shape = [n_classes-1, n_SV] Coefficient of the support vector in the decision function.</td>
</tr>
<tr>
<td>coef_</td>
<td>array, shape = [n_classes-1, n_SV] Weights assigned to the features (coefficients in the primal problem). This is only available in the case of linear kernel. coef_ is readonly property derived from dual_coef_ and support_vectors_.</td>
</tr>
<tr>
<td>intercept_</td>
<td>array, shape = [n_classes-1] Constants in decision function.</td>
</tr>
</tbody>
</table>

Methods

<table>
<thead>
<tr>
<th>Method</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>decision_function(X)</td>
<td>Distance of the samples X to the separating hyperplane.</td>
</tr>
<tr>
<td>fit(X[, sample_weight])</td>
<td>Detects the soft boundary of the set of samples X.</td>
</tr>
<tr>
<td>get_params([deep])</td>
<td>Get parameters for the estimator</td>
</tr>
<tr>
<td>predict(X)</td>
<td>Perform classification or regression samples in X.</td>
</tr>
<tr>
<td>predict_log_proba(X)</td>
<td>Compute the log likelihoods each possible outcomes of samples in X.</td>
</tr>
<tr>
<td>predict_proba(X)</td>
<td>Compute the likelihoods each possible outcomes of samples in T.</td>
</tr>
<tr>
<td>set_params(**params)</td>
<td>Set the parameters of the estimator.</td>
</tr>
</tbody>
</table>

__init__(kernel='rbf', degree=3, gamma=0.0, coef0=0.0, tol=0.001, nu=0.5, shrinking=True, cache_size=200, verbose=False)  

decision_function(X)
Distance of the samples X to the separating hyperplane.

**Parameters** X : array-like, shape = [n_samples, n_features]

**Returns** X : array-like, shape = [n_samples, n_class * (n_class-1) / 2]

Returns the decision function of the sample for each class in the model.

fit(X, sample_weight=None, **params)
Detects the soft boundary of the set of samples X.

**Parameters** X : {array-like, sparse matrix}, shape = [n_samples, n_features]
Set of samples, where n_samples is the number of samples and n_features is the number of features.

Returns self : object

Returns self.

Notes

If X is not a C-ordered contiguous array it is copied.

generate_params (deep=True)

Get parameters for the estimator

Parameters deep: boolean, optional :

If True, will return the parameters for this estimator and contained subobjects that are estimators.

predict (X)

Perform classification or regression samples in X.

For a classification model, the predicted class for each sample in X is returned. For a regression model, the function value of X calculated is returned.

For an one-class model, +1 or -1 is returned.

Parameters X : {array-like, sparse matrix}, shape = [n_samples, n_features]

Returns C : array, shape = [n_samples]

predict_log_proba (X)

Compute the log likehoods each possible outcomes of samples in X.

The model need to have probability information computed at training time: fit with attribute probability set to True.

Parameters X : array-like, shape = [n_samples, n_features]

Returns X : array-like, shape = [n_samples, n_classes]

Returns the log-probabilities of the sample for each class in the model, where classes are ordered by arithmetical order.

Notes

The probability model is created using cross validation, so the results can be slightly different than those obtained by predict. Also, it will meaningless results on very small datasets.

predict_proba (X)

Compute the likehoods each possible outcomes of samples in T.

The model need to have probability information computed at training time: fit with attribute probability set to True.

Parameters X : array-like, shape = [n_samples, n_features]

Returns X : array-like, shape = [n_samples, n_classes]

Returns the probability of the sample for each class in the model, where classes are ordered by arithmetical order.
Notes

The probability model is created using cross validation, so the results can be slightly different than those obtained by predict. Also, it will produce meaningless results on very small datasets.

set_params (**params)

Set the parameters of the estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The former have parameters of the form `<component>__<parameter>` so that it’s possible to update each component of a nested object.

Returns self :

```
svm.l1_min_c(X, y[, loss, fit_intercept, ...])
```

Return the lowest bound for C such that for C in (l1_min_C, infinity) the model is guaranteed not to be empty. This applies to l1 penalized classifiers, such as LinearSVC with penalty='l1' and linear_model.LogisticRegression with penalty='l1'.

This value is valid if class_weight parameter in fit() is not set.

Parameters:

- **X**: array-like or sparse matrix, shape = [n_samples, n_features]
  Training vector, where n_samples is the number of samples and n_features is the number of features.
- **y**: array, shape = [n_samples]
  Target vector relative to X
- **loss**: {'l2', 'log'}, default to 'l2'
  Specifies the loss function. With ‘l2’ it is the l2 loss (a.k.a. squared hinge loss). With ‘log’ it is the loss of logistic regression models.
- **fit_intercept**: bool, default: True
  Specifies if the intercept should be fitted by the model. It must match the fit() method parameter.
- **intercept_scaling**: float, default: 1
  when fit_intercept is True, instance vector x becomes [x, intercept_scaling], i.e. a “synthetic” feature with constant value equals to intercept_scaling is appended to the instance vector. It must match the fit() method parameter.

Returns:

- **l1_min_c**: float
  minimum value for C

Low-level methods

```
svm.libsvm.fit
```

Train the model using libsvm (low-level method)

```
svm.libsvm.decision_function
```

Predict margin (libsvm name for this is predict_values)
Table 1.188 – continued from previous page

<table>
<thead>
<tr>
<th>Method</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>svm.libsvm.predict</code></td>
<td>Predict target values of X given a model (low-level method)</td>
</tr>
<tr>
<td><code>svm.libsvm.predict_proba</code></td>
<td>Predict probabilities svm_model stores all parameters needed to predict a given value.</td>
</tr>
<tr>
<td><code>svm.libsvm.cross_validation</code></td>
<td>Binding of the cross-validation routine (low-level routine)</td>
</tr>
</tbody>
</table>

**sklearn.svm.libsvm.fit**

`sklearn.svm.libsvm.fit()`

Train the model using libsvm (low-level method)

**Parameters**

**X**: array-like, dtype=float64, size=[n_samples, n_features]

**Y**: array, dtype=float64, size=[n_samples]

- `target vector`
- `svm_type`: {0, 1, 2, 3, 4}
  - Type of SVM: C_SVC, NuSVC, OneClassSVM, EpsilonSVR or NuSVR respectively.
- `kernel`: {'linear', 'rbf', 'poly', 'sigmoid', 'precomputed'}
  - Kernel to use in the model: linear, polynomial, RBF, sigmoid or precomputed.
- `degree`: int32
  - Degree of the polynomial kernel (only relevant if kernel is set to polynomial)
- `gamma`: float64
  - Gamma parameter in RBF kernel (only relevant if kernel is set to RBF)
- `coef0`: float64
  - Independent parameter in poly/sigmoid kernel.
- `tol`: float64
  - Stopping criteria.
- `C`: float64
  - C parameter in C-Support Vector Classification
- `nu`: float64
- `cache_size`: float64

**Returns**

- `support`: array, shape=[n_support]
  - index of support vectors
- `support_vectors`: array, shape=[n_support, n_features]
  - support vectors (equivalent to X[support]). Will return an empty array in the case of precomputed kernel.
- `n_class_SV`: array
  - number of support vectors in each class.
- `sv_coef`: array
  - coefficients of support vectors in decision function.
- `intercept`: array

1.8. Reference
intercept in decision function

**label**: labels for different classes (only relevant in classification).

**probA, probB**: array

probability estimates, empty array for probability=False

### sklearn.svm.libsvm.decision_function

**sklearn.svm.libsvm.decision_function()**

Predict margin (libsvm name for this is predict_values)

We have to reconstruct model and parameters to make sure we stay in sync with the python object.

### sklearn.svm.libsvm.predict

**sklearn.svm.libsvm.predict()**

Predict target values of X given a model (low-level method)

**Parameters**

- **X**: array-like, dtype=float, size=[n_samples, n_features]
  - **svm_type**: {0, 1, 2, 3, 4}
    - Type of SVM: C SVC, nu SVC, one class, epsilon SVR, nu SVR
  - **kernel**: {'linear', 'rbf', 'poly', 'sigmoid', 'precomputed'}
    - Kernel to use in the model: linear, polynomial, RBF, sigmoid or precomputed.
  - **degree**: int
    - Degree of the polynomial kernel (only relevant if kernel is set to polynomial)
  - **gamma**: float
    - Gamma parameter in RBF kernel (only relevant if kernel is set to RBF)
  - **coef0**: float
    - Independent parameter in poly/sigmoid kernel.
  - **eps**: float
    - Stopping criteria.
  - **C**: float
    - C parameter in C-Support Vector Classification

**Returns**

- **dec_values**: array
  - predicted values.

**TODO**: probably there’s no point in setting some parameters, like:

**cache_size or weights**: 
sklearn.svm.libsvm.predict_proba

sklearn.svm.libsvm.predict_proba()  
Predict probabilities  
svm_model stores all parameters needed to predict a given value.  
For speed, all real work is done at the C level in function copy_predict (libsvm_helper.c).  
We have to reconstruct model and parameters to make sure we stay in sync with the python object.  
See sklearn.svm.predict for a complete list of parameters.  

Parameters X: array-like, dtype=float :  

Y: array  
target vector  
kernel : {'linear', 'rbf', 'poly', 'sigmoid', 'precomputed'}  

Returns dec_values : array  
predicted values.

sklearn.svm.libsvm.cross_validation

sklearn.svm.libsvm.cross_validation()  
Binding of the cross-validation routine (low-level routine)  

Parameters X: array-like, dtype=float, size=[n_samples, n_features] :

Y: array, dtype=float, size=[n_samples] :

target vector  
svm_type : {0, 1, 2, 3, 4}  
Type of SVM: C SVC, nu SVC, one class, epsilon SVR, nu SVR  
kernel : {'linear', 'rbf', 'poly', 'sigmoid', 'precomputed'}  
Kernel to use in the model: linear, polynomial, RBF, sigmoid or precomputed.  

degree : int  
Degree of the polynomial kernel (only relevant if kernel is set to polynomial)  
gamma : float  
Gamma parameter in RBF kernel (only relevant if kernel is set to RBF)  
coef0 : float  
Independent parameter in poly/sigmoid kernel.  
tol : float  
Stopping criteria.  
C : float  
C parameter in C-Support Vector Classification  
nu : float  
cache_size : float
1.8.27 sklearn.tree: Decision Trees

The sklearn.tree module includes decision tree-based models for classification and regression.

User guide: See the Decision Trees section for further details.

<table>
<thead>
<tr>
<th>Class</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>tree.DecisionTreeClassifier</td>
<td>A decision tree classifier.</td>
</tr>
<tr>
<td>tree.DecisionTreeRegressor</td>
<td>A tree regressor.</td>
</tr>
<tr>
<td>tree.ExtraTreeClassifier</td>
<td>An extremely randomized tree classifier.</td>
</tr>
<tr>
<td>tree.ExtraTreeRegressor</td>
<td>An extremely randomized tree regressor.</td>
</tr>
</tbody>
</table>

### sklearn.tree.DecisionTreeClassifier

class sklearn.tree.DecisionTreeClassifier (criterion='gini', max_depth=None, min_samples_split=1, min_samples_leaf=1, min_density=0.1, max_features=None, compute_importances=False, random_state=None) A decision tree classifier.

Parameters:

- **criterion**: string, optional (default="gini")
  
The function to measure the quality of a split. Supported criteria are “gini” for the Gini impurity and “entropy” for the information gain.

- **max_depth**: integer or None, optional (default=None)
  
The maximum depth of the tree. If None, then nodes are expanded until all leaves are pure or until all leaves contain less than min_samples_split samples.

- **min_samples_split**: integer, optional (default=1)
  
The minimum number of samples required to split an internal node.

- **min_samples_leaf**: integer, optional (default=1)
  
The minimum number of samples required to be at a leaf node.

- **min_density**: float, optional (default=0.1)
  
  This parameter controls a trade-off in an optimization heuristic. It controls the minimum density of the sample_mask (i.e. the fraction of samples in the mask). If the density falls below this threshold the mask is recomputed and the input data is packed which results in data copying. If min_density equals to one, the partitions are always represented as copies of the original data. Otherwise, partitions are represented as bit masks (aka sample masks).

- **max_features**: int, string or None, optional (default=None)
  
  The number of features to consider when looking for the best split. If “auto”, then max_features=sqrt(n_features) on classification tasks and max_features=n_features on regression problems. If “sqrt”, then max_features=sqrt(n_features). If “log2”, then max_features=log2(n_features). If None, then max_features=n_features.

- **compute_importances**: boolean, optional (default=True)
  
  Whether feature importances are computed and stored into the feature_importances_ attribute when calling fit.
**random_state**: int, RandomState instance or None, optional (default=None)

If int, random_state is the seed used by the random number generator; If RandomState instance, random_state is the random number generator; If None, the random number generator is the RandomState instance used by *np.random*.

See Also:

DecisionTreeRegressor

References

[R76], [R77], [R78], [R79]

Examples

```python
>>> from sklearn.datasets import load_iris
>>> from sklearn.cross_validation import cross_val_score
>>> from sklearn.tree import DecisionTreeClassifier

>>> clf = DecisionTreeClassifier(random_state=0)
>>> iris = load_iris()

>>> cross_val_score(clf, iris.data, iris.target, cv=10)
...  ...
...  array([ 1. , 0.93..., 0.86..., 0.93..., 0.93..., 0.93..., 0.93..., 1. , 0.93..., 1. ])
```

Attributes

<table>
<thead>
<tr>
<th><strong>tree_</strong></th>
<th>Tree object</th>
<th>The underlying Tree object.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>feature_importances_</strong></td>
<td>array of shape = [n_features]</td>
<td>The feature importances (the higher, the more important the feature). The importance I(f) of a feature f is computed as the (normalized) total reduction of error brought by that feature. It is also known as the Gini importance [R79].</td>
</tr>
</tbody>
</table>

Methods

- `fit(X, y[, sample_mask, X_argsorted])`: Build a decision tree from the training set (X, y).
- `fit_transform(X[, y])`: Fit to data, then transform it
- `get_params([deep])`: Get parameters for the estimator
- `predict(X)`: Predict class or regression target for X.
- `predict_log_proba(X)`: Predict class log-probabilities of the input samples X.
- `predict_proba(X)`: Predict class probabilities of the input samples X.
- `score(X, y)`: Returns the mean accuracy on the given test data and labels.
- `set_params(**params)`: Set the parameters of the estimator.
- `transform(X[, threshold])`: Reduce X to its most important features.

__init__ (criterion=’gini’, max_depth=None, min_samples_split=1, min_samples_leaf=1, min_density=0.1, max_features=None, compute_importances=False, random_state=None)
**fit** \((X, y, \text{sample\_mask}=\text{None}, \text{X\_argsorted}=\text{None})\)

Build a decision tree from the training set \((X, y)\).

**Parameters**

\(X\) : array-like of shape \([n\_samples, n\_features]\)

The training input samples.

\(y\) : array-like, shape \([n\_samples]\)

The target values (integers that correspond to classes in classification, real numbers in regression).

**Returns**

\(self\) : object

Returns self.

**fit\_transform** \((X, y=\text{None}, **\text{fit\_params})\)

Fit to data, then transform it

Fits transformer to \(X\) and \(y\) with optional parameters \(\text{fit\_params}\) and returns a transformed version of \(X\).

**Parameters**

\(X\) : numpy array of shape \([n\_samples, n\_features]\)

Training set.

\(y\) : numpy array of shape \([n\_samples]\)

Target values.

**Returns**

\(X\_\text{new}\) : numpy array of shape \([n\_samples, n\_features\_new]\)

Transformed array.

**Notes**

This method just calls \text{fit} and \text{transform} consecutively, i.e., it is not an optimized implementation of \text{fit\_transform}, unlike other transformers such as PCA.

**get\_params** \((deep=True)\)

Get parameters for the estimator

**Parameters**

\(deep\) : boolean, optional

If True, will return the parameters for this estimator and contained subobjects that are estimators.

**predict** \((X)\)

Predict class or regression target for \(X\).

For a classification model, the predicted class for each sample in \(X\) is returned. For a regression model, the predicted value based on \(X\) is returned.

**Parameters**

\(X\) : array-like of shape \([n\_samples, n\_features]\)

The input samples.

**Returns**

\(y\) : array of shape \([n\_samples]\)

The predicted classes, or the predict values.

**predict\_log\_proba** \((X)\)

Predict class log-probabilities of the input samples \(X\).

**Parameters**

\(X\) : array-like of shape \([n\_samples, n\_features]\)

The input samples.
Returns $p$ : array of shape = [n_samples, n_classes]

The class log-probabilities of the input samples. Classes are ordered by arithmetical order.

**predict_proba** ($X$)

Predict class probabilities of the input samples $X$.

**Parameters**

$X$ : array-like of shape = [n_samples, n_features]

The input samples.

**Returns**

$p$ : array of shape = [n_samples, n_classes]

The class probabilities of the input samples. Classes are ordered by arithmetical order.

**score** ($X$, $y$)

Returns the mean accuracy on the given test data and labels.

**Parameters**

$X$ : array-like, shape = [n_samples, n_features]

Training set.

$y$ : array-like, shape = [n_samples]

Labels for $X$.

**Returns**

$z$ : float

**set_params** (**params**)

Set the parameters of the estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The former have parameters of the form `<component>__<parameter>` so that it’s possible to update each component of a nested object.

**Returns**

self :

**transform** ($X$, threshold=None)

Reduce $X$ to its most important features.

**Parameters**

$X$ : array or scipy sparse matrix of shape [n_samples, n_features]

The input samples.

threshold : string, float or None, optional (default=None)

The threshold value to use for feature selection. Features whose importance is greater or equal are kept while the others are discarded. If “median” (resp. “mean”), then the threshold value is the median (resp. the mean) of the feature importances. A scaling factor (e.g., “1.25*mean”) may also be used. If None and if available, the object attribute threshold is used. Otherwise, “mean” is used by default.

**Returns**

$X_r$ : array of shape [n_samples, n_selected_features]

The input samples with only the selected features.

### sklearn.tree.DecisionTreeRegressor

A tree regressor.

**class** sklearn.tree.DecisionTreeRegressor (criterion='mse', max_depth=None, min_samples_split=1, min_samples_leaf=1, min_density=0.1, max_features=None, compute_importances=False, random_state=None)

1.8. Reference 647
Parameters
criterion : string, optional (default="mse")
The function to measure the quality of a split. The only supported criterion is “mse” for
the mean squared error.

max_depth : integer or None, optional (default=None)
The maximum depth of the tree. If None, then nodes are expanded until all leaves are
pure or until all leaves contain less than min_samples_split samples.

min_samples_split : integer, optional (default=1)
The minimum number of samples required to split an internal node.

min_samples_leaf : integer, optional (default=1)
The minimum number of samples required to be at a leaf node.

min_density : float, optional (default=0.1)
This parameter controls a trade-off in an optimization heuristic. It controls the minimum
density of the sample_mask (i.e. the fraction of samples in the mask). If the density falls
below this threshold the mask is recomputed and the input data is packed which results
in data copying. If min_density equals to one, the partitions are always represented
as copies of the original data. Otherwise, partitions are represented as bit masks (aka
sample masks).

max_features : int, string or None, optional (default=None)
The number of features to consider when looking for the best split. If “auto”, then
max_features=sqrt(n_features) on classification tasks and max_features=n_features on
regression problems. If “sqrt”, then max_features=sqrt(n_features). If “log2”, then
max_features=log2(n_features). If None, then max_features=n_features.

compute_importances : boolean, optional (default=True)
Whether feature importances are computed and stored into the
feature_importances_ attribute when calling fit.

random_state : int, RandomState instance or None, optional (default=None)
If int, random_state is the seed used by the random number generator; If RandomState
instance, random_state is the random number generator; If None, the random number
generator is the RandomState instance used by np.random.

See Also:
DecisionTreeClassifier

References
[R80], [R81], [R82], [R83]

Examples
>>> from sklearn.datasets import load_boston
>>> from sklearn.cross_validation import cross_val_score
>>> from sklearn.tree import DecisionTreeRegressor

>>> boston = load_boston()
>>> regressor = DecisionTreeRegressor(random_state=0)
R2 scores (a.k.a. coefficient of determination) over 10-folds CV:

```python
>>> cross_val_score(regressor, boston.data, boston.target, cv=10)
...  
...  
array([ 0.61..., 0.57..., -0.34..., 0.41..., 0.75..., 
        0.07..., 0.29..., 0.33..., -1.42..., -1.77...])
```

### Attributes

<table>
<thead>
<tr>
<th>Name</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>tree_</code></td>
<td>Tree object</td>
</tr>
<tr>
<td><code>feature_importances_</code></td>
<td>The feature importances (the higher, the more important the feature). The importance I(f) of a feature f is computed as the (normalized) total reduction of error brought by that feature. It is also known as the Gini importance [R83].</td>
</tr>
</tbody>
</table>

### Methods

- **`fit(X, y[, sample_mask, X_argsorted])`**: Build a decision tree from the training set (X, y).
- **`fit_transform(X[, y])`**: Fit to data, then transform it.
- **`get_params([deep])`**: Get parameters for the estimator.
- **`predict(X)`**: Predict class or regression target for X.
- **`score(X, y)`**: Returns the coefficient of determination $R^2$ of the prediction.
- **`set_params(**params)`**: Set the parameters of the estimator.
- **`transform(X[, threshold])`**: Reduce X to its most important features.

```python
__init__(criterion='mse', max_depth=None, min_samples_split=1, min_samples_leaf=1, min_density=0.1, max_features=None, compute_importances=False, random_state=None)
```

**fit** *(X, y, sample_mask=None, X_argsorted=None)*  
Build a decision tree from the training set (X, y).

**Parameters**  
- **X**: array-like of shape = [n_samples, n_features]  
The training input samples.
- **y**: array-like, shape = [n_samples]  
The target values (integers that correspond to classes in classification, real numbers in regression).

**Returns** **self**: object

Returns self.

```python
fit_transform(X, y=None, **fit_params)
```
Fit to data, then transform it.

Fits transformer to X and y with optional parameters fit_params and returns a transformed version of X.

**Parameters**  
- **X**: numpy array of shape [n_samples, n_features]  
  Training set.
- **y**: numpy array of shape [n_samples]  
  Target values.

**Returns** **X_new**: numpy array of shape [n_samples, n_features_new]
Transformed array.

Notes

This method just calls fit and transform consecutively, i.e., it is not an optimized implementation of fit_transform, unlike other transformers such as PCA.

get_params (deep=True)
Get parameters for the estimator

Parameters deep: boolean, optional:
If True, will return the parameters for this estimator and contained subobjects that are estimators.

predict (X)
Predict class or regression target for X.
For a classification model, the predicted class for each sample in X is returned. For a regression model, the predicted value based on X is returned.

Parameters X : array-like of shape = [n_samples, n_features]
The input samples.

Returns y : array of shape = [n_samples]
The predicted classes, or the predict values.

score (X, y)
Returns the coefficient of determination $R^2$ of the prediction.
The coefficient $R^2$ is defined as $(1 - u/v)$, where $u$ is the regression sum of squares $((y - y_{pred})^2).sum()$ and $v$ is the residual sum of squares $((y_{true} - y_{true}.mean())^2).sum()$. Best possible score is 1.0, lower values are worse.

Parameters X : array-like, shape = [n_samples, n_features]
Training set.
y : array-like, shape = [n_samples]

Returns z : float

set_params (**params)
Set the parameters of the estimator.
The method works on simple estimators as well as on nested objects (such as pipelines). The former have parameters of the form <component>__<parameter> so that it's possible to update each component of a nested object.

Returns self :

transform (X, threshold=None)
Reduce X to its most important features.

Parameters X : array or scipy sparse matrix of shape [n_samples, n_features]
The input samples.

threshold : string, float or None, optional (default=None)
The threshold value to use for feature selection. Features whose importance is greater or equal are kept while the others are discarded. If “median” (resp. “mean”), then the threshold value is the median (resp. the mean) of the feature importances. A scaling factor (e.g., “1.25*mean”) may also be used. If None and if available, the object attribute threshold is used. Otherwise, “mean” is used by default.

**Returns** X_r: array of shape [n_samples, n_selected_features]

The input samples with only the selected features.

**sklearn.tree.ExtraTreeClassifier**

class sklearn.tree.ExtraTreeClassifier (criterion='gini', max_depth=None, min_samples_split=1, min_samples_leaf=1, min_density=0.1, max_features='auto', compute_importances=False, random_state=None)

An extremely randomized tree classifier.

Extra-trees differ from classic decision trees in the way they are built. When looking for the best split to separate the samples of a node into two groups, random splits are drawn for each of the max_features randomly selected features and the best split among those is chosen. When max_features is set 1, this amounts to building a totally random decision tree.

Warning: Extra-trees should only be used within ensemble methods.

**See Also:**
ExtraTreeRegressor, ExtraTreesClassifier, ExtraTreesRegressor

**References**

[R84]

**Methods**

### fit(X, y[, sample_mask, X_argsorted])
Build a decision tree from the training set (X, y).

### fit_transform(X[, y])
Fit to data, then transform it

### get_params([deep])
Get parameters for the estimator

### predict(X)
Predict class or regression target for X.

### predict_log_proba(X)
Predict class log-probabilities of the input samples X.

### predict_proba(X)
Predict class probabilities of the input samples X.

### score(X, y)
Returns the mean accuracy on the given test data and labels.

### set_params(**params)
Set the parameters of the estimator.

### transform(X[, threshold])
Reduce X to its most important features.

### __init__(criterion='gini', max_depth=None, min_samples_split=1, min_samples_leaf=1, min_density=0.1, max_features='auto', compute_importances=False, random_state=None)

fit (X, y, sample_mask=None, X_argsorted=None)
Build a decision tree from the training set (X, y).

**Parameters** X : array-like of shape = [n_samples, n_features]
The training input samples.
y : array-like, shape = [n_samples]
The target values (integers that correspond to classes in classification, real numbers in regression).

Returns self : object
Returns self.

**fit_transform** *(X, y=None, **fit_params)*
Fit to data, then transform it

Fits transformer to X and y with optional parameters fit_params and returns a transformed version of X.

**Parameters**

X : numpy array of shape [n_samples, n_features]
Training set.

y : numpy array of shape [n_samples]
Target values.

**Returns**

X_new : numpy array of shape [n_samples, n_features_new]
Transformed array.

Notes

This method just calls fit and transform consecutively, i.e., it is not an optimized implementation of fit_transform, unlike other transformers such as PCA.

**get_params** *(deep=True)*
Get parameters for the estimator

**Parameters**

deep: boolean, optional

If True, will return the parameters for this estimator and contained subobjects that are estimators.

**predict** *(X)*
Predict class or regression target for X.

For a classification model, the predicted class for each sample in X is returned. For a regression model, the predicted value based on X is returned.

**Parameters**

X : array-like of shape = [n_samples, n_features]
The input samples.

y : array of shape = [n_samples]
The predicted classes, or the predict values.

**predict_log_proba** *(X)*
Predict class log-probabilities of the input samples X.

**Parameters**

X : array-like of shape = [n_samples, n_features]
The input samples.

**Returns**

p : array of shape = [n_samples, n_classes]
The class log-probabilities of the input samples. Classes are ordered by arithmetical order.
**predict_proba** *(X)*  
Predict class probabilities of the input samples X.

**Parameters**

- X : array-like of shape = [n_samples, n_features]  
The input samples.

**Returns**

- p : array of shape = [n_samples, n_classes]  
The class probabilities of the input samples. Classes are ordered by arithmetical order.

**score** *(X, y)*  
Returns the mean accuracy on the given test data and labels.

**Parameters**

- X : array-like, shape = [n_samples, n_features]  
  Training set.
- y : array-like, shape = [n_samples]  
  Labels for X.

**Returns**

- z : float

**set_params** (**params**)  
Set the parameters of the estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The former have parameters of the form `<component>__<parameter>` so that it's possible to update each component of a nested object.

**Returns**

- self :

**transform** *(X, threshold=None)*  
Reduce X to its most important features.

**Parameters**

- X : array or scipy sparse matrix of shape [n_samples, n_features]  
  The input samples.
- threshold : string, float or None, optional (default=None)  
  The threshold value to use for feature selection. Features whose importance is greater or equal are kept while the others are discarded. If “median” (resp. “mean”), then the threshold value is the median (resp. the mean) of the feature importances. A scaling factor (e.g., “1.25*mean”) may also be used. If None and if available, the object attribute `threshold` is used. Otherwise, “mean” is used by default.

**Returns**

- X_r : array of shape [n_samples, n_selected_features]  
  The input samples with only the selected features.

---

**sklearn.tree.ExtraTreeRegressor**

**class** `sklearn.tree.ExtraTreeRegressor` *(criterion='mse', max_depth=None, min_samples_split=1, min_samples_leaf=1, min_density=0.1, max_features='auto', compute_importances=False, random_state=None)*  
An extremely randomized tree regressor.

Extra-trees differ from classic decision trees in the way they are built. When looking for the best split to separate the samples of a node into two groups, random splits are drawn for each of the `max_features` randomly selected features and the best split among those is chosen. When `max_features` is set 1, this amounts to building a totally random decision tree.
Warning: Extra-trees should only be used within ensemble methods.

See Also:

**ExtraTreeClassifier** A classifier base on extremely randomized trees

**sklearn.ensemble.ExtraTreesClassifier** An ensemble of extra-trees for classification

**sklearn.ensemble.ExtraTreesRegressor** An ensemble of extra-trees for regression

References

[R85]

Methods

**fit** *(X, y[, sample_mask, X_argsorted])* Build a decision tree from the training set (X, y).

**fit_transform** *(X[, y])* Fit to data, then transform it

**get_params** *(deep)* Get parameters for the estimator

**predict** *(X)* Predict class or regression target for X.

**score** *(X, y)* Returns the coefficient of determination $R^2$ of the prediction.

**set_params** *(**params)* Set the parameters of the estimator.

**transform** *(X[, threshold])* Reduce X to its most important features.

__init__ *(criterion='mse', max_depth=None, min_samples_split=1, min_samples_leaf=1, min_density=0.1, max_features='auto', compute_importances=False, random_state=None)*

**fit** *(X, y, sample_mask=None, X_argsorted=None)*

Build a decision tree from the training set (X, y).

**Parameters**

- **X**: array-like of shape = [n_samples, n_features]  
  The training input samples.

- **y**: array-like, shape = [n_samples]  
  The target values (integers that correspond to classes in classification, real numbers in regression).

**Returns**

- **self**: object
  Returns self.

**fit_transform** *(X, y=None, **fit_params)*

Fit to data, then transform it

Fits transformer to X and y with optional parameters fit_params and returns a transformed version of X.

**Parameters**

- **X**: numpy array of shape [n_samples, n_features]  
  Training set.

- **y**: numpy array of shape [n_samples]  
  Target values.

**Returns**

- **X_new**: numpy array of shape [n_samples, n_features_new]  
  Transformed array.
Notes

This method just calls fit and transform consecutively, i.e., it is not an optimized implementation of
fit_transform, unlike other transformers such as PCA.

get_params (deep=True)
Get parameters for the estimator

Parameters deep: boolean, optional :

If True, will return the parameters for this estimator and contained subobjects that are
estimators.

predict (X)
Predict class or regression target for X.

For a classification model, the predicted class for each sample in X is returned. For a regression model,
the predicted value based on X is returned.

Parameters X : array-like of shape = [n_samples, n_features]

The input samples.

Returns y : array of shape = [n_samples]

The predicted classes, or the predict values.

score (X, y)
Returns the coefficient of determination R^2 of the prediction.

The coefficient R^2 is defined as (1 - u/v), where u is the regression sum of squares ((y - y_pred) **
2).sum() and v is the residual sum of squares ((y_true - y_true.mean()) ** 2).sum(). Best possible score is
1.0, lower values are worse.

Parameters X : array-like, shape = [n_samples, n_features]

Training set.

y : array-like, shape = [n_samples]

Returns z : float

set_params (**params)
Set the parameters of the estimator.

The method works on simple estimators as well as on nested objects (such as pipelines). The former have
parameters of the form <component>__<parameter> so that it’s possible to update each component
of a nested object.

Returns self :

transform (X, threshold=None)
Reduce X to its most important features.

Parameters X : array or scipy sparse matrix of shape [n_samples, n_features]

The input samples.

threshold : string, float or None, optional (default=None)

The threshold value to use for feature selection. Features whose importance is greater
or equal are kept while the others are discarded. If “median” (resp. “mean”), then the
threshold value is the median (resp. the mean) of the feature importances. A scaling
factor (e.g., “1.25*mean”) may also be used. If None and if available, the object attribute
threshold is used. Otherwise, “mean” is used by default.
Returns $X_r$: array of shape [n_samples, n_selected_features]

The input samples with only the selected features.

tree.export_graphviz(decision_tree[, ...])  
Export a decision tree in DOT format.

sklearn.tree.export_graphviz

sklearn.tree.export_graphviz(decision_tree, out_file=None, feature_names=None)

Export a decision tree in DOT format.

This function generates a GraphViz representation of the decision tree, which is then written into out_file. Once exported, graphical renderings can be generated using, for example:

$ dot -Tps tree.dot -o tree.ps  # (PostScript format)
$ dot -Tpng tree.dot -o tree.png  # (PNG format)

Parameters decision_tree: decision tree classifier

The decision tree to be exported to graphviz.

out: file object or string, optional (default=None)

Handle or name of the output file.

feature_names: list of strings, optional (default=None)

Names of each of the features.

Returns out_file: file object

The file object to which the tree was exported. The user is expected to close() this object when done with it.

Examples

>>> from sklearn.datasets import load_iris
>>> from sklearn import tree

>>> clf = tree.DecisionTreeClassifier()
>>> iris = load_iris()

>>> clf = clf.fit(iris.data, iris.target)
>>> import tempfile

>>> out_file = tree.export_graphviz(clf, out_file=tempfile.TemporaryFile())
>>> out_file.close()

1.8.28 sklearn.utils: Utilities

The sklearn.utils module includes various utilities.

Developer guide: See the Utilities for Developers page for further details.

<table>
<thead>
<tr>
<th>Function</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>utils.check_random_state</td>
<td>Turn seed into a np.random.RandomState instance</td>
</tr>
<tr>
<td>utils.resample(*arrays, **options)</td>
<td>Resample arrays or sparse matrices in a consistent way</td>
</tr>
<tr>
<td>utils.shuffle(*arrays, **options)</td>
<td>Shuffle arrays or sparse matrices in a consistent way</td>
</tr>
</tbody>
</table>
**sklearn.utils.check_random_state**

**sklearn.utils.check_random_state**(seed)

Turn seed into a np.random.RandomState instance

If seed is None, return the RandomState singleton used by np.random. If seed is an int, return a new RandomState instance seeded with seed. If seed is already a RandomState instance, return it. Otherwise raise ValueError.

**sklearn.utils.resample**

**sklearn.utils.resample**(arrays, **options)

Resample arrays or sparse matrices in a consistent way

The default strategy implements one step of the bootstrapping procedure.

**Parameters**

arrays : sequence of arrays or scipy.sparse matrices with same shape[0]
replace : boolean, True by default
    Implements resampling with replacement. If False, this will implement (sliced) random permutations.

n_samples : int, None by default
    Number of samples to generate. If left to None this is automatically set to the first dimension of the arrays.

random_state : int or RandomState instance
    Control the shuffling for reproducible behavior.

**Returns**

Sequence of resampled views of the collections. The original arrays are not impacted.

**See Also:**

sklearn.cross_validation.Bootstrap, sklearn.utils.shuffle

**Examples**

It is possible to mix sparse and dense arrays in the same run:

```python
>>> X = [[1., 0.], [2., 1.], [0., 0.]]
>>> y = np.array([0, 1, 2])
>>> from scipy.sparse import coo_matrix
>>> X_sparse = coo_matrix(X)
>>> from sklearn.utils import resample
>>> X, X_sparse, y = resample(X, X_sparse, y, random_state=0)

array([[1., 0.],
        [2., 1.],
        [1., 0.]])

<3x2 sparse matrix of type '<... 'numpy.float64'>'
with 4 stored elements in Compressed Sparse Row format>
```
>>> X_sparse.toarray()
array([[ 1.,  0.],
       [ 2.,  1.],
       [ 1.,  0.]]

>>> y
array([0, 1, 0])

>>> resample(y, n_samples=2, random_state=0)
array([0, 1])

sklearn.utils.shuffle

sklearn.utils.shuffle(*arrays, **options)

Shuffle arrays or sparse matrices in a consistent way

This is a convenience alias to resample(*arrays, replace=False) to do random permutations of the collections.

- **arrays**: sequence of arrays or scipy.sparse matrices with same shape
- **random_state**: int or RandomState instance
  Control the shuffling for reproducible behavior.
- **n_samples**: int, None by default
  Number of samples to generate. If left to None this is automatically set to the first dimension of the arrays.

Returns Sequence of shuffled views of the collections. The original arrays are not impacted.

See Also:

sklearn.utils.resample

Examples

It is possible to mix sparse and dense arrays in the same run:

>>> X = [[1., 0.], [2., 1.], [0., 0.]]
>>> y = np.array([0, 1, 2])

>>> from scipy.sparse import coo_matrix
>>> X_sparse = coo_matrix(X)

>>> from sklearn.utils import shuffle

>>> X, X_sparse, y = shuffle(X, X_sparse, y, random_state=0)

>>> X
array([[ 0.,  0.],
       [ 2.,  1.],
       [ 1.,  0.]])

>>> X_sparse
<3x2 sparse matrix of type '<... 'numpy.float64''>
  with 3 stored elements in Compressed Sparse Row format>
>>> X_sparse.toarray()
array([[ 0.,  0.],
       [ 2.,  1.],
       [ 1.,  0.]]

>>> y
array([2, 1, 0])

>>> shuffle(y, n_samples=2, random_state=0)
array([0, 1])
EXAMPLE GALLERY

2.1 Examples

2.1.1 General examples

General-purpose and introductory examples for the scikit.

![Plot classification probability]

Figure 2.1: *Plot classification probability*

Plot the classification probability for different classifiers. We use a 3 class dataset, and we classify it with a Support Vector classifier, as well as L1 and L2 penalized logistic regression.

The logistic regression is not a multiclass classifier out of the box. As a result it can identify only the first class.
classif_rate for Linear SVC : 82.000000
classif_rate for L1 logistic : 79.333333
classif_rate for L2 logistic : 76.666667

Python source code: plot_classification_probability.py

```python
print __doc__

# Author: Alexandre Gramfort <alexandre.gramfort@inria.fr>
# License: BSD Style.

import pylab as pl
import numpy as np

from sklearn.linear_model import LogisticRegression
from sklearn.svm import SVC
from sklearn import datasets
```

iris = datasets.load_iris()
X = iris.data[:, 0:2]  # we only take the first two features for visualization
y = iris.target

n_features = X.shape[1]
C = 1.0

# Create different classifiers. The logistic regression cannot do
# multiclass out of the box.
classifiers = {
    'L1 logistic': LogisticRegression(C=C, penalty='l1'),
    'L2 logistic': LogisticRegression(C=C, penalty='l2'),
    'Linear SVC': SVC(kernel='linear', C=C, probability=True),
}

n_classifiers = len(classifiers)

pl.figure(figsize=(3 * 2, n_classifiers * 2))
pl.subplots_adjust(bottom=.2, top=.95)

for index, (name, classifier) in enumerate(classifiers.iteritems()):
    classifier.fit(X, y)
    y_pred = classifier.predict(X)
    classif_rate = np.mean(y_pred.ravel() == y.ravel()) * 100
    print "classif_rate for \%s : \%f " % (name, classif_rate)

    # View probabilities=
    xx = np.linspace(3, 9, 100)
    yy = np.linspace(1, 5, 100).T
    xx, yy = np.meshgrid(xx, yy)
    Xfull = np.c_[xx.ravel(), yy.ravel()]
    probas = classifier.predict_proba(Xfull)
    n_classes = np.unique(y_pred).size
    for k in range(n_classes):
        pl.subplot(n_classifiers, n_classes, index * n_classes + k + 1)
        pl.title("Class \%d" % k)
        if k == 0:
            pl.ylabel(name)
        imshow_handle = pl.imshow(probas[:, k].reshape((100, 100)),
                                extent=(3, 9, 1, 5), origin='lower')
        pl.xticks(())
        pl.yticks(()
        idx = (y_pred == k)
        if idx.any():
            pl.scatter(X[idx, 0], X[idx, 1], marker='o', c='k')

    ax = pl.axes([0.15, 0.04, 0.7, 0.05])
    pl.title("Probability")
    pl.colorbar(imshow_handle, cax=ax, orientation='horizontal')
    pl.show()
Confusion matrix

Example of confusion matrix usage to evaluate the quality of the output of a classifier.

Script output:
Recognizing hand-written digits

An example showing how the scikit-learn can be used to recognize images of hand-written digits. This example is commented in the tutorial section of the user manual.

Figure 2.3: Recognizing hand-written digits
Classification report for classifier SVC(C=1.0, cache_size=200, class_weight=None, coef0=0.0, degree=3, gamma=0.001, kernel=rbf, probability=False, shrinking=True, tol=0.001, verbose=False):

<table>
<thead>
<tr>
<th></th>
<th>precision</th>
<th>recall</th>
<th>f1-score</th>
<th>support</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.99</td>
<td>0.99</td>
<td>0.99</td>
<td>88</td>
</tr>
<tr>
<td></td>
<td>0.99</td>
<td>0.99</td>
<td>0.99</td>
<td>91</td>
</tr>
<tr>
<td></td>
<td>0.98</td>
<td>0.87</td>
<td>0.92</td>
<td>91</td>
</tr>
<tr>
<td></td>
<td>0.99</td>
<td>0.96</td>
<td>0.97</td>
<td>92</td>
</tr>
<tr>
<td></td>
<td>0.95</td>
<td>0.97</td>
<td>0.96</td>
<td>91</td>
</tr>
<tr>
<td></td>
<td>0.99</td>
<td>0.99</td>
<td>0.99</td>
<td>91</td>
</tr>
<tr>
<td></td>
<td>0.96</td>
<td>0.99</td>
<td>0.97</td>
<td>89</td>
</tr>
<tr>
<td></td>
<td>0.94</td>
<td>1.00</td>
<td>0.97</td>
<td>88</td>
</tr>
<tr>
<td></td>
<td>0.93</td>
<td>0.98</td>
<td>0.95</td>
<td>92</td>
</tr>
<tr>
<td>avg / total</td>
<td>0.97</td>
<td>0.97</td>
<td>0.97</td>
<td>899</td>
</tr>
</tbody>
</table>

Confusion matrix:
```
[87  0  0  0  1  0  0  0  0  0]
[ 0 88  1  0  0  0  0  0  1  0]
[ 0  0 85  1  0  0  0  0  0  0]
[ 0  0  0 79  0  3  0  4  5  0]
```
# Author: Gael Varoquaux <gael dot varoquaux at normalesup dot org>
# License: Simplified BSD

# Standard scientific Python imports
import pylab as pl

# Import datasets, classifiers and performance metrics
from sklearn import datasets, svm, metrics

# The digits dataset
digits = datasets.load_digits()

# The data that we are interested in is made of 8x8 images of digits,
# let’s have a look at the first 3 images, stored in the ‘images’
# attribute of the dataset. If we were working from image files, we
# could load them using pylab.imread. For these images know which
# digit they represent: it is given in the ‘target’ of the dataset.
for index, (image, label) in enumerate(zip(digits.images, digits.target)[:4]):
    pl.subplot(2, 4, index + 1)
    pl.axis('off')
    pl.imshow(image, cmap=pl.cm.gray_r, interpolation='nearest')
    pl.title('Training: %i' % label)

# To apply an classifier on this data, we need to flatten the image, to
# turn the data in a (samples, feature) matrix:
    n_samples = len(digits.images)
data = digits.images.reshape((n_samples, -1))

# Create a classifier: a support vector classifier
classifier = svm.SVC(gamma=0.001)

# We learn the digits on the first half of the digits
classifier.fit(data[:n_samples / 2], digits.target[:n_samples / 2])

# Now predict the value of the digit on the second half:
expected = digits.target[n_samples / 2:]
predicted = classifier.predict(data[n_samples / 2:])

print "Classification report for classifier %s:n%s
" % (classifier, metrics.classification_report(expected, predicted))
print "Confusion matrix:n%s" % metrics.confusion_matrix(expected, predicted)

for index, (image, prediction) in enumerate(zip(digits.images[n_samples / 2:], predicted)[:4]):
    pl.subplot(2, 4, index + 5)
    pl.axis('off')
    pl.imshow(image, cmap=pl.cm.gray_r, interpolation='nearest')

Python source code: plot_digits_classification.py

print __doc__
Pipelining: chaining a PCA and a logistic regression

The PCA does an unsupervised dimensionality reduction, while the logistic regression does the prediction. We use a GridSearchCV to set the dimensionality of the PCA.
digits = datasets.load_digits()
X_digits = digits.data
y_digits = digits.target

# Plot the PCA spectrum
pca.fit(X_digits)
pl.figure(1, figsize=(4, 3))
pl.clf()
pl.axes([.2, .2, .7, .7])
pl.plot(pca.explained_variance_, linewidth=2)
pl.axis('tight')
pl.xlabel('n_components')
pl.ylabel('explained_variance_')

from sklearn.grid_search import GridSearchCV

n_components = [20, 40, 64]
Cs = np.logspace(-4, 4, 3)

estimator = GridSearchCV(pipe, dict(pca__n_components=n_components,
                                   logistic__C=Cs))
estimator.fit(X_digits, y_digits)

pl.axvline(estimator.best_estimator_.named_steps['pca'].n_components,
            linestyle=':', label='n_components chosen')
pl.legend(prop=dict(size=12))
pl.show()

Figure 2.5: Univariate Feature Selection

Univariate Feature Selection

An example showing univariate feature selection.

Noisy (non informative) features are added to the iris data and univariate feature selection is applied. For each feature, we plot the p-values for the univariate feature selection and the corresponding weights of an SVM. We can see that univariate feature selection selects the informative features and that these have larger SVM weights.

In the total set of features, only the 4 first ones are significant. We can see that they have the highest score with univariate feature selection. The SVM attributes small weights to these features, but these weight are non zero.
Applying univariate feature selection before the SVM increases the SVM weight attributed to the significant features, and will thus improve classification.

Python source code: `plot_feature_selection.py`

```python
print __doc__
import numpy as np
import pylab as pl
from sklearn import datasets, svm
from sklearn.feature_selection import SelectPercentile, f_classif

# import some data to play with
# The IRIS dataset
iris = datasets.load_iris()

# Some noisy data not correlated
E = np.random.normal(size=(len(iris.data), 35))

# Add the noisy data to the informative features
X = np.hstack((iris.data, E))
y = iris.target
```
Figure 2.6: Demonstration of sampling from HMM

Demonstration of sampling from HMM

This script shows how to sample points from a Hidden Markov Model (HMM): we use a 4-components with specified mean and covariance.

The plot show the sequence of observations generated with the transitions between them. We can see that, as specified by our transition matrix, there are no transition between component 1 and 3.
Python source code: plot_hmm_sampling.py

```python
import numpy as np
import matplotlib.pyplot as plt
from sklearn import hmm

# Prepare parameters for a 3-components HMM
# Initial population probability
start_prob = np.array([0.6, 0.3, 0.1, 0.0])

# The transition matrix, note that there are no transitions possible
# between component 1 and 4
trans_mat = np.array([[0.7, 0.2, 0.0, 0.1],
                      [0.3, 0.5, 0.2, 0.0],
                      [0.0, 0.3, 0.5, 0.2],
                      [0.2, 0.0, 0.2, 0.6]])

# The means of each component
means = np.array([[0.0, 0.0],
                   [0.0, 11.0],
                   [9.0, 10.0],
                   [11.0, -1.0]])

# The covariance of each component
covars = .5 * np.tile(np.identity(2), (4, 1, 1))
```

Chapter 2. Example Gallery
# Build an HMM instance and set parameters
model = hmm.GaussianHMM(4, "full", start_prob, trans_mat,
                         random_state=42)

# Instead of fitting it from the data, we directly set the estimated
# parameters, the means and covariance of the components
model.means_ = means
model.covars_ = covars

# Generate samples
X, Z = model.sample(500)

# Plot the sampled data
plt.plot(X[:, 0], X[:, 1], "-o", label="observations", ms=6,
           mfc="orange", alpha=0.7)

# Indicate the component numbers
for i, m in enumerate(means):
    plt.text(m[0], m[1], 'Component %i' % (i + 1),
             size=17, horizontalalignment='center',
             bbox=dict(alpha=.7, facecolor='w'))

plt.legend(loc='best')
plt.show()

Figure 2.7: Gaussian HMM of stock data

Gaussian HMM of stock data

This script shows how to use Gaussian HMM. It uses stock price data, which can be obtained from yahoo finance. For more information on how to get stock prices with matplotlib, please refer to date_demo1.py of matplotlib.
Script output:

fitting to HMM and decoding ... done

Transition matrix

\[
\begin{bmatrix}
9.76719299e-01 & 1.35417228e-16 & 2.38997332e-03 & 2.08907155e-02 & 1.18773340e-08 \\
8.32867819e-04 & 2.92086856e-02 & 8.20163873e-01 & 1.35374694e-05 & 1.49781036e-01 \\
2.62989391e-01 & 3.24149388e-01 & 3.61148574e-18 & 4.12861221e-01 & 1.07421560e-16 \\
3.94120552e-03 & 1.18350712e-01 & 1.54841511e-11 & 3.55404724e-03 & 7.19312524e-01
\end{bmatrix}
\]

means and vars of each hidden state

0th hidden state

1th hidden state
mean = [ 3.82710228e-02  1.10461347e+08]  var = [ 2.07797740e-01  8.81745732e+14]

2th hidden state
mean = [ 6.45173011e-03 4.91151802e+07]
var = [ 5.33155033e-02 1.09532022e+14]

3th hidden state
mean = [ -7.94680418e-01 1.49185466e+08]
var = [ 6.50069278e+00 1.02490114e+16]

4th hidden state
mean = [ 1.20905487e-02 6.99175140e+07]
var = [ 1.31030113e-01 1.52865824e+14]

Python source code: plot_hmm_stock_analysis.py

```python
print __doc__

import datetime
import numpy as np
import pylab as pl
from matplotlib.finance import quotes_historical_yahoo
from matplotlib.dates import YearLocator, MonthLocator, DateFormatter
from sklearn.hmm import GaussianHMM

def plot_hmm_stock_analysis():
    # Downloading the data
    date1 = datetime.date(1995, 1, 1)  # start date
    date2 = datetime.date(2012, 1, 6)  # end date
    quotes = quotes_historical_yahoo("INTC", date1, date2)
    if len(quotes) == 0:
        raise SystemExit

    # unpack quotes
    dates = np.array([q[0] for q in quotes], dtype=int)
    close_v = np.array([q[2] for q in quotes])
    volume = np.array([q[5] for q in quotes])[1:]

    # take diff of close value
    diff = close_v[1:] - close_v[:-1]
    dates = dates[1:]
    close_v = close_v[1:]

    # pack diff and volume for training
    X = np.column_stack((diff, volume))

    # Run Gaussian HMM
    model = GaussianHMM(n_components, covariance_type="diag", n_iter=1000)
    model.fit([X])

    # predict the optimal sequence of internal hidden state
    hidden_states = model.predict(X)
```

2.1. Examples
print "done"

# print trained parameters and plot
print "Transition matrix"
print model.transmat_
print ""

print "means and vars of each hidden state"
for i in xrange(n_components):
    print "%dth hidden state" % i
    print "mean = ", model.means_[i]
    print "var = ", np.diag(model.covars_[i])
    print ""

years = YearLocator()  # every year
months = MonthLocator()  # every month
yearsFmt = DateFormatter('%Y')
fig = pl.figure()
av = fig.add_subplot(111)

for i in xrange(n_components):
    # use fancy indexing to plot data in each state
    idx = (hidden_states == i)
    ax.plot_date(dates[idx], close_v[idx], 'o', label="%dth hidden state" % i)
ax.legend()

# format the ticks
ax.xaxis.set_major_locator(years)
av.xaxis.set_major_formatter(yearsFmt)
av.xaxis.set_minor_locator(months)
av.autoscale_view()

# format the coords message box
ax.fmt_xdata = DateFormatter('%Y-%m-%d')
av.fmt_ydata = lambda x: '$%1.2f' % x
ax.grid(True)

fig.autofmt_xdate()
pl.show()
print __doc__

# Code source: Gael Varoqueux
# Modified for Documentation merge by Jaques Grobler
# License: BSD

import numpy as np
import pylab as pl
from sklearn import neighbors, datasets, linear_model, svm

# import some data to play with
iris = datasets.load_iris()
X = iris.data[:, :2]  # we only take the first two features.
Y = iris.target

h = .02  # step size in the mesh

classifiers = dict(
    knn=neighbors.KNeighborsClassifier(),
    logistic=linear_model.LogisticRegression(C=1e5),
    svm=svm.LinearSVC(C=1e5, loss='l1'),
)

fignum = 1
# we create an instance of Neighbours Classifier and fit the data.
for name, clf in classifiers.iteritems():
    clf.fit(X, Y)
    # Plot the decision boundary. For that, we will assign a color to each point in the mesh [x_min, m_max]x[y_min, y_max].
    x_min, x_max = X[:, 0].min() - .5, X[:, 0].max() + .5
    y_min, y_max = X[:, 1].min() - .5, X[:, 1].max() + .5
    xx, yy = np.meshgrid(np.arange(x_min, x_max, h),
                         np.arange(y_min, y_max, h))
    Z = clf.predict(np.c_[xx.ravel(), yy.ravel()])
    # Put the result into a color plot
    Z = Z.reshape(xx.shape)
    pl.figure(fignum, figsize=(4, 3))
    pl.pcolormesh(xx, yy, Z, cmap=pl.cm.Paired)
    # Plot also the training points
    pl.scatter(X[:, 0], X[:, 1], c=Y, cmap=pl.cm.Paired)
    pl.xlabel('Sepal length')
    pl.ylabel('Sepal width')
    pl.xlim(xx.min(), xx.max())
    pl.ylim(yy.min(), yy.max())
    pl.xticks(())
    pl.yticks(())
    fignum += 1
    pl.show()

Figure 2.9: Explicit feature map approximation for RBF kernels

Explicit feature map approximation for RBF kernels

An example shows how to use RBFSampler to approximate the feature map of an RBF kernel for classification with an SVM on the digits dataset. Results using a linear SVM in the original space, a linear SVM using the approximate mapping and using a kernelized SVM are compared. Timings and accuracy for varying amounts of Monte Carlo samplings for the approximate mapping are shown.

Sampling more dimensions clearly leads to better classification results, but comes at a greater cost. This means there is a tradeoff between runtime and accuracy, given by the parameter n_components. Note that solving the Linear SVM and also the approximate kernel SVM could be greatly accelerated by using stochastic gradient descent via sklearn.linear_model.SGDClassifier. This is not easily possible for the case of the kernelized SVM.

The second plot visualized the decision surfaces of the RBF kernel SVM and the linear SVM with approximate kernel map. The plot shows decision surfaces of the classifiers projected onto the first two principal components of the data. This visualization should be taken with a grain of salt since it is just an interesting slice through the decision surface in 64 dimensions. In particular note that a datapoint (represented as a dot) does not necessarily be classified into the
region it is lying in, since it will not lie on the plane that the first two principal components span.

The usage of `RBFSampler` is described in detail in *Kernel Approximation*.

![Classification accuracy](image)

![Training times](image)

**Python source code:** plot_kernel_approximation.py

```python
print __doc__

# Author: Gael Varoquaux <gael dot varoquaux at normalesup dot org>
# modified Andreas Mueller
# License: Simplified BSD

# Standard scientific Python imports
import pylab as pl
import numpy as np
from time import time

# Import datasets, classifiers and performance metrics
from sklearn import datasets, svm, pipeline
from sklearn.kernel_approximation import RBFSampler
from sklearn.decomposition import PCA

# The digits dataset
digits = datasets.load_digits(n_class=9)

# To apply an classifier on this data, we need to flatten the image, to
# turn the data in a (samples, feature) matrix:
n_samples = len(digits.data)
data = digits.data / 16.
data -= data.mean(axis=0)

# We learn the digits on the first half of the digits
data_train, targets_train = data[:n_samples / 2], digits.target[:n_samples / 2]

# Now predict the value of the digit on the second half:
data_test, targets_test = data[n_samples / 2:], digits.target[n_samples / 2:]
#data_test = scaler.transform(data_test)

# Create a classifier: a support vector classifier
kernel_svm = svm.SVC(gamma=.2)
linear_svm = svm.LinearSVC()

# create pipeline from kernel approximation
# and linear svm
feature_map = RBFSampler(gamma=.2, random_state=1)
approx_kernel_svm = pipeline.Pipeline([('feature_map', feature_map),
                                        ('svm', svm.LinearSVC())])

# fit and predict using linear and kernel svm:
kernel_svm_time = time()
kernel_svm.fit(data_train, targets_train)
kernel_svm_score = kernel_svm.score(data_test, targets_test)
kernel_svm_time = time() - kernel_svm_time

linear_svm_time = time()
linear_svm.fit(data_train, targets_train)
linear_svm_score = linear_svm.score(data_test, targets_test)
linear_svm_time = time() - linear_svm_time

sample_sizes = 50 * np.arange(1, 10)
approx_kernel_scores = []
approx_kernel_times = []

for D in sample_sizes:
    approx_kernel_svm.set_params(feature_map__n_components=D)
    approx_kernel_timing = time()
    approx_kernel_svm.fit(data_train, targets_train)
    approx_kernel_times.append(time() - approx_kernel_timing)
    score = approx_kernel_svm.score(data_test, targets_test)
    approx_kernel_scores.append(score)

# plot the results:
accuracy = pl.subplot(211)
# second y axis for timeings
timescale = pl.subplot(212)

accuracy.plot(sample_sizes, approx_kernel_scores, label="approx. kernel")
timescale.plot(sample_sizes, approx_kernel_times, '--', label='approx. kernel')

# horizontal lines for exact rbf and linear kernels:
accuracy.plot([sample_sizes[0], sample_sizes[-1]], [linear_svm_score, linear_svm_score], label="linear svm")
timescale.plot([sample_sizes[0], sample_sizes[-1]], [linear_svm_time, linear_svm_time], '--', label='linear svm')

accuracy.plot([sample_sizes[0], sample_sizes[-1]], [kernel_svm_score, kernel_svm_score], label="rbf svm")
timescale.plot([sample_sizes[0], sample_sizes[-1]], [kernel_svm_time, kernel_svm_time], '--', label='rbf svm')

# vertical line for dataset dimensionality = 64
accuracy.plot([64, 64], [0.7, 1], label="n_features")

# legends and labels
accuracy.set_title("Classification accuracy")
timescale.set_title("Training times")
accuracy.set_xlim(sample_sizes[0], sample_sizes[-1])
accuracy.set_xticks(()
accuracy.set_ylim(np.min(approx_kernel_scores), 1)
timescale.set_xlabel("Sampling steps = transformed feature dimension")
accuracy.set_ylabel("Classification accuracy")
timescale.set_xlabel("Training time in seconds")
accuracy.legend(loc='best')
timescale.legend(loc='best')

# visualize the decision surface, projected down to the first
# two principal components of the dataset
pca = PCA(n_components=8).fit(data_train)
X = pca.transform(data_train)

# Generate grid along first two principal components
multiples = np.arange(-2, 2, 0.1)
# steps along first component
first = multiples[:, np.newaxis] * pca.components_[0, :]
# steps along second component
second = multiples[:, np.newaxis] * pca.components_[1, :]
# combine
grid = first[np.newaxis, :, :] + second[:, np.newaxis, :]
flat_grid = grid.reshape(-1, data.shape[1])

# title for the plots
titles = ['SVC with rbf kernel',
          'SVC (linear kernel) with rbf feature map
           n n_components=100']
pl.figure(figsize=(12, 5))

# predict and plot
for i, clf in enumerate((kernel_svm, approx_kernel_svm)):
    # Plot the decision boundary. For that, we will assign a color to each
    # point in the mesh [x_min, m_max]x[y_min, y_max].
    pl.subplot(1, 2, i + 1)
    Z = clf.predict(flat_grid)
    pl.contourf(multiples, multiples, Z, cmap=pl.cm.Paired)
    pl.axis('off')
    # Plot also the training points
    pl.scatter(X[:, 0], X[:, 1], c=targets_train, cmap=pl.cm.Paired)
    pl.title(titles[i])
pl.show()

**Linear and Quadratic Discriminant Analysis with confidence ellipsoid**

Plot the confidence ellipsoids of each class and decision boundary

2.1. Examples
Figure 2.10: Linear and Quadratic Discriminant Analysis with confidence ellipsoid

Python source code: plot_lda_qda.py

```python
print __doc__

from scipy import linalg
import numpy as np
import pylab as pl
import matplotlib as mpl
from matplotlib import colors
from sklearn.lda import LDA
from sklearn.qda import QDA
```

# Linear Discriminant Analysis
# Quadratic Discriminant Analysis

Data with fixed covariance

Data with varying covariances

LDA vs QDA
# colormap

cmap = colors.LinearSegmentedColormap('red_blue_classes',
    {'red': [(0, 1, 1), (1, 0.7, 0.7)],
     'green': [(0, 0.7, 0.7), (1, 0.7, 0.7)],
     'blue': [(0, 0.7, 0.7), (1, 1, 1)]})

pl.cm.register_cmap(cmap=cmap)

# generate datasets

def dataset_fixed_cov():
    '''Generate 2 Gaussians samples with the same covariance matrix'''
    n, dim = 300, 2
    np.random.seed(0)
    C = np.array([[0., -0.23], [0.83, .23]])
    X = np.r_[np.dot(np.random.randn(n, dim), C),
             np.dot(np.random.randn(n, dim), C) + np.array([1, 1])]
    y = np.hstack((np.zeros(n), np.ones(n)))
    return X, y

def dataset_cov():
    '''Generate 2 Gaussians samples with different covariance matrices'''
    n, dim = 300, 2
    np.random.seed(0)
    C = np.array([[0., -1.], [2.5, .7]]) * 2.
    X = np.r_[np.dot(np.random.randn(n, dim), C),
             np.dot(np.random.randn(n, dim), C.T) + np.array([1, 4])]
    y = np.hstack((np.zeros(n), np.ones(n)))
    return X, y

# plot functions

def plot_data(lda, X, y, y_pred, fig_index):
    splot = pl.subplot(2, 2, fig_index)
    if fig_index == 1:
        pl.title('Linear Discriminant Analysis')
        pl.ylabel('Data with fixed covariance')
    elif fig_index == 2:
        pl.title('Quadratic Discriminant Analysis')
    elif fig_index == 3:
        pl.ylabel('Data with varying covariances')
    tp = (y == y_pred)  # True Positive
    tp0, tp1 = tp[y == 0], tp[y == 1]
    X0, X1 = X[y == 0], X[y == 1]
    X0_tp, X0_fp = X0[tp0], X0[tp0 != True]
    X1_tp, X1_fp = X1[tp1], X1[tp1 != True]
    xmin, xmax = X[:, 0].min(), X[:, 0].max()
    ymin, ymax = X[:, 1].min(), X[:, 1].max()
    # class 0: dots
    pl.plot(X0_tp[:, 0], X0_tp[:, 1], 'o', color='red')
    pl.plot(X0_fp[:, 0], X0_fp[:, 1], '.', color='#990000')  # dark red
    # class 1: dots
    pl.plot(X1_tp[:, 0], X1_tp[:, 1], 'o', color='blue')

2.1. Examples
# class 0 and 1 : areas
nx, ny = 200, 100
x_min, x_max = pl.xlim()
y_min, y_max = pl.ylim()
xx, yy = np.meshgrid(np.linspace(x_min, x_max, nx),
                    np.linspace(y_min, y_max, ny))
Z = lda.predict_proba(np.c_[xx.ravel(), yy.ravel()])
Z = Z[:, 1].reshape(xx.shape)
pl.pcolormesh(xx, yy, Z, cmap='red_blue_classes',
              norm=colors.Normalize(0., 1.))
pl.contour(xx, yy, Z, [0.5], linewidths=2., colors='k')
# means
pl.plot(lda.means_[0][0], lda.means_[0][1],
        'o', color='black', markersize=10)
pl.plot(lda.means_[1][0], lda.means_[1][1],
        'o', color='black', markersize=10)

return splot

def plot_ellipse(splot, mean, cov, color):
    v, w = linalg.eigh(cov)
    u = w[0] / linalg.norm(w[0])
    angle = np.arctan(u[1] / u[0])
    angle = 180 * angle / np.pi # convert to degrees
    # filled gaussian at 2 standard deviation
    ell = mpl.patches.Ellipse(mean, 2 * v[0] ** 0.5, 2 * v[1] ** 0.5,
                               180 + angle, color=color)
    ell.set_clip_box(splot.bbox)
    ell.set_alpha(0.5)
    splot.add_artist(ell)
    splot.set_xticks(())
    splot.set_yticks(())

def plot_lda_cov(lda, splot):
    plot_ellipse(splot, lda.means_[0], lda.covariance_, 'red')
    plot_ellipse(splot, lda.means_[1], lda.covariance_, 'blue')

def plot_qda_cov(qda, splot):
    plot_ellipse(splot, qda.means_[0], qda.covariances_[0], 'red')
    plot_ellipse(splot, qda.means_[1], qda.covariances_[1], 'blue')

for i, (X, y) in enumerate([dataset_fixed_cov(), dataset_cov()]):
    # LDA
    lda = LDA()
y_pred = lda.fit(X, y, store_covariance=True).predict(X)
splot = plot_data(lda, X, y, y_pred, fig_index=2 * i + 1)
plot_lda_cov(lda, splot)
pl.axis('tight')

    # QDA
    qda = QDA()
```python
y_pred = qda.fit(X, y, store_covariances=True).predict(X)
splot = plot_data(qda, X, y, y_pred, fig_index=2 * i + 2)
plot_qda_cov(qda, splot)
pl.axis(‘tight’)
pl.suptitle(‘LDA vs QDA’)
pl.show()
```

Figure 2.11: Multilabel classification

**Multilabel classification**

This example simulates a multi-label document classification problem. The dataset is generated randomly based on the following process:

- pick the number of labels: $n \sim \text{Poisson}(n_{\text{labels}})$
- $n$ times, choose a class $c$: $c \sim \text{Multinomial}(\theta)$
- pick the document length: $k \sim \text{Poisson}(\text{length})$
- $k$ times, choose a word: $w \sim \text{Multinomial}(\theta_c)$

In the above process, rejection sampling is used to make sure that $n$ is more than 2, and that the document length is never zero. Likewise, we reject classes which have already been chosen. The documents that are assigned to both classes are plotted surrounded by two colored circles.

The classification is performed by projecting to the first two principal components found by PCA and CCA for visualisation purposes, followed by using the `sklearn.multiclass.OneVsRestClassifier` metaclassifier using two SVCs with linear kernels to learn a discriminative model for each class. Note that PCA is used to perform an unsupervised dimensionality reduction, while CCA is used to perform a supervised one.

Note: in the plot, “unlabeled samples” does not mean that we don’t know the labels (as in semi-supervised learning) but that the samples simply do not have a label.
```python
print(__doc__)

import numpy as np
import matplotlib.pyplot as pl

from sklearn.datasets import make_multilabel_classification
from sklearn.multiclass import OneVsRestClassifier
from sklearn.svm import SVC
from sklearn.preprocessing import LabelBinarizer
from sklearn.decomposition import PCA
from sklearn.pls import CCA

def plot_hyperplane(clf, min_x, max_x, linestyle, label):
    # get the separating hyperplane
    w = clf.coef_[0]
    a = -w[0] / w[1]
    xx = np.linspace(min_x - 5, max_x + 5)  # make sure the line is long enough
    yy = a * xx - (clf.intercept_[0]) / w[1]
    pl.plot(xx, yy, linestyle, label=label)

def plot_subfigure(X, Y, subplot, title, transform):
    if transform == "pca":
```

X = PCA(n_components=2).fit_transform(X)

elif transform == "cca":
    # Convert list of tuples to a class indicator matrix first
    Y_indicator = LabelBinarizer().fit(Y).transform(Y)
    X = CCA(n_components=2).fit(X, Y_indicator).transform(X)

else:
    raise ValueError

min_x = np.min(X[:, 0])
max_x = np.max(X[:, 0])

classif = OneVsRestClassifier(SVC(kernel='linear'))
classif.fit(X, Y)

pl.subplot(2, 2, subplot)
pl.title(title)

zero_class = np.where([0 in y for y in Y])
one_class = np.where([1 in y for y in Y])
pl.scatter(X[:, 0], X[:, 1], s=40, c='gray')
pl.scatter(X[zero_class, 0], X[zero_class, 1], s=160, edgecolors='b',
           facecolors='none', linewidths=2, label='Class 1')
pl.scatter(X[one_class, 0], X[one_class, 1], s=80, edgecolors='orange',
           facecolors='none', linewidths=2, label='Class 2')
pl.axis('tight')

plot_hyperplane(classif.estimators_[0], min_x, max_x, 'k--',
                 'Boundary
for class 1')
plot_hyperplane(classif.estimators_[1], min_x, max_x, 'k-.',
                 'Boundary
for class 2')
pl.xticks(())
pl.yticks(())

if subplot == 2:
    pl.xlim(min_x - 5, max_x)
    pl.xlabel('First principal component')
    pl.ylabel('Second principal component')
    pl.legend(loc="upper left")

pl.figure(figsize=(8, 6))

X, Y = make_multilabel_classification(n_classes=2, n_labels=1,
                                       allow_unlabeled=True,
                                       random_state=1)

plot_subfigure(X, Y, 1, "With unlabeled samples + CCA", "cca")
plot_subfigure(X, Y, 2, "With unlabeled samples + PCA", "pca")

X, Y = make_multilabel_classification(n_classes=2, n_labels=1,
                                       allow_unlabeled=False,
                                       random_state=1)

plot_subfigure(X, Y, 3, "Without unlabeled samples + CCA", "cca")
plot_subfigure(X, Y, 4, "Without unlabeled samples + PCA", "pca")

pl.subplots_adjust(.04, .02, .97, .94, .09, .2)
pl.show()
Test with permutations the significance of a classification score

In order to test if a classification score is significative a technique in repeating the classification procedure after randomizing, permuting, the labels. The p-value is then given by the percentage of runs for which the score obtained is greater than the classification score obtained in the first place.

Script output:

Classification score 0.393333333333 (pvalue : 0.0792079207921)

Python source code: plot_permutation_test_for_classification.py

# Author: Alexandre Gramfort <alexandre.gramfort@inria.fr>
# License: BSD
print __doc__

import numpy as np
import pylab as pl

from sklearn.svm import SVC
from sklearn.cross_validation import StratifiedKFold, permutation_test_score
from sklearn import datasets
from sklearn.metrics import zero_one_score

##############################################################################
# Loading a dataset
iris = datasets.load_iris()
X = iris.data
y = iris.target
n_classes = np.unique(y).size

# Some noisy data not correlated
random = np.random.RandomState(seed=0)
E = random.normal(size=(len(X), 2200))

# Add noisy data to the informative features for make the task harder
X = np.c_[X, E]

svm = SVC(kernel='linear')
cv = StratifiedKFold(y, 2)

score, permutation_scores, pvalue = permutation_test_score(svm, X, y,
zero_one_score, cv=cv,
n_permutations=100, n_jobs=1)

print "Classification score %s (pvalue : %s)" % (score, pvalue)

##############################################################################
# View histogram of permutation scores
pl.hist(permutation_scores, 20, label='Permutation scores')
ylim = pl.ylim()

# BUG: vlines(..., linestyle='--') fails on older versions of matplotlib
#pl.vlines(score, ylim[0], ylim[1], linestyle='--',
#          color='g', linewidth=3, label='Classification Score'
#          (pvalue %s) % pvalue)
#pl.vlines(1.0 / n_classes, ylim[0], ylim[1], linestyle='--',
#          color='k', linewidth=3, label='Luck')
pl.plot(2 * [score], ylim, '--g', linewidth=3,
       label='Classification Score'
       (pvalue %s) % pvalue)
pl.plot(2 * [1. / n_classes], ylim, '--k', linewidth=3, label='Luck')

pl.ylim(ylim)
pl.legend()
pl.xlabel('Score')
pl.show()
PLS Partial Least Squares

Simple usage of various PLS flavor:
- **PLSCanonical** - PLSRegression, with multivariate response, a.k.a. PLS2
- **PLSRegression**, with univariate response, a.k.a. PLS1
- **CCA**

Given 2 multivariate covarying two-dimensional datasets, X, and Y, PLS extracts the ‘directions of covariance’, i.e. the components of each datasets that explain the most shared variance between both datasets. This is apparent on the scatterplot matrix display: components 1 in dataset X and dataset Y are maximally correlated (points lie around the first diagonal). This is also true for components 2 in both dataset, however, the correlation across datasets for different components is weak: the point cloud is very spherical.

---

**Script output:**
Corr(X)
[[ 1. 0.5 -0.07 0.04]
 [0.5 1. 0.07 0.06]
 [-0.07 0.07 1. 0.5]
 [0.04 0.06 0.5 1.]]

Corr(Y)
[[ 1. 0.46 -0.04 0.01]
 [0.46 1. -0.04 -0.02]
 [-0.04 -0.04 1. 0.54]
 [0.01 -0.02 0.54 1.]]

True B (such that: Y = XB + Err)
[[1 1 1]
 [2 2 2]
 [0 0 0]
 [0 0 0]
 [0 0 0]
 [0 0 0]
 [0 0 0]
 [0 0 0]
 [0 0 0]
 [0 0 0]]

Estimated B
[[ 1. 1. 1.]
 [2.1 2.1 2.1]
 [0. -0. 0.]
 [-0. 0. -0.]
 [-0.1 0. -0.1]
 [0. -0. 0.]
 [0. 0. -0.]
 [0. 0. 0.]
 [-0. 0. 0.]
 [-0. -0. -0.]]

Estimated betas
[[ 1.]
 [2.]
 [0.]
 [-0.]
 [-0.]
 [-0.]
 [0.]
 [0.]
 [0.]]

Python source code: plot_pls.py

```python
import numpy as np
import pylab as pl
from sklearn.pls import PLSCanonical, PLSRegression, CCA

# Dataset based latent variables model

n = 500
# 2 latent vars:
l1 = np.random.normal(size=n)
```

2.1. Examples
12 = np.random.normal(size=n)

latents = np.array([l1, l1, l2, l2]).T
X = latents + np.random.normal(size=4 * n).reshape((n, 4))
Y = latents + np.random.normal(size=4 * n).reshape((n, 4))

X_train = X[:n / 2]
Y_train = Y[:n / 2]
X_test = X[n / 2:]
Y_test = Y[n / 2:]

print "Corr(X)"
print np.round(np.corrcoef(X.T), 2)
print "Corr(Y)"
print np.round(np.corrcoef(Y.T), 2)

# Canonical (symmetric) PLS

# Transform data
plsca = PLSCanonical(n_components=2)
plsca.fit(X_train, Y_train)
X_train_r, Y_train_r = plsca.transform(X_train, Y_train)
X_test_r, Y_test_r = plsca.transform(X_test, Y_test)

# Scatter plot of scores
# ~~~~~~~~~~~~~~~~~~~~~~
# 1) On diagonal plot X vs Y scores on each components
pl.subplot(221)
pl.plot(X_train_r[:, 0], Y_train_r[:, 0], "ob", label="train")
pl.plot(X_test_r[:, 0], Y_test_r[:, 0], "or", label="test")
pl.xlabel("x scores")
pl.ylabel("y scores")
pl.title('Comp. 1: X vs Y (test corr = %.2f)'
        % np.corrcoef(X_test_r[:, 0], Y_test_r[:, 0])[0, 1])
pl.legend()

pl.subplot(222)
pl.plot(X_train_r[:, 1], Y_train_r[:, 1], "ob", label="train")
pl.plot(X_test_r[:, 1], Y_test_r[:, 1], "or", label="test")
pl.xlabel("x scores")
pl.ylabel("y scores")
pl.title('Comp. 2: X vs Y (test corr = %.2f)'
        % np.corrcoef(X_test_r[:, 1], Y_test_r[:, 1])[0, 1])
pl.legend()

# 2) Off diagonal plot components 1 vs 2 for X and Y
pl.subplot(223)
pl.plot(X_train_r[:, 0], X_train_r[:, 1], "*b", label="train")
pl.plot(X_test_r[:, 0], X_test_r[:, 1], "*r", label="test")
pl.xlabel("X comp. 1")
pl.ylabel("X comp. 2")
pl.title('X comp. 1 vs X comp. 2 (test corr = %.2f)'
        % np.corrcoef(X_test_r[:, 0], X_test_r[:, 1])[0, 1])
pl.legend()

pl.subplot(224)
pl.plot(Y_train_r[:, 0], Y_train_r[:, 1], "*b", label="train")
pl.plot(Y_test_r[:, 0], Y_test_r[:, 1], "*r", label="test")
pl.xlabel("Y comp. 1")
pl.ylabel("Y comp. 2")
pl.title("Y comp. 1 vs Y comp. 2 , (test corr = %.2f)" % 
np.corrcoef(Y_test_r[:, 0], Y_test_r[:, 1])[0, 1])
pl.legend()
pl.show()

# PLS regression, with multivariate response, a.k.a. PLS2
n = 1000
q = 3
p = 10
X = np.random.normal(size=n * p).reshape((n, p))
B = np.array([[1, 2] + [0] * (p - 2)] * q).T
# each Yj = 1*X1 + 2*X2 + noize
Y = np.dot(X, B) + np.random.normal(size=n * q).reshape((n, q)) + 5
pls2 = PLSRegression(n_components=3)
pls2.fit(X, Y)
print "True B (such that: Y = XB + Err)"
print B
# compare pls2.coefs with B
print "Estimated B"
print np.round(pls2.coefs, 1)
pls2.predict(X)

# PLS regression, with univariate response, a.k.a. PLS1
n = 1000
p = 10
X = np.random.normal(size=n * p).reshape((n, p))
y = X[:, 0] + 2 * X[:, 1] + np.random.normal(size=n * 1) + 5
pls1 = PLSRegression(n_components=3)
pls1.fit(X, y)
# note that the number of components exceeds 1 (the dimension of y)
print "Estimated betas"
print np.round(pls1.coefs, 1)

# CCA (PLS mode B with symetric deflation)
cca = CCA(n_components=2)
cca.fit(X_train, Y_train)
X_train_r, Y_train_r = plsca.transform(X_train, Y_train)
X_test_r, Y_test_r = plsca.transform(X_test, Y_test)

Precision-Recall

Example of Precision-Recall metric to evaluate the quality of the output of a classifier.
Figure 2.14: Precision-Recall

Script output:
Area Under Curve: 0.82

Python source code: plot_precision_recall.py

```
print __doc__

import random
import pylab as pl
import numpy as np
from sklearn import svm, datasets
from sklearn.metrics import precision_recall_curve
from sklearn.metrics import auc
```
# import some data to play with
iris = datasets.load_iris()
X = iris.data
y = iris.target
X, y = X[y != 2], y[y != 2]  # Keep also 2 classes (0 and 1)
n_samples, n_features = X.shape
p = range(n_samples)  # Shuffle samples
random.seed(0)
random.shuffle(p)
X, y = X[p], y[p]
half = int(n_samples / 2)

# Add noisy features
np.random.seed(0)
X = np.c_[X, np.random.randn(n_samples, 200 * n_features)]

# Run classifier
classifier = svm.SVC(kernel='linear', probability=True)
probas_ = classifier.fit(X[:half], y[:half]).predict_proba(X[half:]).
# Compute Precision-Recall and plot curve
precision, recall, thresholds = precision_recall_curve(y[half:], probas_[:, 1])
area = auc(recall, precision)
print "Area Under Curve: $0.2f" % area

pl.clf()
pl.plot(recall, precision, label='Precision-Recall curve')
pl.xlabel('Recall')
pl.ylabel('Precision')
pl.ylim([0.0, 1.05])
pl.xlim([0.0, 1.0])
pl.title('Precision-Recall example: AUC=$0.2f$ % area')
pl.legend(loc="lower left")
pl.show()
from sklearn.svm import SVC
from sklearn.datasets import load_digits
from sklearn.feature_selection import RFE

# Load the digits dataset
digits = load_digits()
X = digits.images.reshape((len(digits.images), -1))
y = digits.target

# Create the RFE object and rank each pixel
svc = SVC(kernel="linear", C=1)
rfe = RFE(estimator=svc, n_features_to_select=1, step=1)
rfe.fit(X, y)
ranking = rfe.ranking_.reshape(digits.images[0].shape)
Recursive feature elimination with cross-validation

A recursive feature elimination example with automatic tuning of the number of features selected with cross-validation.

Figure 2.16: *Recursive feature elimination with cross-validation*

Script output:
Optimal number of features: 5

**Python source code:** plot_rfe_with_cross_validation.py

```python
print __doc__

from sklearn.svm import SVC
from sklearn.cross_validation import StratifiedKFold
from sklearn.feature_selection import RFECV
from sklearn.datasets import make_classification
from sklearn.metrics import zero_one

# Build a classification task using 3 informative features
X, y = make_classification(n_samples=1000, n_features=25, n_informative=3,
                           n_redundant=2, n_repeated=0, n_classes=8, n_clusters_per_class=1,
                           random_state=0)

# Create the RFE object and compute a cross-validated score.
svc = SVC(kernel="linear")
rfecv = RFECV(estimator=svc, step=1, cv=StratifiedKFold(y, 2),
              loss_func=zero_one)
rfecv.fit(X, y)

print "Optimal number of features : %d" % rfecv.n_features_

# Plot number of features VS. cross-validation scores
import pylab as pl
pl.figure()
pl.xlabel("Number of features selected")
pl.ylabel("Cross validation score (nb of misclassifications)")
pl.plot(xrange(1, len(rfecv.cv_scores_) + 1), rfecv.cv_scores_)
pl.show()
```

---

**Receiver operating characteristic (ROC)**

Example of Receiver operating characteristic (ROC) metric to evaluate the quality of the output of a classifier.
Script output:

Area under the ROC curve : 0.794686

Python source code: plot_roc.py

```python
print __doc__

import numpy as np
import pylab as pl
from sklearn import svm, datasets
from sklearn.utils import shuffle
from sklearn.metrics import roc_curve, auc

random_state = np.random.RandomState(0)

# Import some data to play with
iris = datasets.load_iris()
X = iris.data
y = iris.target

# Make it a binary classification problem by removing the third class
X, y = X[y != 2], y[y != 2]
n_samples, n_features = X.shape

# Add noisy features to make the problem harder
X = np.c_[X, random_state.randn(n_samples, 200 * n_features)]
```

2.1. Examples
# shuffle and split training and test sets
X, y = shuffle(X, y, random_state=random_state)
half = int(n_samples / 2)
X_train, X_test = X[:half], X[half:]
y_train, y_test = y[:half], y[half:]

# Run classifier
classifier = svm.SVC(kernel='linear', probability=True)
probas_ = classifier.fit(X_train, y_train).predict_proba(X_test)

# Compute ROC curve and area the curve
fpr, tpr, thresholds = roc_curve(y_test, probas_[:, 1])
roc_auc = auc(fpr, tpr)
print "Area under the ROC curve : $f" % roc_auc

# Plot ROC curve
pl.clf()
pl.plot(fpr, tpr, label='ROC curve (area = $0.2f$) % roc_auc)
pl.plot([0, 1], [0, 1], 'k--')
pl.xlim([0.0, 1.0])
pl.ylim([0.0, 1.0])
pl.xlabel('False Positive Rate')
pl.ylabel('True Positive Rate')
pl.title('Receiver operating characteristic example')
pl.legend(loc="lower right")
pl.show()

Figure 2.18: Receiver operating characteristic (ROC) with cross validation

Receiver operating characteristic (ROC) with cross validation

Example of Receiver operating characteristic (ROC) metric to evaluate the quality of the output of a classifier using cross-validation.
import numpy as np
from scipy import interp
import matplotlib.pyplot as plt

from sklearn import svm, datasets
from sklearn.metrics import roc_curve, auc
from sklearn.cross_validation import StratifiedKFold

# Data IO and generation
# import some data to play with
iris = datasets.load_iris()
X = iris.data
y = iris.target
X, y = X[y != 2], y[y != 2]
n_samples, n_features = X.shape

# Add noisy features
X = np.c_[X, np.random.randn(n_samples, 200 * n_features)]
# Classification and ROC analysis

# Run classifier with crossvalidation and plot ROC curves

cv = StratifiedKFold(y, k=6)
classifier = svm.SVC(kernel='linear', probability=True)

mean_tpr = 0.0
mean_fpr = np.linspace(0, 1, 100)
all_tpr = []

for i, (train, test) in enumerate(cv):
    probas_ = classifier.fit(X[train], y[train]).predict_proba(X[test])
    # Compute ROC curve and area the curve
    fpr, tpr, thresholds = roc_curve(y[test], probas_[:, 1])
    mean_tpr += interp(mean_fpr, fpr, tpr)
    mean_tpr[0] = 0.0
    roc_auc = auc(fpr, tpr)
    pl.plot(fpr, tpr, lw=1, label='ROC fold %d (area = %0.2f)' % (i, roc_auc))

pl.plot([0, 1], [0, 1], '--', color=(0.6, 0.6, 0.6), label='Luck')

mean_tpr /= len(cv)
mean_tpr[-1] = 1.0
mean_auc = auc(mean_fpr, mean_tpr)
pl.plot(mean_fpr, mean_tpr, 'k--',
        label='Mean ROC (area = %0.2f)' % mean_auc, lw=2)

pl.xlim([-0.05, 1.05])
pl.ylim([-0.05, 1.05])
pl.xlabel('False Positive Rate')
pl.ylabel('True Positive Rate')
pl.title('Receiver operating characteristic example')
pl.legend(loc="lower right")
pl.show()

Figure 2.19: Train error vs Test error

Train error vs Test error

Illustration of how the performance of an estimator on unseen data (test data) is not the same as the performance on training data. As the regularization increases the performance on train decreases while the performance on test is optimal within a range of values of the regularization parameter. The example with an Elastic-Net regression model and the performance is measured using the explained variance a.k.a. $R^2$. 

702 Chapter 2. Example Gallery
scikit-learn user guide, Release 0.12-git

Script output:
Optimal regularization parameter : 0.000335292414925

Python source code: plot_train_error_vs_test_error.py
print __doc__
# Author: Alexandre Gramfort <alexandre.gramfort@inria.fr>
# License: BSD Style.
import numpy as np
from sklearn import linear_model
###############################################################################
# Generate sample data
n_samples_train, n_samples_test, n_features = 75, 150, 500
np.random.seed(0)
coef = np.random.randn(n_features)
coef[50:] = 0.0 # only the top 10 features are impacting the model
X = np.random.randn(n_samples_train + n_samples_test, n_features)
y = np.dot(X, coef)
# Split train and test data
X_train, X_test = X[:n_samples_train], X[n_samples_train:]
y_train, y_test = y[:n_samples_train], y[n_samples_train:]

2.1. Examples

703


Compute train and test errors
alphas = np.logspace(-5, 1, 60)
enet = linear_model.ElasticNet(rho=0.7)
train_errors = list()
test_errors = list()
for alpha in alphas:
enet.set_params(alpha=alpha)
enet.fit(X_train, y_train)
train_errors.append(enet.score(X_train, y_train))
test_errors.append(enet.score(X_test, y_test))

i_alpha_optim = np.argmax(test_errors)
alpha_optim = alphas[i_alpha_optim]
print "Optimal regularization parameter : %s" % alpha_optim

Estimate the coef_ on full data with optimal regularization parameter
enet.set_params(alpha=alpha_optim)
coef_ = enet.fit(X, y).coef_

Plot results functions
import pylab as pl
pl.subplot(2, 1, 1)
pl.semilogx(alphas, train_errors, label='Train')
pl.semilogx(alphas, test_errors, label='Test')
pl.vlines(alpha_optim, pl.ylim()[0], np.max(test_errors),
          color='k', linewidth=3, label='Optimum on test')
pl.legend(loc='lower left')
pl.ylim([0, 1.2])
pl.xlabel('Regularization parameter')
pl.ylabel('Performance')

Show estimated coef_ vs true coef
pl.subplot(2, 1, 2)
pl.plot(coef, label='True coef')
pl.plot(coef_, label='Estimated coef')
pl.legend()
pl.subplots_adjust(0.09, 0.04, 0.94, 0.94, 0.26, 0.26)
pl.show()

Figure 2.20: Classification of text documents using sparse features
Classification of text documents using sparse features

This is an example showing how the scikit-learn can be used to classify documents by topics using a bag-of-words approach. This example uses a scipy.sparse matrix to store the features instead of standard numpy arrays and demos various classifiers that can efficiently handle sparse matrices.

The dataset used in this example is the 20 newsgroups dataset which will be automatically downloaded and then cached.

You can adjust the number of categories by giving their names to the dataset loader or setting them to None to get the 20 of them.

Python source code: document_classification_20newsgroups.py

```python
import logging
import numpy as np
from optparse import OptionParser
import time
import pylab as pl

from sklearn.datasets import fetch_20newsgroups
from sklearn.feature_extraction.text import TfidfVectorizer
from sklearn.feature_selection import SelectKBest, chi2
from sklearn.linear_model import RidgeClassifier
from sklearn.svm import LinearSVC
from sklearn.linear_model import SGDClassifier
from sklearn.linear_model import Perceptron
from sklearn.naive_bayes import BernoulliNB, MultinomialNB
from sklearn.neighbors import KNeighborsClassifier
from sklearn.neighbors import NearestCentroid
from sklearn.utils.extmath import density
from sklearn import metrics

# Display progress logs on stdout
logging.basicConfig(level=logging.INFO,
                    format='%(asctime)s %(levelname)s %(message)s')

# parse commandline arguments
op = OptionParser()
op.add_option("--report", 
    action="store_true", dest="print_report",
    help="Print a detailed classification report.")
op.add_option("--chi2_select",
    action="store", type="int", dest="select_chi2",
    help="Select some number of features using a chi-squared test")
op.add_option("--confusion_matrix",
    action="store_true", dest="print_cm",
    help="Print the confusion matrix.")
op.add_option("--top10",
    action="store_true", dest="print_top10",
```
help="Print ten most discriminative terms per class" 
" for every classifier.")

(opts, args) = op.parse_args()
if len(args) > 0:
op.error("this script takes no arguments.")
sys.exit(1)

print __doc__
op.print_help()

print

#############################################################################
# Load some categories from the training set

categories = ['alt.atheism',
              'talk.religion.misc',
              'comp.graphics',
              'sci.space',
          ]
# Uncomment the following to do the analysis on all the categories
#categories = None

print "Loading 20 newsgroups dataset for categories:

print categories
if categories else "all"

data_train = fetch_20newsgroups(subset='train', categories=categories,
                                shuffle=True, random_state=42)

print ‘data loaded’

categories = data_train.target_names # for case categories == None

print "%d documents (training set)" % len(data_train.data)
print "%d documents (testing set)" % len(data_test.data)
print "%d categories" % len(categories)
print

# split a training set and a test set
y_train, y_test = data_train.target, data_test.target

print "Extracting features from the training dataset using a sparse vectorizer"
t0 = time()
vectorizer = TfidfVectorizer(sublinear_tf=True, max_df=0.5,
                             stop_words='english')

X_train = vectorizer.fit_transform(data_train.data)
print "done in %fs" % (time() - t0)
print "n_samples: %d, n_features: %d" % X_train.shape

print "Extracting features from the test dataset using the same vectorizer"
t0 = time()
X_test = vectorizer.transform(data_test.data)
print "done in %fs" % (time() - t0)
print "n_samples: %d, n_features: %d" % X_test.shape
scikit-learn user guide, Release 0.12-git

print
if opts.select_chi2:
print ("Extracting %d best features by a chi-squared test" %
opts.select_chi2)
t0 = time()
ch2 = SelectKBest(chi2, k=opts.select_chi2)
X_train = ch2.fit_transform(X_train, y_train)
X_test = ch2.transform(X_test)
print "done in %fs" % (time() - t0)
print

def trim(s):
"""Trim string to fit on terminal (assuming 80-column display)"""
return s if len(s) <= 80 else s[:77] + "..."

# mapping from integer feature name to original token string
feature_names = vectorizer.get_feature_names()

###############################################################################
# Benchmark classifiers
def benchmark(clf):
print 80 * ’_’
print "Training: "
print clf
t0 = time()
clf.fit(X_train, y_train)
train_time = time() - t0
print "train time: %0.3fs" % train_time
t0 = time()
pred = clf.predict(X_test)
test_time = time() - t0
print "test time: %0.3fs" % test_time
score = metrics.f1_score(y_test, pred)
print "f1-score:
%0.3f" % score
if hasattr(clf, ’coef_’):
print "dimensionality: %d" % clf.coef_.shape[1]
print "density: %f" % density(clf.coef_)
if opts.print_top10:
print "top 10 keywords per class:"
for i, category in enumerate(categories):
top10 = np.argsort(clf.coef_[i])[-10:]
print trim("%s: %s" % (
category, " ".join(feature_names[top10])))
print
if opts.print_report:
print "classification report:"
print metrics.classification_report(y_test, pred,
target_names=categories)

2.1. Examples

707


if opts.print_cm:
    print "confusion matrix:"
    print metrics.confusion_matrix(y_test, pred)

print
clf_descr = str(clf).split('()')[0]
return clf_descr, score, train_time, test_time

results = []
for clf, name in ((RidgeClassifier(tol=1e-1), "Ridge Classifier"),
                   (Perceptron(n_iter=50), "Perceptron"),
                   (KNeighborsClassifier(n_neighbors=10), "kNN")):
    print 80 * '='
    print name
    results.append(benchmark(clf))

for penalty in ["l2", "l1"]:
    print 80 * '='
    print "%s penalty" % penalty.upper()
    # Train Liblinear model
    results.append(benchmark(LinearSVC(loss='l2', penalty=penalty,
                                       dual=False, tol=1e-3)))
    # Train SGD model
    results.append(benchmark(SGDClassifier(alpha=.0001, n_iter=50,
                                            penalty=penalty)))

# Train SGD with Elastic Net penalty
print 80 * '='
print "Elastic-Net penalty"
results.append(benchmark(SGDClassifier(alpha=.0001, n_iter=50,
                                       penalty="elasticnet")))

# Train NearestCentroid without threshold
print 80 * '='
print "NearestCentroid (aka Rocchio classifier)"
results.append(benchmark(NearestCentroid()))

# Train sparse Naive Bayes classifiers
print 80 * '='
print "Naive Bayes"
results.append(benchmark(MultinomialNB(alpha=.01)))
results.append(benchmark(BernoulliNB(alpha=.01)))

class L1LinearSVC(LinearSVC):

    def fit(self, X, y):
        # The smaller C, the stronger the regularization.
        # The more regularization, the more sparsity.
        self.transformer_ = LinearSVC(penalty="l1",
                                       dual=False, tol=1e-3)
        X = self.transformer_.fit_transform(X, y)
        return LinearSVC.fit(self, X, y)

    def predict(self, X):
        X = self.transformer_.transform(X)
return LinearSVC.predict(self, X)

print 80 * '='
print "LinearSVC with L1-based feature selection"
results.append(benchmark(L1LinearSVC()))

# make some plots
indices = np.arange(len(results))
results = [[x[i] for x in results] for i in xrange(4)]
clf_names, score, training_time, test_time = results
pl.title("Score")
pl.barh(indices, score, .2, label="score", color='r')
pl.barh(indices + .3, training_time, .2, label="training time", color='g')
pl.barh(indices + .6, test_time, .2, label="test time", color='b')
pl.yticks(()
pl.legend(loc='best')
pl.subplots_adjust(left=.25)
for i, c in zip(indices, clf_names):
    pl.text(-.3, i, c)
pl.show()

Figure 2.21: Clustering text documents using k-means

Clustering text documents using k-means

This is an example showing how the scikit-learn can be used to cluster documents by topics using a bag-of-words approach. This example uses a scipy.sparse matrix to store the features instead of standard numpy arrays.

Two algorithms are demoed: ordinary k-means and its faster cousin minibatch k-means.

Python source code: document_clustering.py

from sklearn.datasets import fetch_20newsgroups
from sklearn.feature_extraction.text import TfidfVectorizer
from sklearn import metrics
from sklearn.cluster import KMeans, MiniBatchKMeans

import logging
from optparse import OptionParser
import sys
from time import time

import numpy as np

# Display progress logs on stdout
logging.basicConfig(level=logging.INFO,
                    format='%(asctime)s %(levelname)s %(message)s')

# parse commandline arguments
op = OptionParser()
op.add_option('--no-minibatch',
              action='store_false', dest='minibatch', default=True,
              help='Use ordinary k-means algorithm."

print __doc__
op.print_help()

(opts, args) = op.parse_args()
if len(args) > 0:
    op.error("this script takes no arguments.
    sys.exit(1)

# Load some categories from the training set
categories = ['alt.atheism',
              'talk.religion.misc',
              'comp.graphics',
              'sci.space',
              ]
# Uncomment the following to do the analysis on all the categories
# categories = None

print "Loading 20 newsgroups dataset for categories:"
print categories

dataset = fetch_20newsgroups(subset='all', categories=categories,
                             shuffle=True, random_state=42)

print "%d documents" % len(dataset.data)
print "%d categories" % len(dataset.target_names)

print

labels = dataset.target
ture_k = np.unique(labels).shape[0]

print "Extracting features from the training dataset using a sparse vectorizer"
t0 = time()
vectorizer = TfidfVectorizer(max_df=0.5, max_features=10000,
                             stop_words='english')
X = vectorizer.fit_transform(dataset.data)
print "done in \$fs\ % (time() - t0)
print "n_samples: \$d, n_features: \$d" % X.shape
print

# Do the actual clustering
if opts.minibatch:
    km = MiniBatchKMeans(n_clusters=true_k, init='k-means++', n_init=1,
                         init_size=1000,
                         batch_size=1000, verbose=1)
else:
    km = KMeans(n_clusters=true_k, init='random', max_iter=100, n_init=1,
                verbose=1)

print "Clustering sparse data with \$s\ % km
t0 = time()
km.fit(X)
print "done in \$0.3fs\ % (time() - t0)
print
print "Homogeneity: \$0.3f\ % metrics.homogeneity_score(labels, km.labels_)
print "Completeness: \$0.3f\ % metrics.completeness_score(labels, km.labels_)
print "V-measure: \$0.3f\ % metrics.v_measure_score(labels, km.labels_)
print "Adjusted Rand-Index: \$3f\ % metrics.adjusted_rand_score(labels, km.labels_)
print "Silhouette Coefficient: \$0.3f\ % metrics.silhouette_score(
    X, labels, sample_size=1000)

print

Figure 2.22: Pipeline Anova SVM

Pipeline Anova SVM

Simple usage of Pipeline that runs successively a univariate feature selection with anova and then a C-SVM of the selected features.

Python source code: feature_selection_pipeline.py

from sklearn import svm
from sklearn.datasets import samples_generator
from sklearn.feature_selection import SelectKBest, f_regression

2.1. Examples
from sklearn.pipeline import Pipeline

# import some data to play with
X, y = samples_generator.make_classification(
    n_features=20, n_informative=3, n_redundant=0,
    n_classes=4, n_clusters_per_class=2)

# ANOVA SVM-C
# 1) anova filter, take 3 best ranked features
anova_filter = SelectKBest(f_regression, k=3)
# 2) svm
clf = svm.SVC(kernel='linear')

anova_svm = Pipeline([('anova', anova_filter), ('svm', clf)])
anova_svm.fit(X, y)
anova_svm.predict(X)

Figure 2.23: Parameter estimation using grid search with a nested cross-validation

Parameter estimation using grid search with a nested cross-validation

The classifier is optimized by “nested” cross-validation using the sklearn.grid_search.GridSearchCV object on a development set that comprises only half of the available labeled data.

The performance of the selected hyper-parameters and trained model is then measured on a dedicated evaluation set that was not used during the model selection step.

More details on tools available for model selection can be found in the sections on Cross-Validation: evaluating estimator performance and Grid Search: setting estimator parameters.

Python source code: grid_search_digits.py

print __doc__

from sklearn import datasets
from sklearn.cross_validation import train_test_split
from sklearn.grid_search import GridSearchCV
from sklearn.metrics import classification_report
from sklearn.metrics import precision_score
from sklearn.metrics import recall_score
from sklearn.svm import SVC

# Loading the Digits dataset
digits = datasets.load_digits()

# To apply an classifier on this data, we need to flatten the image, to
# turn the data in a (samples, feature) matrix:
n_samples = len(digits.images)
X = digits.images.reshape((n_samples, -1))
y = digits.target

# Split the dataset in two equal parts
X_train, X_test, y_train, y_test = train_test_split(
    X, y, test_fraction=0.5, random_state=0)

# Set the parameters by cross-validation

tuned_parameters = [{'kernel': ['rbf'], 'gamma': [1e-3, 1e-4],
                    'C': [1, 10, 100, 1000]},
                   {'kernel': ['linear'], 'C': [1, 10, 100, 1000]}]

scores = [('precision', precision_score), ('recall', recall_score),
          ('f1', f1_score)]

for score_name, score_func in scores:
    print("# Tuning hyper-parameters for \$s\" % score_name)
    print
    clf = GridSearchCV(SVC(C=1), tuned_parameters, score_func=score_func)
    clf.fit(X_train, y_train, cv=5)

    print("Best parameters set found on development set:"
          "
          print clf.best_estimator_
    print "Grid scores on development set:"
    print
    for params, mean_score, scores in clf.grid_scores_:
        print "%0.3f (+/-%0.03f) for %r" % (mean_score, scores.std() / 2, params)
    print

    print "Detailed classification report:"
    print "The model is trained on the full development set."
    print "The scores are computed on the full evaluation set."
    print
    y_true, y_pred = y_test, clf.predict(X_test)
    print classification_report(y_true, y_pred)
    print

# Note the problem is too easy: the hyperparameter plateau is too flat and the
# output model is the same for precision and recall with ties in quality.

Sample pipeline for text feature extraction and evaluation

The dataset used in this example is the 20 newsgroups dataset which will be automatically downloaded and then cached
and reused for the document classification example.

You can adjust the number of categories by giving there name to the dataset loader or setting them to None to get the
20 of them.

Here is a sample output of a run on a quad-core machine:
Figure 2.24: *Sample pipeline for text feature extraction and evaluation*

Loading 20 newsgroups dataset for categories:
['alt.atheism', 'talk.religion.misc']
1427 documents
2 categories

Performing grid search...
pipeline: ['vect', 'tfidf', 'clf']
parameters:
{'clf__alpha': (1.0000000000000001e-05, 9.9999999999999995e-07),
 'clf__n_iter': (10, 50, 80),
 'clf__penalty': ('l2', 'elasticnet'),
 'tfidf__use_idf': (True, False),
 'vect__max_n': (1, 2),
 'vect__max_df': (0.5, 0.75, 1.0),
 'vect__max_features': (None, 5000, 10000, 50000)}
done in 1737.030s

Best score: 0.940
Best parameters set:
clf__alpha: 9.9999999999999995e-07
clf__n_iter: 50
clf__penalty: 'elasticnet'
tfidf__use_idf: True
vect__max_n: 2
vect__max_df: 0.75
vect__max_features: 50000

Python source code: grid_search_text_feature_extraction.py

```python
print __doc__

# Author: Olivier Grisel <olivier.grisel@ensta.org>
# Peter Prettenhofer <peter.prettenhofer@gmail.com>
# Mathieu Blondel <mathieu@mblondel.org>
# License: Simplified BSD

from pprint import pprint
from time import time
import logging

from sklearn.datasets import fetch_20newsgroups
from sklearn.feature_extraction.text import CountVectorizer
from sklearn.feature_extraction.text import TfidfTransformer
from sklearn.linear_model import SGDClassifier
from sklearn.grid_search import GridSearchCV
from sklearn.pipeline import Pipeline
```
# Display progress logs on stdout
logging.basicConfig(level=logging.INFO,
                     format='%(asctime)s %(levelname)s %(message)s')

# Load some categories from the training set
categories = [  
    'alt.atheism',  
    'talk.religion.misc',
]

# Uncomment the following to do the analysis on all the categories
# categories = None

print("Loading 20 newsgroups dataset for categories:")
print(categories)
data = fetch_20newsgroups(subset='train', categories=categories)

print("%d documents" % len(data.filenames))
print("%d categories" % len(data.target_names))

# define a pipeline combining a text feature extractor with a simple classifier
pipeline = Pipeline([  
    ('vect', CountVectorizer()),  
    ('tfidf', TfidfTransformer()),  
    ('clf', SGDClassifier()),
])

parameters = {
    # uncommenting more parameters will give better exploring power but will increase processing time in a combinatorial way
    'vect__max_df': (0.5, 0.75, 1.0),
    # 'vect__max_features': (None, 5000, 10000, 50000),
    # 'vect__max_n': (1, 2),  # words or bigrams
    # 'tfidf__use_idf': (True, False),
    # 'tfidf__norm': ('l1', 'l2'),
    'clf__alpha': (0.00001, 0.000001),
    'clf__penalty': ('l2', 'elasticnet'),
    # 'clf__n_iter': (10, 50, 80),
}

if __name__ == '__main__':
    # multiprocessing requires the fork to happen in a __main__ protected block
    # find the best parameters for both the feature extraction and the classifier
    grid_search = GridSearchCV(pipeline, parameters, n_jobs=-1, verbose=1)

    print("Performing grid search...")
    print("pipeline:")
    pprint([name for name, _ in pipeline.steps])
    print("parameters:")
    pprint(parameters)
    t0 = time()
    grid_search.fit(data.data, data.target)
Classification of text documents: using a MLComp dataset

This is an example showing how the scikit-learn can be used to classify documents by topics using a bag-of-words approach. This example uses a scipy.sparse matrix to store the features instead of standard numpy arrays.

The dataset used in this example is the 20 newsgroups dataset and should be downloaded from the http://mlcomp.org (free registration required):

http://mlcomp.org/datasets/379

Once downloaded unzip the archive somewhere on your filesystem. For instance in:

% mkdir -p ~/data/mlcomp
% cd ~/data/mlcomp
% unzip /path/to/dataset-379-20news-18828_XXXXX.zip

You should get a folder ~/data/mlcomp/379 with a file named metadata and subfolders raw, train and test holding the text documents organized by newsgroups.

Then set the MLCOMP_DATASETS_HOME environment variable pointing to the root folder holding the uncompressed archive:

% export MLCOMP_DATASETS_HOME="~/data/mlcomp"

Then you are ready to run this example using your favorite python shell:

% ipython examples/mlcomp_sparse_document_classification.py

**Python source code:** mlcomp_sparse_document_classification.py

```
print "done in $0.3fs" % (time() - t0)
print
print "Best score: $0.3f" % grid_search.best_score
print "Best parameters set:"
best_parameters = grid_search.best_estimator.get_params()
for param_name in sorted(parameters.keys()):
    print \t%s: %r % (param_name, best_parameters[param_name])
```

**Figure 2.25:** Classification of text documents: using a MLComp dataset
import os
import numpy as np
import scipy.sparse as sp
import pylab as pl

from sklearn.datasets import load_mlcomp
from sklearn.feature_extraction.text import TfidfVectorizer
from sklearn.linear_model import SGDClassifier
from sklearn.metrics import confusion_matrix
from sklearn.metrics import classification_report
from sklearn.naive_bayes import MultinomialNB

if 'MLCOMP_DATASETS_HOME' not in os.environ:
    sys.exit(0)

# Load the training set
print "Loading 20 newsgroups training set... 
news_train = load_mlcomp('20news-18828', 'train')
print news_train.DESCR
print "%d documents" % len(news_train.filenames)
print "%d categories" % len(news_train.target_names)

print "Extracting features from the dataset using a sparse vectorizer"
t0 = time()
vectorizer = TfidfVectorizer(charset='latin1')
X_train = vectorizer.fit_transform((open(f).read() for f in news_train.filenames))
assert sp.issparse(X_train)
y_train = news_train.target

print "Loading 20 newsgroups test set... 
news_test = load_mlcomp('20news-18828', 'test')
t0 = time()
print "done in %fs" % (time() - t0)

print "Predicting the labels of the test set..."
print "%d documents" % len(news_test.filenames)
print "%d categories" % len(news_test.target_names)

print "Extracting features from the dataset using the same vectorizer"
t0 = time()
X_test = vectorizer.transform((open(f).read() for f in news_test.filenames))
y_test = news_test.target
print "done in %fs" % (time() - t0)

# Benchmark classifiers

def benchmark(clf_class, params, name):
    print "parameters!", params
    t0 = time()
    clf = clf_class(**params).fit(X_train, y_train)
    print "done in %fs" % (time() - t0)
if hasattr(clf, 'coef_'):
    print "Percentage of non zeros coef: %f" % (np.mean(clf.coef_ != 0) * 100)

print "Predicting the outcomes of the testing set"
t0 = time()
pred = clf.predict(X_test)
print "done in %fs" % (time() - t0)

print "Classification report on test set for classifier:"
print clf
print
print classification_report(y_test, pred,
    target_names=news_test.target_names)

cm = confusion_matrix(y_test, pred)
print "Confusion matrix:"
print cm

# Show confusion matrix
pl.matshow(cm)
pl.title('Confusion matrix of the %s classifier' % name)
pl.colorbar()

print "Testbenching a linear classifier..."
parameters = {
    'loss': 'hinge',
    'penalty': 'l2',
    'n_iter': 50,
    'alpha': 0.00001,
    'fit_intercept': True,
}

benchmark(SGDClassifier, parameters, 'SGD')

print "Testbenching a MultinomialNB classifier...
parameters = {'alpha': 0.01}

benchmark(MultinomialNB, parameters, 'MultinomialNB')

pl.show()

2.1.2 Examples based on real world datasets

Applications to real world problems with some medium sized datasets or interactive user interface.

Outlier detection on a real data set

This example illustrates the need for robust covariance estimation on a real data set. It is useful both for outlier detection and for a better understanding of the data structure.

We selected two sets of two variables from the boston housing data set as an illustration of what kind of analysis can be done with several outlier detection tools. For the purpose of visualization, we are working with two-dimensional examples, but one should be aware that things are not so trivial in high-dimensional, as it will be pointed out.
In both examples below, the main result is that the empirical covariance estimate, as a non-robust one, is highly influenced by the heterogeneous structure of the observations. Although the robust covariance estimate is able to focus on the main mode of the data distribution, it sticks to the assumption that the data should be Gaussian distributed, yielding some biased estimation of the data structure, but yet accurate to some extent. The One-Class SVM algorithm

**First example**

The first example illustrates how robust covariance estimation can help concentrating on a relevant cluster when another one exists. Here, many observations are confounded into one and break down the empirical covariance estimation. Of course, some screening tools would have pointed out the presence of two clusters (Support Vector Machines, Gaussian Mixture Models, univariate outlier detection, ...). But had it been a high-dimensional example, none of these could be applied that easily.

**Second example**

The second example shows the ability of the Minimum Covariance Determinant robust estimator of covariance to concentrate on the main mode of the data distribution: the location seems to be well estimated, although the covariance is hard to estimate due to the banana-shaped distribution. Anyway, we can get rid of some outlying observations. The One-Class SVM is able to capture the real data structure, but the difficulty is to adjust its kernel bandwith parameter so as to obtain a good compromise between the shape of the data scatter matrix and the risk of over-fitting the data.
import numpy as np
from sklearn.covariance import EllipticEnvelope
from sklearn.svm import OneClassSVM
import matplotlib.pyplot as plt
import matplotlib.font_manager
from sklearn.datasets import load_boston

# Get data
X1 = load_boston()['data'][:, [8, 10]]  # two clusters
X2 = load_boston()['data'][:, [5, 12]]  # "banana"-shaped

# Define "classifiers" to be used
classifiers = {
    "Empirical Covariance": EllipticEnvelope(support_fraction=1., contamination=0.261),
    "Robust Covariance (Minimum Covariance Determinant)": EllipticEnvelope(contamination=0.261),
    "OCSVM": OneClassSVM(nu=0.261, gamma=0.05)}
colors = ['m', 'g', 'b']
legend1 = {}
legend2 = {}

# Learn a frontier for outlier detection with several classifiers
xx1, yy1 = np.meshgrid(np.linspace(-8, 28, 500), np.linspace(3, 40, 500))
xx2, yy2 = np.meshgrid(np.linspace(3, 10, 500), np.linspace(-5, 45, 500))
for i, (clf_name, clf) in enumerate(classifiers.iteritems()):
    plt.figure(1)
    clf.fit(X1)
    Z1 = clf.decision_function(np.c_[xx1.ravel(), yy1.ravel()])
    Z1 = Z1.reshape(xx1.shape)
    legend1[clf_name] = plt.contour(xx1, yy1, Z1, levels=[0], linewidths=2, colors=colors[i])
    plt.figure(2)
    clf.fit(X2)
    Z2 = clf.decision_function(np.c_[xx2.ravel(), yy2.ravel()])
    Z2 = Z2.reshape(xx2.shape)
    legend2[clf_name] = plt.contour(xx2, yy2, Z2, levels=[0], linewidths=2, colors=colors[i])
Species distribution modeling

Modeling species' geographic distributions is an important problem in conservation biology. In this example we model the geographic distribution of two south american mammals given past observations and 14 environmental variables. Since we have only positive examples (there are no unsuccessful observations), we cast this problem as a density estimation problem and use the OneClassSVM provided by the package sklearn.svm as our modeling tool. The dataset is provided by Phillips et. al. (2006). If available, the example uses basemap to plot the coast lines and national

---

2.1. Examples
boundaries of South America.

The two species are:

- "Bradypus variegatus", the Brown-throated Sloth.
- "Microryzomys minutus", also known as the Forest Small Rice Rat, a rodent that lives in Peru, Colombia, Ecuador, Peru, and Venezuela.

References


Script output:

```
Modeling distribution of species 'bradypus variegatus'
- fit OneClassSVM ... done.
- plot coastlines from coverage
- predict species distribution

Area under the ROC curve : 0.865318

Modeling distribution of species 'microryzomys minutus'
- fit OneClassSVM ... done.
```
- plot coastlines from coverage
- predict species distribution

Area under the ROC curve : 0.993919

time elapsed: 15.32s

**Python source code:** plot_species_distribution_modeling.py

```python
# Authors: Peter Prettenhofer <peter.prettenhofer@gmail.com>
# Jake Vanderplas <vanderplas@astro.washington.edu>
#
# License: BSD Style.

from time import time
import numpy as np
import pylab as pl
from sklearn.datasets.base import Bunch
from sklearn.datasets import fetch_species_distributions
from sklearn.datasets.species_distributions import construct_grids
from sklearn import svm, metrics

# if basemap is available, we’ll use it.
# otherwise, we’ll improvise later...
try:
    from mpl_toolkits.basemap import Basemap
    basemap = True
except ImportError:
    basemap = False

print __doc__

def create_species_bunch(species_name,
                          train, test,
                          coverages, xgrid, ygrid):
    
    ""
    create a bunch with information about a particular organism
    
    This will use the test/train record arrays to extract the
data specific to the given species name.
    ""
    bunch = Bunch(name=' '.join(species_name.split('_')[:2]))

    points = dict(test=test, train=train)

    for label, pts in points.iteritems():
        # choose points associated with the desired species
        pts = pts[pts['species'] == species_name]
        bunch['pts_%s' % label] = pts

        # determine coverage values for each of the training & testing points
        ix = np.searchsorted(xgrid, pts['dd long'])
        iy = np.searchsorted(ygrid, pts['dd lat'])
        bunch['cov_%s' % label] = coverages[:, -iy, ix].T
```

2.1. Examples
def plot_species_distribution(species=["bradypus_variegatus_0", "microryzomys_minutus_0"])::
    
    Plot the species distribution.

    if len(species) > 2:
        print("Note: when more than two species are provided, only "
              "the first two will be used")

    t0 = time()

    # Load the compressed data
    data = fetch_species_distributions()

    # Set up the data grid
    xgrid, ygrid = construct_grids(data)

    # The grid in x,y coordinates
    X, Y = np.meshgrid(xgrid, ygrid[::-1])

    # create a bunch for each species
    BV_bunch = create_species_bunch(species[0],
                                     data.train, data.test,
                                     data.coverages, xgrid, ygrid)
    MM_bunch = create_species_bunch(species[1],
                                     data.train, data.test,
                                     data.coverages, xgrid, ygrid)

    # background points (grid coordinates) for evaluation
    np.random.seed(13)
    background_points = np.c_[np.random.randint(low=0, high=data.Ny,
                                     size=10000),
                             np.random.randint(low=0, high=data.Nx,
                                     size=10000)].T

    # We'll make use of the fact that coverages[6] has measurements at all
    # land points. This will help us decide between land and water.
    land_reference = data.coverages[6]

    # Fit, predict, and plot for each species.
    for i, species in enumerate([BV_bunch, MM_bunch]):
        print "_" * 80
        print "Modeling distribution of species '%s'" % species.name

        # Standardize features
        mean = species.cov_train.mean(axis=0)
        std = species.cov_train.std(axis=0)
        train_cover_std = (species.cov_train - mean) / std

        # Fit OneClassSVM
        print " - fit OneClassSVM ... ",
        clf = svm.OneClassSVM(nu=0.1, kernel="rbf", gamma=0.5)
        clf.fit(train_cover_std)
        print "done. "

return bunch
# Plot map of South America
pl.subplot(1, 2, i + 1)
if basemap:
    print " - plot coastlines using basemap"
    m = Basemap(projection='cyl', llcrnrlat=Y.min(),
                 urcrnrlat=Y.max(), llcrnrlon=X.min(),
                 urcrnrlon=X.max(), resolution='c')
    m.drawcoastlines()
m.drawcountries()
else:
    print " - plot coastlines from coverage"
    pl.contour(X, Y, land_reference,
                levels=[-9999], colors="k",
                linestyles="solid")
    pl.xticks([])
    pl.yticks([])
print " - predict species distribution"

# Predict species distribution using the training data
Z = np.ones((data.Ny, data.Nx), dtype=np.float64)
# We’ll predict only for the land points.
idx = np.where(land_reference > -9999)
coverages_land = data.coverages[:, idx[0], idx[1]].T
pred = clf.decision_function((coverages_land - mean) / std)[:, 0]
Z *= pred.min()
Z[idx[0], idx[1]] = pred
levels = np.linspace(Z.min(), Z.max(), 25)
Z[land_reference == -9999] = -9999

# plot contours of the prediction
pl.contourf(X, Y, Z, levels=levels, cmap=pl.cm.Reds)
pl.colorbar(format='%.2f')
# scatter training/testing points
pl.scatter(species.pts_train['dd long'], species.pts_train['dd lat'],
           s=2 ** 2, marker='^', label='train')
pl.scatter(species.pts_test['dd long'], species.pts_test['dd lat'],
           s=2 ** 2, marker='x', label='test')
pl.legend()
pl.title(species.name)
pl.axis('equal')

# Compute AUC w.r.t. background points
pred_background = Z[background_points[0], background_points[1]]
pred_test = clf.decision_function((species.cov_test - mean)
                                   / std)[:, 0]
scores = np.r_[pred_test, pred_background]
y = np.r_[np.ones(pred_test.shape), np.zeros(pred_background.shape)]
fpr, tpr, thresholds = metrics.roc_curve(y, scores)
roc_auc = metrics.auc(fpr, tpr)
pl.text(-35, -70, "AUC: %.3f" % roc_auc, ha="right")
print "\nArea under the ROC curve : %.2f" % roc_auc
Visualizing the stock market structure

This example employs several unsupervised learning techniques to extract the stock market structure from variations in historical quotes.

The quantity that we use is the daily variation in quote price: quotes that are linked tend to cofluctuate during a day.

Learning a graph structure

We use sparse inverse covariance estimation to find which quotes are correlated conditionally on the others. Specifically, sparse inverse covariance gives us a graph, that is a list of connection. For each symbol, the symbols that it is connected too are those useful to expain its fluctuations.

Clustering

We use clustering to group together quotes that behave similarly. Here, amongst the various clustering techniques available in the scikit-learn, we use Affinity propagation as it does not enforce equal-size clusters, and it can choose automatically the number of clusters from the data.

Note that this gives us a different indication than the graph, as the graph reflects conditional relations between variables, while the clustering reflects marginal properties: variables clustered together can be considered as having a similar impact at the level of the full stock market.

Embedding in 2D space

For visualization purposes, we need to lay out the different symbols on a 2D canvas. For this we use Manifold learning techniques to retrieve 2D embedding.

Visualization

The output of the 3 models are combined in a 2D graph where nodes represents the stocks and edges the:
• cluster labels are used to define the color of the nodes
• the sparse covariance model is used to display the strength of the edges
• the 2D embedding is used to position the nodes in the plan

This example has a fair amount of visualization-related code, as visualization is crucial here to display the graph. One of the challenge is to position the labels minimizing overlap. For this we use an heuristic based on the direction of the nearest neighbor along each axis.

**Script output:**

Cluster 1: Pepsi, Coca Cola, Kellogg
Cluster 2: Apple, Amazon, Yahoo
Cluster 3: GlaxoSmithKline, Novartis, Sanofi-Aventis
Cluster 4: Comcast, Time Warner, Cablevision
Cluster 5: ConocoPhillips, Chevron, Total, Valero Energy, Exxon
Cluster 6: Walgreen, CVS
Cluster 7: Kraft Foods
Cluster 8: Navistar, Sony, Marriott, Caterpillar, Canon, Toyota, Honda, Mitsubishi, Xerox, Unilever
Cluster 9: Kimberly-Clark, Colgate-Palmolive, Procter Gamble
Cluster 10: American express, Ryder, Goldman Sachs, Wal-Mart, General Electrics, Pfizer, 3M, Wells Fargo
Cluster 11: Microsoft, SAP, IBM, Texas instruments, HP, Dell, Cisco
Cluster 12: Raytheon, Boeing, Lookheed Martin, General Dynamics, Northrop Grumman
print __doc__

# Author: Gael Varoquaux gael.varoquaux@normalesup.org
# License: BSD

import datetime

import numpy as np
import pylab as pl
from matplotlib import finance
from matplotlib.collections import LineCollection
from sklearn import cluster, covariance, manifold

# Retrieve the data from Internet
# Choose a time period reasonably calm (not too long ago so that we get
# high-tech firms, and before the 2008 crash)
d1 = datetime.datetime(2003, 01, 01)
d2 = datetime.datetime(2008, 01, 01)
symbol_dict = {
    'TOT': 'Total',
    'XOM': 'Exxon',
    'CVX': 'Chevron',
    'COP': 'ConocoPhillips',
    'VLO': 'Valero Energy',
    'MSFT': 'Microsoft',
    'IBM': 'IBM',
    'TWX': 'Time Warner',
    'CMCSA': 'Comcast',
    'CVC': 'Cablevision',
    'YHOO': 'Yahoo',
    'DELL': 'Dell',
    'HPQ': 'HP',
    'AMZN': 'Amazon',
    'TM': 'Toyota',
    'CAJ': 'Canon',
    'MTU': 'Mitsubishi',
    'SNE': 'Sony',
    'F': 'Ford',
    'HMC': 'Honda',
    'NAV': 'Navistar',
    'NOC': 'Northrop Grumman',
    'BA': 'Boeing',
    'KO': 'Coca Cola',
    'MMM': '3M',
    'MCD': 'Mc Donalds',
    'PEP': 'Pepsi',
    'KFT': 'Kraft Foods',
    'K': 'Kellogg',
    'UN': 'Unilever',
    'MAR': 'Marriott',
    'PG': 'Procter Gamble',
    'CL': 'Colgate-Palmolive',
    'NWS': 'News Corp',
}
symbols, names = np.array(symbol_dict.items()).T

quotes = [finance.quotes_historical_yahoo(symbol, d1, d2, asobject=True)
    for symbol in symbols]

open = np.array([q.open for q in quotes]).astype(np.float)
close = np.array([q.close for q in quotes]).astype(np.float)

# The daily variations of the quotes are what carry most information
variation = close - open

# Learn a graphical structure from the correlations
edge_model = covariance.GraphLassoCV()

# standardize the time series: using correlations rather than covariance
# is more efficient for structure recovery
X = variation.copy().T
X /= X.std(axis=0)
edge_model.fit(X)

# Cluster using affinity propagation
_, labels = cluster.affinity_propagation(edge_model.covariance_)
n_labels = labels.max()

for i in range(n_labels + 1):
    print 'Cluster %i: %s' % ((i + 1), ', '.join(names[labels == i]))
Find a low-dimension embedding for visualization: find the best position of
the nodes (the stocks) on a 2D plane

We use a dense eigen_solver to achieve reproducibility (arpack is
initiated with random vectors that we don’t control). In addition, we
use a large number of neighbors to capture the large-scale structure.

```python
node_position_model = manifold.LocallyLinearEmbedding(
    n_components=2, eigen_solver='dense', n_neighbors=6)
embedding = node_position_model.fit_transform(X.T).T
```

Visualization

```python
pl.figure(1, facecolor='w', figsize=(10, 8))
pl.clf()
ax = pl.axes([0., 0., 1., 1.])
pl.axis('off')
# Display a graph of the partial correlations
partial_correlations = edge_model.precision_.copy()
d = 1 / np.sqrt(np.diag(partial_correlations))
partial_correlations *= d
partial_correlations *= d[:, np.newaxis]
non_zero = (np.abs(np.triu(partial_correlations, k=1)) > 0.02)
# Plot the nodes using the coordinates of our embedding
pl.scatter(embedding[0], embedding[1], s=100 * d ** 2, c=labels,
cmap=pl.cm.spectral)
# Plot the edges
start_idx, end_idx = np.where(non_zero)
segments = [[embedding[:, start], embedding[:, stop]]
    for start, stop in zip(start_idx, end_idx)]
values = np.abs(partial_correlations[non_zero])
lc = LineCollection(segments,
zorder=0, cmap=pl.cm.hot_r,
norm=pl.Normalize(0, .7 * values.max()))
lc.set_array(values)
lc.set_linewidths(15 * values)
ax.add_collection(lc)
# Add a label to each node. The challenge here is that we want to
# position the labels to avoid overlap with other labels
for index, (name, label, (x, y)) in enumerate(zip(names, labels, embedding.T)):
    dx = x - embedding[0]
dx[index] = 1
dy = y - embedding[1]
    dy[index] = 1
this_dx = dx[np.argmin(np.abs(dy))]
this_dy = dy[np.argmin(np.abs(dx))]
    if this_dx > 0:
        horizontalalignment = 'left'
x = x + .002
```

730 Chapter 2. Example Gallery
else:
    horizontalalignment = 'right'
    x = x - .002
if this_dy > 0:
    verticalalignment = 'bottom'
y = y + .002
else:
    verticalalignment = 'top'
y = y - .002
pl.text(x, y, name, size=10,
horizontalalignment=horizontalalignment,
verticalalignment=verticalalignment,
bbox=dict(facecolor='w',
edgecolor=pl.cm.spectral(label / float(n_labels)),
       alpha=.6))

pl.xlim(embedding[0].min() - .15 * embedding[0].ptp(),
        embedding[0].max() + .10 * embedding[0].ptp(),)
pl.ylim(embedding[1].min() - .03 * embedding[1].ptp(),
        embedding[1].max() + .03 * embedding[1].ptp())

pl.show()

Figure 2.29: Compressive sensing: tomography reconstruction with L1 prior (Lasso)

Compressive sensing: tomography reconstruction with L1 prior (Lasso)

This example shows the reconstruction of an image from a set of parallel projections, acquired along different angles. Such a dataset is acquired in computed tomography (CT).

Without any prior information on the sample, the number of projections required to reconstruct the image is of the order of the linear size $l$ of the image (in pixels). For simplicity we consider here a sparse image, where only pixels on the boundary of objects have a non-zero value. Such data could correspond for example to a cellular material. Note however that most images are sparse in a different basis, such as the Haar wavelets. Only $l/7$ projections are acquired, therefore it is necessary to use prior information available on the sample (its sparsity): this is an example of compressive sensing.

The tomography projection operation is a linear transformation. In addition to the data-fidelity term corresponding to a linear regression, we penalize the L1 norm of the image to account for its sparsity. The resulting optimization problem is called the Lasso. We use the class `sklearn.linear_model.sparse.Lasso`, that uses the coordinate descent algorithm. Importantly, this implementation is more computationally efficient on a sparse matrix, as the projection operator used here.

The reconstruction with L1 penalization gives a result with zero error (all pixels are successfully labeled with 0 or 1), even if noise was added to the projections. In comparison, an L2 penalization (`sklearn.linear_model.Ridge`) produces a large number of labeling errors for the pixels. Important artifacts are observed on the reconstructed image, contrary to the L1 penalization. Note in particular the circular artifact separating the pixels in the corners, that have contributed to fewer projections than the central disk.
Python source code: plot_tomography_l1_reconstruction.py

```python
print __doc__

# Author: Emmanuelle Gouillart <emmanuelle.gouillart@nsup.org>
# License: Simplified BSD

import numpy as np
from scipy import sparse
from scipy import ndimage
from sklearn.linear_model.sparse import Lasso
from sklearn.linear_model import Ridge
import matplotlib.pyplot as plt

def _weights(x, dx=1, orig=0):
    x = np.ravel(x)
    floor_x = np.floor((x - orig) / dx)
    alpha = (x - orig - floor_x * dx) / dx
    return np.hstack((floor_x, floor_x + 1)), np.hstack((1 - alpha, alpha))

def _generate_center_coordinates(l_x):
    l_x = float(l_x)
    X, Y = np.mgrid[:l_x, :l_x]
    center = l_x / 2.
    X += 0.5 - center
    Y += 0.5 - center
    return X, Y

def build_projection_operator(l_x, n_dir):
    """Compute the tomography design matrix."

    Parameters
    ----------

    l_x : int
        linear size of image array
```
n_dir : int
    number of angles at which projections are acquired.

Returns
-----

p : sparse matrix of shape (n_dir * x, l_x**2)

    X, Y = _generate_center_coordinates(l_x)
    angles = np.linspace(0, np.pi, n_dir, endpoint=False)
    data_inds, weights, camera_inds = [], [], []
    data_unravel_indices = np.arange(l_x ** 2)
    data_unravel_indices = np.hstack((data_unravel_indices,
                                       data_unravel_indices))
    for i, angle in enumerate(angles):
        Xrot = np.cos(angle) * X - np.sin(angle) * Y
        inds, w = _weights(Xrot, dx=1, orig=X.min())
        mask = np.logical_and(inds >= 0, inds < l_x)
        weights += list(w[mask])
        camera_inds += list(inds[mask] + i * l_x)
        data_inds += list(data_unravel_indices[mask])
    proj_operator = sparse.coo_matrix((weights, (camera_inds, data_inds)))
    return proj_operator

def generate_synthetic_data():
    """ Synthetic binary data ""
    rs = np.random.RandomState(0)
    n_pts = 36.
    x, y = np.ogrid[0:l, 0:l]
    mask_outer = (x - l / 2) ** 2 + (y - l / 2) ** 2 < (l / 2) ** 2
    mask = np.zeros((l, l))
    points = l * rs.rand(2, n_pts)
    mask[(points[0]).astype(np.int), (points[1]).astype(np.int)] = 1
    mask = ndimage.gaussian_filter(mask, sigma=l / n_pts)
    res = np.logical_and(mask > mask.mean(), mask_outer)
    return res - ndimage.binary_erosion(res)

# Generate synthetic images, and projections
l = 128
proj_operator = build_projection_operator(l, l / 7.)
data = generate_synthetic_data()
proj = proj_operator * data.ravel()[:, np.newaxis]
proj += 0.15 * np.random.randn(*proj.shape)

# Reconstruction with L2 (Ridge) penalization
rgr_ridge = Ridge(alpha=0.2)
rgr_ridge.fit(proj_operator, proj.ravel())
rec_l2 = rgr_ridge.coef_.reshape(l, l)

# Reconstruction with L1 (Lasso) penalization
# the best value of alpha was determined using cross validation
# with LassoCV
rgr_lasso = Lasso(alpha=0.001)
rgr_lasso.fit(proj_operator, proj.ravel())
rec_l1 = rgr_lasso.coef_.reshape(l, l)

plt.figure(figsize=(8, 3.3))

2.1. Examples
Figure 2.30: Faces recognition example using eigenfaces and SVMs

Faces recognition example using eigenfaces and SVMs

The dataset used in this example is a preprocessed excerpt of the “Labeled Faces in the Wild”, aka LFW:


Expected results for the top 5 most represented people in the dataset:

<table>
<thead>
<tr>
<th>Name</th>
<th>Precision</th>
<th>Recall</th>
<th>F1-score</th>
<th>Support</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gerhard_Schroeder</td>
<td>0.91</td>
<td>0.75</td>
<td>0.82</td>
<td>28</td>
</tr>
<tr>
<td>Donald_Rumsfeld</td>
<td>0.84</td>
<td>0.82</td>
<td>0.83</td>
<td>33</td>
</tr>
<tr>
<td>Tony_Blair</td>
<td>0.65</td>
<td>0.82</td>
<td>0.73</td>
<td>34</td>
</tr>
<tr>
<td>Colin_Powell</td>
<td>0.78</td>
<td>0.88</td>
<td>0.83</td>
<td>58</td>
</tr>
<tr>
<td>George_W_Bush</td>
<td>0.93</td>
<td>0.86</td>
<td>0.90</td>
<td>129</td>
</tr>
</tbody>
</table>

avg / total            | 0.86      | 0.84   | 0.85     | 282     |

Python source code: face_recognition.py

```python
print __doc__

from time import time
import logging
import matplotlib.pyplot as plt
from sklearn.cross_validation import train_test_split
```
scikit-learn user guide, Release 0.12-git

from
from
from
from
from
from

sklearn.datasets import fetch_lfw_people
sklearn.grid_search import GridSearchCV
sklearn.metrics import classification_report
sklearn.metrics import confusion_matrix
sklearn.decomposition import RandomizedPCA
sklearn.svm import SVC

# Display progress logs on stdout
logging.basicConfig(level=logging.INFO, format=’%(asctime)s %(message)s’)

###############################################################################
# Download the data, if not already on disk and load it as numpy arrays
lfw_people = fetch_lfw_people(min_faces_per_person=70, resize=0.4)
# introspect the images arrays to find the shapes (for plotting)
n_samples, h, w = lfw_people.images.shape
# fot machine learning we use the 2 data directly (as relative pixel
# positions info is ignored by this model)
X = lfw_people.data
n_features = X.shape[1]
# the label to predict is the id of the person
y = lfw_people.target
target_names = lfw_people.target_names
n_classes = target_names.shape[0]
print
print
print
print

"Total dataset size:"
"n_samples: %d" % n_samples
"n_features: %d" % n_features
"n_classes: %d" % n_classes

###############################################################################
# Split into a training set and a test set using a stratified k fold
# split into a training and testing set
X_train, X_test, y_train, y_test = train_test_split(
X, y, test_fraction=0.25)

###############################################################################
# Compute a PCA (eigenfaces) on the face dataset (treated as unlabeled
# dataset): unsupervised feature extraction / dimensionality reduction
n_components = 150
print "Extracting the top %d eigenfaces from %d faces" % (
n_components, X_train.shape[0])
t0 = time()
pca = RandomizedPCA(n_components=n_components, whiten=True).fit(X_train)
print "done in %0.3fs" % (time() - t0)
eigenfaces = pca.components_.reshape((n_components, h, w))
print "Projecting the input data on the eigenfaces orthonormal basis"
t0 = time()

2.1. Examples

735


X_train_pca = pca.transform(X_train)
X_test_pca = pca.transform(X_test)
print "done in %0.3fs" % (time() - t0)

# Train a SVM classification model
print "Fitting the classifier to the training set"
t0 = time()
param_grid = {
    'C': [1e3, 5e3, 1e4, 5e4, 1e5],
    'gamma': [0.0001, 0.0005, 0.001, 0.005, 0.01, 0.1],
}
clf = GridSearchCV(SVC(kernel='rbf', class_weight='auto'), param_grid)
clf = clf.fit(X_train_pca, y_train)
print "done in %0.3fs" % (time() - t0)
print "Best estimator found by grid search:"
print clf.best_estimator_

# Quantitative evaluation of the model quality on the test set
print "Predicting the people names on the testing set"
t0 = time()
y_pred = clf.predict(X_test_pca)
print "done in %0.3fs" % (time() - t0)
print classification_report(y_test, y_pred, target_names=target_names)
print confusion_matrix(y_test, y_pred, labels=range(n_classes))

# Qualitative evaluation of the predictions using matplotlib

def plot_gallery(images, titles, h, w, n_row=3, n_col=4):
    """Helper function to plot a gallery of portraits""
    pl.figure(figsize=(1.8 * n_col, 2.4 * n_row))
    pl.subplots_adjust(bottom=0, left=.01, right=.99, top=.90, hspace=.35)
    for i in range(n_row * n_col):
        pl.subplot(n_row, n_col, i + 1)
        pl.imshow(images[i].reshape((h, w)), cmap=pl.cm.gray)
        pl.title(titles[i], size=12)
        pl.xticks(())
        pl.yticks(())

# plot the result of the prediction on a portion of the test set

def title(y_pred, y_test, target_names, i):
    pred_name = target_names[y_pred[i]].rsplit(' ', 1)[-1]
    true_name = target_names[y_test[i]].rsplit(' ', 1)[-1]
    return 'predicted: %s
true: %s' % (pred_name, true_name)
prediction_titles = [title(y_pred, y_test, target_names, i)
    for i in range(y_pred.shape[0])]

736 Chapter 2. Example Gallery
plot_gallery(X_test, prediction_titles, h, w)

# plot the gallery of the most significative eigenfaces

eigenface_titles = ["eigenface %d" % i for i in range(eigenfaces.shape[0])]
plot_gallery(eigenfaces, eigenface_titles, h, w)

pl.show()

Figure 2.31: Libsvm GUI

Libsvm GUI

A simple graphical frontend for Libsvm mainly intended for didactic purposes. You can create data points by point and click and visualize the decision region induced by different kernels and parameter settings.

To create positive examples click the left mouse button; to create negative examples click the right button.

If all examples are from the same class, it uses a one-class SVM.

Python source code: svm_gui.py

from __future__ import division

print __doc__

# Author: Peter Prettenhofer <peter.prettenhofer@gmail.com>
#
# License: BSD Style.

import matplotlib
matplotlib.use('TkAgg')

from matplotlib.backends.backend_tkagg import FigureCanvasTkAgg,
from matplotlib.backends.backend_tkagg import NavigationToolbar2TkAgg
from matplotlib.figure import Figure
from matplotlib.contour import ContourSet

import Tkinter as Tk
import sys
import numpy as np

from sklearn import svm
from sklearn.datasets import dump_svmlight_file

y_min, y_max = -50, 50
x_min, x_max = -50, 50

2.1. Examples
class Model(object):
    """The Model which hold the data. It implements the observable in the observer pattern and notifies the registered observers on change event. """
    def __init__(self):
        self.observers = []
        self.surface = None
        self.data = []
        self.cls = None
        self.surface_type = 0
    def changed(self, event):
        """Notify the observers."""
        for observer in self.observers:
            observer.update(event, self)
    def add_observer(self, observer):
        """Register an observer."""
        self.observers.append(observer)
    def set_surface(self, surface):
        self.surface = surface
    def dump_svmlight_file(self, file):
        data = np.array(self.data)
        X = data[:, 0:2]
        y = data[:, 2]
        dump_svmlight_file(X, y, file)

class Controller(object):
    def __init__(self, model):
        self.model = model
        self.kernel = Tk.IntVar()
        self.surface_type = Tk.IntVar()
        # Whether or not a model has been fitted
        self.fitted = False
    def fit(self):
        print "fit the model"
        train = np.array(self.model.data)
        X = train[:, 0:2]
        y = train[:, 2]
        C = float(self.complexity.get())
        gamma = float(self.gamma.get())
        coef0 = float(self.coef0.get())
        degree = int(self.degree.get())
        kernel_map = {0: "linear", 1: "rbf", 2: "poly"}
        if len(np.unique(y)) == 1:
            clf = svm.OneClassSVM(kernel=kernel_map[self.kernel.get()],
                                  gamma=gamma, coef0=coef0, degree=degree)
            clf.fit(X)
        else:
            clf = svm.SVC(kernel=kernel_map[self.kernel.get()], C=C,
gamma=gamma, coef0=coef0, degree=degree)
clf.fit(X, y)
if hasattr(clf, 'score'):
    print "Accuracy:", clf.score(X, y) * 100
X1, X2, Z = self.decision_surface(clf)
self.model.clf = clf
self.model.set_surface((X1, X2, Z))
self.model.surface_type = self.surface_type.get()
self.fitted = True
self.model.changed("surface")

def decision_surface(self, cls):
    delta = 1
    x = np.arange(x_min, x_max + delta, delta)
    y = np.arange(y_min, y_max + delta, delta)
    X1, X2 = np.meshgrid(x, y)
    Z = cls.decision_function(np.c_[X1.ravel(), X2.ravel()])
    Z = Z.reshape(X1.shape)
    return X1, X2, Z

def clear_data(self):
    self.model.data = []
    self.fitted = False
    self.model.changed("clear")

def add_example(self, x, y, label):
    self.model.data.append((x, y, label))
    self.model.changed("example_added")

    # update decision surface if already fitted.
    self.refit()

def refit(self):
    """Refit the model if already fitted. ""
    if self.fitted:
        self.fit()

class View(object):
    """Test docstring. ""
    def __init__(self, root, controller):
        f = Figure()
        ax = f.add_subplot(111)
        ax.set_xticks([])
        ax.set_yticks([])
        ax.set_xlim((x_min, x_max))
        ax.set_ylim((y_min, y_max))
        canvas = FigureCanvasTkAgg(f, master=root)
        canvas.show()
        canvas.get_tk_widget().pack(side=Tk.TOP, fill=Tk.BOTH, expand=1)
        toolbar = NavigationToolbar2TkAgg(canvas, root)
        toolbar.update()
        self.controllbar = ControllBar(root, controller)
        self.f = f
        self.ax = ax
        self.canvas = canvas

2.1. Examples
```python
self.controller = controller  
self.contours = []  
self.c_labels = None  
self.plot_kernels()

def plot_kernels(self):
    self.ax.text(-50, -60, r"Linear: $u^T v$"")
    self.ax.text(-20, -60, r"RBF: $\exp (-\gamma \| u-v \|^2)$")
    self.ax.text(10, -60, r"Poly: $(\gamma \, u^T v + r)^d$" )

def onclick(self, event):
    if event.xdata and event.ydata:
        if event.button == 1:
            self.controller.add_example(event.xdata, event.ydata, 1)
        elif event.button == 3:
            self.controller.add_example(event.xdata, event.ydata, -1)

def update_example(self, model, idx):
    x, y, l = model.data[idx]
    if l == 1:
        color = 'w'
    elif l == -1:
        color = 'k'
    self.ax.plot([x], [y], '%so' % color, scalex=0.0, scaley=0.0)

def update(self, event, model):
    if event == "examples_loaded":  
        for i in xrange(len(model.data)):
            self.update_example(model, i)

    if event == "example_added":  
        self.update_example(model, -1)

    if event == "clear":  
        self.ax.clear()  
        self.ax.set_xticks([])  
        self.ax.set_yticks([])  
        self.contours = []  
        self.c_labels = None  
        self.plot_kernels()

    if event == "surface":  
        self.remove_surface()  
        self.plot_support_vectors(model.clf.support_vectors_)  
        self.plot_decision_surface(model.surface, model.surface_type)

    self.canvas.draw()  

def remove_surface(self):
    """Remove old decision surface."""
    if len(self.contours) > 0:  
        for contour in self.contours:
            if isinstance(contour, ContourSet):
                for lineset in contour.collections:
                    lineset.remove()
            else:
                contour.remove()
        self.contours = []
```

Chapter 2. Example Gallery
def plot_support_vectors(self, support_vectors):
    """Plot the support vectors by placing circles over the corresponding data points and adds the circle collection to the contours list."""
    cs = self.ax.scatter(support_vectors[:, 0], support_vectors[:, 1], s=80, edgecolors="k", facecolors="none")
    self.contours.append(cs)

def plot_decision_surface(self, surface, type):
    X1, X2, Z = surface
    if type == 0:
        levels = [-1.0, 0.0, 1.0]
        linestyles = ["dashed", "solid", "dashed"]
        colors = "k"
        self.contours.append(self.ax.contour(X1, X2, Z, levels, colors=colors, linestyles=linestyles))
    elif type == 1:
        self.contours.append(self.ax.contourf(X1, X2, Z, 10, cmap=matplotlib.cm.bone, origin="lower", alpha=0.85))
        self.contours.append(self.ax.contour(X1, X2, Z, 
            [0.0], colors="k", linestyles=["solid"]))
    else:
        raise ValueError("surface type unknown")

class ControllBar(object):
    def __init__(self, root, controller):
        fm = Tk.Frame(root)
        kernel_group = Tk.Frame(fm)
        Tk.Radiobutton(kernel_group, text="Linear", variable=controller.kernel, value=0, command=controller.refit).pack(anchor=Tk.W)
        Tk.Radiobutton(kernel_group, text="RBF", variable=controller.kernel, value=1, command=controller.refit).pack(anchor=Tk.W)
        Tk.Radiobutton(kernel_group, text="Poly", variable=controller.kernel, value=2, command=controller.refit).pack(anchor=Tk.W)
        kernel_group.pack(side=Tk.LEFT)

        valbox = Tk.Frame(fm)
        controller.complexity = Tk.StringVar()
        controller.complexity.set("1.0")
        c = Tk.Frame(valbox)
        Tk.Label(c, text="C:", anchor="e", width=7).pack(side=Tk.LEFT)
        Tk.Entry(c, width=6, textvariable=controller.complexity).pack(side=Tk.LEFT)
        c.pack()

        controller.gamma = Tk.StringVar()
        controller.gamma.set("0.01")
        g = Tk.Frame(valbox)
        Tk.Label(g, text="gamma:", anchor="e", width=7).pack(side=Tk.LEFT)
        Tk.Entry(g, width=6, textvariable=controller.gamma).pack(side=Tk.LEFT)
        g.pack()

        controller.degree = Tk.StringVar()
controller.degree.set("3")
d = Tk.Frame(valbox)
Tk.Label(d, text="degree:", anchor="e", width=7).pack(side=Tk.LEFT)
Tk.Entry(d, width=6, textvariable=controller.degree).pack(side=Tk.LEFT)
d.pack()

controller.coef0 = Tk.StringVar()
controller.coef0.set("0")
r = Tk.Frame(valbox)
Tk.Label(r, text="coef0:", anchor="e", width=7).pack(side=Tk.LEFT)
Tk.Entry(r, width=6, textvariable=controller.coef0).pack(side=Tk.LEFT)
r.pack()
valbox.pack(side=Tk.LEFT)

cmap_group = Tk.Frame(fm)
Tk.Radiobutton(cmap_group, text="Hyperplanes",
variable=controller.surface_type, value=0,
command=controller.refit).pack(anchor=Tk.W)
Tk.Radiobutton(cmap_group, text="Surface",
variable=controller.surface_type, value=1,
command=controller.refit).pack(anchor=Tk.W)
cmap_group.pack(side=Tk.LEFT)

train_button = Tk.Button(fm, text='Fit', width=5,
command=controller.fit)
train_button.pack()
fm.pack(side=Tk.LEFT)
Tk.Button(fm, text='Clear', width=5,
command=controller.clear_data).pack(side=Tk.LEFT)

def get_parser():
    from optparse import OptionParser
    op = OptionParser()
    op.add_option("--output",
        action="store", type="str", dest="output",
        help="Path where to dump data.")
    return op

def main(argv):
    op = get_parser()
    opts, args = op.parse_args(argv[1:])
    root = Tk.Tk()
    model = Model()
    controller = Controller(model)
    root.wm_title("Scikit-learn Libsvm GUI")
    view = View(root, controller)
    model.add_observer(view)
    Tk.mainloop()
    if opts.output:
        model.dump_svmlight_file(opts.output)
if __name__ == "__main__":
    main(sys.argv)
Topics extraction with Non-Negative Matrix Factorization

This is a proof of concept application of Non Negative Matrix Factorization of the term frequency matrix of a corpus of documents so as to extract an additive model of the topic structure of the corpus.

The default parameters (n_samples / n_features / n_topics) should make the example runnable in a couple of tens of seconds. You can try to increase the dimensions of the problem be ware than the time complexity is polynomial.

Here are some sample extracted topics that look quite good:

Topic #0: god people bible israel jesus christian true moral think christians believe don say human israeli church life children jewish

Topic #1: drive windows card drivers video scsi software pc thanks vga graphics help disk uni dos file ide controller work

Topic #2: game team nhl games ca hockey players buffal oedu cc year play university teams baseball columbia league player toronto

Topic #3: window manager application mit motif size display widget program xlib windows user color event information use events x11r5 values

Topic #4: pitt gordon banks cs science pittsburgh univ computer soon disease edu reply pain health david article medical medicine

**Python source code:** topics_extraction_with_nmf.py

```python
# Author: Olivier Grisel <olivier.grisel@ensta.org>
# License: Simplified BSD

from time import time
from sklearn.feature_extraction import text
from sklearn import decomposition
from sklearn import datasets

n_samples = 1000
n_features = 1000
n_topics = 10
n_top_words = 20

t0 = time()

print "Loading dataset and extracting TF-IDF features..."
dataset = datasets.fetch_20newsgroups(shuffle=True, random_state=1)

vectorizer = text.CountVectorizer(max_df=0.95, max_features=n_features)
```

2.1. Examples
counts = vectorizer.fit_transform(dataset.data[:n_samples])
tfidf = text.TfidfTransformer().fit_transform(counts)
print "done in %0.3fs." % (time() - t0)

# Fit the NMF model
print "Fitting the NMF model on with n_samples=%d and n_features=%d..." % (n_samples, n_features)
nmf = decomposition.NMF(n_components=n_topics).fit(tfidf)
print "done in %0.3fs." % (time() - t0)

# Inverse the vectorizer vocabulary to be able
feature_names = vectorizer.get_feature_names()

for topic_idx, topic in enumerate(nmf.components_):
    print "Topic #%d:" % topic_idx
    print " ".join([feature_names[i]
      for i in topic.argsort()[::-n_top_words - 1:-1]])

print

Figure 2.33: Wikipedia principal eigenvector

Wikipedia principal eigenvector

A classical way to assert the relative importance of vertices in a graph is to compute the principal eigenvector of the adjacency matrix so as to assign to each vertex the values of the components of the first eigenvector as a centrality score:


On the graph of webpages and links those values are called the PageRank scores by Google.
The goal of this example is to analyze the graph of links inside wikipedia articles to rank articles by relative importance according to this eigenvector centrality.
The traditional way to compute the principal eigenvector is to use the power iteration method:

http://en.wikipedia.org/wiki/Power_iteration

Here the computation is achieved thanks to Martinsson’s Randomized SVD algorithm implemented in the scikit.
The graph data is fetched from the DBpedia dumps. DBpedia is an extraction of the latent structured data of the Wikipedia content.

Python source code: wikipedia_principal_eigenvector.py

print __doc__

# Author: Olivier Grisel <olivier.grisel@ensta.org>
# License: Simplified BSD
from bz2 import BZ2File
import os
from datetime import datetime
from pprint import pprint
from time import time
import numpy as np
from scipy import sparse
from sklearn.utils.extmath import randomized_svd
from sklearn.externals.joblib import Memory

#############################################################################
# Where to download the data, if not already on disk
redirects_url = "http://downloads.dbpedia.org/3.5.1/en/redirects_en.nt.bz2"
redirects_filename = redirects_url.rsplit("/", 1)[1]

page_links_url = "http://downloads.dbpedia.org/3.5.1/en/page_links_en.nt.bz2"
page_links_filename = page_links_url.rsplit("/", 1)[1]

resources = [
    (redirects_url, redirects_filename),
    (page_links_url, page_links_filename),
]

for url, filename in resources:
    if not os.path.exists(filename):
        import urllib
        print "Downloading data from '%s', please wait..." % url
        opener = urllib.urlopen(url)
        open(filename, 'wb').write(opener.read())
        print

#############################################################################
# Loading the redirect files
memory = Memory(cachedir=".")

def index(redirects, index_map, k):
    """Find the index of an article name after redirect resolution"""
    k = redirects.get(k, k)
    return index_map.setdefault(k, len(index_map))

DBPEDIA_RESOURCE_PREFIX_LEN = len("http://dbpedia.org/resource/")
SHORTNAME_SLICE = slice(DBPEDIARESOURCE_PREFIX_LEN + 1, -1)

def short_name(nt_uri):
    """Remove the < and > URI markers and the common URI prefix"""
    return nt_uri[SHORTNAME_SLICE]

def get_redirects(redirects_filename):
Parse the redirections and build a transitively closed map out of it

```python
redirects = {}
print "Parsing the NT redirect file"
for l, line in enumerate(BZ2File(redirects_filename)):
    split = line.split()
    if len(split) != 4:
        print "ignoring malformed line: " + line
        continue
    redirects[short_name(split[0])] = short_name(split[2])
    if l % 1000000 == 0:
        print "[%s] line: %08d" % (datetime.now().isoformat(), l)

# compute the transitive closure
print "Computing the transitive closure of the redirect relation"
for l, source in enumerate(redirects.keys()):
    transitive_target = None
    target = redirects[source]
    seen = set({source})
    while True:
        transitive_target = target
        target = redirects.get(target)
        if target is None or target in seen:
            break
        seen.add(target)
    redirects[source] = transitive_target
    if l % 1000000 == 0:
        print "[%s] line: %08d" % (datetime.now().isoformat(), l)
return redirects
```

# disabling joblib as the pickling of large dicts seems much too slow
# @memory.cache
def get_adjacency_matrix(redirects_filename, page_links_filename, limit=None):
    
    print "Computing the redirect map"
    redirects = get_redirects(redirects_filename)

    print "Computing the integer index map"
    index_map = dict()
    links = list()
    for l, line in enumerate(BZ2File(page_links_filename)):
        split = line.split()
        if len(split) != 4:
            print "ignoring malformed line: " + line
            continue
        i = index(redirects, index_map, short_name(split[0]))
        j = index(redirects, index_map, short_name(split[2]))
        links.append((i, j))
        if l % 1000000 == 0:
```
print "[$s] line: $08d" % (datetime.now().isoformat(), l)

if limit is not None and l >= limit - 1:
    break

print "Computing the adjacency matrix"
X = sparse.lil_matrix((len(index_map), len(index_map)), dtype=np.float32)
for i, j in links:
    X[i, j] = 1.0
del links
print "Converting to CSR representation"
X = X.tocsr()
print "CSR conversion done"
return X, redirects, index_map

# stop after 5M links to make it possible to work in RAM
X, redirects, index_map = get_adjacency_matrix(redirects_filename, page_links_filename, limit=5000000)
names = dict((i, name) for name, i in index_map.iteritems())

print "Computing the principal singular vectors using randomized_svd"
t0 = time()
U, s, V = randomized_svd(X, 5, n_iterations=3)
print "done in $0.3fs" % (time() - t0)

# print the names of the wikipedia related strongest components of the the # principal singular vector which should be similar to the highest eigenvector
print "Top wikipedia pages according to principal singular vectors"
pprint([names[i] for i in np.abs(U.T[0]).argsort()[-10:]]
pprint([names[i] for i in np.abs(V[0]).argsort()[-10:]]

def centrality_scores(X, alpha=0.85, max_iter=100, tol=1e-10):
    """Power iteration computation of the principal eigenvector
    This method is also known as Google PageRank and the implementation
    is based on the one from the NetworkX project (BSD licensed too)
    with copyrights by:
    Aric Hagberg <hagberg@lanl.gov>
    Dan Schult <dschult@colgate.edu>
    Pieter Swart <swart@lanl.gov>
    """
    n = X.shape[0]
    X = X.copy()
    incoming_counts = np.asarray(X.sum(axis=1)).ravel()

    print "Normalizing the graph"
    for i in incoming_counts.nonzero()[0]:
        X.data[X.indptr[i]:X.indptr[i + 1]] *= 1.0 / incoming_counts[i]
    dangle = np.asarray(np.where(X.sum(axis=1) == 0, 1.0 / n, 0)).ravel()

    scores = np.ones(n, dtype=np.float32) / n
    # initial guess
    for i in range(max_iter):
        print "power iteration #%d" % i
        prev_scores = scores
        scores = (alpha * (scores * X + np.dot(dangle, prev_scores)))

    return scores

2.1. Examples  747
print "Computing principal eigenvector score using a power iteration method"

t0 = time()
scores = centrality_scores(X, max_iter=100, tol=1e-10)

print "done in %0.3f s" % (time() - t0)
pprint([names[i] for i in np.abs(scores).argsort()[-10:]])

2.1.3 Clustering

Examples concerning the sklearn.cluster package.

Figure 2.34: Adjustment for chance in clustering performance evaluation

Adjustment for chance in clustering performance evaluation

The following plots demonstrate the impact of the number of clusters and number of samples on various clustering performance evaluation metrics.

Non-adjusted measures such as the V-Measure show a dependency between the number of clusters and the number of samples: the mean V-Measure of random labeling increases significantly as the number of clusters is closer to the total number of samples used to compute the measure.

Adjusted for chance measure such as ARI display some random variations centered around a mean score of 0.0 for any number of samples and clusters.

Only adjusted measures can hence safely be used as a consensus index to evaluate the average stability of clustering algorithms for a given value of k on various overlapping sub-samples of the dataset.
Script output:

Computing adjusted_rand_score for 10 values of n_clusters and n_samples=100
done in 0.253s
Computing v_measure_score for 10 values of n_clusters and n_samples=100
done in 2.110s
Computing adjusted_mutual_info_score for 10 values of n_clusters and n_samples=100
done in 7.992s
Computing mutual_info_score for 10 values of n_clusters and n_samples=100
done in 0.033s
Computing adjusted_rand_score for 10 values of n_clusters and n_samples=1000
done in 0.432s
Computing v_measure_score for 10 values of n_clusters and n_samples=1000
done in 0.932s
Computing adjusted_mutual_info_score for 10 values of n_clusters and n_samples=1000
done in 21.824s
Computing mutual_info_score for 10 values of n_clusters and n_samples=1000
done in 0.155s

Python source code: plot_adjusted_for_chance_measures.py

```python
print __doc__

# Author: Olivier Grisel <olivier.grisel@ensta.org>
# License: Simplified BSD

import numpy as np
import pylab as pl
from time import time
```

2.1. Examples
from sklearn import metrics

def uniform_labelings_scores(score_func, n_samples, n_clusters_range,
        fixed_n_classes=None, n_runs=5, seed=42):
    """Compute score for 2 random uniform cluster labelings.
    Both random labelings have the same number of clusters for each value
    possible value in ``n_clusters_range``.
    When fixed_n_classes is not None the first labeling is considered a ground
    truth class assignment with fixed number of classes.
    """
    random_labels = np.random.RandomState(seed).random_integers
    scores = np.zeros((len(n_clusters_range), n_runs))
    if fixed_n_classes is not None:
        labels_a = random_labels(low=0, high=fixed_n_classes - 1,
            size=n_samples)
        for i, k in enumerate(n_clusters_range):
            for j in range(n_runs):
                if fixed_n_classes is None:
                    labels_a = random_labels(low=0, high=k - 1, size=n_samples)
                labels_b = random_labels(low=0, high=k - 1, size=n_samples)
                scores[i, j] = score_func(labels_a, labels_b)
    return scores

score_funcs = [
    metrics.adjusted_rand_score,
    metrics.v_measure_score,
    metrics.adjusted_mutual_info_score,
    metrics.mutual_info_score,
]

# 2 independent random clusterings with equal cluster number
n_samples = 100
n_clusters_range = np.linspace(2, n_samples, 10).astype(np.int)

fig = plt.figure()
plots = []
names = []
for score_func in score_funcs:
    print "Computing %s for %d values of n_clusters and n_samples=%d" % (
        score_func.__name__, len(n_clusters_range), n_samples)
    t0 = time()
    scores = uniform_labelings_scores(score_func, n_samples, n_clusters_range)
    print "done in %.3fs" % (time() - t0)
    plots.append(pl.errorbar(n_clusters_range, np.median(scores, axis=1), scores.std(axis=1)))[0])
    names.append(score_func.__name__)

fig.title("Clustering measures for 2 random uniform labelings\n" "with equal number of clusters")
fig.xlabel('Number of clusters (Number of samples is fixed to %d)' % n_samples)
# Random labeling with varying n_clusters against ground class labels
# with fixed number of clusters

n_samples = 1000
n_clusters_range = np.linspace(2, 100, 10).astype(np.int)
n_classes = 10

pl.figure(2)

plots = []
names = []

for score_func in score_funcs:
    print "Computing %s for %d values of n_clusters and n_samples=%d" % (score_func.__name__, len(n_clusters_range), n_samples)
    t0 = time()
    scores = uniform_labelings_scores(score_func, n_samples, n_clusters_range, fixed_n_classes=n_classes)
    print "done in %0.3fs" % (time() - t0)
    plots.append(pl.errorbar(n_clusters_range, scores.mean(axis=1), scores.std(axis=1))[0])
    names.append(score_func.__name__)

pl.title("Clustering measures for random uniform labeling\n" "against reference assignment with %d classes" % n_classes)
pl.xlabel('Number of clusters (Number of samples is fixed to %d)' % n_samples)
pl.ylabel('Score value')
pl.ylim(ymin=-0.05, ymax=1.05)
pl.legend(plots, names)
pl.show()

Figure 2.35: Demo of affinity propagation clustering algorithm

Demo of affinity propagation clustering algorithm

Script output:
Estimated number of clusters: 3
Homogeneity: 0.885
Completeness: 0.885
V-measure: 0.885
Adjusted Rand Index: 0.922
Adjusted Mutual Information: 0.884
Silhouette Coefficient: 0.774

Python source code: plot_affinity_propagation.py
print __doc__

import numpy as np
from sklearn.cluster import AffinityPropagation
from sklearn import metrics
from sklearn.datasets.samples_generator import make_blobs

##############################################################################
# Generate sample data
centers = [[1, 1], [-1, -1], [1, -1]]
X, labels_true = make_blobs(n_samples=300, centers=centers, cluster_std=0.5)

##############################################################################
# Compute similarities
X_norms = np.sum(X ** 2, axis=1)
S = - X_norms[:, np.newaxis] - X_norms[np.newaxis, :] + 2 * np.dot(X, X.T)
p = 10 * np.median(S)

# Compute Affinity Propagation
af = AffinityPropagation().fit(S, p)
cluster_centers_indices = af.cluster_centers_indices_
labels = af.labels_

n_clusters_ = len(cluster_centers_indices)
print('Estimated number of clusters: %d' % n_clusters_)
print('Homogeneity: %0.3f' % metrics.homogeneity_score(labels_true, labels))
print('Completeness: %0.3f' % metrics.completeness_score(labels_true, labels))
print('V-measure: %0.3f' % metrics.v_measure_score(labels_true, labels))
print('Adjusted Rand Index: %0.3f' % metrics.adjusted_rand_score(labels_true, labels))
print('Adjusted Mutual Information: %0.3f' % metrics.adjusted_mutual_info_score(labels_true, labels))
D = (S / np.min(S))
print('Silhouette Coefficient: %0.3f' % metrics.silhouette_score(D, labels, metric='precomputed'))

# Plot result
import pylab as pl
from itertools import cycle
pl.close('all')
pl.figure(1)
pl.clf()

colors = cycle(['bgrcmykbgrcmykbgrcmykbgrcmyk'])
for k, col in zip(range(n_clusters_), colors):
    class_members = labels == k
    cluster_center = X[cluster_centers_indices[k]]
    pl.plot(X[class_members, 0], X[class_members, 1], col + '.
    pl.plot(cluster_center[0], cluster_center[1], 'o', markerfacecolor=col,
            markeredgecolor='k', markersize=14)
    for x in X[class_members]:
        pl.plot([cluster_center[0], x[0]], [cluster_center[1], x[1]], col)

pl.title('Estimated number of clusters: %d' % n_clusters_)
pl.show()

Comparing different clustering algorithms on toy datasets

This example aims at showing characteristics of different clustering algorithms on datasets that are “interesting” but still in 2D. The last dataset is an example of a ‘null’ situation for clustering: the data is homogeneous, and there is no good clustering.

While these examples give some intuition about the algorithms, this intuition might not apply to very high dimensional data.

The results could be improved by tweaking the parameters for each clustering strategy, for instance setting the number of clusters for the methods that needs this parameter specified. Note that affinity propagation has a tendency to create many clusters. Thus in this example its two parameters (damping and per-point preference) were set to to mitigate this
Figure 2.36: Comparing different clustering algorithms on toy datasets

behavior.

Python source code: plot_cluster_comparison.py

```python
print __doc__

import time

import numpy as np
import pylab as pl

from sklearn import cluster, datasets
from sklearn.metrics import euclidean_distances
```
from sklearn.neighbors import kneighbors_graph
from sklearn.preprocessing import Scaler

np.random.seed(0)

# Generate datasets. We choose the size big enough to see the scalability
# of the algorithms, but not too big to avoid too long running times
n_samples = 1500
noisy_circles = datasets.make_circles(n_samples=n_samples, factor=.5,
    noise=.05)
noisy_moons = datasets.make_moons(n_samples=n_samples, noise=.05)
blobs = datasets.make_blobs(n_samples=n_samples, random_state=8)
no_structure = np.random.rand(n_samples, 2), None

colors = np.array([x for x in 'bgrcmykbgrcmykbgrcmykbgrcmyk'])
colors = np.hstack([colors] * 20)

pl.figure(figsize=(14, 9.5))
pl.subplots_adjust(left=.001, right=.999, bottom=.001, top=.96, wspace=.05,
    hspace=.01)

plot_num = 1
for i_dataset, dataset in enumerate([noisy_circles, noisy_moons, blobs,
    no_structure]):
    X, y = dataset
    # normalize dataset for easier parameter selection
    X = Scaler().fit_transform(X)

    # estimate bandwidth for mean shift
    bandwidth = cluster.estimate_bandwidth(X, quantile=0.3)

    # connectivity matrix for structured Ward
    connectivity = kneighbors_graph(X, n_neighbors=10)
    # make connectivity symmetric
    connectivity = 0.5 * (connectivity + connectivity.T)

    # Compute distances
    distances = euclidean_distances(X)

    # create clustering estimators
    ms = cluster.MeanShift(bandwidth=bandwidth, bin_seeding=True)
two_means = cluster.MiniBatchKMeans(n_clusters=2)
ward_five = cluster.Ward(n_clusters=2, connectivity=connectivity)
spectral = cluster.SpectralClustering(n_clusters=2, mode='arpack')
dbscan = cluster.DBSCAN(eps=.2)
affinity_propagation = cluster.AffinityPropagation(damping=.9)

    for algorithm in [two_means, affinity_propagation, ms, spectral,
        ward_five, dbscan]:
        # predict cluster memberships
        t0 = time.time()
        if algorithm == spectral:
            algorithm.fit(connectivity)
        elif algorithm == affinity_propagation:
            # Set a low preference to avoid creating too many
            # clusters. This parameter is hard to set in practice
            algorithm.fit(-distances, p=-50 * distances.max())
        else:
    pl.subplot(2, 5, plot_num + i_dataset)
    if algorithm == spectral:
        algorithm.fit(connectivity)
    elif algorithm == affinity_propagation:
        algorithm.fit(-distances, p=-50 * distances.max())
    else:
        algorithm.fit(X, y)
        labels = algorithm.labels_
        if algorithm == ms:
            colors = colors[labels]
        else:
            colors = colors
        pl.scatter(X[:,0], X[:,1], c=colors, s=10)
        pl.axis('off')
    plot_num += 1

    pl.title(i_dataset[0].title())

pl.show()
algorithm.fit(X)
t1 = time.time()
if hasattr(algorithm, 'labels_ '):
y_pred = algorithm.labels_.astype(np.int)
else:
y_pred = algorithm.predict(X)

# plot
pl.subplot(4, 6, plot_num)
if i_dataset == 0:
    pl.title(str(algorithm).split('(')[0], size=18)
    pl.scatter(X[:, 0], X[:, 1], color=colors[y_pred].tolist(), s=10)
if hasattr(algorithm, 'cluster_centers_ '):
    centers = algorithm.cluster_centers_
    center_colors = colors[:len(centers)]
    pl.scatter(centers[:, 0], centers[:, 1], s=100, c=center_colors)
pl.xlim(-2, 2)
pl.ylim(-2, 2)
pl.xticks(())
pl.yticks(())
pl.text(.99, .01, ('%.2fs' % (t1 - t0)).lstrip('0'),
         transform=pl.gca().transAxes, size=15,
         horizontalalignment='right')
plot_num += 1

pl.show()

Figure 2.37: K-means Clustering

K-means Clustering

The plots display firstly what a K-means algorithm would yield using three clusters. It is then shown what the effect of a bad initialization is on the classification process: By setting n_init to only 1 (default is 10), the amount of times that the algorithm will be run with different centroid seeds is reduced. The next plot displays what using eight clusters would deliver and finally the ground truth.
Python source code: plot_cluster_iris.py

print __doc__

# Code source: Gael Varoquaux
# Modified for Documentation merge by Jaques Grobler
# License: BSD

import numpy as np
import pylab as pl
from mpl_toolkits.mplot3d import Axes3D

from sklearn.cluster import KMeans
from sklearn import datasets

np.random.seed(5)

centers = [[1, 1], [-1, -1], [1, -1]]
iris = datasets.load_iris()
X = iris.data
y = iris.target

estimators = {'k_means_iris_3': KMeans(n_clusters=3),
              'k_means_iris_8': KMeans(n_clusters=8),
              'k_means_iris_bad_init': KMeans(n_clusters=3, n_init=1, init='random'),}
fignum = 1
for name, est in estimators.iteritems():
    fig = pl.figure(fignum, figsize=(4, 3))
    pl.clf()
    ax = Axes3D(fig, rect=[0, 0, .95, 1], elev=48, azim=134)
    pl.cla()
    est.fit(X)
    labels = est.labels_
    ax.scatter(X[:, 3], X[:, 0], X[:, 2], c=labels.astype(np.float))
    ax.w_xaxis.set_ticklabels([])
    ax.w_yaxis.set_ticklabels([])
    ax.w_zaxis.set_ticklabels([])
    ax.set_xlabel('Petal width')
    ax.set_ylabel('Sepal length')
    ax.set_zlabel('Petal length')
    fignum = fignum + 1

# Plot the ground truth
fig = pl.figure(fignum, figsize=(4, 3))
pl.clf()
ax = Axes3D(fig, rect=[0, 0, .95, 1], elev=48, azim=134)
pl.cla()
for name, label in [('Setosa', 0), ('Versicolour', 1), ('Virginica', 2):]
    ax.text3D(X[y == label, 3].mean(),
              X[y == label, 0].mean() + 1.5,
              X[y == label, 2].mean(), name,
              horizontalalignment='center',
              bbox=dict(alpha=.5, edgecolor='w', facecolor='w'),
              )
# Reorder the labels to have colors matching the cluster results
y = np.choose(y, [1, 2, 0]).astype(np.float)
ax.scatter(X[:, 3], X[:, 0], X[:, 2], c=y)

ax.w_xaxis.set_ticklabels([])
ax.w_yaxis.set_ticklabels([])
ax.w_zaxis.set_ticklabels([])
ax.set_xlabel('Petal width')
ax.set_ylabel('Sepal length')
ax.set_zlabel('Petal length')
pl.show()

**Color Quantization using K-Means**

Performs a pixel-wise Vector Quantization (VQ) of an image of the summer palace (China), reducing the number of colors required to show the image from 96,615 unique colors to 64, while preserving the overall appearance quality.

In this example, pixels are represented in a 3D-space and K-means is used to find 64 color clusters. In the image processing literature, the codebook obtained from K-means (the cluster centers) is called the color palette. Using a single byte, up to 256 colors can be addressed, whereas an RGB encoding requires 3 bytes per pixel. The GIF file
Figure 2.38: **Color Quantization using K-Means**

format, for example, uses such a palette. For comparison, a quantized image using a random codebook (colors picked up randomly) is also shown.
Quantized image (64 colors, K-Means)

Script output:

Fitting estimator on a small sub-sample of the data
done in 1.165s.
Predicting color indices on the full image (k-means)
done in 1.024s.
Predicting color indices on the full image (random)
done in 1.131s.

Python source code: plot_color_quantization.py

# Authors: Robert Layton <robertlayton@gmail.com>
# Olivier Grisel <olivier.grisel@ensta.org>
# Mathieu Blondel <mathieu@mblondel.org>
#
# License: BSD

print __doc__
import numpy as np
import matplotlib as pl
from sklearn.cluster import KMeans
from sklearn.metrics import euclidean_distances
from sklearn.datasets import load_sample_image
from sklearn.utils import shuffle
from time import time

n_colors = 64

# Load the Summer Palace photo
china = load_sample_image("china.jpg")

# Convert to floats instead of the default 8 bits integer coding. Dividing by
# 255 is important so that pl.imshow behaves well on float data (need to
# be in the range [0-1])
china = np.array(china, dtype=np.float64) / 255

# Load Image and transform to a 2D numpy array.
w, h, d = original_shape = tuple(china.shape)
assert d == 3
image_array = np.reshape(china, (w * h, d))

print "Fitting estimator on a small sub-sample of the data"
t0 = time()
image_array_sample = shuffle(image_array, random_state=0)[::1000]
kmeans = KMeans(n_clusters=n_colors, random_state=0).fit(image_array_sample)
print "done in \$0.3fs." % (time() - t0)

# Get labels for all points
print "Predicting color indices on the full image (k-means)"
t0 = time()
labels = kmeans.predict(image_array)
print "done in \$0.3fs." % (time() - t0)

codebook_random = shuffle(image_array, random_state=0)[:n_colors + 1]
print "Predicting color indices on the full image (random)"
t0 = time()
dist = euclidean_distances(codebook_random, image_array, squared=True)
labels_random = dist.argmin(axis=0)
print "done in \$0.3fs." % (time() - t0)

def recreate_image(codebook, labels, w, h):
    """Recreate the (compressed) image from the code book & labels""
    d = codebook.shape[1]
    image = np.zeros((w, h, d))
    label_idx = 0
    for i in range(w):
        for j in range(h):
            image[i][j] = codebook[labels[label_idx]]
            label_idx += 1
    return image

# Display all results, alongside original image
pl.figure(1)
pl.clf()
ar = pl.axes([0, 0, 1, 1])
pl.axis('off')
pl.title('Original image (96,615 colors)')
pl.imshow(china)
pl.figure(2)
pl.clf()
ar = pl.axes([0, 0, 1, 1])
pl.axis('off')
pl.title('Quantized image (64 colors, K-Means)')
pl.imshow(recreate_image(kmeans.cluster_centers_, labels, w, h))
pl.figure(3)
pl.clf()
ar = pl.axes([0, 0, 1, 1])
pl.axis('off')
pl.title('Quantized image (64 colors, Random)')
pl.imshow(recreate_image(codebook_random, labels_random, w, h))
pl.show()

Demo of DBSCAN clustering algorithm

Finds core samples of high density and expands clusters from them.
Figure 2.39: Demo of DBSCAN clustering algorithm

Script output:
Estimated number of clusters: 2
Homogeneity: 0.517
Completeness: 0.660
V-measure: 0.580
Adjusted Rand Index: 0.501
Adjusted Mutual Information: 0.516
Silhouette Coefficient: 0.381

Python source code: plot_dbscan.py

```python
print __doc__

import numpy as np
```
```python
from scipy.spatial import distance
from sklearn.cluster import DBSCAN
from sklearn import metrics
from sklearn.datasets.samples_generator import make_blobs

# Generate sample data
centers = [[1, 1], [-1, -1], [1, -1]]
X, labels_true = make_blobs(n_samples=750, centers=centers, cluster_std=0.4)

# Compute similarities
D = distance.squareform(distance.pdist(X))
S = 1 - (D / np.max(D))

# Compute DBSCAN
db = DBSCAN(eps=0.95, min_samples=10).fit(S)
core_samples = db.core_sample_indices_
labels = db.labels_

# Number of clusters in labels, ignoring noise if present.
n_clusters_ = len(set(labels)) - (1 if -1 in labels else 0)

print('Estimated number of clusters: %d' % n_clusters_)
print('Homogeneity: %0.3f' % metrics.homogeneity_score(labels_true, labels))
print('Completeness: %0.3f' % metrics.completeness_score(labels_true, labels))
print('V-measure: %0.3f' % metrics.v_measure_score(labels_true, labels))
print('Adjusted Rand Index: %0.3f' % metrics.adjusted_rand_score(labels_true, labels))
print('Adjusted Mutual Information: %0.3f' % metrics.adjusted_mutual_info_score(labels_true, labels))
print('Silhouette Coefficient: %0.3f' % metrics.silhouette_score(D, labels, metric='precomputed'))

# Plot result
import matplotlib.pyplot as plt
from itertools import cycle

plt.close('all')
plt.figure(1)
plt.clf()

# Black removed and is used for noise instead.
colors = cycle(['b', 'g', 'r', 'c', 'm', 'y', 'b', 'g', 'r', 'c', 'm', 'y'])
for k, col in zip(set(labels), colors):
    if k == -1:
        # Black used for noise.
        col = 'k'
        markersize = 6
    class_members = [index[0] for index in np.argwhere(labels == k)]
    cluster_core_samples = [index for index in core_samples if labels[index] == k]
    for index in class_members:
        x = X[index]
        if index in core_samples and k != -1:
```

2.1. Examples

---

763
markersize = 14
else:
    markersize = 6
pl.plot(x[0], x[1], 'o', markerfacecolor=col,
    markeredgecolor='k', markersize=markersize)

pl.title('Estimated number of clusters: %d' % n_clusters_)
pl.show()

Figure 2.40: Feature agglomeration

Feature agglomeration

These images show similar features are merged together using feature agglomeration.

Python source code: plot_digits_agglomeration.py

import numpy as np
import pylab as pl

# Code source: Gael Varoquaux
# Modified for Documentation merge by Jaques Grobler
# License: BSD

print __doc__
from sklearn import datasets, cluster
from sklearn.feature_extraction.image import grid_to_graph

digits = datasets.load_digits()
images = digits.images
X = np.reshape(images, (len(images), -1))
connectivity = grid_to_graph(*images[0].shape)

agglo = cluster.WardAgglomeration(connectivity=connectivity,
                                   n_clusters=32)

agglo.fit(X)
X_reduced = agglo.transform(X)

X_restored = agglo.inverse_transform(X_reduced)
images_restored = np.reshape(X_restored, images.shape)

pl.figure(1, figsize=(4, 3.5))
pl.clf()
pl.subplots_adjust(left=.01, right=.99, bottom=.01, top=.91)
for i in range(4):
    pl.subplot(3, 4, i + 1)
    pl.imshow(images[i], cmap=pl.cm.gray,
              vmax=16, interpolation='nearest')
    pl.xticks(())
    pl.yticks(())
    if i == 1:
        pl.title('Original data')
    pl.subplot(3, 4, 4 + i + 1)
    pl.imshow(images_restored[i],
              cmap=pl.cm.gray, vmax=16, interpolation='nearest')
    if i == 1:
        pl.title('Agglomerated data')
    pl.xticks(())
    pl.yticks(())
pl.subplot(3, 4, 10)
pl.imshow(np.reshape(agglo.labels_, images[0].shape),
          interpolation='nearest', cmap=pl.cm.spectral)
pl.xticks(())
pl.yticks(())
pl.title('Labels')

Figure 2.41: Feature agglomeration vs. univariate selection

Feature agglomeration vs. univariate selection

This example compares 2 dimensionality reduction strategies:

• univariate feature selection with Anova
• feature agglomeration with Ward hierarchical clustering

Both methods are compared in a regression problem using a BayesianRidge as supervised estimator.
Script output:

```
[Memory] Calling sklearn.cluster.hierarchical.ward_tree...
ward_tree(array([[-0.451933, ..., -0.675318],
    ...
[ 0.275706, ..., -1.085711]]),
<1600x1600 sparse matrix of type '<type 'numpy.int32'>'
    with 7840 stored elements in COOrdinate format>, copy=True, n_components=1)
________________________________________________________ward_tree - 0.3s, 0.0min

[Memory] Calling sklearn.cluster.hierarchical.ward_tree...
ward_tree(array([[ 0.905206, ..., 0.161245],
    ...
[-0.849835, ..., -1.091621]]),
<1600x1600 sparse matrix of type '<type 'numpy.int32'>'
    with 7840 stored elements in COOrdinate format>, copy=True, n_components=1)
________________________________________________________ward_tree - 0.3s, 0.0min

[Memory] Calling sklearn.cluster.hierarchical.ward_tree...
ward_tree(array([[ 0.905206, ..., -0.849835],
    ...
[ 0.161245, ..., -1.091621]]),
<1600x1600 sparse matrix of type '<type 'numpy.int32'>'
    with 7840 stored elements in COOrdinate format>, copy=True, n_components=1)
________________________________________________________ward_tree - 0.3s, 0.0min

[Memory] Calling sklearn.feature_selection.univariate_selection.f_regression...
f_regression(array([[-0.451933, ..., 0.275706],
    ...
[-0.675318, ..., -1.085711]]),
array([ 25.267703, ..., -25.026711]))
_____________________________________________________f_regression - 0.0s, 0.0min

[Memory] Calling sklearn.feature_selection.univariate_selection.f_regression...
f_regression(array([[ 0.905206, ..., -0.849835],
    ...
[ 0.161245, ..., -1.091621]]),
array([ -27.447268, ..., -112.638768]))
_____________________________________________________f_regression - 0.0s, 0.0min

[Memory] Calling sklearn.feature_selection.univariate_selection.f_regression...
f_regression(array([[ 0.905206, ..., -0.849835],
    ...
[ 0.161245, ..., -1.091621]]),
array([ 25.267703, ..., -25.026711]))
_____________________________________________________f_regression - 0.0s, 0.0min
```

Chapter 2. Example Gallery
array([-27.447268, ..., -25.026711]))

Python source code: plot_feature_agglomeration_vs_univariate_selection.py

```python
# Author: Alexandre Gramfort <alexandre.gramfort@inria.fr>
# License: BSD Style.

import shutil
import tempfile
import numpy as np
import pylab as pl
from scipy import linalg, ndimage
from sklearn.feature_extraction.image import grid_to_graph
from sklearn import feature_selection
from sklearn.cluster import WardAgglomeration
from sklearn.linear_model import BayesianRidge
from sklearn.pipeline import Pipeline
from sklearn.grid_search import GridSearchCV
from sklearn.externals.joblib import Memory
from sklearn.cross_validation import KFold

# Generate data
n_samples = 200
size = 40  # image size
roi_size = 15
snr = 5.
np.random.seed(0)
mask = np.ones([size, size], dtype=np.bool)
coef = np.zeros((size, size))
coef[0:roi_size, 0:roi_size] = -1.
coef[-roi_size:, -roi_size:] = 1.
X = np.random.randn(n_samples, size ** 2)
for x in X:
    # smooth data
    x[:] = ndimage.gaussian_filter(x.reshape(size, size), sigma=1.0).ravel()
X -= X.mean(axis=0)
X /= X.std(axis=0)
y = np.dot(X, coef.ravel())
noise = np.random.randn(y.shape[0])
noise_coef = (linalg.norm(y, 2) / np.exp(snr / 20.)) / linalg.norm(noise, 2)
y += noise_coef * noise  # add noise

# Compute the coefs of a Bayesian Ridge with GridSearch
cv = KFold(len(y), 2)  # cross-validation generator for model selection
ridge = BayesianRidge()
cachedir = tempfile.mkdtemp()
mem = Memory(cachedir=cachedir, verbose=1)
```

2.1. Examples
# Ward agglomeration followed by BayesianRidge

\[
A = \text{grid	extunderscore to	extunderscore graph}(n\_x=size, n\_y=size) \\
\text{ward} = \text{WardAgglomeration}(n\_clusters=10, connectivity=A, memory=mem, n\_components=1) \\
\text{clf} = \text{Pipeline}([('ward', ward), ('ridge', ridge)])
\]

# Select the optimal number of parcels with grid search

\[
\text{clf} = \text{GridSearchCV}(\text{clf}, \{'\text{ward\_n\_clusters}': [10, 20, 30]\}, n\_jobs=1, cv=cv)
\]

\[
\text{clf}.\text{fit}(X, y) \quad \# \text{set the best parameters}
\]

\[
\text{coef\_} = \text{clf}.\text{best\_estimator\_steps[-1][1].coef\_}
\]

\[
\text{coef\_} = \text{clf}.\text{best\_estimator\_steps[0][1].inverse\_transform(coef\_)}
\]

\[
\text{coef\_agglomeration\_} = \text{coef\_}\text{.reshape(size, size)}
\]

# Anova univariate feature selection followed by BayesianRidge

\[
f\_\text{regression} = \text{mem}.\text{cache}(\text{feature	extunderscore selection.}f\_\text{regression}) \quad \# \text{caching function}
\]

\[
\text{anova} = \text{feature	extunderscore selection.}\text{SelectPercentile}(f\_\text{regression})
\]

\[
\text{clf} = \text{Pipeline}([('anova', anova), ('ridge', ridge)])
\]

# Select the optimal percentage of features with grid search

\[
\text{clf} = \text{GridSearchCV}(\text{clf}, \{'\text{anova\_percentile}': [5, 10, 20]\}, cv=cv)
\]

\[
\text{clf}.\text{fit}(X, y) \quad \# \text{set the best parameters}
\]

\[
\text{coef\_} = \text{clf}.\text{best\_estimator\_steps[-1][1].coef\_}
\]

\[
\text{coef\_} = \text{clf}.\text{best\_estimator\_steps[0][1].inverse\_transform(coef\_)}
\]

\[
\text{coef\_selection\_} = \text{coef\_}\text{.reshape(size, size)}
\]

# Inverse the transformation to plot the results on an image

\[
\text{pl.close('all')} \\
\text{pl.figure(figsize=(7.3, 2.7))} \\
\text{pl.subplot(1, 3, 1)} \\
\text{pl.imshow(coef, interpolation="nearest", cmap=pl.cm.RdBu_r)} \\
\text{pl.title("True weights")]}
\]

\[
\text{pl.subplot(1, 3, 2)} \\
\text{pl.imshow(coef\_selection\_, interpolation="nearest", cmap=pl.cm.RdBu_r)} \\
\text{pl.title("Feature Selection")}
\]

\[
\text{pl.subplot(1, 3, 3)} \\
\text{pl.imshow(coef\_agglomeration\_, interpolation="nearest", cmap=pl.cm.RdBu_r)} \\
\text{pl.title("Feature Agglomeration")}
\]

\[
\text{pl.subplots_adjust(0.04, 0.0, 0.98, 0.94, 0.16, 0.26)} \\
\text{pl.show()}
\]

# Attempt to remove the temporary cachedir, but don’t worry if it fails

\[
\text{shutil.rmtree(cachedir, ignore\_errors=True)}
\]

Figure 2.42: A demo of K-Means clustering on the handwritten digits data

**A demo of K-Means clustering on the handwritten digits data**

In this example with compare the various initialization strategies for K-means in terms of runtime and quality of the results.
As the ground truth is known here, we also apply different cluster quality metrics to judge the goodness of fit of the cluster labels to the ground truth.

Cluster quality metrics evaluated (see *Clustering performance evaluation* for definitions and discussions of the metrics):

<table>
<thead>
<tr>
<th>Shorthand</th>
<th>full name</th>
</tr>
</thead>
<tbody>
<tr>
<td>homo</td>
<td>homogeneity score</td>
</tr>
<tr>
<td>compl</td>
<td>completeness score</td>
</tr>
<tr>
<td>v-meas</td>
<td>V measure</td>
</tr>
<tr>
<td>ARI</td>
<td>adjusted Rand index</td>
</tr>
<tr>
<td>AMI</td>
<td>adjusted mutual information</td>
</tr>
<tr>
<td>silhouette</td>
<td>silhouette coefficient</td>
</tr>
</tbody>
</table>

**K-means clustering on the digits dataset (PCA-reduced data)**

Centroids are marked with white cross

---

Script output:

```
n_digits: 10, n_samples 1797, n_features 64
```

<table>
<thead>
<tr>
<th></th>
<th>init time</th>
<th>inertia</th>
<th>homo</th>
<th>compl</th>
<th>v-meas</th>
<th>ARI</th>
<th>AMI</th>
<th>silhouette</th>
</tr>
</thead>
<tbody>
<tr>
<td>k-means++</td>
<td>1.90s</td>
<td>69432</td>
<td>0.602</td>
<td>0.650</td>
<td>0.625</td>
<td>0.465</td>
<td>0.598</td>
<td>0.146</td>
</tr>
<tr>
<td>random</td>
<td>1.80s</td>
<td>69694</td>
<td>0.669</td>
<td>0.710</td>
<td>0.689</td>
<td>0.553</td>
<td>0.666</td>
<td>0.147</td>
</tr>
<tr>
<td>PCA-based</td>
<td>0.14s</td>
<td>71820</td>
<td>0.673</td>
<td>0.715</td>
<td>0.693</td>
<td>0.567</td>
<td>0.670</td>
<td>0.150</td>
</tr>
</tbody>
</table>

Python source code: plot_kmeans_digits.py

---

2.1. Examples
```python
from time import time
import numpy as np
import pylab as pl

from sklearn import metrics
from sklearn.cluster import KMeans
from sklearn.datasets import load_digits
from sklearn.decomposition import PCA
from sklearn.preprocessing import scale

np.random.seed(42)

digits = load_digits()
data = scale(digits.data)
n_samples, n_features = data.shape
n_digits = len(np.unique(digits.target))
labels = digits.target

sample_size = 300

print "n_digits: %d, n_samples %d, n_features %d" % (n_digits, n_samples, n_features)

print 79 * '_'

print ('% 9s %i %.3f %.3f %.3f %.3f %.3f %.3f' % (n_digits, n_samples, n_features))

print 79 * '_'

print ('% 9s %i %.3f %.3f %.3f %.3f %.3f %.3f' % (n_digits, n_samples, n_features))

print 79 * '_'

print ('% 9s %i %.3f %.3f %.3f %.3f %.3f %.3f' % (n_digits, n_samples, n_features))

print 79 * '_'

print ('% 9s %i %.3f %.3f %.3f %.3f %.3f %.3f' % (n_digits, n_samples, n_features))

print 79 * '_'

print ('% 9s %i %.3f %.3f %.3f %.3f %.3f %.3f' % (n_digits, n_samples, n_features))

print 79 * '_'

print ('% 9s %i %.3f %.3f %.3f %.3f %.3f %.3f' % (n_digits, n_samples, n_features))

print 79 * '_'

print ('% 9s %i %.3f %.3f %.3f %.3f %.3f %.3f' % (n_digits, n_samples, n_features))

print 79 * '_'

print (name, ((time() - t0), estimator.inertia_,)
metrics.homogeneity_score(labels, estimator.labels_),
metrics.completeness_score(labels, estimator.labels_),
metrics.v_measure_score(labels, estimator.labels_),
metrics.adjusted_rand_score(labels, estimator.labels_),
metrics.adjusted_mutual_info_score(labels, estimator.labels_),
metrics.silhouette_score(data, estimator.labels_,
metric='euclidean',
sample_size=sample_size),)

bench_k_means(KMeans(init='k-means++', n_clusters=n_digits, n_init=10),
name="k-means++", data=data)

bench_k_means(KMeans(init='random', n_clusters=n_digits, n_init=10),
name="random", data=data)

# in this case the seeding of the centers is deterministic, hence we run the
# kmeans algorithm only once with n_init=1
pca = PCA(n_components=n_digits).fit(data)
bench_k_means(KMeans(init=pca.components_, n_clusters=n_digits, n_init=1),
name="PCA-based")
```

data=data
print 79 * '－'

# Visualize the results on PCA-reduced data
reduced_data = PCA(n_components=2).fit_transform(data)
kmeans = KMeans(init='k-means++', n_clusters=n_digits, n_init=10)
kmeans.fit(reduced_data)

# Step size of the mesh. Decrease to increase the quality of the VQ.
h = .02 # point in the mesh [x_min, m_max)x[y_min, y_max].

# Plot the decision boundary. For that, we will asign a color to each
x_min, x_max = reduced_data[:, 0].min() + 1, reduced_data[:, 0].max() - 1
y_min, y_max = reduced_data[:, 1].min() + 1, reduced_data[:, 1].max() - 1
xx, yy = np.meshgrid(np.arange(x_min, x_max, h), np.arange(y_min, y_max, h))

# Obtain labels for each point in mesh. Use last trained model.
Z = kmeans.predict(np.c_[xx.ravel(), yy.ravel()])

# Put the result into a color plot
Z = Z.reshape(xx.shape)
pl.figure(1)
pl.clf()
pl.imshow(Z, interpolation='nearest',
        extent=(xx.min(), xx.max(), yy.min(), yy.max()),
        cmap=pl.cm.Paired,
        aspect='auto', origin='lower')

pl.plot(reduced_data[:, 0], reduced_data[:, 1], 'k.', markersize=2)
pl.plot(centroids[:, 0], centroids[:, 1], 'x', markersize=15)

pl.title('K-means clustering on the digits dataset (PCA-reduced data)

Centroids are marked with white cross')
pl.xlim(x_min, x_max)
pl.ylim(y_min, y_max)
pl.xticks(())
pl.yticks(())
pl.show()
Empirical evaluation of the impact of k-means initialization

Evaluate the ability of k-means initializations strategies to make the algorithm convergence robust as measured by the relative standard deviation of the inertia of the clustering (i.e. the sum of distances to the nearest cluster center).

The first plot shows the best inertia reached for each combination of the model (KMeans or MiniBatchKMeans) and the init method (init="random" or init="kmeans++") for increasing values of the n_init parameter that controls the number of initializations.

The second plot demonstrate one single run of the MiniBatchKMeans estimator using a init="random" and n_init=1. This run leads to a bad convergence (local optimum) with estimated centers between stucked between ground truth clusters.

The dataset used for evaluation is a 2D grid of isotropic gaussian clusters widely spaced.

Script output:

Evaluation of KMeans with k-means++ init
Evaluation of KMeans with random init
Evaluation of MiniBatchKMeans with k-means++ init
Evaluation of MiniBatchKMeans with random init

Python source code: plot_kmeans_stability_low_dim_dense.py

```python
print __doc__

# Author: Olivier Grisel <olivier.grisel@ensta.org>
# License: Simplified BSD

import numpy as np
```
import pylab as pl
import matplotlib.cm as cm

from sklearn.utils import shuffle
from sklearn.utils import check_random_state
from sklearn.cluster import MiniBatchKMeans
from sklearn.cluster import KMeans

random_state = np.random.RandomState(0)

# Number of run (with randomly generated dataset) for each strategy so as
# to be able to compute an estimate of the standard deviation
n_runs = 5

# k-means models can do several random inits so as to be able to trade
# CPU time for convergence robustness
n_init_range = np.array([1, 5, 10, 15, 20])

# Datasets generation parameters
n_samples_per_center = 100
grid_size = 3
scale = 0.1
n_clusters = grid_size ** 2

def make_data(random_state, n_samples_per_center, grid_size, scale):
    random_state = check_random_state(random_state)
    centers = np.array([[i, j]
                        for i in range(grid_size)
                        for j in range(grid_size)])
    n_clusters_true, n_features = centers.shape

    noise = random_state.normal(
        scale=scale, size=(n_samples_per_center, centers.shape[1]))

    X = np.concatenate([c + noise for c in centers])
    y = np.concatenate([[i] * n_samples_per_center
                        for i in range(n_clusters_true)])
    return shuffle(X, y, random_state=random_state)

# Part 1: Quantitative evaluation of various init methods

fig = pl.figure()
plots = []
legends = []
cases = [
    (KMeans, 'k-means++', {}),
    (KMeans, 'random', {}),
    (MiniBatchKMeans, 'k-means++', {'max_no_improvement': 3}),
    (MiniBatchKMeans, 'random', {'max_no_improvement': 3, 'init_size': 500}),
]

for factory, init, params in cases:
    print "Evaluation of $s with $s init" % (factory.__name__, init)
    inertia = np.empty((len(n_init_range), n_runs))

    for run_id in range(n_runs):
X, y = make_data(run_id, n_samples_per_center, grid_size, scale)
for i, n_init in enumerate(n_init_range):
    km = factory(n_clusters=n_clusters, init=init, random_state=run_id,
                 n_init=n_init, **params).fit(X)
    inertia[i, run_id] = km.inertia_
    p = pl.errorbar(n_init_range, inertia.mean(axis=1), inertia.std(axis=1))
    plots.append(p[0])
    legends.append("%s with $s init" % (factory.__name__, init))
pl.xlabel('n_init')
pl.ylabel('inertia')
pl.legend(plots, legends)
pl.title("Mean inertia for various k-means init across $d runs" % n_runs)

# Part 2: Qualitative visual inspection of the convergence

X, y = make_data(random_state, n_samples_per_center, grid_size, scale)
km = MiniBatchKMeans(n_clusters=n_clusters, init='random', n_init=1,
                     random_state=random_state).fit(X)
fig = pl.figure()
for k in range(n_clusters):
    my_members = km.labels_ == k
    color = cm.spectral(float(k) / n_clusters, 1)
    pl.plot(X[my_members, 0], X[my_members, 1], 'o', marker='.', c=color)
    cluster_center = km.cluster_centers_[k]
    pl.plot(cluster_center[0], cluster_center[1], 'o',
            markerfacecolor=color, markeredgecolor='k', markersize=6)
pl.title("Example cluster allocation with a single random init
         "with MiniBatchKMeans")
pl.show()
Python source code: plot_lena_compress.py

```python
print __doc__

# Code source: Gael Varoqueux
# Modified for Documentation merge by Jaques Grobler
# License: BSD

import numpy as np
import scipy as sp
import pylab as pl

from sklearn import cluster

n_clusters = 5
np.random.seed(0)

try:
    lena = sp.lena()
except AttributeError:
    # Newer versions of scipy have lena in misc
    from scipy import misc
    lena = misc.lena()

X = lena.reshape((-1, 1))  # We need an (n_sample, n_feature) array
k_means = cluster.KMeans(n_clusters=n_clusters, n_init=4)
k_means.fit(X)
values = k_means.cluster_centers_.squeeze()
labels = k_means.labels_

# create an array from labels and values
lena_compressed = np.choose(labels, values)
lena_compressed.shape = lena.shape

vmin = lena.min()
vmax = lena.max()
```

2.1. Examples 775
Segmenting the picture of Lena in regions

This example uses Spectral clustering on a graph created from voxel-to-voxel difference on an image to break this image into multiple partly-homogenous regions.

This procedure (spectral clustering on an image) is an efficient approximate solution for finding normalized graph cuts.
Python source code: plot_lena_segmentation.py

print __doc__

# Author: Gael Varoquaux <gael.varoquaux@normalesup.org>
# License: BSD

import numpy as np
import scipy as sp
import pylab as pl

from sklearn.feature_extraction import image
from sklearn.cluster import spectral_clustering

lena = sp.misc.lena()
# Downsample the image by a factor of 4
lena = lena[:,::2] + lena[:,1::2] + lena[::2, ::2] + lena[1::2, ::2] + lena[::2, 1::2] + lena[1::2, 1::2]

# Convert the image into a graph with the value of the gradient on the edges.
graph = image.img_to_graph(lena)

# Take a decreasing function of the gradient: an exponential
# The smaller beta is, the more independent the segmentation is of the

2.1. Examples
# actual image. For beta=1, the segmentation is close to a voronoi
beta = 5
eps = 1e-6
graph.data = np.exp(-beta * graph.data / lena.std()) + eps

# Apply spectral clustering (this step goes much faster if you have pyamg
# installed)
N_REGIONS = 11
labels = spectral_clustering(graph, n_clusters=N_REGIONS)
labels = labels.reshape(lena.shape)

# Visualize the resulting regions
pl.figure(figsize=(5, 5))
pl.imshow(lena, cmap=pl.cm.gray)
for l in range(N_REGIONS):
    pl.contour(labels == l, contours=1,
               colors=[pl.cm.spectral(l / float(N_REGIONS)), ],
               )
pl.xticks(())
pl.yticks(())
pl.show()

Figure 2.46: A demo of structured Ward hierarchical clustering on Lena image

A demo of structured Ward hierarchical clustering on Lena image

Compute the segmentation of a 2D image with Ward hierarchical clustering. The clustering is spatially constrained in order for each segmented region to be in one piece.
Script output:
Compute structured hierarchical clustering...
Elapsed time: 18.7588250637
Number of pixels: 65536
Number of clusters: 15

Python source code: plot_lena_ward_segmentation.py

```python
# Author : Vincent Michel, 2010
# Alexandre Gramfort, 2011
# License: BSD Style.

print __doc__

import time as time
import numpy as np
import scipy as sp
import pylab as pl
from sklearn.feature_extraction.image import grid_to_graph
from sklearn.cluster import Ward

# Generate data
lena = sp.misc.lena()
# Downsampling the image by a factor of 4
```
lenaf = lena[::2, ::2] + lena[1::2, ::2] + lena[::2, 1::2] + lena[1::2, 1::2]
X = np.reshape(lena, (-1, 1))

# Define the structure A of the data. Pixels connected to their neighbors.
connectivity = grid_to_graph(*lena.shape)

# Compute structured hierarchical clustering
print "Compute structured hierarchical clustering..."
st = time.time()
n_clusters = 15  # number of regions
ward = Ward(n_clusters=n_clusters, connectivity=connectivity).fit(X)
label = np.reshape(ward.labels_, lena.shape)
print "Elapsed time: ", time.time() - st
print "Number of pixels: ", label.size
print "Number of clusters: ", np.unique(label).size

# Plot the results on an image
pl.figure(figsize=(5, 5))
pl.imshow(lena, cmap=pl.cm.gray)
for l in range(n_clusters):
    pl.contour(label == l, contours=1,
               colors=[pl.cm.spectral(l / float(n_clusters)), ])
pl.xticks(())
pl.yticks(())
pl.show()

Figure 2.47: A demo of the mean-shift clustering algorithm

A demo of the mean-shift clustering algorithm

Reference:
number of estimated clusters : 3

Python source code: plot_mean_shift.py

```python
from sklearn.cluster import MeanShift, estimate_bandwidth
from sklearn.datasets.samples_generator import make_blobs
import numpy as np

centers = [[1, 1], [-1, -1], [1, -1]]
X, _ = make_blobs(n_samples=10000, centers=centers, cluster_std=0.6)

bandwidth = estimate_bandwidth(X, quantile=0.2, n_samples=500)
ms = MeanShift(bandwidth=bandwidth, bin_seeding=True)
ms.fit(X)
labels = ms.labels_
cluster_centers = ms.cluster_centers_
```

*2.1. Examples*
labels_unique = np.unique(labels)
n_clusters_ = len(labels_unique)

print "number of estimated clusters : %d" % n_clusters_

# Plot result
import matplotlib.pyplot as plt
from itertools import cycle

plt.figure(1)
plt.clf()

colors = cycle(['b', 'g', 'r', 'c', 'm', 'y', 'k'])
for k, col in zip(range(n_clusters_), colors):
    my_members = labels == k
    cluster_center = cluster_centers[k]
    plt.plot(X[my_members, 0], X[my_members, 1], col + '.
    plt.plot(cluster_center[0], cluster_center[1], 'o', markerfacecolor=col,
             markeredgecolor='k', markersize=14)

plt.title('Estimated number of clusters: %d' % n_clusters_)
plt.show()

Figure 2.48: A demo of the K Means clustering algorithm

A demo of the K Means clustering algorithm

We want to compare the performance of the MiniBatchKMeans and KMeans: the MiniBatchKMeans is faster, but gives slightly different results (see Mini Batch K-Means).

We will cluster a set of data, first with KMeans and then with MiniBatchKMeans, and plot the results. We will also plot the points that are labelled differently between the two algorithms.

Python source code: plot_mini_batch_kmeans.py
import time

import numpy as np
import scipy as sp

from sklearn.cluster import MiniBatchKMeans, KMeans
from sklearn.metrics.pairwise import euclidean_distances
from sklearn.datasets.samples_generator import make_blobs

# Generate sample data
np.random.seed(0)
batch_size = 45
centers = [[1,1], [-1,-1], [1,-1]]
n_clusters = len(centers)
X, labels_true = make_blobs(n_samples=3000, centers=centers, cluster_std=0.7)

# Compute clustering with Means
k_means = KMeans(init='k-means++', n_clusters=3, n_init=10)
t0 = time.time()
k_means.fit(X)
t_batch = time.time() - t0
k_means_labels = k_means.labels_
k_means_cluster_centers = k_means.cluster_centers_
k_means_labels_unique = np.unique(k_means_labels)

# Compute clustering with MiniBatchKMeans
mbk = MiniBatchKMeans(init='k-means++', n_clusters=3, batch_size=batch_size,
                        n_init=10, max_no_improvement=10, verbose=0)
t0 = time.time()
mbk.fit(X)
t_mini_batch = time.time() - t0
mbk_means_labels = mbk.labels_
mbk_means_cluster_centers = mbk.cluster_centers_
mbk_means_labels_unique = np.unique(mbk_means_labels)

# Plot result
fig = plt.figure(figsize=(8, 3))
fig.subplots_adjust(left=0.02, right=0.98, bottom=0.05, top=0.9)

# We want to have the same colors for the same cluster from the
# MiniBatchKMeans and the KMeans algorithm. Let’s pair the cluster centers per
# closest one.

distance = euclidean_distances(k_means_cluster_centers,
                                mbk_means_cluster_centers,
                                squared=True)

order = distance.argmin(axis=1)
Spectral clustering for image segmentation

In this example, an image with connected circles is generated and Spectral clustering is used to separate the circles.

In these settings, the spectral clustering approach solves the problem known as ‘normalized graph cuts’: the image is seen as a graph of connected voxels, and the spectral clustering algorithm amounts to choosing graph cuts defining regions while minimizing the ratio of the gradient along the cut, and the volume of the region.

As the algorithm tries to balance the volume (ie balance the region sizes), if we take circles with different sizes, the
Figure 2.49: *Spectral clustering for image segmentation*

Segmentation fails.

In addition, as there is no useful information in the intensity of the image, or its gradient, we choose to perform the spectral clustering on a graph that is only weakly informed by the gradient. This is close to performing a Voronoi partition of the graph.

In addition, we use the mask of the objects to restrict the graph to the outline of the objects. In this example, we are interested in separating the objects one from the other, and not from the background.
Python source code: plot_segmentation_toy.py

```
print __doc__

# Authors: Emmanuelle Gouillart <emmanuelle.gouillart@normalesup.org>
#          Gael Varoquaux <gael.varoquaux@normalesup.org>
# License: BSD

import numpy as np
import pylab as pl

from sklearn.feature_extraction import image
from sklearn.cluster import spectral_clustering

l = 100
x, y = np.indices((l, l))

center1 = (28, 24)
center2 = (40, 50)
```
center3 = (67, 58)
center4 = (24, 70)

radius1, radius2, radius3, radius4 = 16, 14, 15, 14

circle1 = (x - center1[0]) ** 2 + (y - center1[1]) ** 2 < radius1 ** 2
circle2 = (x - center2[0]) ** 2 + (y - center2[1]) ** 2 < radius2 ** 2
circle3 = (x - center3[0]) ** 2 + (y - center3[1]) ** 2 < radius3 ** 2
circle4 = (x - center4[0]) ** 2 + (y - center4[1]) ** 2 < radius4 ** 2

# Convert the image into a graph with the value of the gradient on the # edges.
graph = image.img_to_graph(img, mask=mask)

# Take a decreasing function of the gradient: we take it weakly # dependant from the gradient the segmentation is close to a voronoi graph.data = np.exp(-graph.data / graph.data.std())

# Force the solver to be arpack, since amg is numerically # unstable on this example
labels = spectral_clustering(graph, n_clusters=4, mode='arpack')
label_im = -np.ones(mask.shape)
label_im[mask] = labels

pl.matshow(img)
pl.matshow(label_im)

# Convert the image into a graph with the value of the gradient on the # edges.
graph = image.img_to_graph(img, mask=mask)

# Take a decreasing function of the gradient: we take it weakly # dependant from the gradient the segmentation is close to a voronoi graph.data = np.exp(-graph.data / graph.data.std())

# Force the solver to be arpack, since amg is numerically # unstable on this example
labels = spectral_clustering(graph, n_clusters=2, mode='arpack')
label_im = -np.ones(mask.shape)
label_im[mask] = labels

pl.matshow(img)
pl.matshow(label_im)
pl.show()
Hierarchical clustering: structured vs unstructured ward

Example builds a swiss roll dataset and runs Hierarchical clustering on their position.

In a first step, the hierarchical clustering without connectivity constraints on structure, solely based on distance, whereas in a second step clustering restricted to the k-Nearest Neighbors graph: it’s a hierarchical clustering with structure prior.

Some of the clusters learned without connectivity constraints do not respect the structure of the swiss roll and extend across different folds of the manifolds. On the opposite, when opposing connectivity constraints, the clusters form a nice parcellation of the swiss roll.

Script output:
Compute unstructured hierarchical clustering...
Elapsed time: 0.912338972092
Number of points: 1000
Compute structured hierarchical clustering...
Elapsed time: 0.172855138779
Number of points: 1000

**Python source code:** plot_ward_structured_vs_unstructured.py

```python
import time as time
import numpy as np
import pylab as pl
import mpl_toolkits.mplot3d.axes3d as p3
from sklearn.cluster import Ward
from sklearn.datasets.samples_generator import make_swiss_roll

# Generate data (swiss roll dataset)
n_samples = 1000
noise = 0.05
X, _ = make_swiss_roll(n_samples, noise)
# Make it thinner
X[:, 1] *= .5

# Compute clustering
print "Compute unstructured hierarchical clustering..."
st = time.time()
ward = Ward(n_clusters=6).fit(X)
label = ward.labels_
print "Elapsed time: ", time.time() - st
print "Number of points: ", label.size

# Plot result
fig = pl.figure()
ax = p3.Axes3D(fig)
ax.view_init(7, -80)
for l in np.unique(label):
    ax.plot3D(X[label == l, 0], X[label == l, 1], X[label == l, 2],
              'o', color=pl.cm.jet(np.float(l) / np.max(label + 1)))
pl.title('Without connectivity constraints')

# Define the structure A of the data. Here a 10 nearest neighbors
from sklearn.neighbors import kneighbors_graph
connectivity = kneighbors_graph(X, n_neighbors=10)

# Compute clustering
print "Compute structured hierarchical clustering..."
st = time.time()
ward = Ward(n_clusters=6, connectivity=connectivity).fit(X)
```

2.1. Examples
label = ward.labels_
print "Elapsed time: ", time.time() - st
print "Number of points: ", label.size

# Plot result
fig = pl.figure()
ax = p3.Axes3D(fig)
ax.view_init(7, -80)
for l in np.unique(label):
    ax.plot3D(X[label == l, 0], X[label == l, 1], X[label == l, 2], 'o', color=pl.cm.jet(float(l) / np.max(label + 1)))
pl.title('With connectivity constraints')
pl.show()

2.1.4 Covariance estimation

Examples concerning the sklearn.covariance package.

Figure 2.51: Ledoit-Wolf vs Covariance simple estimation

Ledoit-Wolf vs Covariance simple estimation

The usual covariance maximum likelihood estimate can be regularized using shrinkage. Ledoit and Wolf proposed a close formula to compute the asymptotical optimal shrinkage parameter (minimizing a MSE criterion), yielding the Ledoit-Wolf covariance estimate.

Chen et al. proposed an improvement of the Ledoit-Wolf shrinkage parameter, the OAS coefficient, whose convergence is significantly better under the assumption that the data are gaussian.

In this example, we compute the likelihood of unseen data for different values of the shrinkage parameter, highlighting the LW and OAS estimates. The Ledoit-Wolf estimate stays close to the likelihood criterion optimal value, which is an artifact of the method since it is asymptotic and we are working with a small number of observations. The OAS estimate deviates from the likelihood criterion optimal value but better approximate the MSE optimal value, especially for a small number a observations.
print __doc__

import numpy as np
import pylab as pl
from scipy import linalg

# Generate sample data
n_features, n_samples = 30, 20
base_X_train = np.random.normal(size=(n_samples, n_features))
base_X_test = np.random.normal(size=(n_samples, n_features))

# Color samples
coloring_matrix = np.random.normal(size=(n_features, n_features))
X_train = np.dot(base_X_train, coloring_matrix)
X_test = np.dot(base_X_test, coloring_matrix)

# Compute Ledoit-Wolf and Covariances on a grid of shrinkages
from sklearn.covariance import LedoitWolf, OAS, ShrunkCovariance,
                      log_likelihood, empirical_covariance

# Ledoit-Wolf optimal shrinkage coefficient estimate
Ledoit-Wolf vs OAS estimation

The usual covariance maximum likelihood estimate can be regularized using shrinkage. Ledoit and Wolf proposed a close formula to compute the asymptotical optimal shrinkage parameter (minimizing a MSE criterion), yielding the
Figure 2.52: Ledoit-Wolf vs OAS estimation

Ledoit-Wolf covariance estimate.

Chen et al. proposed an improvement of the Ledoit-Wolf shrinkage parameter, the OAS coefficient, whose convergence is significantly better under the assumption that the data are gaussian.

This example, inspired from Chen’s publication [1], shows a comparison of the estimated MSE of the LW and OAS methods, using gaussian distributed data.


Python source code: plot_lw_vs_oas.py

print __doc__
import numpy as np
import matplotlib as mpl
from scipy.linalg import toeplitz, cholesky

from sklearn.covariance import LedoitWolf, OAS

n_features = 100
# simulation covariance matrix (AR(1) process)
r = 0.1
real_cov = toeplitz(r ** np.arange(n_features))
coloring_matrix = cholesky(real_cov)

n_samples_range = np.arange(6, 31, 1)
repeat = 100
lw_mse = np.zeros((n_samples_range.size, repeat))
oa_mse = np.zeros((n_samples_range.size, repeat))
lw_shrinkage = np.zeros((n_samples_range.size, repeat))
oa_shrinkage = np.zeros((n_samples_range.size, repeat))

for i, n_samples in enumerate(n_samples_range):
    for j in range(repeat):
        X = np.dot(
            np.random.normal(size=(n_samples, n_features)),
            coloring_matrix.T)

        lw = LedoitWolf(store_precision=False)
        lw.fit(X, assume_centered=True)
        lw_mse[i, j] = lw.error_norm(real_cov, scaling=False)
        lw_shrinkage[i, j] = lw.shrinkage_

        oa = OAS(store_precision=False)
        oa.fit(X, assume_centered=True)
        oa_mse[i, j] = oa.error_norm(real_cov, scaling=False)
        oa_shrinkage[i, j] = oa.shrinkage_

# plot MSE
pl.subplot(2, 1, 1)
pl.errorbar(n_samples_range, lw_mse.mean(1), yerr=lw_mse.std(1),
            label='Ledoit-Wolf', color='g')
pl.errorbar(n_samples_range, oa_mse.mean(1), yerr=oa_mse.std(1),
            label='OAS', color='r')
pl.ylabel("Squared error")
pl.legend(loc="upper right")
pl.title("Comparison of covariance estimators")
pl.xlim(5, 31)

# plot shrinkage coefficient
pl.subplot(2, 1, 2)
pl.errorbar(n_samples_range, lw_shrinkage.mean(1), yerr=lw_shrinkage.std(1),
            label='Ledoit-Wolf', color='g')
pl.errorbar(n_samples_range, oa_shrinkage.mean(1), yerr=oa_shrinkage.std(1),
            label='OAS', color='r')
pl.xlabel("n_samples")
pl.ylabel("Shrinkage")
pl.legend(loc="lower right")
pl.ylim(pl.ylim()[0], 1. + (pl.ylim()[1] - pl.ylim()[0]) / 10.)
pl.xlim(5, 31)

pl.show()
Robust covariance estimation and Mahalanobis distances relevance

For Gaussian distributed data, the distance of an observation \( x_i \) to the mode of the distribution can be computed using its Mahalanobis distance:

\[
d_{(\mu, \Sigma)}(x_i)^2 = (x_i - \mu)' \Sigma^{-1} (x_i - \mu)
\]

where \( \mu \) and \( \Sigma \) are the location and the covariance of the underlying gaussian distribution.

In practice, \( \mu \) and \( \Sigma \) are replaced by some estimates. The usual covariance maximum likelihood estimate is very sensitive to the presence of outliers in the data set and therefore, the corresponding Mahalanobis distances are. One would better have to use a robust estimator of covariance to guarantee that the estimation is resistant to “erroneous” observations in the data set and that the associated Mahalanobis distances accurately reflect the true organisation of the observations.

The Minimum Covariance Determinant estimator is a robust, high-breakdown point (i.e. it can be used to estimate the covariance matrix of highly contaminated datasets, up to \( \frac{n_{\text{samples}} - n_{\text{features}} - 1}{2} \) outliers) estimator of covariance. The idea is to find \( \frac{n_{\text{samples}} + n_{\text{features}} + 1}{2} \) observations whose empirical covariance has the smallest determinant, yielding a “pure” subset of observations from which to compute standards estimates of location and covariance.

The Minimum Covariance Determinant estimator (MCD) has been introduced by P.J.Rousseeuw in [1].

This example illustrates how the Mahalanobis distances are affected by outlying data: observations drawn from a contaminating distribution are not distinguishable from the observations coming from the real, Gaussian distribution that one may want to work with. Using MCD-based Mahalanobis distances, the two populations become distinguishable. Associated applications are outliers detection, observations ranking, clustering, ... For visualisation purpose, the cubic root of the Mahalanobis distances are represented in the boxplot, as Wilson and Hilferty suggest [2]

```python
import numpy as np
import matplotlib.pyplot as plt
from sklearn.covariance import EmpiricalCovariance, MinCovDet

n_samples = 125
n_outliers = 25
n_features = 2

# generate data
gen_cov = np.eye(n_features)
gen_cov[0, 0] = 2.
X = np.dot(np.random.randn(n_samples, n_features), gen_cov)

# add some outliers
outliers_cov = np.eye(n_features)
outliers_cov[np.arange(1, n_features), np.arange(1, n_features)] = 7.
X[-n_outliers:] = np.dot(np.random.randn(n_outliers, n_features), outliers_cov)

# fit a Minimum Covariance Determinant (MCD) robust estimator to data
robust_cov = MinCovDet().fit(X)

# compare estimators learnt from the full data set with true parameters
```

Python source code: plot_mahalanobis_distances.py

print(__doc__)

import numpy as np
import matplotlib.pyplot as plt

from sklearn.covariance import EmpiricalCovariance, MinCovDet

n_samples = 125
n_outliers = 25
n_features = 2

# generate data
gen_cov = np.eye(n_features)
gen_cov[0, 0] = 2.
X = np.dot(np.random.randn(n_samples, n_features), gen_cov)
# add some outliers
outliers_cov = np.eye(n_features)
outliers_cov[np.arange(1, n_features), np.arange(1, n_features)] = 7.
X[-n_outliers:] = np.dot(np.random.randn(n_outliers, n_features), outliers_cov)

# fit a Minimum Covariance Determinant (MCD) robust estimator to data
robust_cov = MinCovDet().fit(X)

# compare estimators learnt from the full data set with true parameters
em_cov = EmpiricalCovariance().fit(X)

# Display results
fig = plt.figure()
pl.subplots_adjust(hspace=-.1, wspace=.4, top=.95, bottom=.05)

# Show data set
subfig1 = pl.subplot(3, 1, 1)
inlier_plot = subfig1.scatter(X[:, 0], X[:, 1],
    color='black', label='inliers')
outlier_plot = subfig1.scatter(X[:, 0][-n_outliers:], X[:, 1][-n_outliers:],
    color='red', label='outliers')
subfig1.set_xlim(subfig1.get_xlim()[0], 11.)
subfig1.set_title("Mahalanobis distances of a contaminated data set:"

# Show contours of the distance functions
xx, yy = np.meshgrid(np.linspace(pl.xlim()[0], pl.xlim()[1], 100),
    np.linspace(pl.ylim()[0], pl.ylim()[1], 100))
zz = np.c_[xx.ravel(), yy.ravel()]
mahal_emp_cov = emp_cov.mahalanobis(zz)
mahal_emp_cov = mahal_emp_cov.reshape(xx.shape)
emp_cov_contour = subfig1.contour(xx, yy, np.sqrt(mahal_emp_cov),
    cmap=pl.cm.PuBu_r,
    linestyles='dashed')
mahal_robust_cov = robust_cov.mahalanobis(zz)
mahal_robust_cov = mahal_robust_cov.reshape(xx.shape)
robust_contour = subfig1.contour(xx, yy, np.sqrt(mahal_robust_cov),
    cmap=pl.cm.YlOrBr_r,
    linestyles='dotted')

subfig1.legend([emp_cov_contour.collections[1],
    robust_contour.collections[1], inlier_plot, outlier_plot],
    ['MLE dist', 'robust dist', 'inliers', 'outliers'],
    loc="upper right", borderaxespad=0)
pl.xticks(()
pl.yticks(()

# Plot the scores for each point
emp_mahal = emp_cov.mahalanobis(X - np.mean(X, 0)) ** (0.33)
subfig2 = pl.subplot(2, 2, 3)
subfig2.boxplot([emp_mahal[:n_outliers], emp_mahal[n_outliers:]],
    widths=.25)
subfig2.plot(1.26 * np.ones(n_samples-n_outliers), empiricalPlot[0], '+k', markeredgewidth=1)
subfig2.plot(2.26 * np.ones(n_outliers),
    empiricalPlot[1], '+k', markeredgewidth=1)
subfig2.set_xticklabels(('inliers', 'outliers'), size=15)
subfig2.set_ylabel(r"$\sqrt[3]{\text{(Mahal. dist.)}}$", size=16)
subfig2.set_title("1. from non-robust estimates \
"Maximum Likelihood")
pl.xticks(()
pl.yticks()

robust_mahal = robust_cov.mahalanobis(X - robust_cov.location_) ** (0.33)
subfig3 = pl.subplot(2, 2, 4)
subfig3.boxplot([robust_mahal[:n_outliers], robust_mahal[n_outliers:]],
    widths=.25)
subfig3.plot(1.26 * np.ones(n_samples-n_outliers), robustPlot[0], '+k', markeredgewidth=1)
subfig3.plot(2.26 * np.ones(n_outliers),
    robustPlot[1], '+k', markeredgewidth=1)
subfig3.set_xticklabels(('inliers', 'outliers'), size=15)
subfig3.set_ylabel(r"$\sqrt[3]{\text{(Mahal. dist.)}}$", size=16)
subfig3.set_title("2. from robust estimates")
pl.xticks(()
pl.yticks()
Outlier detection with several methods.

This example illustrates two ways of performing Novelty and Outlier Detection when the amount of contamination is known:

- based on a robust estimator of covariance, which is assuming that the data are Gaussian distributed and performs better than the One-Class SVM in that case.
- using the One-Class SVM and its ability to capture the shape of the data set, hence performing better when the data is strongly non-Gaussian, i.e. with two well-separated clusters;

The ground truth about inliers and outliers is given by the points colors while the orange-filled area indicates which points are reported as outliers by each method.

Here, we assume that we know the fraction of outliers in the datasets. Thus rather than using the ‘predict’ method of the objects, we set the threshold on the decision function to separate out the corresponding fraction.
Python source code: plot_outlier_detection.py

```python
print __doc__

import numpy as np
import pylab as pl
import matplotlib.font_manager
from scipy import stats
from sklearn import svm
from sklearn.covariance import EllipticEnvelope

# Example settings
n_samples = 200
outliers_fraction = 0.25
clusters_separation = [0, 1, 2]

# define two outlier detection tools to be compared
classifiers = {
    "One-Class SVM": svm.OneClassSVM(nu=0.95 + outliers_fraction + 0.05,
                                          kernel="rbf", gamma=0.1),
    "robust covariance estimator": EllipticEnvelope(contamination=.1),
}

# Compare given classifiers under given settings
xx, yy = np.meshgrid(np.linspace(-7, 7, 500), np.linspace(-7, 7, 500))
n_inliers = int((1. - outliers_fraction) * n_samples)
n_outliers = int(outliers_fraction * n_samples)
ground_truth = np.ones(n_samples, dtype=int)
ground_truth[-n_outliers:] = 0

# Fit the problem with varying cluster separation
for i, offset in enumerate(clusters_separation):
    np.random.seed(42)
    # Data generation
    X1 = 0.3 * np.random.randn(0.5 * n_inliers, 2) - offset
    X2 = 0.3 * np.random.randn(0.5 * n_inliers, 2) + offset
    X = np.r_[X1, X2]
    # Add outliers
    X = np.r_[X, np.random.uniform(low=-6, high=6, size=(n_outliers, 2))]
    # Fit the model with the One-Class SVM
    pl.figure(figsize=(10, 5))
    for i, (clf_name, clf) in enumerate(classifiers.iteritems()):
        # fit the data and tag outliers
        clf.fit(X)
        y_pred = clf.decision_function(X).ravel()
        threshold = stats.scoreatpercentile(y_pred,
```

2.1. Examples
Robust vs Empirical covariance estimate

The usual covariance maximum likelihood estimate is very sensitive to the presence of outliers in the data set. In such a case, one would have better to use a robust estimator of covariance to guarantee that the estimation is resistant to “erroneous” observations in the data set.

The Minimum Covariance Determinant estimator is a robust, high-breakdown point (i.e. it can be used to estimate the covariance matrix of highly contaminated datasets, up to \( \frac{n_{\text{samples}}-n_{\text{features}}-1}{2} \) outliers) estimator of covariance. The idea is to find \( \frac{1}{2} \) observations whose empirical covariance has the smallest determinant, yielding a “pure” subset of observations from which to compute standards estimates of location and covariance. After a correction step aiming at compensating the fact the the estimates were learnt from only a portion of the initial data, we end up with robust estimates of the data set location and covariance.

The Minimum Covariance Determinant estimator (MCD) has been introduced by P.J.Rousseeuw in [1].

In this example, we compare the estimation errors that are made when using three types of location and covariance estimates on contaminated gaussian distributed data sets:
• The mean and the empirical covariance of the full dataset, which break down as soon as there are outliers in the
data set
• The robust MCD, that has a low error provided n_samples > 5 * n_features
• The mean and the empirical covariance of the observations that are known to be good ones. This can be consid-
ered as a “perfect” MCD estimation, so one can trust our implementation by comparing to this case.


Python source code: plot_robust_vs_empirical_covariance.py

```python
print __doc__

import numpy as np
import pylab as pl
import matplotlib.font_manager

from sklearn.covariance import EmpiricalCovariance, MinCovDet

# example settings
n_samples = 80
n_features = 5
repeat = 10
```

2.1. Examples
range_n_outliers = np.concatenate((
    np.linspace(0, n_samples / 8, 5),
    np.linspace(n_samples / 8, n_samples / 2, 5)[1:-1]))

# definition of arrays to store results
err_loc_mcd = np.zeros((range_n_outliers.size, repeat))
err_cov_mcd = np.zeros((range_n_outliers.size, repeat))
err_loc_emp_full = np.zeros((range_n_outliers.size, repeat))
err_cov_emp_full = np.zeros((range_n_outliers.size, repeat))
err_loc_emp_pure = np.zeros((range_n_outliers.size, repeat))
err_cov_emp_pure = np.zeros((range_n_outliers.size, repeat))

# computation
for i, n_outliers in enumerate(range_n_outliers):
    for j in range(repeat):
        # generate data
        X = np.random.randn(n_samples, n_features)
        # add some outliers
        outliers_index = np.random.permutation(n_samples) [:n_outliers]
        outliers_offset = 10. * \
            (np.random.randint(2, size=(n_outliers, n_features)) - 0.5)
        X[outliers_index] += outliers_offset
        inliers_mask = np.ones(n_samples).astype(bool)
        inliers_mask[outliers_index] = False

        # fit a Minimum Covariance Determinant (MCD) robust estimator to data
        S = MinCovDet().fit(X)
        # compare raw robust estimates with the true location and covariance
        err_loc_mcd[i, j] = np.sum(S.location_ ** 2)
        err_cov_mcd[i, j] = S.error_norm(np.eye(n_features))

        # compare estimators learnt from the full data set with true parameters
        err_loc_emp_full[i, j] = np.sum(X.mean(0) ** 2)
        err_cov_emp_full[i, j] = EmpiricalCovariance().fit(X).error_norm(
            np.eye(n_features))

        # compare with an empirical covariance learnt from a pure data set
        # (i.e. "perfect" MCD)
        pure_X = X[inliers_mask]
        pure_location = pure_X.mean(0)
        pure_emp_cov = EmpiricalCovariance().fit(pure_X)
        err_loc_emp_pure[i, j] = np.sum(pure_location ** 2)
        err_cov_emp_pure[i, j] = pure_emp_cov.error_norm(np.eye(n_features))

# Display results
font_prop = matplotlib.font_manager.FontProperties(size=11)
pl.subplot(2, 1, 1)
pl.errorbar(range_n_outliers, err_loc_mcd.mean(1),
    yerr=err_loc_mcd.std(1) / np.sqrt(repeat),
    label="Robust location", color='m')
pl.errorbar(range_n_outliers, err_loc_emp_full.mean(1),
    yerr=err_loc_emp_full.std(1) / np.sqrt(repeat),
    label="Full data set mean", color='green')
pl.errorbar(range_n_outliers, err_loc_emp_pure.mean(1),
    yerr=err_loc_emp_pure.std(1) / np.sqrt(repeat),
    label="Pure data set mean", color='black')
pl.title("Influence of outliers on the location estimation")
pl.ylabel(r"Error $(\|\mu - \hat{\mu}\|_2^2)$")
pl.legend(loc="upper left", prop=font_prop)
pl.subplot(2, 1, 2)
x_size = range_n_outliers.size
pl.errorbar(range_n_outliers, err_cov_mcd.mean(1),
    yerr=err_cov_mcd.std(1),
    label="Robust covariance (MCD)", color='m')
pl.errorbar(range_n_outliers[:((x_size / 5) + 1)],
    err_cov_emp_full.mean(1)[:((x_size / 5) + 1)],
    yerr=err_cov_emp_full.std(1)[:((x_size / 5) + 1)],
    label="Full data set empirical covariance", color='green')
pl.plot(range_n_outliers[((x_size / 5)):((x_size / 2) - 1)],
    err_cov_emp_full.mean(1)((x_size / 5):((x_size / 2) - 1)],
    color='green', ls='--')
pl.errorbar(range_n_outliers, err_cov_emp_pure.mean(1),
    yerr=err_cov_emp_pure.std(1),
    label="Pure data set empirical covariance", color='black')
pl.title("Influence of outliers on the covariance estimation")
pl.xlabel("Amount of contamination (%)")
pl.ylabel("RMSE")
pl.legend(loc="upper center", prop=font_prop)
pl.show()

Figure 2.56: Sparse inverse covariance estimation

Sparse inverse covariance estimation

Using the GraphLasso estimator to learn a covariance and sparse precision from a small number of samples.

To estimate a probabilistic model (e.g. a Gaussian model), estimating the precision matrix, that is the inverse covariance matrix, is as important as estimating the covariance matrix. Indeed a Gaussian model is parametrized by the precision matrix.

To be in favorable recovery conditions, we sample the data from a model with a sparse inverse covariance matrix. In addition, we ensure that the data is not too much correlated (limiting the largest coefficient of the precision matrix) and that there are no small coefficients in the precision matrix that cannot be recovered. In addition, with a small number of observations, it is easier to recover a correlation matrix rather than a covariance, thus we scale the time series.

Here, the number of samples is slightly larger than the number of dimensions, thus the empirical covariance is still invertible. However, as the observations are strongly correlated, the empirical covariance matrix is ill-conditioned and as a result its inverse—the empirical precision matrix—is very far from the ground truth.

If we use l2 shrinkage, as with the Ledoit-Wolf estimator, as the number of samples is small, we need to shrink a lot. As a result, the Ledoit-Wolf precision is fairly close to the ground truth precision, that is not far from being diagonal, but the off-diagonal structure is lost.

The l1-penalized estimator can recover part of this off-diagonal structure. It learns a sparse precision. It is not able to recover the exact sparsity pattern: it detects too many non-zero coefficients. However, the highest non-zero coefficients of the l1 estimated correspond to the non-zero coefficients in the ground truth. Finally, the coefficients of
the l1 precision estimate are biased toward zero: because of the penalty, they are all smaller than the corresponding ground truth value, as can be seen on the figure.

Note that, the color range of the precision matrices is tweaked to improve readability of the figure. The full range of values of the empirical precision is not displayed.

The alpha parameter of the GraphLasso setting the sparsity of the model is set by internal cross-validation in the GraphLassoCV. As can be seen on figure 2, the grid to compute the cross-validation score is iteratively refined in the neighborhood of the maximum.

Python source code: plot_sparse_cov.py

```python
print __doc__
# author: Gael Varoquaux <gael.varoquaux@inria.fr>
# License: BSD Style
# Copyright: INRIA

import numpy as np
from scipy import linalg
from sklearn.datasets import make_sparse_spd_matrix
from sklearn.covariance import GraphLassoCV, ledoit_wolf
import pylab as pl

# Generate the data
n_samples = 60
n_features = 20

prng = np.random.RandomState(1)
prec = make_sparse_spd_matrix(n_features, alpha=.98,
                           smallest_coef=.4,
                           largest_coef=.7,
                           random_state=prng)

cov = linalg.inv(prec)
d = np.sqrt(np.diag(cov))
cov /= d
cov /= d[:, np.newaxis]
```
prec *= d
prec *= d[:, np.newaxis]
X = prng.multivariate_normal(np.zeros(n_features), cov, size=n_samples)
X -= X.mean(axis=0)
X /= X.std(axis=0)

# Estimate the covariance
emp_cov = np.dot(X.T, X) / n_samples
model = GraphLassoCV()
model.fit(X)
cov_ = model.covariance_
prec_ = model.precision_

lw_cov_, _ = ledoit_wolf(X)
lw_prec_ = linalg.inv(lw_cov_)

# Plot the results
pl.figure(figsize=(10, 6))
pl.subplots_adjust(left=0.02, right=0.98)

# plot the covariances
covs = [('Empirical', emp_cov), ('Ledoit-Wolf', lw_cov_),
        ('GraphLasso', cov_), ('True', cov_)]
vmax = cov_.max()
for i, (name, this_cov) in enumerate(covs):
    pl.subplot(2, 4, i + 1)
    pl.imshow(this_cov, interpolation='nearest', vmin=-vmax, vmax=vmax,
               cmap=pl.cm.RdBu_r)
    pl.xticks(())
    pl.yticks(())
    pl.title(' %s covariance' % name)

# plot the precisions
precs = [('Empirical', linalg.inv(emp_cov)), ('Ledoit-Wolf', lw_prec_),
         ('GraphLasso', prec_), ('True', prec_)]
vmax = .9 * prec_.max()
for i, (name, this_prec) in enumerate(precs):
    ax = pl.subplot(2, 4, i + 5)
    pl.imshow(np.ma.masked_equal(this_prec, 0),
              interpolation='nearest', vmin=-vmax, vmax=vmax,
              cmap=pl.cm.RdBu_r)
    pl.xticks(())
    pl.yticks(())
    pl.title(' %s precision' % name)
    ax.set_axis_bgcolor('0.7')

# plot the model selection metric
pl.figure(figsize=(4, 3))
pl.axes([.2, .15, .75, .7])
pl.plot(model.cv_alphas_, np.mean(model.cv_scores, axis=1), 'o-')
pl.axvline(model.alpha_, color='.5')
pl.title('Model selection')
pl.ylabel('Cross-validation score')
pl.xlabel('alpha')

2.1. Examples
2.1.5 Dataset examples

Examples concerning the `sklearn.datasets` package.

**The Digit Dataset**

This dataset is made up of 1797 8x8 images. Each image, like the one shown below, is of a hand-written digit. In order to utilise an 8x8 figure like this, we'd have to first transform it into a feature vector with length 64.

See here for more information about this dataset.

```python
# Code source: Gael Varoqueux
# Modified for Documentation merge by Jaques Grobler
# License: BSD

from sklearn import datasets
import pylab as pl

#Load the digits dataset
```

*Python source code:* `plot_digits_last_image.py`

```python
print __doc__

```
digits = datasets.load_digits()

#Display the first digit
pl.figure(1, figsize=(3, 3))
pl.imshow(digits.images[-1], cmap=pl.cm.gray_r, interpolation='nearest')
pl.show()

Figure 2.58: The Iris Dataset

The Iris Dataset

This data set consists of 3 different types of irises’ (Setosa, Versicolour, and Virginica) petal and sepal length, stored in a 150x4 numpy.ndarray.

The rows being the samples and the columns being: Sepal Length, Sepal Width, Petal Length and Petal Width.

The below plot uses the first two features. See here for more information on this dataset.

Python source code: plot_iris_dataset.py

print __doc__

# Code source: Gael Varoqueux
# Modified for Documentation merge by Jaques Grobler
# License: BSD

import pylab as pl
from sklearn import datasets

# import some data to play with
iris = datasets.load_iris()
X = iris.data[:, :2]  # we only take the first two features.
Y = iris.target

x_min, x_max = X[:, 0].min() -.5, X[:, 0].max() + .5
y_min, y_max = X[:, 1].min() -.5, X[:, 1].max() + .5

pl.figure(1, figsize=(4, 3))
pl.clf()

# Plot also the training points
pl.scatter(X[:, 0], X[:, 1], c=Y, cmap=pl.cm.Paired)
pl.xlabel('Sepal length')
pl.ylabel('Sepal width')
pl.xticks(())
pl.yticks(())
pl.show()

Figure 2.59: Plot randomly generated classification dataset

Plot randomly generated classification dataset

Plot several randomly generated 2D classification datasets. This example illustrates the datasets.make_classification function.

Three binary and two multi-class classification datasets are generated, with different numbers of informative features and clusters per class.
Python source code: plot_random_dataset.py

```
print __doc__

import pylab as pl

from sklearn.datasets import make_classification

pl.figure(figsize=(8, 6))
pl.subplots_adjust(bottom=.05, top=.9, left=.05, right=.95)

pl.subplot(221)
pl.title("One informative feature, one cluster", fontsize='small')
X1, Y1 = make_classification(n_features=2, n_redundant=0, n_informative=1,
                           n_clusters_per_class=1)
pl.scatter(X1[:, 0], X1[:, 1], marker='o', c=Y1)

pl.subplot(222)
pl.title("Two informative features, one cluster", fontsize='small')
X1, Y1 = make_classification(n_features=2, n_redundant=0, n_informative=2,
                           n_clusters_per_class=1)
pl.scatter(X1[:, 0], X1[:, 1], marker='o', c=Y1)

pl.subplot(223)
pl.title("Two informative features, two clusters", fontsize='small')
X2, Y2 = make_classification(n_features=2, n_redundant=0, n_informative=2)
```

2.1. Examples
pl.scatter(X2[:, 0], X2[:, 1], marker='o', c=Y2)

pl.subplot(224)
pl.title("Multi-class, two informative features, one cluster",
    fontsize='small')
X1, Y1 = make_classification(n_features=2, n_redundant=0, n_informative=2,
    n_clusters_per_class=1, n_classes=3)
pl.scatter(X1[:, 0], X1[:, 1], marker='o', c=Y1)
pl.show()

2.1.6 Decomposition

Examples concerning the sklearn.decomposition package.

Figure 2.60: Faces dataset decompositions

Faces dataset decompositions

This example applies to The Olivetti faces dataset different unsupervised matrix decomposition (dimension reduction) methods from the module sklearn.decomposition (see the documentation chapter Decomposing signals in components (matrix factorization problems)).

Independent components - FastICA - Train time 2.0s
Script output:

Dataset consists of 400 faces
Extracting the top 6 Eigenfaces - RandomizedPCA...
done in 0.491s
Extracting the top 6 Non-negative components - NMF...
done in 2.500s
Extracting the top 6 Independent components - FastICA...
done in 1.971s
Extracting the top 6 Sparse comp. - MiniBatchSparsePCA...
done in 1.763s
Extracting the top 6 MiniBatchDictionaryLearning...
done in 1.350s
Extracting the top 6 Cluster centers - MiniBatchKMeans...
done in 0.463s

Python source code: plot_faces_decomposition.py

```python
print __doc__

# Authors: Vlad Niculae, Alexandre Gramfort
# License: BSD

import logging
from time import time

from numpy.random import RandomState
import pylab as pl

from sklearn.datasets import fetch_olivetti_faces
```
```python
from sklearn.cluster import MiniBatchKMeans
from sklearn import decomposition

# Display progress logs onstdout
logging.basicConfig(level=logging.INFO,
                     format='%(asctime)s %(levelname)s %(message)s')
n_row, n_col = 2, 3
n_components = n_row * n_col
image_shape = (64, 64)
rng = RandomState(0)

# Load faces data
dataset = fetch_olivetti_faces(shuffle=True, random_state=rng)
faces = dataset.data
n_samples, n_features = faces.shape

def plot_gallery(title, images):
    pl.figure(figsize=(2. * n_col, 2.26 * n_row))
    pl.suptitle(title, size=16)
    for i, comp in enumerate(images):
        pl.subplot(n_row, n_col, i + 1)
        vmax = max(comp.max(), -comp.min())
        pl.imshow(comp.reshape(image_shape), cmap=pl.cm.gray,
                  interpolation='nearest',
                  vmin=-vmax, vmax=vmax)
        pl.xticks(())
        pl.yticks(())
    pl.subplots_adjust(0.01, 0.05, 0.99, 0.93, 0.04, 0.)

estimators = [('Eigenfaces - RandomizedPCA',
               decomposition.RandomizedPCA(n_components=n_components, whiten=True,
               True),
               ('Non-negative components - NMF',
               decomposition.NMF(n_components=n_components, init='nndsvda', beta=5.0,
               tol=5e-3, sparseness='components'),
               False),
               ('Independent components - FastICA',
               decomposition.FastICA(n_components=n_components, whiten=True,
               max_iter=10),
               True)]
```

2.1. Examples
('Sparse comp. - MiniBatchSparsePCA',
decomposition.MiniBatchSparsePCA(n_components=n_components, alpha=0.8,
    n_iter=100, chunk_size=3,
    random_state=rng),
    True),

('MiniBatchDictionaryLearning',
decomposition.MiniBatchDictionaryLearning(n_atoms=15, alpha=0.1,
    n_iter=50, chunk_size=3,
    random_state=rng),
    True),

('Cluster centers - MiniBatchKMeans',
MiniBatchKMeans(n_clusters=n_components, tol=1e-3, batch_size=20,
    max_iter=50, random_state=rng),
    True)
]

# Plot a sample of the input data
plot_gallery("First centered Olivetti faces", faces_centered[:n_components])

# Do the estimation and plot it
for name, estimator, center in estimators:
    print "Extracting the top %d %s..." % (n_components, name)
    t0 = time()
    data = faces
    if center:
        data = faces_centered
    estimator.fit(data)
    train_time = (time() - t0)
    print "done in %0.3f s" % train_time
    if hasattr(estimator, 'cluster_centers_'):
        components_ = estimator.cluster_centers_
    else:
        components_ = estimator.components_
    plot_gallery('%s - Train time %.1fs' % (name, train_time),
        components_[:n_components])
pl.show()
Blind source separation using FastICA

Independent component analysis (ICA) is used to estimate sources given noisy measurements. Imagine 2 instruments playing simultaneously and 2 microphones recording the mixed signals. ICA is used to recover the sources ie. what is played by each instrument.

Python source code: plot_ica_blind_source_separation.py

```python
import numpy as np
import pylab as pl
from sklearn.decomposition import FastICA

# Generate sample data
np.random.seed(0)
n_samples = 2000
time = np.linspace(0, 10, n_samples)
s1 = np.sin(2 * time)  # Signal 1 : sinusoidal signal
s2 = np.sign(np.sin(3 * time))  # Signal 2 : square signal
S = np.c_[s1, s2]
S += 0.2 * np.random.normal(size=S.shape)  # Add noise
S /= S.std(axis=0)  # Standardize data
```

2.1. Examples
A = np.array([[1, 1], [0.5, 2]])  # Mixing matrix
X = np.dot(S, A.T)  # Generate observations
# Compute ICA
ica = FastICA()
S_ = ica.fit(X).transform(X)  # Get the estimated sources
A_ = ica.get_mixing_matrix()  # Get estimated mixing matrix
assert np.allclose(X, np.dot(S_, A_.T))

# Plot results
pl.figure()
pl.subplot(3, 1, 1)
pl.plot(S)
pl.title('True Sources')
pl.subplot(3, 1, 2)
pl.plot(X)
pl.title('Observations (mixed signal)')
pl.subplot(3, 1, 3)
pl.plot(S_)
pl.title('ICA estimated sources')
pl.subplots_adjust(0.09, 0.04, 0.94, 0.94, 0.26, 0.36)
pl.show()
Python source code: plot_ica_vs_pca.py

```
print __doc__

# Authors: Alexandre Gramfort, Gael Varoquaux  
# License: BSD

import numpy as np
import pylab as pl

from sklearn.decomposition import PCA, FastICA

# Generate sample data
rng = np.random.RandomState(42)
S = rng.standard_t(1.5, size=(20000, 2))
S[:, 0] *= 2.

# Mix data
A = np.array([[1, 1], [0, 2]])  
# Mixing matrix

X = np.dot(S, A.T)  
# Generate observations

pca = PCA()  
S_pca_ = pca.fit(X).transform(X)
```

2.1. Examples
ica = FastICA(random_state=rng)
S_ica_ = ica.fit(X).transform(X)  # Estimate the sources
S_ica_ /= S_ica_.std(axis=0)

#########################  # Plot results

def plot_samples(S, axis_list=None):
    pl.scatter(S[:, 0], S[:, 1], s=2, marker='o', linewidths=0, zorder=10)
    if axis_list is not None:
        colors = [(0, 0.6, 0), (0.6, 0, 0)]
    for color, axis in zip(colors, axis_list):
        axis /= axis.std()
        x_axis, y_axis = axis
        # Trick to get legend to work
        pl.plot(0.1 * x_axis, 0.1 * y_axis, linewidth=2, color=color)
        # pl.quiver(x_axis, y_axis, x_axis, y_axis, zorder=11, width=0.01,
        pl.quiver(0, 0, x_axis, y_axis, zorder=11, width=0.01,
                  scale=6, color=color)
    pl.hlines(0, -3, 3)
    pl.vlines(0, -3, 3)
    pl.xlim(-3, 3)
    pl.ylim(-3, 3)
    pl.xlabel('x')
    pl.ylabel('y')

pl.subplot(2, 2, 1)
plot_samples(S / S.std())
pl.title('True Independent Sources')
axis_list = [pca.components_.T, ica.get_mixing_matrix()]
pl.subplot(2, 2, 2)
plot_samples(X / np.std(X), axis_list=axis_list)
pl.legend(['PCA', 'ICA'], loc='upper left')
pl.title('Observations')
pl.subplot(2, 2, 3)
plot_samples(S_pca_ / np.std(S_pca_, axis=0))
pl.title('PCA scores')
pl.subplot(2, 2, 4)
plot_samples(S_ica_ / np.std(S_ica_))
pl.title('ICA estimated sources')

pl.subplots_adjust(0.09, 0.04, 0.94, 0.94, 0.26, 0.26)
pl.show()

**Image denoising using dictionary learning**

An example comparing the effect of reconstructing noisy fragments of Lena using online *Dictionary Learning* and various transform methods.

The dictionary is fitted on the non-distorted left half of the image, and subsequently used to reconstruct the right half.
A common practice for evaluating the results of image denoising is by looking at the difference between the reconstruction and the original image. If the reconstruction is perfect this will look like gaussian noise.

It can be seen from the plots that the results of Orthogonal Matching Pursuit (OMP) with two non-zero coefficients is a bit less biased than when keeping only one (the edges look less prominent). It is in addition closer from the ground truth in Frobenius norm.

The result of Least Angle Regression is much more strongly biased: the difference is reminiscent of the local intensity value of the original image.

Thresholding is clearly not useful for denoising, but it is here to show that it can produce a suggestive output with very high speed, and thus be useful for other tasks such as object classification, where performance is not necessarily related to visualisation.
Script output:
Distorting image...
Extracting clean patches...
done in 0.27s.
Learning the dictionary...
done in 9.67s.
Extracting noisy patches...
done in 0.19s.
Orthogonal Matching Pursuit
1 atom ...
done in 6.15s.
Orthogonal Matching Pursuit
2 atoms ...
done in 9.46s.
Least-angle regression
5 atoms ...
done in 50.56s.
Thresholding
alpha=0.1 ...
done in 0.97s.

Python source code: plot_image_denoising.py

```python
print __doc__

from time import time
```
scikit-learn user guide, Release 0.12-git

import pylab as pl
import numpy as np
from scipy.misc import lena
from sklearn.decomposition import MiniBatchDictionaryLearning
from sklearn.feature_extraction.image import extract_patches_2d
from sklearn.feature_extraction.image import reconstruct_from_patches_2d
###############################################################################
# Load Lena image and extract patches
lena = lena() / 256.0
# downsample for higher speed
lena = lena[::2, ::2] + lena[1::2, ::2] + lena[::2, 1::2] + lena[1::2, 1::2]
lena /= 4.0
height, width = lena.shape
# Distort the right half of the image
print ’Distorting image...’
distorted = lena.copy()
distorted[:, height / 2:] += 0.075 * np.random.randn(width, height / 2)
# Extract all clean patches from the left half of the image
print ’Extracting clean patches...’
t0 = time()
patch_size = (7, 7)
data = extract_patches_2d(distorted[:, :height / 2], patch_size)
data = data.reshape(data.shape[0], -1)
data -= np.mean(data, axis=0)
data /= np.std(data, axis=0)
print ’done in %.2fs.’ % (time() - t0)
###############################################################################
# Learn the dictionary from clean patches
print ’Learning the dictionary... ’
t0 = time()
dico = MiniBatchDictionaryLearning(n_atoms=100, alpha=1, n_iter=500)
V = dico.fit(data).components_
dt = time() - t0
print ’done in %.2fs.’ % dt
pl.figure(figsize=(4.2, 4))
for i, comp in enumerate(V[:100]):
pl.subplot(10, 10, i + 1)
pl.imshow(comp.reshape(patch_size), cmap=pl.cm.gray_r,
interpolation=’nearest’)
pl.xticks(())
pl.yticks(())
pl.suptitle(’Dictionary learned from Lena patches\n’ +
’Train time %.1fs on %d patches’ % (dt, len(data)),
fontsize=16)
pl.subplots_adjust(0.08, 0.02, 0.92, 0.85, 0.08, 0.23)

###############################################################################

2.1. Examples

821


# Display the distorted image

```python
def show_with_diff(image, reference, title):
    """Helper function to display denoising"""
    pl.figure(figsize=(5, 3.3))
    pl.subplot(1, 2, 1)
    pl.title('Image')
    pl.imshow(image, vmin=0, vmax=1, cmap=pl.cm.gray, interpolation='nearest')
    pl.xticks(())
    pl.yticks(())
    pl.subplot(1, 2, 2)
    difference = image - reference
    pl.title('Difference (norm: %.2f)' % np.sqrt(np.sum(difference ** 2)))
    pl.imshow(difference, vmin=-0.5, vmax=0.5, cmap=pl.cm.PuOr,
              interpolation='nearest')
    pl.xticks(())
    pl.yticks(())
    pl.suptitle(title, size=16)
    pl.subplots_adjust(0.02, 0.02, 0.98, 0.79, 0.02, 0.2)
    show_with_diff(distorted, lena, 'Distorted image')
```

# Extract noisy patches and reconstruct them using the dictionary

```python
print 'Extracting noisy patches... ' 
for title, transform_algorithm, kwargs in transform_algorithms:
    print title, '... ' 
    reconstructions[title] = lena.copy()
    t0 = time()
    dico.set_params(transform_algorithm=transform_algorithm, **kwargs)
    code = dico.transform(data)
    patches = np.dot(code, V)
    if transform_algorithm == 'threshold':
        patches -= patches.min()
        patches /= patches.max()
    patches += intercept
    patches = patches.reshape(len(data), *patch_size)
```
if transform_algorithm == 'threshold':
    patches -= patches.min()
    patches /= patches.max()
    reconstructions[title][:, height / 2:] = reconstruct_from_patches_2d(
        patches, (width, height / 2))
    dt = time() - t0
    print 'done in %.2fs.' % dt
    show_with_diff(reconstructions[title], lena,
                  title + ' (time: %.1fs)' % dt)

pl.show()

Figure 2.64: Kernel PCA

Kernel PCA

This example shows that Kernel PCA is able to find a projection of the data that makes data linearly separable.
print __doc__

# Authors: Mathieu Blondel
# Andreas Mueller
# License: BSD

import numpy as np
import pylab as pl

from sklearn.decomposition import PCA, KernelPCA
from sklearn.datasets import make_circles

np.random.seed(0)

X, y = make_circles(n_samples=400, factor=.3, noise=.05)

kpca = KernelPCA(kernel="rbf", fit_inverse_transform=True, gamma=10)
X_kpca = kpca.fit_transform(X)
X_back = kpca.inverse_transform(X_kpca)
pca = PCA()
X_pca = pca.fit_transform(X)

# Plot results
These figures aid in illustrating how a the point cloud can be very flat in one direction - which is where PCA would come in to choose a direction that is not flat.
import numpy as np
from scipy import stats
from mpl_toolkits.mplot3d import Axes3D

e = np.exp(1)
np.random.seed(4)
def pdf(x):
    return 0.5 * (stats.norm(scale=0.25 / e).pdf(x) + stats.norm(scale=4 / e).pdf(x))

y = np.random.normal(scale=0.5, size=(30000))
x = np.random.normal(scale=0.5, size=(30000))
z = np.random.normal(scale=0.1, size=len(x))
density = pdf(x) * pdf(y)
pdf_z = pdf(5 * z)
density *= pdf_z

a = x + y
b = 2 * y
c = a - b + z

norm = np.sqrt(a.var() + b.var())
a /= norm
b /= norm
### Plot the figures

```python
# Plot the figures

def plot_figs(fig_num, elev, azim):
    fig = pl.figure(fig_num, figsize=(4, 3))
    pl.clf()
    ax = Axes3D(fig, rect=[0, 0, .95, 1], elev=elev, azim=azim)

    ax.scatter(a[:10], b[:10], c[:10], c=density, marker='+',
               alpha=.4)
    Y = np.c_[a, b, c]
    U, pca_score, V = np.linalg.svd(Y, full_matrices=False)
    x_pca_axis, y_pca_axis, z_pca_axis = V.T * pca_score / pca_score.min()
    x_pca_axis, y_pca_axis, z_pca_axis = 3 * V.T
    x_pca_plane = np.r_[x_pca_axis[:2], -x_pca_axis[1::-1]]
    y_pca_plane = np.r_[y_pca_axis[:2], -y_pca_axis[1::-1]]
    z_pca_plane = np.r_[z_pca_axis[:2], -z_pca_axis[1::-1]]
    x_pca_plane.shape = (2, 2)
    y_pca_plane.shape = (2, 2)
    z_pca_plane.shape = (2, 2)
    ax.plot_surface(x_pca_plane, y_pca_plane, z_pca_plane)
    ax_w_xaxis.set_ticklabels([])
    ax_w_yaxis.set_ticklabels([])
    ax_w_zaxis.set_ticklabels([])

    elev = -40
    azim = -80
    plot_figs(1, elev, azim)

    elev = 30
    azim = 20
    plot_figs(2, elev, azim)

    pl.show()
```

Figure 2.66: **PCA example with Iris Data-set**
PCA example with Iris Data-set

Python source code: plot_pca_iris.py

```python
print __doc__

# Code source: Gael Varoqueux
# License: BSD

import numpy as np
import matplotlib as pl
from mpl_toolkits.mplot3d import Axes3D

from sklearn import decomposition
from sklearn import datasets

np.random.seed(5)

centers = [[1, 1], [-1, -1], [1, -1]]
iris = datasets.load_iris()
X = iris.data
y = iris.target

fig = pl.figure(1, figsize=(4, 3))
pl.clf()
ax = Axes3D(fig, rect=[0, 0, .95, 1], elev=48, azim=134)

pca = decomposition.PCA(n_components=3)
pca.fit(X)
X = pca.transform(X)

for name, label in [('Setosa', 0), ('Versicolour', 1), ('Virginica', 2)]:
    ax.text3D(X[y == label, 0].mean(),
              X[y == label, 1].mean() + 1.5,
              X[y == label, 2].mean(), name,
```

828 Chapter 2. Example Gallery
# Reorder the labels to have colors matching the cluster results
y = np.choose(y, [1, 2, 0]).astype(np.float)
ax.scatter(X[:, 0], X[:, 1], X[:, 2], c=y, cmap=pl.cm.spectral)

x_surf = [X[:, 0].min(), X[:, 0].max(), X[:, 0].min(), X[:, 0].max()]
y_surf = [X[:, 0].max(), X[:, 0].max(), X[:, 0].min(), X[:, 0].min()]
x_surf = np.array(x_surf)
y_surf = np.array(y_surf)
v0 = pca.transform(pca.components_[0])
v0 /= v0[-1]
v1 = pca.transform(pca.components_[1])
v1 /= v1[-1]

ax.w_xaxis.set_ticklabels([])
ax.w_yaxis.set_ticklabels([])
ax.w_zaxis.set_ticklabels([])

pl.show()

---

**Figure 2.67:** Comparison of LDA and PCA 2D projection of Iris dataset

**Comparison of LDA and PCA 2D projection of Iris dataset**

The Iris dataset represents 3 kind of Iris flowers (Setosa, Versicolour and Virginica) with 4 attributes: sepal length, sepal width, petal length and petal width.

Principal Component Analysis (PCA) applied to this data identifies the combination of attributes (principal components, or directions in the feature space) that account for the most variance in the data. Here we plot the different samples on the 2 first principal components.

Linear Discriminant Analysis (LDA) tries to identify attributes that account for the most variance between classes. In particular, LDA, in constrast to PCA, is a supervised method, using known class labels.
explained variance ratio (first two components): [ 0.92461621 0.05301557]

Python source code: plot_pca_vs_lda.py

```python
import pylab as pl

from sklearn import datasets
from sklearn.decomposition import PCA
from sklearn.lda import LDA

iris = datasets.load_iris()
X = iris.data
y = iris.target
target_names = iris.target_names

pca = PCA(n_components=2)
X_r = pca.fit(X).transform(X)

lda = LDA(n_components=2)
X_r2 = lda.fit(X, y).transform(X)

# Percentage of variance explained for each components
print 'explained variance ratio (first two components):', 
     pca.explained_variance_ratio_
Sparse coding with a precomputed dictionary

Transform a signal as a sparse combination of Ricker wavelets. This example visually compares different sparse coding methods using the `sklearn.decomposition.SparseCoder` estimator. The Ricker (also known as mexican hat or the second derivative of a gaussian) is not a particularly good kernel to represent piecewise constant signals like this one. It can therefore be seen how much adding different widths of atoms matters and it therefore motivates learning the dictionary to best fit your type of signals.

The richer dictionary on the right is not larger in size, heavier subsampling is performed in order to stay on the same order of magnitude.
```python
print __doc__

import numpy as np
import matplotlib.pylab as pl

from sklearn.decomposition import SparseCoder

def ricker_function(resolution, center, width):
    """Discrete sub-sampled Ricker (mexican hat) wavelet""
    x = np.linspace(0, resolution - 1, resolution)
    x = (2 / ((np.sqrt(3 * width) * np.pi ** 1 / 4))) * (
        1 - ((x - center) ** 2 / width ** 2)) * np.exp(
        -(x - center) ** 2 / (2 * width ** 2))
    return x

def ricker_matrix(width, resolution, n_atoms):
    """Dictionary of Ricker (mexican hat) wavelets""
    centers = np.linspace(0, resolution - 1, n_atoms)
    D = np.empty((n_atoms, resolution))
    for i, center in enumerate(centers):
        D[i] = ricker_function(resolution, center, width)
    D /= np.sqrt(np.sum(D ** 2, axis=1))[:, np.newaxis]
    return D

resolution = 1024
subsampling = 3  # subsampling factor
width = 100
n_atoms = resolution / subsampling

# Compute a wavelet dictionary
D_fixed = ricker_matrix(width=width, resolution=resolution, n_atoms=n_atoms)
D_multi = np.r_[tuple(ricker_matrix(width=w, resolution=resolution,
                                n_atoms=np.floor(n_atoms / 5))
                    for w in (10, 50, 100, 500, 1000))]

# Generate a signal
y = np.linspace(0, resolution - 1, resolution)
first_quarter = y < resolution / 4
y[first_quarter] = 3.
y[np.logical_not(first_quarter)] = -1.

# List the different sparse coding methods in the following format:
# (title, transform_algorithm, transform_alpha, transform_n_nozero_coefs)
estimators = [('OMP', 'omp', None, 15),
              ('Lasso', 'lasso_cd', 2, None),
              ]

pl.figure(figsize=(13, 6))
for subplot, (D, title) in enumerate(zip((D_fixed, D_multi),
                                         ('fixed width', 'multiple widths'))):
    pl.subplot(1, 2, subplot + 1)
    pl.title('Sparse coding against %s dictionary' % title)
    pl.plot(y, ls='dotted', label='Original signal')
    # Do a wavelet approximation
```

---

**Python source code:** plot_sparse_coding.py

```
print __doc__

import numpy as np
import matplotlib.pylab as pl

from sklearn.decomposition import SparseCoder

def ricker_function(resolution, center, width):
    """Discrete sub-sampled Ricker (mexican hat) wavelet""
    x = np.linspace(0, resolution - 1, resolution)
    x = (2 / ((np.sqrt(3 * width) * np.pi ** 1 / 4))) * (
        1 - ((x - center) ** 2 / width ** 2)) * np.exp(
        -(x - center) ** 2 / (2 * width ** 2))
    return x

def ricker_matrix(width, resolution, n_atoms):
    """Dictionary of Ricker (mexican hat) wavelets""
    centers = np.linspace(0, resolution - 1, n_atoms)
    D = np.empty((n_atoms, resolution))
    for i, center in enumerate(centers):
        D[i] = ricker_function(resolution, center, width)
    D /= np.sqrt(np.sum(D ** 2, axis=1))[:, np.newaxis]
    return D

resolution = 1024
subsampling = 3  # subsampling factor
width = 100
n_atoms = resolution / subsampling

# Compute a wavelet dictionary
D_fixed = ricker_matrix(width=width, resolution=resolution, n_atoms=n_atoms)
D_multi = np.r_[tuple(ricker_matrix(width=w, resolution=resolution,
                                n_atoms=np.floor(n_atoms / 5))
                    for w in (10, 50, 100, 500, 1000))]

# Generate a signal
y = np.linspace(0, resolution - 1, resolution)
first_quarter = y < resolution / 4
y[first_quarter] = 3.
y[np.logical_not(first_quarter)] = -1.

# List the different sparse coding methods in the following format:
# (title, transform_algorithm, transform_alpha, transform_n_nozero_coefs)
estimators = [('OMP', 'omp', None, 15),
              ('Lasso', 'lasso_cd', 2, None),
              ]

pl.figure(figsize=(13, 6))
for subplot, (D, title) in enumerate(zip((D_fixed, D_multi),
                                         ('fixed width', 'multiple widths'))):
    pl.subplot(1, 2, subplot + 1)
    pl.title('Sparse coding against %s dictionary' % title)
    pl.plot(y, ls='dotted', label='Original signal')
    # Do a wavelet approximation
```
for title, algo, alpha, n_nonzero in estimators:
    coder = SparseCoder(dictionary=D, transform_n_nonzero_coefs=n_nonzero,
                         transform_alpha=alpha, transform_algorithm=algo)
    x = coder.transform(y)
    density = len(np.flatnonzero(x))
    x = np.ravel(np.dot(x, D))
    squared_error = np.sum((y - x) ** 2)
    pl.plot(x, label='%s: %s nonzero coefs, %.2f error' %
             (title, density, squared_error))

# Soft thresholding debiasing
    coder = SparseCoder(dictionary=D, transform_algorithm='threshold',
                         transform_alpha=20)
    x = coder.transform(y)
    _, idx = np.where(x != 0)
    x[0, idx], _, _, _ = np.linalg.lstsq(D[idx, :].T, y)
    x = np.ravel(np.dot(x, D))
    squared_error = np.sum((y - x) ** 2)
    pl.plot(x,
            label='Thresholding w/ debiasing: %d nonzero coefs, %.2f error' %
                  (len(idx), squared_error))
pl.axis('tight')
pl.legend()
pl.subplots_adjust(.04, .07, .97, .90, .09, .2)
pl.show()

2.1.7 Ensemble methods

Examples concerning the sklearn.ensemble package.

Figure 2.69: Feature importances with forests of trees

Feature importances with forests of trees

This examples shows the use of forests of trees to evaluate the importance of features on an artificial classification task. The red plots are the feature importances of each individual tree, and the blue plot is the feature importance of the whole forest.

As expected, the knee in the blue plot suggests that 3 features are informative, while the remaining are not.
Script output:

Feature ranking:
1. feature 1 (0.245865)
2. feature 0 (0.194416)
3. feature 2 (0.174455)
4. feature 7 (0.057138)
5. feature 8 (0.055967)
6. feature 4 (0.055516)
7. feature 5 (0.055179)
8. feature 9 (0.054639)
9. feature 3 (0.053921)
10. feature 6 (0.052904)

Python source code: plot_forest_importances.py

```python
print __doc__

import numpy as np

from sklearn.datasets import make_classification
from sklearn.ensemble import ExtraTreesClassifier

# Build a classification task using 3 informative features
X, y = make_classification(n_samples=1000,
                           n_features=10,
                           ...```
n_informative=3,
n_redundant=0,
n_repeated=0,
n_classes=2,
random_state=0,
shuffle=False)

# Build a forest and compute the feature importances
forest = ExtraTreesClassifier(n_estimators=250,
                             compute_importances=True,
                             random_state=0)
forest.fit(X, y)
importances = forest.feature_importances_
indices = np.argsort(importances)[::-1]

# Print the feature ranking
print "Feature ranking:

for f in xrange(10):
    print "%d feature %d (%f)" % (f + 1, indices[f], importances[indices[f]])

# Plot the feature importances of the trees and of the forest
import pylab as pl
pl.figure()
pl.title("Feature importances")
for tree in forest.estimators_:
    pl.plot(xrange(10), tree.feature_importances_[indices], "r")
pl.plot(xrange(10), importances[indices], "b")
pl.show()

Figure 2.70: Pixel importances with a parallel forest of trees

Pixel importances with a parallel forest of trees

This example shows the use of forests of trees to evaluate the importance of the pixels in an image classification task (faces). The hotter the pixel, the more important.

The code below also illustrates how the construction and the computation of the predictions can be parallelized within multiple jobs.
Script output:

Fitting ExtraTreesClassifier on faces data with 1 cores...
done in 25.886s

Python source code: plot_forest_importances_faces.py

```python
print __doc__

from time import time
import pylab as pl

from sklearn.datasets import fetch_olivetti_faces
from sklearn.ensemble import ExtraTreesClassifier

# Number of cores to use to perform parallel fitting of the forest model
n_jobs = 1

# Loading the digits dataset
```
data = fetch_olivetti_faces()
X = data.images.reshape((len(data.images), -1))
y = data.target

mask = y < 5  # Limit to 5 classes
X = X[mask]
y = y[mask]

# Build a forest and compute the pixel importances
print "Fitting ExtraTreesClassifier on faces data with %d cores..." % n_jobs
t0 = time()
forest = ExtraTreesClassifier(n_estimators=1000,
                               max_features=128,
                               compute_importances=True,
                               n_jobs=n_jobs,
                               random_state=0)

forest.fit(X, y)
print "done in %0.3fs" % (time() - t0)
importances = forest.feature_importances_
importances = importances.reshape(data.images[0].shape)

# Plot pixel importances
pl.matshow(importances, cmap=pl.cm.hot)
pl.title("Pixel importances with forests of trees")
pl.show()

Figure 2.71: Plot the decision surfaces of ensembles of trees on the iris dataset

Plot the decision surfaces of ensembles of trees on the iris dataset

Plot the decision surfaces of forests of randomized trees trained on pairs of features of the iris dataset.

This plot compares the decision surfaces learned by a decision tree classifier (first column), by a random forest classifier (second column) and by an extra-trees classifier (third column).

In the first row, the classifiers are built using the sepal width and the sepal length features only, on the second row using the petal length and sepal length only, and on the third row using the petal width and the petal length only.
Decision surfaces of a decision tree, of a random forest, and of an extra-trees classifier

Python source code: plot_forest_iris.py

```python
print __doc__

import numpy as np
import pylab as pl

from sklearn import clone
from sklearn.datasets import load_iris
from sklearn.ensemble import RandomForestClassifier, ExtraTreesClassifier
from sklearn.tree import DecisionTreeClassifier

# Parameters
n_classes = 3
n_estimators = 30
plot_colors = "bry"
plot_step = 0.02

# Load data
iris = load_iris()

plot_idx = 1

for pair in ([0, 1], [0, 2], [2, 3]):
    for model in (DecisionTreeClassifier(), RandomForestClassifier(n_estimators=n_estimators),
                  ExtraTreesClassifier(n_estimators=n_estimators),):
ExtraTreesClassifier(n_estimators=n_estimators):

# We only take the two corresponding features
X = iris.data[:, pair]
y = iris.target

# Shuffle
idx = np.arange(X.shape[0])
np.random.seed(13)
np.random.shuffle(idx)
X = X[idx]
y = y[idx]

# Standardize
mean = X.mean(axis=0)
std = X.std(axis=0)
X = (X - mean) / std

# Train
clf = clone(model)
clf = model.fit(X, y)

# Plot the decision boundary
pl.subplot(3, 3, plot_idx)
xx, yy = np.meshgrid(np.arange(x_min, x_max, plot_step),
np.arange(y_min, y_max, plot_step))

if isinstance(model, DecisionTreeClassifier):
    Z = model.predict(np.c_[xx.ravel(), yy.ravel()])
    Z = Z.reshape(xx.shape)
    cs = pl.contourf(xx, yy, Z, cmap=pl.cm.Paired)
else:
    for tree in model.estimators_:
        Z = tree.predict(np.c_[xx.ravel(), yy.ravel()])
        Z = Z.reshape(xx.shape)
        cs = pl.contourf(xx, yy, Z, alpha=0.1, cmap=pl.cm.Paired)

pl.axis("tight")

# Plot the training points
for i, c in zip(xrange(n_classes), plot_colors):
    idx = np.where(y == i)
    pl.scatter(X[idx, 0], X[idx, 1], c=c, label=iris.target_names[i],
              cmap=pl.cm.Paired)

pl.axis("tight")

plot_idx += 1

pl.suptitle("Decision surfaces of a decision tree, of a random forest, and of "
            "an extra-trees classifier")
pl.show()
Gradient Boosting regression

Demonstrate Gradient Boosting on the boston housing dataset.

This example fits a Gradient Boosting model with least squares loss and 500 regression trees of depth 4.

Script output:

MSE: 6.2736

Python source code: plot_gradient_boosting_regression.py

```
print __doc__

# Author: Peter Prettenhofer <peter.prettenhofer@gmail.com>
#
# License: BSD

import numpy as np
import matplotlib.pyplot as plt
from sklearn import ensemble
from sklearn import datasets
from sklearn.utils import shuffle
from sklearn.metrics import mean_squared_error
```
# Load data
boston = datasets.load_boston()
X, y = shuffle(boston.data, boston.target, random_state=13)
X = X.astype(np.float32)
offset = int(X.shape[0] * 0.9)
X_train, y_train = X[:offset], y[:offset]
X_test, y_test = X[offset:], y[offset:]

# Fit regression model
params = {'n_estimators': 500, 'max_depth': 4, 'min_samples_split': 1,
           'learning_rate': 0.01, 'loss': 'ls'}
clf = ensemble.GradientBoostingRegressor(**params)
clf.fit(X_train, y_train)
mse = mean_squared_error(y_test, clf.predict(X_test))
print("MSE: %.4f" % mse)

# Plot training deviance
# compute test set deviance
test_score = np.zeros((params['n_estimators']), dtype=np.float64)
for i, y_pred in enumerate(clf.staged_decision_function(X_test)):
    test_score[i] = clf.loss_(y_test, y_pred)
pl.figure(figsize=(12, 6))
pl.subplot(1, 2, 1)
pl.title('Deviance')
pl.plot(np.arange(params['n_estimators']) + 1, clf.train_score_, 'b-',
        label='Training Set Deviance')
pl.plot(np.arange(params['n_estimators']) + 1, test_score, 'r-',
        label='Test Set Deviance')
pl.legend(loc='upper right')
pl.xlabel('Boosting Iterations')
pl.ylabel('Deviance')

# Plot feature importance
feature_importance = clf.feature_importances_
# make importances relative to max importance
feature_importance = 100.0 * (feature_importance / feature_importance.max())
sorted_idx = np.argsort(feature_importance)
pos = np.arange(sorted_idx.shape[0]) + .5
pl.xticks(pos, boston.feature_names[sorted_idx], rotation=90)
pl.barh(pos, feature_importance[sorted_idx], align='center')
pl.xlabel('Relative Importance')
pl.title('Variable Importance')
pl.show()

**Gradient Boosting regularization**

Illustration of the effect of different regularization strategies for Gradient Boosting. The example is taken from Hastie et al 2009.
The loss function used is binomial deviance. In combination with shrinkage, stochastic gradient boosting (Sample 0.5) can produce more accurate models. Subsampling without shrinkage usually does poorly.

Python source code: plot_gradient_boosting_regularization.py

```python
print __doc__

# Author: Peter Prettenhofer <peter.prettenhofer@gmail.com>
#
# License: BSD

import numpy as np
import pylab as pl
from sklearn import ensemble
```
from sklearn import datasets

X, y = datasets.make_hastie_10_2(n_samples=12000, random_state=1)
X = X.astype(np.float32)

X_train, X_test = X[:2000], X[2000:]
y_train, y_test = y[:2000], y[2000:]

original_params = {'n_estimators': 1000, 'max_depth': 2, 'random_state': 1, 'min_samples_split': 5}

pl.figure()
for label, color, setting in [('No shrinkage', 'orange', {'learn_rate': 1.0, 'subsample': 1.0}),
                              ('Shrink=0.1', 'turquoise', {'learn_rate': 0.1, 'subsample': 1.0}),
                              ('Sample=0.5', 'blue', {'learn_rate': 1.0, 'subsample': 0.5}),
                              ('Shrink=0.1, Sample=0.5', 'gray', {'learn_rate': 0.1, 'subsample': 0.5})]:
    params = dict(original_params)
    params.update(setting)
    clf = ensemble.GradientBoostingClassifier(**params)
    clf.fit(X_train, y_train)
    # compute test set deviance
    test_deviance = np.zeros((params['n_estimators']), dtype=np.float64)
    for i, y_pred in enumerate(clf.staged_decision_function(X_test)):
        test_deviance[i] = clf.loss_(y_test, y_pred)
    pl.plot(np.arange(test_deviance.shape[0]) + 1, test_deviance, '-', color=color, label=label)

pl.title('Deviance')
pl.legend(loc='upper left')
pl.xlabel('Boosting Iterations')
pl.ylabel('Test Set Deviance')
pl.show()

2.1.8 Tutorial exercices

Exercises for the tutorials

Figure 2.74: Cross-validation on diabetes Dataset Exercise
Cross-validation on diabetes Dataset Exercise

This exercise is used in the Cross-validated estimators part of the Model selection: choosing estimators and their parameters section of the A tutorial on statistical-learning for scientific data processing.

Script output:

[0.10000000000000001, 0.10000000000000001, 0.10000000000000001]

Python source code: plot_cv_diabetes.py

```python
print __doc__

import numpy as np
import pylab as pl

from sklearn import cross_validation, datasets, linear_model

diabetes = datasets.load_diabetes()
X = diabetes.data
y = diabetes.target

lasso = linear_model.Lasso()

alphas = np.logspace(-4, -1, 20)

scores = list()
scores_std = list()

for alpha in alphas:
    lasso.alpha = alpha
    this_scores = cross_validation.cross_val_score(lasso, X, y, n_jobs=1)
    scores.append(np.mean(this_scores))
    scores_std.append(np.std(this_scores))

pl.figure(1, figsize=(2.5, 2))
pl.clf()
pl.axes([.1, .25, .8, .7])
pl.semilogx(alphas, scores)
pl.semilogx(alphas, np.array(scores) + np.array(scores_std) / 20, 'b--')
pl.semilogx(alphas, np.array(scores) - np.array(scores_std) / 20, 'b--')
pl.yticks(())
pl.ylabel('CV score')
pl.xlabel('alpha')
pl.axhline(np.max(scores), linestyle='--', color='.5')
```
Cross-validation on Digits Dataset Exercise

This exercise is used in the Cross-validation generators part of the Model selection: choosing estimators and their parameters section of the A tutorial on statistical-learning for scientific data processing.

Python source code: plot_cv_digits.py

```python
import numpy as np
from sklearn import cross_validation, datasets, svm
digits = datasets.load_digits()
X = digits.data
y = digits.target
svc = svm.SVC()
C_s = np.logspace(1, 10, 10)
scores = list()
scores_std = list()
for C in C_s:
    svc.C = C
    this_scores = cross_validation.cross_val_score(svc, X, y, n_jobs=1)
    scores.append(np.mean(this_scores))
    scores_std.append(np.std(this_scores))
```

Figure 2.75: Cross-validation on Digits Dataset Exercise
pl.clf()
pl.axes([.1, .25, .8, .7])
pl.semilogx(C_s, scores)
pl.semilogx(C_s, np.array(scores) + np.array(scores_std), 'b--')
pl.semilogx(C_s, np.array(scores) - np.array(scores_std), 'b--')
pl.yticks(()
pl.ylabel('CV score')
pl.xlabel('Parameter C')
pl.ylim(0, 1.1)
#pl.axhline(np.max(scores), linestyle='--', color='0.5')
pl.text(C_s[np.argmax(scores)], .9 * np.max(scores), '
pl.show()

Figure 2.76: Digits Classification Exercise

Digits Classification Exercise

This exercise is used in the Classification part of the Supervised learning: predicting an output variable from high-dimensional observations section of the A tutorial on statistical-learning for scientific data processing.

Script output:
KNN score: 0.961111
LogisticRegression score: 0.938889

Python source code: plot_digits_classification_exercise.py

```python
print __doc__

from sklearn import datasets, neighbors, linear_model
digits = datasets.load_digits()
X_digits = digits.data
y_digits = digits.target
n_samples = len(X_digits)
X_train = X_digits[:n_samples]
y_train = y_digits[:n_samples]
X_test = X_digits[n_samples:]
y_test = y_digits[n_samples:]
knn = neighbors.KNeighborsClassifier()
logistic = linear_model.LogisticRegression()

print ('KNN score: %f' %

```
```python
eknn.fit(X_train, y_train).score(X_test, y_test))
print('LogisticRegression score: {:.2f} %
logistic.fit(X_train, y_train).score(X_test, y_test))
```

Figure 2.77: SVM Exercise

**SVM Exercise**

This exercise is used in the *Using kernels* part of the *Supervised learning: predicting an output variable from high-dimensional observations* section of the *A tutorial on statistical-learning for scientific data processing*.

Python source code: `plot_iris_exercise.py`

```python
import numpy as np
import pylab as pl
```

2.1. Examples
from sklearn import datasets, svm

iris = datasets.load_iris()
X = iris.data
y = iris.target

X = X[y != 0, :2]
y = y[y != 0]

n_sample = len(X)

np.random.seed(0)
order = np.random.permutation(n_sample)
X = X[order]
y = y[order].astype(np.float)

X_train = X[:.9 * n_sample]
y_train = y[:.9 * n_sample]
X_test = X[.9 * n_sample:]
y_test = y[.9 * n_sample:]

# fit the model
for fig_num, kernel in enumerate(('linear', 'rbf', 'poly')):
    clf = svm.SVC(kernel=kernel, gamma=10)
    clf.fit(X_train, y_train)

    pl.figure(fig_num)
    pl.clf()
    pl.scatter(X[:, 0], X[:, 1], c=y, zorder=10, cmap=pl.cm.Paired)
    # Circle out the test data
    pl.scatter(X_test[:, 0], X_test[:, 1],
               s=80, facecolors='none', zorder=10)
    pl.axis('tight')
x_min = X[:, 0].min()
x_max = X[:, 0].max()
y_min = X[:, 1].min()
y_max = X[:, 1].max()

    XX, YY = np.mgrid[x_min:x_max:200j, y_min:y_max:200j]
    Z = clf.decision_function(np.c_[XX.ravel(), YY.ravel()])
    Z = Z.reshape(XX.shape)
    pl.pcolormesh(XX, YY, Z > 0, cmap=pl.cm.Paired)
    pl.contour(XX, YY, Z, colors=['k', 'k', 'k'],
               linestyles=['--', '-', '--'],
               levels=[-.5, 0, .5])

    pl.title(kernel)
    pl.show()
Gaussian Processes classification example: exploiting the probabilistic output

A two-dimensional regression exercise with a post-processing allowing for probabilistic classification thanks to the Gaussian property of the prediction.

The figure illustrates the probability that the prediction is negative with respect to the remaining uncertainty in the prediction. The red and blue lines corresponds to the 95% confidence interval on the prediction of the zero level set.

Python source code: plot_gp_probabilistic_classification_after_regression.py

# Author: Vincent Dubourg <vincent.dubourg@gmail.com>
# License: BSD style
import numpy as np
from scipy import stats
from sklearn.gaussian_process import GaussianProcess
from matplotlib import pyplot as pl
from matplotlib import cm

# Standard normal distribution functions
phi = stats.distributions.norm().pdf
PHI = stats.distributions.norm().cdf
PHIinv = stats.distributions.norm().ppf

# A few constants
lim = 8

def g(x):
    
    """The function to predict (classification will then consist in predicting
    whether g(x) <= 0 or not)"
    ""
    return 5. - x[:, 1] - .5 * x[:, 0] ** 2.

# Design of experiments
X = np.array([[[-4.61611719, -6.00099547],
               [4.10469096, 5.32782448],
               [0.00000000, -0.50000000],
               [-6.17289014, -4.6984743],
               [1.3109306, -6.93271427],
               [-6.17289014, 3.10584743],
               [-5.03823144, -5.03823144],
               [5.21301203, 4.26386883]]
             )

# Observations
y = g(X)

# Instantiate and fit Gaussian Process Model
gp = GaussianProcess(theta0=5e-1)

# Don’t perform MLE or you’ll get a perfect prediction for this simple example!
gp.fit(X, y)

# Evaluate real function, the prediction and its MSE on a grid
res = 50
x1, x2 = np.meshgrid(np.linspace(- lim, lim, res),
                     np.linspace(- lim, lim, res))
xx = np.vstack([x1.reshape(x1.size), x2.reshape(x2.size)]).T

y_true = g(xx)
y_pred, MSE = gp.predict(xx, eval_MSE=True)
sigma = np.sqrt(MSE)
y_true = y_true.reshape((res, res))
y_pred = y_pred.reshape((res, res))
sigma = sigma.reshape((res, res))
k = PHIinv(.975)

# Plot the probabilistic classification iso-values using the Gaussian property
# of the prediction
fig = pl.figure(1)
ax = fig.add_subplot(111)
ax.axes.set_aspect('equal')
A simple one-dimensional regression exercise computed in two different ways:

1. A noise-free case with a cubic correlation model
2. A noisy case with a squared Euclidean correlation model

In both cases, the model parameters are estimated using the maximum likelihood principle.

The figures illustrate the interpolating property of the Gaussian Process model as well as its probabilistic nature in the form of a pointwise 95% confidence interval.
Note that the parameter `nugget` is applied as a Tikhonov regularization of the assumed covariance between the training points. In the special case of the squared euclidean correlation model, nugget is mathematically equivalent to a normalized variance: That is

```python
# Author: Vincent Dubourg <vincent.dubourg@gmail.com>
# Jake Vanderplas <vanderplas@astro.washington.edu>
# License: BSD style

import numpy as np
from sklearn.gaussian_process import GaussianProcess
from matplotlib import pyplot as pl

np.random.seed(1)

def f(x):
    """The function to predict.""
    return x * np.sin(x)

# First the noiseless case
X = np.atleast_2d([1., 3., 5., 6., 7., 8.]).T
y = f(X).ravel()

Python source code: plot_gp_regression.py

print __doc__
```

```
Mesh the input space for evaluations of the real function, the prediction and its MSE

\[
x = \text{np.atleast}_2d(\text{np.linspace}(0, 10, 1000)).T
\]

Instanciate a Gaussian Process model

\[
gp = \text{GaussianProcess}(\text{corr}='\text{cubic}', \text{theta0}=1e-2, \text{thetaL}=1e-4, \text{thetaU}=1e-1, \text{random}\_\text{start}=100)
\]

Fit to data using Maximum Likelihood Estimation of the parameters

\[
gp.\text{fit}(X, y)
\]

Make the prediction on the meshed x-axis (ask for MSE as well)

\[
y\_\text{pred}, \text{MSE} = gp.\text{predict}(x, \text{eval}\_\text{MSE}=\text{True})
\]

sigma = np.sqrt(MSE)

Plot the function, the prediction and the 95% confidence interval based on the MSE

\[
\text{fig = pl.figure()}
\]
\[
\text{pl.plot(x, f(x), 'r:', label=u'$f(x) = x\ \sin(x)$')}
\]
\[
\text{pl.plot(X, y, 'r.', markersize=10, label=u'Observations')}
\]
\[
\text{pl.plot(x, y\_pred, 'b-', label=u'Prediction')}
\]
\[
\text{pl.fill(np.concatenate([x, x[::-1]]),}
\]
\[
\text{np.concatenate([y\_pred - 1.9600 * sigma,}
\]
\[
\text{(y\_pred + 1.9600 * sigma)[::-1]])},
\]
\[
\text{alpha=.5, fc='b', ec='None', label='95\% confidence interval')}
\]
\[
\text{pl.xlabel('$x$')}
\]
\[
\text{pl.ylabel('$f(x)$')}
\]
\[
\text{pl.ylim(-10, 20)}
\]
\[
\text{pl.legend(loc='upper left')}
\]

now the noisy case

X = np.linspace(0.1, 9.9, 20)
X = np.atleast_2d(X).T

Observations and noise

y = f(X).ravel()

\[
dy = 0.5 + 1.0 * \text{np.random.random(y.shape)}
\]

\[
n\text{oise} = \text{np.random.normal}(0, dy)
\]

y += noise

Mesh the input space for evaluations of the real function, the prediction and its MSE

\[
x = \text{np.atleast}_2d(\text{np.linspace}(0, 10, 1000)).T
\]

Instanciate a Gaussian Process model

\[
gp = \text{GaussianProcess}(\text{corr}='\text{squared}\_\text{exponential}', \text{theta0}=1e-1,
\]
\[
\text{thetaL}=1e-3, \text{thetaU}=1,
\]
\[
\text{nugget}=(dy / y) ** 2,
\]
\[
\text{random}\_\text{start}=100)
\]

Fit to data using Maximum Likelihood Estimation of the parameters

\[
gp.\text{fit}(X, y)
\]

Make the prediction on the meshed x-axis (ask for MSE as well)

\[
y\_\text{pred}, \text{MSE} = gp.\text{predict}(x, \text{eval}\_\text{MSE}=\text{True})
\]

sigma = np.sqrt(MSE)
Gaussian Processes regression: goodness-of-fit on the ‘diabetes’ dataset

This example consists in fitting a Gaussian Process model onto the diabetes dataset.

The correlation parameters are determined by means of maximum likelihood estimation (MLE). An anisotropic squared exponential correlation model with a constant regression model are assumed. We also used a nugget = 1e-2 in order to account for the (strong) noise in the targets.

We compute then compute a cross-validation estimate of the coefficient of determination (R2) without reperforming MLE, using the set of correlation parameters found on the whole dataset.

Python source code: gp_diabetes_dataset.py

```python
print __doc__
```

# Author: Vincent Dubourg <vincent.dubourg@gmail.com>
# License: BSD style

```python
from sklearn import datasets
from sklearn.gaussian_process import GaussianProcess
from sklearn.cross_validation import cross_val_score, KFold

diabetes = datasets.load_diabetes()
X, y = diabetes.data, diabetes.target

# Instanciate a GP model
gp = GaussianProcess(regr='constant', corr='absolute_exponential',
```

# Fit the GP model to the data performing maximum likelihood estimation
gp.fit(X, y)

# Deactivate maximum likelihood estimation for the cross-validation loop
gp.theta0 = gp.theta  # Given correlation parameter = MLE
gp.thetaL, gp.thetaU = None, None  # None bounds deactivate MLE

# Perform a cross-validation estimate of the coefficient of determination using
# the cross_validation module using all CPUs available on the machine
K = 20  # folds
R2 = cross_val_score(gp, X, y=y, cv=KFold(y.size, K), n_jobs=1).mean()
print("The %d-Folds estimate of the coefficient of determination is R2 = %s" % (K, R2))

2.1.10 Generalized Linear Models

Examples concerning the sklearn.linear_model package.

Figure 2.81: Automatic Relevance Determination Regression (ARD)

Automatic Relevance Determination Regression (ARD)

Fit regression model with Bayesian Ridge Regression.

Compared to the OLS (ordinary least squares) estimator, the coefficient weights are slightly shifted toward zeros, which stabilises them.

The histogram of the estimated weights is very peaked, as a sparsity-inducing prior is implied on the weights.

The estimation of the model is done by iteratively maximizing the marginal log-likelihood of the observations.
Python source code: plot_ard.py

```python
print __doc__

import numpy as np
import pylab as pl
from scipy import stats

from sklearn.linear_model import ARDRegression, LinearRegression
```
# Generating simulated data with Gaussian weights

# Parameters of the example
np.random.seed(0)
n_samples, n_features = 100, 100

# Create gaussian data
X = np.random.randn(n_samples, n_features)

# Create weights with a precision lambda_ of 4.
lambda_ = 4.
w = np.zeros(n_features)

# Only keep 10 weights of interest
relevant_features = np.random.randint(0, n_features, 10)

for i in relevant_features:
    w[i] = stats.norm.rvs(loc=0, scale=1. / np.sqrt(lambda_))

# Create noise with a precision alpha of 50.
alpha_ = 50.
noise = stats.norm.rvs(loc=0, scale=1. / np.sqrt(alpha_), size=n_samples)

# Create the target
y = np.dot(X, w) + noise

# Fit the ARD Regression
clf = ARDRegression(compute_score=True)
clf.fit(X, y)

ols = LinearRegression()
ols.fit(X, y)

# Plot the true weights, the estimated weights and the histogram of the weights
pl.figure(figsize=(6, 5))
pl.title("Weights of the model")
pl.plot(clf.coef_, 'b-', label="ARD estimate")
pl.plot(ols.coef_, 'r--', label="OLS estimate")
pl.plot(w, 'g-', label="Ground truth")
pl.xlabel("Features")
pl.ylabel("Values of the weights")
pl.legend(loc=1)

pl.figure(figsize=(6, 5))
pl.title("Histogram of the weights")
pl.hist(clf.coef_, bins=n_features, log=True)
pl.plot(clf.coef_[relevant_features], 5 * np.ones(len(relevant_features)), 'ro', label="Relevant features")
pl.xlabel("Features")
pl.ylabel("Values of the weights")
pl.legend(loc=1)

pl.figure(figsize=(6, 5))
pl.title("Marginal log-likelihood")
pl.plot(clf.scores_)
pl.xlabel("Iterations")
pl.ylabel("Score")
pl.show()
Bayesian Ridge Regression

Computes a Bayesian Ridge Regression on a synthetic dataset.

Compared to the OLS (ordinary least squares) estimator, the coefficient weights are slightly shifted toward zeros, which stabilises them.

As the prior on the weights is a Gaussian prior, the histogram of the estimated weights is Gaussian.

The estimation of the model is done by iteratively maximizing the marginal log-likelihood of the observations.
```python
import numpy as np
import pylab as pl
from scipy import stats
from sklearn.linear_model import BayesianRidge, LinearRegression

# Generating simulated data with Gaussian weights
np.random.seed(0)
n_samples, n_features = 100, 100
X = np.random.randn(n_samples, n_features)  # Create gaussian data

# Create weights with a precision lambda_ of 4.
lambda_ = 4.
w = np.zeros(n_features)
# Only keep 10 weights of interest
relevant_features = np.random.randint(0, n_features, 10)
for i in relevant_features:
    w[i] = stats.norm.rvs(loc=0, scale=1. / np.sqrt(lambda_))  # Create noise with a precision alpha of 50.
alpha_ = 50.
noise = stats.norm.rvs(loc=0, scale=1. / np.sqrt(alpha_), size=n_samples)  # Create the target
y = np.dot(X, w) + noise

# Fit the Bayesian Ridge Regression and an OLS for comparison
clf = BayesianRidge(compute_score=True)
clf.fit(X, y)

ols = LinearRegression()
ols.fit(X, y)

# Plot true weights, estimated weights and histogram of the weights
pl.figure(figsize=(6, 5))
pl.title("Weights of the model")
pl.plot(clf.coef_, 'b-', label="Bayesian Ridge estimate")
pl.plot(w, 'g-', label="Ground truth")
pl.plot(ols.coef_, 'r--', label="OLS estimate")
```

**Python source code:** plot_bayesian_ridge.py

```python
print __doc__

import numpy as np
import pylab as pl
from scipy import stats
from sklearn.linear_model import BayesianRidge, LinearRegression

np.random.seed(0)
n_samples, n_features = 100, 100
X = np.random.randn(n_samples, n_features)  # Create gaussian data

# Create weights with a precision lambda_ of 4.
lambda_ = 4.
w = np.zeros(n_features)
# Only keep 10 weights of interest
relevant_features = np.random.randint(0, n_features, 10)
for i in relevant_features:
    w[i] = stats.norm.rvs(loc=0, scale=1. / np.sqrt(lambda_))  # Create noise with a precision alpha of 50.
alpha_ = 50.
noise = stats.norm.rvs(loc=0, scale=1. / np.sqrt(alpha_), size=n_samples)  # Create the target
y = np.dot(X, w) + noise

# Fit the Bayesian Ridge Regression and an OLS for comparison
clf = BayesianRidge(compute_score=True)
clf.fit(X, y)

ols = LinearRegression()
ols.fit(X, y)

# Plot true weights, estimated weights and histogram of the weights
pl.figure(figsize=(6, 5))
pl.title("Weights of the model")
pl.plot(clf.coef_, 'b-', label="Bayesian Ridge estimate")
pl.plot(w, 'g-', label="Ground truth")
pl.plot(ols.coef_, 'r--', label="OLS estimate")
```
Logistic Regression 3-class Classifier

Show below is a logistic-regression classifiers decision boundaries on the iris dataset. The datapoints are colored according to their labels.

Python source code: plot_iris_logistic.py

print __doc__
import numpy as np
import matplotlib.pylab as pl
from sklearn import linear_model, datasets

# import some data to play with
iris = datasets.load_iris()
X = iris.data[:, :2]  # we only take the first two features.
Y = iris.target

h = .02  # step size in the mesh

logreg = linear_model.LogisticRegression(C=1e5)

# we create an instance of Neighbours Classifier and fit the data.
logreg.fit(X, Y)

# Plot the decision boundary. For that, we will assign a color to each
# point in the mesh [x_min, m_max]x[y_min, y_max].
x_min, x_max = X[:, 0].min() - .5, X[:, 0].max() + .5
y_min, y_max = X[:, 1].min() - .5, X[:, 1].max() + .5
xx, yy = np.meshgrid(np.arange(x_min, x_max, h), np.arange(y_min, y_max, h))
Z = logreg.predict(np.c_[xx.ravel(), yy.ravel()])

# Put the result into a color plot
Z = Z.reshape(xx.shape)
pl.figure(1, figsize=(4, 3))
pl.pcolormesh(xx, yy, Z, cmap=pl.cm.Paired)

# Plot also the training points
pl.scatter(X[:, 0], X[:, 1], c=Y, edgecolors='k', cmap=pl.cm.Paired)
pl.xlabel('Sepal length')
pl.ylabel('Sepal width')
pl.xlim(xx.min(), xx.max())
pl.ylim(yy.min(), yy.max())
pl.xticks(())
pl.yticks(())
pl.show()
Lasso and Elastic Net for Sparse Signals

Script output:

Lasso(alpha=0.1, copy_X=True, fit_intercept=True, max_iter=1000, normalize=False, positive=False, precompute=auto, tol=0.0001, warm_start=False)
r^2 on test data : 0.384710
ElasticNet(alpha=0.1, copy_X=True, fit_intercept=True, max_iter=1000, normalize=False, positive=False, precompute=auto, rho=0.7, tol=0.0001, warm_start=False)
r^2 on test data : 0.240176

Python source code: plot_lasso_and_elasticnet.py

```
print __doc__

import numpy as np
import pylab as pl

from sklearn.metrics import r2_score

np.random.seed(42)
```
n_samples, n_features = 50, 200
X = np.random.randn(n_samples, n_features)
coef = 3 * np.random.randn(n_features)
inds = np.arange(n_features)
np.random.shuffle(inds)
coef[inds[10:]] = 0  # sparsify coef
y = np.dot(X, coef)

# add noise
y += 0.01 * np.random.normal((n_samples,))

# Split data in train set and test set
n_samples = X.shape[0]
X_train, y_train = X[:n_samples // 2], y[:n_samples // 2]
X_test, y_test = X[n_samples // 2:], y[n_samples // 2:]

# Lasso
from sklearn.linear_model import Lasso
alpha = 0.1
lasso = Lasso(alpha=alpha)
y_pred_lasso = lasso.fit(X_train, y_train).predict(X_test)
r2_score_lasso = r2_score(y_test, y_pred_lasso)
print lasso
print "\text{r}^2 \text{ on test data : } \%f\" % r2_score_lasso

# ElasticNet
from sklearn.linear_model import ElasticNet
enet = ElasticNet(alpha=alpha, rho=0.7)
y_pred_enet = enet.fit(X_train, y_train).predict(X_test)
r2_score_enet = r2_score(y_test, y_pred_enet)
print enet
print "\text{r}^2 \text{ on test data : } \%f\" % r2_score_enet

pl.plot(enet.coef_, label='Elastic net coefficients')
pl.plot(lasso.coef_, label='Lasso coefficients')
pl.plot(coef, '--', label='original coefficients')
pl.legend(loc='best')
pl.title("Lasso R^2: %f, Elastic Net R^2: %f \% (r2_score_lasso, r2_score_enet)"
pl.show()
Lasso and Elastic Net

Lasso and elastic net (L1 and L2 penalisation) implemented using a coordinate descent. The coefficients can be forced to be positive.

Script output:
Computing regularization path using the lasso...
Computing regularization path using the positive lasso...
Computing regularization path using the elastic net...
Computing regularization path using the positive elastic net...

Python source code: plot_lasso_coordinate_descent_path.py
print __doc__

# Author: Alexandre Gramfort <alexandre.gramfort@inria.fr>
# License: BSD Style.

import numpy as np
import pylab as pl

from sklearn.linear_model import lasso_path, enet_path
from sklearn import datasets

diabetes = datasets.load_diabetes()
X = diabetes.data
y = diabetes.target

X /= X.std(0)  # Standardize data (easier to set the rho parameter)

#############################################################################
# Compute paths
eps = 5e-3  # the smaller it is the longer is the path

print "Computing regularization path using the lasso..."
models = lasso_path(X, y, eps=eps)
alphas_lasso = np.array([model.alpha for model in models])
coefs_lasso = np.array([model.coef_ for model in models])

print "Computing regularization path using the positive lasso..."
models = lasso_path(X, y, eps=eps, positive=True)
alphas_positive_lasso = np.array([model.alpha for model in models])
coefs_positive_lasso = np.array([model.coef_ for model in models])

print "Computing regularization path using the elastic net..."
models = enet_path(X, y, eps=eps, rho=0.8)
alphas_enet = np.array([model.alpha for model in models])
coefs_enet = np.array([model.coef_ for model in models])

print "Computing regularization path using the positive elastic net..."
models = enet_path(X, y, eps=eps, rho=0.8, positive=True)
alphas_positive_enet = np.array([model.alpha for model in models])
coefs_positive_enet = np.array([model.coef_ for model in models])

#############################################################################
# Display results
pl.figure(1)
x = pl.gca()
x.set_color_cycle(2 * ["b", "r", "g", "c", "k"])
l1 = pl.plot(coefs_lasso)
l2 = pl.plot(coefs_enet, linestyle="--")

pl.xlabel("-Log(lambda)"
pl.ylabel("weights")
pl.title("Lasso and Elastic-Net Paths")
pl.legend((l1[-1], l2[-1]), ("Lasso", "Elastic-Net"), loc="lower left")
pl.axis("tight")
Lasso path using LARS

Computes Lasso Path along the regularization parameter using the LARS algorithm on the diabetes dataset. Each color represents a different feature of the coefficient vector, and this is displayed as a function of the regularization parameter.
Computing regularization path using the LARS ... 

Python source code: plot_lasso_lars.py

```python
print __doc__

# Author: Fabian Pedregosa <fabian.pedregosa@inria.fr>
# Alexandre Gramfort <alexandre.gramfort@inria.fr>
# License: BSD Style.

import numpy as np
import pylab as pl

from sklearn import linear_model
from sklearn import datasets

diabetes = datasets.load_diabetes()
X = diabetes.data
y = diabetes.target

print "Computing regularization path using the LARS ..."
alphas, _, coefs = linear_model.lars_path(X, y, method='lasso', verbose=True)
xx = np.sum(np.abs(coefs.T), axis=1)
```
xx /= xx[-1]

pl.plot(xx, coefs.T)
ymin, ymax = pl.ylim()
pl.vlines(xx, ymin, ymax, linestyle='dashed')
pl.xlabel('|coef| / max|coef|')
pl.ylabel('Coefficients')
pl.title('LASSO Path')
pl.axis('tight')
pl.show()

Figure 2.87: Lasso model selection: Cross-Validation / AIC / BIC

Lasso model selection: Cross-Validation / AIC / BIC

Use the Akaike information criterion (AIC), the Bayes Information criterion (BIC) and cross-validation to select an optimal value of the regularization parameter alpha of the Lasso estimator.

Results obtained with LassoLarsIC are based on AIC/BIC criteria.

Information-criterion based model selection is very fast, but it relies on a proper estimation of degrees of freedom, are derived for large samples (asymptotic results) and assume the model is correct, i.e. that the data are actually generated by this model. They also tend to break when the problem is badly conditioned (more features than samples).

For cross-validation, we use 20-fold with 2 algorithms to compute the Lasso path: coordinate descent, as implemented by the LassoCV class, and Lars (least angle regression) as implemented by the LassoLarsCV class. Both algorithms give roughly the same results. They differ with regards to their execution speed and sources of numerical errors.

Lars computes a path solution only for each kink in the path. As a result, it is very efficient when there are only of few kinks, which is the case if there are few features or samples. Also, it is able to compute the full path without setting any meta parameter. On the opposite, coordinate descent compute the path points on a pre-specified grid (here we use the default). Thus it is more efficient if the number of grid points is smaller than the number of kinks in the path. Such a strategy can be interesting if the number of features is really large and there are enough samples to select a large amount. In terms of numerical errors, for heavily correlated variables, Lars will accumulate more errors, while the coordinate descent algorithm will only sample the path on a grid.

Note how the optimal value of alpha varies for each fold. This illustrates why nested-cross validation is necessary when trying to evaluate the performance of a method for which a parameter is chosen by cross-validation: this choice of parameter may not be optimal for unseen data.
Script output:

Computing regularization path using the coordinate descent lasso...
Computing regularization path using the Lars lasso...

Python source code: plot_lasso_model_selection.py

```python
print __doc__

# Author: Olivier Grisel, Gael Varoquaux, Alexandre Gramfort
# License: BSD Style.

import time
```

2.1. Examples
import numpy as np
import matplotlib as pl

from sklearn.linear_model import LassoCV, LassoLarsCV, LassoLarsIC
from sklearn import datasets
diabetes = datasets.load_diabetes()
X = diabetes.data
y = diabetes.target
rng = np.random.RandomState(42)
X = np.c_[X, rng.randn(X.shape[0], 14)]  # add some bad features

# normalize data as done by Lars to allow for comparison
X /= np.sqrt(np.sum(X ** 2, axis=0))

# LassoLarsIC: least angle regression with BIC/AIC criterion
model_bic = LassoLarsIC(criterion='bic')
t1 = time.time()
model_bic.fit(X, y)
t_bic = time.time() - t1
alpha_bic_ = model_bic.alpha_

model_aic = LassoLarsIC(criterion='aic')
model_aic.fit(X, y)
alpha_aic_ = model_aic.alpha_

def plot_ic_criterion(model, name, color):
    alpha_ = model.alpha_
    alphas_ = model.alphas_
    criterion_ = model.criterion_
    pl.plot(-np.log10(alphas_), criterion_, '--', color=color,
            linewidth=3, label='

    pl.xlabel('-log(lambda)')
    pl.ylabel('criterion')

pl.figure()
plot_ic_criterion(model_aic, 'AIC', 'b')
plot_ic_criterion(model_bic, 'BIC', 'r')
pl.legend()
pl.title('Information-criterion for model selection (training time %.3fs)' % t_bic)

# LassoCV: coordinate descent

# Compute paths
print "Computing regularization path using the coordinate descent lasso..."
t1 = time.time()
model = LassoCV(cv=20).fit(X, y)
t_lasso_cv = time.time() - t1

# Display results
m_log_alphas = -np.log10(model.alphas)

pl.figure()
ymin, ymax = 2300, 3800
pl.plot(m_log_alphas, model.mse_path_, ':')
pl.plot(m_log_alphas, model.mse_path_.mean(axis=-1), 'k',
label='Average across the folds', linewidth=2)
pl.axvline(-np.log10(model.alpha), linestyle='--', color='k',
label='alpha: CV estimate')
pl.legend()

pl.xlabel('log(lambda)')
pl.ylabel('Mean square error')
pl.title('Mean square error on each fold: coordinate descent '(train time: %.2fs)' % t_lasso_cv)
pl.axis('tight')
pl.ylim(ymin, ymax)
pl.show()

# LassoLarsCV: least angle regression

# Compute paths
print "Computing regularization path using the Lars lasso..."
t1 = time.time()
model = LassoLarsCV(cv=20).fit(X, y)
t_lasso_lars_cv = time.time() - t1

# Display results
m_log_alphas = -np.log10(model.cv_alphas)

pl.figure()
pl.plot(m_log_alphas, model.cv_mse_path_, ':')
pl.plot(m_log_alphas, model.cv_mse_path_.mean(axis=-1), 'k',
label='Average across the folds', linewidth=2)
pl.axvline(-np.log10(model.alpha), linestyle='--', color='k',
label='alpha CV')
pl.legend()

pl.xlabel('log(lambda)')
pl.ylabel('Mean square error')
pl.title('Mean square error on each fold: Lars (train time: %.2fs)' %
t_lasso_lars_cv)
pl.axis('tight')
pl.ylim(ymin, ymax)
pl.show()

Figure 2.88: Logit function
Logit function

Show in the plot is how the logistic regression would, in this synthetic dataset, classify values as either 0 or 1, i.e. class one or two, using the logit-curve.

Python source code: plot_logistic.py

```python
print __doc__

# Code source: Gael Varoqueux
# License: BSD

import numpy as np
import pylab as pl

from sklearn import linear_model

# this is our test set, it’s just a straight line with some
# gaussian noise
xmin, xmax = -5, 5
n_samples = 100
np.random.seed(0)
X = np.random.normal(size=n_samples)
y = (X > 0).astype(np.float)
X[X > 0] *= 4
X += .3 * np.random.normal(size=n_samples)

X = X[:, np.newaxis]

# run the classifier
clf = linear_model.LogisticRegression(C=1e5)
clf.fit(X, y)

# and plot the result
pl.figure(1, figsize=(4, 3))
pl.clf()
pl.scatter(X.ravel(), y, color='black', zorder=20)
X_test = np.linspace(-5, 10, 300)
```

872 Chapter 2. Example Gallery
def model(x):
    return 1 / (1 + np.exp(-x))
loss = model(X_test * clf.coef_ + clf.intercept_).ravel()
pl.plot(X_test, loss, color='blue', linewidth=3)

ols = linear_model.LinearRegression()
ols.fit(X, y)
pl.plot(X_test, ols.coef_ * X_test + ols.intercept_, linewidth=1)
pl.axhline(.5, color='.5')

pl.ylabel('y')
pl.xlabel('X')
pl.xticks()()
pl.yticks()()
pl.ylim(-.25, 1.25)
pl.xlim(-4, 10)
pl.show()

Figure 2.89: L1 Penalty and Sparsity in Logistic Regression

L1 Penalty and Sparsity in Logistic Regression

Comparison of the sparsity (percentage of zero coefficients) of solutions when L1 and L2 penalty are used for different values of C. We can see that large values of C give more freedom to the model. Conversely, smaller values of C constrain the model more. In the L1 penalty case, this leads to sparser solutions.

We classify 8x8 images of digits into two classes: 0-4 against 5-9. The visualization shows coefficients of the models for varying C.
Script output:

C=10.000000
Sparsity with L1 penalty: 6.250000
score with L1 penalty: 0.910406
Sparsity with L2 penalty: 4.687500
score with L2 penalty: 0.909293
C=100.000000
Sparsity with L1 penalty: 4.687500
score with L1 penalty: 0.908737
Sparsity with L2 penalty: 4.687500
score with L2 penalty: 0.909850
C=1000.000000
Sparsity with L1 penalty: 4.687500
score with L1 penalty: 0.910406
Sparsity with L2 penalty: 4.687500
score with L2 penalty: 0.909850

Python source code: plot_logistic_l1_l2_sparsity.py

print __doc__

# Authors: Alexandre Gramfort <alexandre.gramfort@inria.fr>
# Mathieu Blondel <mathieu@mblondel.org>
# Andreas Mueller <amueller@ais.uni-bonn.de>
# License: BSD Style.
import numpy as np
import pylab as pl

from sklearn.linear_model import LogisticRegression
from sklearn import datasets
from sklearn.preprocessing import Scaler

digits = datasets.load_digits()

X, y = digits.data, digits.target
X = Scaler().fit_transform(X)

# classify small against large digits
y = (y > 4).astype(np.int)

# Set regularization parameter
for i, C in enumerate(10. ** np.arange(1, 4):
    # turn down tolerance for short training time
    clf_l1_LR = LogisticRegression(C=C, penalty='l1', tol=0.01)
    clf_l2_LR = LogisticRegression(C=C, penalty='l2', tol=0.01)
    clf_l1_LR.fit(X, y)
    clf_l2_LR.fit(X, y)

    coef_l1_LR = clf_l1_LR.coef_.ravel()
    coef_l2_LR = clf_l2_LR.coef_.ravel()

    # coef_l1_LR contains zeros due to the
    # L1 sparsity inducing norm
    sparsity_l1_LR = np.mean(coef_l1_LR == 0) * 100
    sparsity_l2_LR = np.mean(coef_l2_LR == 0) * 100

    print "C=%f" % C
    print "Sparsity with L1 penalty: %f" % sparsity_l1_LR
    print "score with L1 penalty: %f" % clf_l1_LR.score(X, y)
    print "Sparsity with L2 penalty: %f" % sparsity_l2_LR
    print "score with L2 penalty: %f" % clf_l2_LR.score(X, y)

l1_plot = pl.subplot(3, 2, 2 * i + 1)
l2_plot = pl.subplot(3, 2, 2 * (i + 1))
if i == 0:
    l1_plot.set_title("L1 penalty")
l2_plot.set_title("L2 penalty")

l1_plot.imshow(np.abs(coef_l1_LR.reshape(8, 8)), interpolation='nearest',
cmap='binary', vmin=1, vmax=0)
l2_plot.imshow(np.abs(coef_l2_LR.reshape(8, 8)), interpolation='nearest',
cmap='binary', vmin=1, vmax=0)
pl.text(-8, 3, "C = %d" % C)

l1_plot.set_xticks({})
l1_plot.set_yticks({})
l2_plot.set_xticks({})
l2_plot.set_yticks({})

pl.show()
Figure 2.90: Path with L1-Logistic Regression

Path with L1-Logistic Regression

Computes path on IRIS dataset.

Script output:
Computing regularization path ...
This took 0:00:00.024074

Python source code: plot_logistic_path.py

print __doc__

# Author: Alexandre Gramfort <alexandre.gramfort@inria.fr>
# License: BSD Style.
```python
from datetime import datetime
import numpy as np
import pylab as pl

from sklearn import linear_model
from sklearn import datasets
from sklearn.svm import l1_min_c

iris = datasets.load_iris()
X = iris.data
y = iris.target

X = X[y != 2]
y = y[y != 2]
X -= np.mean(X, 0)

# Demo path functions

cs = l1_min_c(X, y, loss='log') * np.logspace(0, 3)

print "Computing regularization path ..."
start = datetime.now()
clf = linear_model.LogisticRegression(C=1.0, penalty='l1', tol=1e-6)
coefs_ = []
for c in cs:
    clf.set_params(C=c)
    clf.fit(X, y)
    coefs_.append(clf.coef_.ravel().copy())
print "This took ", datetime.now() - start

coefs_ = np.array(coefs_)
pl.plot(np.log10(cs), coefs_)
ymin, ymax = pl.ylim()
pl.xlabel('log(C)')
pl.ylabel('Coefficients')
pl.title('Logistic Regression Path')
pl.axis('tight')
pl.show()
```

Figure 2.91: Linear Regression Example
Linear Regression Example

This example uses the only the first feature of the diabetes dataset, in order to illustrate a two-dimensional plot of this regression technique. The straight line can be seen in the plot, showing how linear regression attempts to draw a straight line that will best minimize the residual sum of squares between the observed responses in the dataset, and the responses predicted by the linear approximation.

The coefficients, the residual sum of squares and the variance score are also calculated.

Script output:

Coefficients:
[ 938.23786125]
Residual sum of squares: 2548.07
Variance score: 0.47

Python source code: plot_ols.py

```
print __doc__

# Code source: Jaques Grobler
# License: BSD

import matplotlib.pyplot as pl
import numpy as np
```

Chapter 2. Example Gallery
from sklearn import datasets, linear_model

# Load the diabetes dataset
diabetes = datasets.load_diabetes()

# Use only one feature
diabetes_X = diabetes.data[:, np.newaxis]
        diabetes_X_temp = diabetes_X[:, :, 2]

# Split the data into training/testing sets
        diabetes_X_train = diabetes_X_temp[:-20]
        diabetes_X_test = diabetes_X_temp[-20:]

from sklearn.datasets.samples_generator import make_regression

# this is our test set, it’s just a straight line with some
# gaussian noise
X, Y = make_regression(n_samples=100, n_features=1, n_informative=1,
        random_state=0, noise=35)

# Split the targets into training/testing sets
        diabetes_y_train = diabetes.target[:-20]
        diabetes_y_test = diabetes.target[-20:]

# Create linear regression object
regr = linear_model.LinearRegression()

# Train the model using the training sets
            regr.fit(diabetes_X_train, diabetes_y_train)

# The coefficients
            print ('Coefficients: 
', regr.coef_)

# The mean square error
            print ('Residual sum of squares: %2f' %
                  np.mean((regr.predict(diabetes_X_test) - diabetes_y_test) ** 2))

# Explained variance score: 1 is perfect prediction
            print ('Variance score: %2f' % regr.score(diabetes_X_test, diabetes_y_test))

# Plot outputs
            pl.scatter(diabetes_X_test, diabetes_y_test, color='black')
            pl.plot(diabetes_X_test, regr.predict(diabetes_X_test), color='blue',
                    linewidth=3)

            pl.xticks(())
            pl.yticks()

            pl.show()

Figure 2.92: Sparsity Example: Fitting only features 1 and 2
Sparsity Example: Fitting only features 1 and 2

Features 1 and 2 of the diabetes-dataset are fitted and plotted below. It illustrates that although feature 2 has a strong coefficient on the full model, it does not give us much regarding $y$ when compared to just feature 1.

Python source code: plot_ols_3d.py

```
import sys
print __doc__

import matplotlib.pyplot as plt
import numpy as np
from mpl_toolkits.mplot3d import Axes3D

from sklearn import datasets, linear_model

diabetes = datasets.load_diabetes()
indices = (0, 1)

X_train = diabetes.data[:-20, indices]
X_test = diabetes.data[-20:, indices]
y_train = diabetes.target[:-20]
y_test = diabetes.target[-20:]

ols = linear_model.LinearRegression()
```
ols.fit(X_train, y_train)

# Plot the figure

```python
def plot_figs(fig_num, elev, azim, X_train, clf):
    fig = plt.figure(fig_num, figsize=(4, 3))
    pl.clf()
    ax = Axes3D(fig, elev=elev, azim=azim)

    ax.scatter(X_train[:, 0], X_train[:, 1], y_train, c='k', marker='+')
    ax.plot_surface(np.array([[-.1, -.1], [.15, .15]]),
                    np.array([[-.1, .15], [-.1, .15]]),
                    clf.predict(np.array([[-.1, -.1, .15, .15],
                                          [-.1, .15, -.1, .15]]).T
                                      ).reshape((2, 2)),
                    alpha=.5)
    ax.set_xlabel('X_1')
    ax.set_ylabel('X_2')
    ax.set_zlabel('Y')
    ax.w_xaxis.set_ticklabels([])
    ax.w_yaxis.set_ticklabels([])
    ax.w_zaxis.set_ticklabels([])

# Generate the three different figures from different views

elev = 43.5
azim = -110
plot_figs(1, elev, azim, X_train, ols)

elev = -.5
azim = 0
plot_figs(2, elev, azim, X_train, ols)

elev = -.5
azim = 90
plot_figs(3, elev, azim, X_train, ols)

pl.show()
```

Figure 2.93: Ordinary Least Squares and Ridge Regression Variance

**Ordinary Least Squares and Ridge Regression Variance**

Due to the few points in each dimension and the straight line that linear regression uses to follow these points as well as it can, noise on the observations will cause great variance as shown in the first plot. Every line’s slope can vary quite a bit for each prediction due to the noise induced in the observations.

Ridge regression is basically minimizing a penalised version of the least-squared function. The penalising shrinks the value of the regression coefficients. Despite the few data points in each dimension, the slope of the prediction is much more stable and the variance in the line itself is greatly reduced, in comparison to that of the standard linear regression
```python
import numpy as np
import matplotlib.pyplot as plt
from sklearn import linear_model

X_train = np.c_[0.5, 1].T
y_train = [0.5, 1]
X_test = np.c_[0, 2].T

np.random.seed(0)

classifiers = dict(
    ols=linear_model.LinearRegression(),
    ridge=linear_model.Ridge(alpha=.1)
)

fignum = 1
for name, clf in classifiers.items():
    fig = plt.figure(fignum, figsize=(4, 3))
    clf.fit(X_train, y_train)
    ax = fig.add_subplot(1, 1, 1)
    for _ in range(6):
        this_X = .1 * np.random.normal(size=(2, 1)) + X_train
        clf.fit(this_X, y_train)
        ax.plot(X_test, clf.predict(X_test), color='.5')
        ax.scatter(this_X, y_train, s=3, c='.5', marker='o', zorder=10)
```

---

Python source code: plot_ols_ridge_variance.py

```python
print __doc__

# Code source: Gael Varoqueux
# Modified for Documentation merge by Jaques Grobler
# License: BSD

import numpy as np
import matplotlib.pyplot as plt
from sklearn import linear_model

X_train = np.c_[0.5, 1].T
y_train = [0.5, 1]
X_test = np.c_[0, 2].T

np.random.seed(0)

classifiers = dict(
    ols=linear_model.LinearRegression(),
    ridge=linear_model.Ridge(alpha=.1)
)

fignum = 1
for name, clf in classifiers.items():
    fig = plt.figure(fignum, figsize=(4, 3))
    clf.fit(X_train, y_train)
    ax = fig.add_subplot(1, 1, 1)
    for _ in range(6):
        this_X = .1 * np.random.normal(size=(2, 1)) + X_train
        clf.fit(this_X, y_train)
        ax.plot(X_test, clf.predict(X_test), color='.5')
        ax.scatter(this_X, y_train, s=3, c='.5', marker='o', zorder=10)
```

---

Chapter 2. Example Gallery
clf.fit(X_train, y_train)
ax.plot(X_test, clf.predict(X_test), linewidth=2, color='blue')
ax.scatter(X_train, y_train, s=30, c='r', marker='+', zorder=10)

ax.set_xticks(())
ax.set_yticks(())
ax.set_ylim((0, 1.6))
ax.set_xlabel('X')
ax.set_ylabel('y')
ax.set_xlim(0, 2)
fignum += 1

pl.show()
Python source code: plot_omp.py

```python
print __doc__

import pylab as pl
import numpy as np
from sklearn.linear_model import orthogonal_mp
from sklearn.datasets import make_sparse_coded_signal

n_components, n_features = 512, 100
n_atoms = 17

# generate the data

# \( y = Dx \)
# \(|x|_0 = n_{atoms} \)

y, D, x = make_sparse_coded_signal(n_samples=1,
                                    n_components=n_components,
                                    n_features=n_features,
                                    n_nonzero_coefs=n_atoms,
                                    random_state=0)

idx, = x.nonzero()
```

884 Chapter 2. Example Gallery
# distort the clean signal
##################################
y_noisy = y + 0.05 * np.random.randn(len(y))

# plot the sparse signal
########################
pl.subplot(3, 1, 1)
pl.xlim(0, 512)
pl.title("Sparse signal")
pl.stem(idx, x[idx])

# plot the noise-free reconstruction
####################################
x_r = orthogonal_mp(D, y, n_atoms)
idx_r, = x_r.nonzero()
pl.subplot(3, 1, 2)
pl.xlim(0, 512)
pl.title("Recovered signal from noise-free measurements")
pl.stem(idx_r, x_r[idx_r])

# plot the noisy reconstruction
###############################
x_r = orthogonal_mp(D, y_noisy, n_atoms)
idx_r, = x_r.nonzero()
pl.subplot(3, 1, 3)
pl.xlim(0, 512)
pl.title("Recovered signal from noisy measurements")
pl.stem(idx_r, x_r[idx_r])
pl.subplots_adjust(0.06, 0.04, 0.94, 0.90, 0.20, 0.38)
pl.suptitle('Sparse signal recovery with Orthogonal Matching Pursuit',
            fontsize=16)
pl.show()

Figure 2.95: Polynomial interpolation

Polynomial interpolation

This example demonstrates how to approximate a function with a polynomial of degree n_degree by using ridge regression. Concretely, from n_samples 1d points, it suffices to build the Vandermonde matrix, which is n_samples x n_degree+1 and has the following form:

\[
[1, x_1, x_1^2, x_1^3, \ldots], [1, x_2, x_2^2, x_2^3, \ldots], \ldots
\]

Intuitively, this matrix can be interpreted as a matrix of pseudo features (the points raised to some power). The matrix is akin to (but different from) the matrix induced by a polynomial kernel.
This example shows that you can do non-linear regression with a linear model, by manually adding non-linear features. Kernel methods extend this idea and can induce very high (even infinite) dimensional feature spaces.

Python source code: plot_polynomial_interpolation.py

```python
print __doc__

# Author: Mathieu Blondel
# License: BSD Style.

import numpy as np
import pylab as pl

from sklearn.linear_model import Ridge

def f(x):
    """ function to approximate by polynomial interpolation""
    return x * np.sin(x)

# generate points used to plot
x_plot = np.linspace(0, 10, 100)

# generate points and keep a subset of them
x = np.linspace(0, 10, 100)
rng = np.random.RandomState(0)
```
rng.shuffle(x)
x = np.sort(x[:20])
y = f(x)

pl.plot(x_plot, f(x_plot), label="ground truth")
pl.scatter(x, y, label="training points")

for degree in [3, 4, 5]:
    ridge = Ridge()
    ridge.fit(np.vander(x, degree + 1), y)
    pl.plot(x_plot, ridge.predict(np.vander(x_plot, degree + 1)),
            label="degree %d" % degree)

pl.legend(loc='lower left')

pl.show()

Figure 2.96: Plot Ridge coefficients as a function of the regularization

Plot Ridge coefficients as a function of the regularization

Shows the effect of collinearity in the coefficients or the Ridge. Each color represents a different feature of the coefficient vector, and this is displayed as a function of the regularization parameter.

At the end of the path, as alpha tends toward zero and the solution tends towards the ordinary least squares, coefficients exhibit big oscillations.
Python source code: plot_ridge_path.py

```
# Author: Fabian Pedregosa -- <fabian.pedregosa@inria.fr>
# License: BSD Style.

print(__doc__)

import numpy as np
import matplotlib.pyplot as pl
from sklearn.linear_model import Ridge

# X is the 10x10 Hilbert matrix
X = 1. / (np.arange(1, 11) + np.arange(0, 10)[:, np.newaxis])
y = np.ones(10)

# Compute paths
n_alphas = 200
alphas = np.logspace(-10, -2, n_alphas)
clf = linear_model.Ridge(fit_intercept=False)

coefs = []
for a in alphas:
    clf.set_params(alpha=a)
    clf.fit(X, y)
```

---

**Ridge coefficients as a function of the regularization**

![Ridge coefficients graph](image.png)

---

888 Chapter 2. Example Gallery
coefs.append(clf.coef_)

# Display results
ax = pl.gca()
ax.set_color_cycle(['b', 'r', 'g', 'c', 'k', 'y', 'm'])

ax.plot(alphas, coefs)
ax.set_xscale('log')
ax.set_xlim(ax.get_xlim()[::-1])  # reverse axis
pl.xlabel('alpha')
pl.ylabel('weights')
pl.title('Ridge coefficients as a function of the regularization')
pl.axis('tight')
pl.show()

Figure 2.97: Plot multi-class SGD on the iris dataset

Plot multi-class SGD on the iris dataset

Plot decision surface of multi-class SGD on iris dataset. The hyperplanes corresponding to the three one-versus-all (OVA) classifiers are represented by the dashed lines.
```python
import numpy as np
import pylab as pl
from sklearn import datasets
from sklearn.linear_model import SGDClassifier

# import some data to play with
iris = datasets.load_iris()
X = iris.data[:, :2]  # we only take the first two features. We could
# avoid this ugly slicing by using a two-dim dataset
y = iris.target
colors = "bry"

# shuffle
idx = np.arange(X.shape[0])
np.random.seed(13)
np.random.shuffle(idx)
X = X[idx]
y = y[idx]

# standardize
mean = X.mean(axis=0)
std = X.std(axis=0)
```

Python source code: `plot_sgd_iris.py`
X = (X - mean) / std

h = .02  # step size in the mesh

clf = SGDClassifier(alpha=0.001, n_iter=100).fit(X, y)

# create a mesh to plot in
x_min, x_max = X[:, 0].min() - 1, X[:, 0].max() + 1
y_min, y_max = X[:, 1].min() - 1, X[:, 1].max() + 1
xx, yy = np.meshgrid(np.arange(x_min, x_max, h),
                     np.arange(y_min, y_max, h))

# Plot the decision boundary. For that, we will assign a color to each
# point in the mesh [x_min, m_max]x[y_min, y_max].
Z = clf.predict(np.c_[xx.ravel(), yy.ravel()])

# Put the result into a color plot
Z = Z.reshape(xx.shape)
cs = pl.contourf(xx, yy, Z, cmap=pl.cm.Paired)
pl.axis('tight')

# Plot also the training points
for i, color in zip(clf.classes_, colors):
    idx = np.where(y == i)
    pl.scatter(X[idx, 0], X[idx, 1], c=color, label=iris.target_names[i],
               cmap=pl.cm.Paired)
pl.title("Decision surface of multi-class SGD")
pl.axis('tight')

# Plot the three one-against-all classifiers
xmin, xmax = pl.xlim()
ymin, ymax = pl.ylim()
coef = clf.coef_
intercept = clf.intercept_

def plot_hyperplane(c, color):
    def line(x0):
        return -(x0 * coef[c, 0]) - intercept[c] / coef[c, 1]

    pl.plot([xmin, xmax], [line(xmin), line(xmax)],
            ls="--", color=color)

for i, color in zip(clf.classes_, colors):
    plot_hyperplane(i, color)
pl.legend()
pl.show()

Figure 2.98: SGD: Convex Loss Functions
SGD: Convex Loss Functions

Plot the convex loss functions supported by `sklearn.linear_model.stochastic_gradient`.

Python source code: plot_sgd_loss_functions.py

```python
print __doc__

import numpy as np
import pylab as pl
from sklearn.linear_model.sgd_fast import Hinge, 
    ModifiedHuber, SquaredLoss

# Define loss functions
xmin, xmax = -3, 3
hinge = Hinge(1)
log_loss = lambda z, p: np.log2(1.0 + np.exp(-z))
modified_huber = ModifiedHuber()
squared_loss = SquaredLoss()

# Plot loss functions
xx = np.linspace(xmin, xmax, 100)
pl.plot([xmin, 0, 0, xmax], [1, 1, 0, 0], 'k-',
    label="Zero-one loss")
```

Chapter 2. Example Gallery
Simple Ordinary Least Squares example with stochastic gradient descent, we draw the linear least squares solution for a random set of points in the plane.

**Ordinary Least Squares with SGD**
```python
print(__doc__)

import pylab as pl

from sklearn.linear_model import SGDRegressor
from sklearn.datasets.samples_generator import make_regression

# this is our test set, it's just a straight line with some
# gaussian noise
X, Y = make_regression(n_samples=100, n_features=1, n_informative=1,
                       random_state=0, noise=35)

# run the classifier
clf = SGDRegressor(alpha=0.1, n_iter=20)
clf.fit(X, Y)

# and plot the result
pl.scatter(X, Y, color='black')
pl.plot(X, clf.predict(X), color='blue', linewidth=3)
pl.show()
```
SGD: Penalties

Plot the contours of the three penalties supported by `sklearn.linear_model.stochastic_gradient`.

Python source code: `plot_sgd_penalties.py`

```python
from __future__ import division
print __doc__

import numpy as np
import pylab as pl

def l1(xs):
```

2.1. Examples
```python
return np.array([np.sqrt((1 - np.sqrt(x ** 2.0)) ** 2.0) for x in xs])

def l2(xs):
    return np.array([np.sqrt(1.0 - x ** 2.0) for x in xs])

def el(xs, z):
    return np.array(
        [(2 - 2 * x - 2 * z + 4 * x * z -
          4 * z ** 2
          - 8 * x * z ** 2
          + 8 * x ** 2 * z ** 2
          - 16 * x ** 2 * z ** 3
          + 8 * x * z ** 3 + 4 * x ** 2 * z ** 4) ** (1. / 2)
          - 2 * x * z ** 2) / (2 - 4 * z) for x in xs])

def cross(ext):
    pl.plot([-ext, ext], [0, 0], "k-")
    pl.plot([0, 0], [-ext, ext], "k-")

xs = np.linspace(0, 1, 100)
alpha = 0.501  # 0.5 division through zero

cross(1.2)

pl.plot(xs, l1(xs), "r-", label="L1")
pl.plot(xs, -1.0 * l1(xs), "r-")
pl.plot(-1 * xs, l1(xs), "r-")
pl.plot(-1 * xs, -1.0 * l1(xs), "r-")

pl.plot(xs, l2(xs), "b-", label="L2")
pl.plot(xs, -1.0 * l2(xs), "b-")
pl.plot(-1 * xs, l2(xs), "b-")
pl.plot(-1 * xs, -1.0 * l2(xs), "b-")

pl.plot(xs, el(xs, alpha), "y-", label="Elastic Net")
pl.plot(xs, -1.0 * el(xs, alpha), "y-")
pl.plot(-1 * xs, el(xs, alpha), "y-")
pl.plot(-1 * xs, -1.0 * el(xs, alpha), "y-")

pl.xlabel(r"$w_0$")
pl.ylabel(r"$w_1$")
pl.legend()
pl.axis("equal")
pl.show()
```

**SGD: Maximum margin separating hyperplane**

Plot the maximum margin separating hyperplane within a two-class separable dataset using a linear Support Vector Machines classifier trained using SGD.
Figure 2.101: *SGD: Maximum margin separating hyperplane*

Python source code: `plot_sgd_separating_hyperplane.py`

```python
print __doc__

import numpy as np
import pylab as pl
from sklearn.linear_model import SGDClassifier
from sklearn.datasets.samples_generator import make_blobs

# we create 50 separable points
X, Y = make_blobs(n_samples=50, centers=2, random_state=0, cluster_std=0.60)

# fit the model
clf = SGDClassifier(loss="hinge", alpha=0.01, n_iter=200, fit_intercept=True)
```

2.1. Examples
clf.fit(X, Y)

# plot the line, the points, and the nearest vectors to the plane
xx = np.linspace(-1, 5, 10)
yy = np.linspace(-1, 5, 10)

X1, X2 = np.meshgrid(xx, yy)
Z = np.empty(X1.shape)
for (i, j), val in np.ndenumerate(X1):
    x1 = val
    x2 = X2[i, j]
    p = clf.decision_function([x1, x2])
    Z[i, j] = p[0]
levels = [-1.0, 0.0, 1.0]
linestyles = ['dashed', 'solid', 'dashed']
colors = 'k'
pl.contour(X1, X2, Z, levels, colors=colors, linestyles=linestyles)
pl.scatter(X[:, 0], X[:, 1], c=Y, cmap=pl.cm.Paired)

pl.axis('tight')
pl.show()

Figure 2.102: SGD: Separating hyperplane with weighted classes

SGD: Separating hyperplane with weighted classes

Fit linear SVMs with and without class weighting. Allows to handle problems with unbalanced classes.
import numpy as np
import matplotlib.pyplot as plt
from sklearn.linear_model import SGDClassifier

# we create 40 separable points
np.random.seed(0)
n_samples_1 = 1000
n_samples_2 = 100
X = np.r_[1.5 * np.random.randn(n_samples_1, 2),
          0.5 * np.random.randn(n_samples_2, 2) + [2, 2]]
y = np.array([0] * (n_samples_1) + [1] * (n_samples_2), dtype=np.float64)
idx = np.arange(y.shape[0])
np.random.shuffle(idx)
X = X[idx]
y = y[idx]
mean = X.mean(axis=0)
std = X.std(axis=0)
X = (X - mean) / std

# fit the model and get the separating hyperplane
clf = SGDClassifier(n_iter=100, alpha=0.01)
clf.fit(X, y)
w = clf.coef_.ravel()
a = -w[0] / w[1]
xx = np.linspace(-5, 5)
yy = a * xx - clf.intercept_ / w[1]

# get the separating hyperplane using weighted classes
wclf = SGDClassifier(n_iter=100, alpha=0.01, class_weight={1: 10})
wclf.fit(X, y)

ww = wclf.coef_.ravel()
wa = -ww[0] / ww[1]
wyy = wa * xx - wclf.intercept_ / ww[1]

# plot separating hyperplanes and samples
h0 = pl.plot(xx, yy, 'k-', label='no weights')
h1 = pl.plot(xx, wyy, 'k--', label='with weights')
pl.scatter(X[:, 0], X[:, 1], c=y, cmap=pl.cm.Paired)
pl.legend()
pl.axis('tight')
pl.show()

Figure 2.103: SGD: Weighted samples

SGD: Weighted samples

Plot decision function of a weighted dataset, where the size of points is proportional to its weight.
Python source code: plot_sgd_weighted_samples.py

```python
import numpy as np
import pylab as pl
from sklearn import linear_model

# we create 20 points
np.random.seed(0)
X = np.r_[np.random.randn(10, 2) + [1, 1], np.random.randn(10, 2)]
y = [1] * 10 + [-1] * 10
sample_weight = 100 * np.abs(np.random.randn(20))
# and assign a bigger weight to the last 10 samples
sample_weight[:10] *= 10

# plot the weighted data points
xx, yy = np.meshgrid(np.linspace(-4, 5, 500), np.linspace(-4, 5, 500))
pl.figure()
pl.scatter(X[:, 0], X[:, 1], c=y, s=sample_weight, alpha=0.9, cmap=pl.cm.bone)

## fit the unweighted model
clf = linear_model.SGDClassifier(alpha=0.01, n_iter=100)
clf.fit(X, y)
Z = clf.decision_function(np.c_[xx.ravel(), yy.ravel()])
```

2.1. Examples
Z = Z.reshape(xx.shape)
no_weights = pl.contour(xx, yy, Z, levels=[0], linestyles=['solid'])

## fit the weighted model
clf = linear_model.SGDClassifier(alpha=0.01, n_iter=100)
clf.fit(X, y, sample_weight=sample_weight)
Z = clf.decision_function(np.c_[xx.ravel(), yy.ravel()])
Z = Z.reshape(xx.shape)
samples_weights = pl.contour(xx, yy, Z, levels=[0], linestyles=['dashed'])

pl.legend([no_weights.collections[0], samples_weights.collections[0]],
           ['no weights', 'with weights'], loc='lower left')
pl.xticks(())
pl.yticks(())
pl.show()
2.1. Examples
Python source code: plot_sparse_recovery.py

```python
print __doc__

# Author: Alexandre Gramfort and Gael Varoquaux
# License: BSD

import pylab as pl
import numpy as np
from scipy.linalg import RandomizedLasso, lasso_stability_path,
                     LassoLarsCV
from sklearn.linear_model import f_regression
from sklearn.feature_selection import mutual_incoherence
from sklearn.preprocessing import Scaler
from sklearn.metrics import auc, precision_recall_curve
from sklearn.ensemble import ExtraTreesRegressor

def mutual_incoherence(X_relevant, X_irelevant):
    """Mutual incoherence, as defined by formula (26a) of [Wainwright2006]."
    projector = np.dot(
        np.dot(X_irelevant.T, X_relevant),
        linalg.pinv(np.dot(X_relevant.T, X_relevant))
    )
    return np.max(np.abs(projector).sum(axis=1))

for conditionning in (1, 1e-4):
    # Simulate regression data with a correlated design
    n_features = 501
    n_relevant_features = 3
    noise_level = .2
    coef_min = .2
    # The Donoho-Tanner phase transition is around n_samples=25: below we
    # will completely fail to recover in the well-conditionned case
    n_samples = 25
    block_size = n_relevant_features

    rng = np.random.RandomState(42)
    # The coefficients of our model
    coef = np.zeros(n_features)
    coef[:n_relevant_features] = coef_min + rng.rand(n_relevant_features)

    # The correlation of our design: variables correlated by blocs of 3
    corr = np.zeros((n_features, n_features))
    for i in range(0, n_features, block_size):
        corr[i:i + block_size, i:i + block_size] = 1 - conditionning
    corr.flat[:n_relevant_features + 1] = 1
    corr = linalg.cholesky(corr)

    # Our design
    X = rng.normal(size=(n_samples, n_features))
    X = np.dot(X, corr)
    # Keep [Wainwright2006] (26c) constant
    X[:n_relevant_features] /= np.abs(coef)[:n_relevant_features]
```

---

904 Chapter 2. Example Gallery
scikit-learn user guide, Release 0.12-git

scikit-learn user guide, Release 0.12-git

linalg.svdvals(X[:n_relevant_features])).max()
X = Scaler().fit_transform(X.copy())

# The output variable
y = np.dot(X, coef)
y /= np.std(y)
# We scale the added noise as a function of the average correlation
# between the design and the output variable
y += noise_level * rng.normal(size=n_samples)
mi = mutual_incoherence(X[:, :n_relevant_features],
                        X[:, n_relevant_features:],)

# Plot stability selection path, using a high eps for early stopping
# of the path, to save computation time
alpha_grid, scores_path = lasso_stability_path(X, y,
                                             random_state=42, eps=0.05)

pl.figure()
# We plot the path as a function of alpha/alpha_max to the power 1/3: the
# power 1/3 scales the path less brutally than the log, and enables to
# see the progression along the path
hg = pl.plot(alpha_grid[1:] ** .333, scores_path[coef != 0].T[1:], 'r')
hb = pl.plot(alpha_grid[1:] ** .333, scores_path[coef == 0].T[1:], 'k')
ymin, ymax = pl.ylim()
pl.xlabel(r'$(\alpha / \alpha_{max})^{1/3}$')
pl.ylabel('Stability score: proportion of times selected')
pl.title('Stability Scores Path - Mutual incoherence: %.1f' % mi)
pl.axis('tight')
pl.legend((hg[0], hb[0]), ('relevant features', 'irrelevant features'),
          loc='best')

# Plot the estimated stability scores for a given alpha
# Use 6-fold cross-validation rather than the default 3-fold: it leads to
# a better choice of alpha:
lars_cv = LassoLarsCV(cv=6).fit(X, y)

# Run the RandomizedLasso: we use a paths going down to .1*alpha_max
# to avoid exploring the regime in which very noisy variables enter
# the model
alphas = np.linspace(lars_cv.alphas_[0], .1 * lars_cv.alphas_[0], 6)
clf = RandomizedLasso(alpha=alphas, random_state=42).fit(X, y)
trees = ExtraTreesRegressor(100, compute_importances=True).fit(X, y)
# Compare with F-score
F, _ = f_regression(X, y)

pl.figure()
for name, score in [('F-test', F),
                    ('Stability selection', clf.scores_),
                    ('Lasso coefs', np.abs(lars_cv.coef_)),
                    ('Trees', trees.feature_importances_),]
    precision, recall, thresholds = precision_recall_curve(coef != 0, score)
    pl.semilogy(np.maximum(score / np.max(score), 1e-4),
                label='%s. AUC: %.3f' % (name, auc(recall, precision)))

2.1. Examples

905
Figure 2.105: Lasso on dense and sparse data

Lasso on dense and sparse data

We show that linear_model.Lasso and linear_model.sparse.Lasso provide the same results and that in the case of sparse data linear_model.sparse.Lasso improves the speed.

Python source code: lasso_dense_vs_sparse_data.py

```python
from time import time
from scipy import sparse
from scipy import linalg

from sklearn.datasets.samples_generator import make_regression
from sklearn.linear_model.sparse import Lasso as SparseLasso
from sklearn.linear_model import Lasso as DenseLasso

# The two Lasso implementations on Dense data
X, y = make_regression(n_samples=200, n_features=5000, random_state=0)
alpha = 1
sparse_lasso = SparseLasso(alpha=alpha, fit_intercept=False, max_iter=1000)
dense_lasso = DenseLasso(alpha=alpha, fit_intercept=False, max_iter=1000)

print "Sparse Lasso done in %fs" % (time() - t0)
```

```
pl.plot(np.where(coef != 0)[0], [2e-4] * n_relevant_features, 'mo',
        label="Ground truth")
pl.xlabel("Features")
pl.ylabel("Score")
# Plot only the 100 first coefficients
pl.xlim(0, 100)
pl.legend(loc='best')
pl.title('Feature selection scores - Mutual incoherence: %.1f' % mi)

pl.show()
```
2.1.11 Manifold learning

Examples concerning the sklearn.manifold package.

Figure 2.106: Comparison of Manifold Learning methods

Comparison of Manifold Learning methods

An illustration of dimensionality reduction on the S-curve dataset with various manifold learning methods.
For a discussion and comparison of these algorithms, see the manifold module page.

Note that the purpose of the MDS is to find a low-dimensional representation of the data (here 2D) in which the distances respect well the distances in the original high-dimensional space, unlike other manifold-learning algorithms, it does not seeks an isotropic representation of the data in the low-dimensional space.

---

**Script output:**

```
standard: 0.2 sec
ltsa: 0.59 sec
hessian: 0.56 sec
modified: 0.49 sec
Isomap: 0.83 sec
MDS: 15 sec
```

**Python source code:** plot_compare_methods.py

```python
# Author: Jake Vanderplas -- <vanderplas@astro.washington.edu>

print __doc__

from time import time
import matplotlib.pyplot as plt
from mpl_toolkits.mplot3d import Axes3D
from matplotlib.ticker import NullFormatter
from sklearn import manifold, datasets
from sklearn.metrics import euclidean_distances

# Next line to silence pyflakes. This import is needed.
Axes3D

n_points = 1000
X, color = datasets.samples_generator.make_s_curve(n_points)
n_neighbors = 10
```
n_components = 2

fig = pl.figure(figsize=(15, 8))
pl.suptitle("Manifold Learning with %i points, %i neighbors" % (1000, n_neighbors), fontsize=14)

try:
    # compatibility matplotlib < 1.0
    ax = fig.add_subplot(241, projection='3d')
    ax.scatter(X[:, 0], X[:, 1], X[:, 2], c=color, cmap=pl.cm.Spectral)
    ax.view_init(4, -72)
except:
    ax = fig.add_subplot(241, projection='3d')
    pl.scatter(X[:, 0], X[:, 2], c=color, cmap=pl.cm.Spectral)

methods = ['standard', 'ltsa', 'hessian', 'modified']
labels = ['LLE', 'LTSA', 'Hessian LLE', 'Modified LLE']

for i, method in enumerate(methods):
    t0 = time()
    Y = manifold.LocallyLinearEmbedding(n_neighbors, n_components,
                                        eigen_solver='auto',
                                        method=method).fit_transform(X)
    t1 = time()
    print "%s: %.2g sec" % (methods[i], t1 - t0)
    ax = fig.add_subplot(242 + i)
    pl.scatter(Y[:, 0], Y[:, 1], c=color, cmap=pl.cm.Spectral)
    pl.title("%s (%.2g sec)" % (labels[i], t1 - t0))
    ax.xaxis.set_major_formatter(NullFormatter())
    ax.yaxis.set_major_formatter(NullFormatter())
    pl.axis('tight')

    t0 = time()
    Y = manifold.Isomap(n_neighbors, n_components).fit_transform(X)
    t1 = time()
    print "Isomap: %.2g sec" % (t1 - t0)
    ax = fig.add_subplot(246)
    pl.scatter(Y[:, 0], Y[:, 1], c=color, cmap=pl.cm.Spectral)
    pl.title("Isomap (%.2g sec)" % (t1 - t0))
    ax.xaxis.set_major_formatter(NullFormatter())
    ax.yaxis.set_major_formatter(NullFormatter())
    pl.axis('tight')

    t0 = time()
    mds = manifold.MDS(n_components, max_iter=100, n_init=1)
    Y = mds.fit_transform(euclidean_distances(X))
    t1 = time()
    print "MDS: %.2g sec" % (t1 - t0)
    ax = fig.add_subplot(247)
    pl.scatter(Y[:, 0], Y[:, 1], c=color, cmap=pl.cm.Spectral)
    pl.title("MDS (%.2g sec)" % (t1 - t0))
    ax.xaxis.set_major_formatter(NullFormatter())
    ax.yaxis.set_major_formatter(NullFormatter())
    pl.axis('tight')

2.1. Examples
Figure 2.107: Manifold learning on handwritten digits: Locally Linear Embedding, Isomap...

Manifold learning on handwritten digits: Locally Linear Embedding, Isomap...

An illustration of various embeddings on the digits dataset.
2.1. Examples
Script output:

Computing random projection
Computing PCA projection
Computing LDA projection
Computing Isomap embedding
Done.
Computing LLE embedding
Done. Reconstruction error: 1.28555e-06
Computing modified LLE embedding
Done. Reconstruction error: 0.359782
Computing Hessian LLE embedding
Done. Reconstruction error: 0.212118
Computing LTSA embedding
Already done.

Python source code: `plot_lle_digits.py`

```python
import numpy as np
import matplotlib.pyplot as plt
from matplotlib import offsetbox

# Authors: Fabian Pedregosa <fabian.pedregosa@inria.fr>
#          Olivier Grisel <olivier.grisel@ensta.org>
#          Mathieu Blondel <mathieu@mblondel.org>
# License: BSD, (C) INRIA 2011

digits = datasets.load_digits(n_class=6)
X = digits.data
y = digits.target
n_samples, n_features = X.shape
n_neighbors = 30

# Scale and visualize the embedding vectors

def plot_embedding(X, title=None):
    x_min, x_max = np.min(X, 0), np.max(X, 0)
    X = (X - x_min) / (x_max - x_min)

    plt.figure()
    ax = plt.subplot(111)
    for i in range(X.shape[0]):
        plt.text(X[i, 0], X[i, 1], str(digits.target[i]),
                 color=pl.cm.Set1(y[i] / 10.),
                 fontdict={'weight': 'bold', 'size': 9})

    if hasattr(offsetbox, 'AnnotationBbox'):
        # only print thumbnails with matplotlib > 1.0
        shown_images = np.array([[1., 1.]])
        # just something big
        for i in range(digits.data.shape[0]):
            dist = np.sum((X[i] - shown_images) ** 2, 1)
            if np.min(dist) < 4e-3:
                # don't show points that are too close
                continue
            shown_images = np.r_[shown_images, [X[i]]]
            imagebox = offsetbox.AnnotationBbox(
                offsetbox.OffsetImage(digits.images[i], cmap=pl.cm.gray_r),
                X[i])
            ax.add_artist(imagebox)

    plt.xticks([]), plt.yticks([])
    if title is not None:
        plt.title(title)
```

2.1. Examples
# Plot images of the digits
N = 20
img = np.zeros((10 * N, 10 * N))
for i in range(N):
    ix = 10 * i + 1
    for j in range(N):
        iy = 10 * j + 1
        img[ix:ix + 8, iy:iy + 8] = X[i * N + j].reshape((8, 8))
pl.imshow(img, cmap=pl.cm.binary)
pl.xticks([])
pl.yticks([])
pl.title('A selection from the 64-dimensional digits dataset')

# Random 2D projection using a random unitary matrix
print "Computing random projection"
rng = np.random.RandomState(42)
Q, _ = qr_economic(rng.normal(size=(n_features, 2)))
X_projected = np.dot(Q.T, X.T).T
plot_embedding(X_projected, "Random Projection of the digits")

# Projection on to the first 2 principal components
print "Computing PCA projection"
t0 = time()
X_pca = decomposition.RandomizedPCA(n_components=2).fit_transform(X)
plot_embedding(X_pca, "Principal Components projection of the digits (time %.2fs)" %
    (time() - t0))

# Projection on to the first 2 linear discriminant components
print "Computing LDA projection"
X2 = X.copy()
X2.flat[:X.shape[1] + 1] += 0.01  # Make X invertible
t0 = time()
X_lda = lda.LDA(n_components=2).fit_transform(X2, y)
plot_embedding(X_lda, "Linear Discriminant projection of the digits (time %.2fs)" %
    (time() - t0))

# Isomap projection of the digits dataset
print "Computing Isomap embedding"
t0 = time()
X_iso = manifold.Isomap(n_neighbors, n_components=2).fit_transform(X)
print "Done."
plot_embedding(X_iso, "Isomap projection of the digits (time %.2fs)" %
    (time() - t0))
# Locally linear embedding of the digits dataset
print "Computing LLE embedding"
c = manifold.LocallyLinearEmbedding(n_neighbors, n_components=2,
                                    method='standard')
t0 = time()
X lle = c.fit_transform(X)
print "Done. Reconstruction error: %.2f" % c.reconstruction_error_
plot_embedding(X lle,
        "Locally Linear Embedding of the digits (time %.2fs)" %
        (time() - t0))

# Modified Locally linear embedding of the digits dataset
print "Computing modified LLE embedding"
c = manifold.LocallyLinearEmbedding(n_neighbors, n_components=2,
                                    method='modified')
t0 = time()
X m lle = c.fit_transform(X)
print "Done. Reconstruction error: %.2f" % c.reconstruction_error_
plot_embedding(X m lle,
        "Modified Locally Linear Embedding of the digits (time %.2fs)" %
        (time() - t0))

# HLLE embedding of the digits dataset
print "Computing Hessian LLE embedding"
c = manifold.LocallyLinearEmbedding(n_neighbors, n_components=2,
                                    method='hessian')
t0 = time()
X h lle = c.fit_transform(X)
print "Done. Reconstruction error: %.2f" % c.reconstruction_error_
plot_embedding(X h lle,
        "Hessian Locally Linear Embedding of the digits (time %.2fs)" %
        (time() - t0))

# LTSA embedding of the digits dataset
print "Computing LTSA embedding"
c = manifold.LocallyLinearEmbedding(n_neighbors, n_components=2,
                                    method='ltsa')
t0 = time()
X ltsa = c.fit_transform(X)
print "Done. Reconstruction error: %.2f" % c.reconstruction_error_
plot_embedding(X ltsa,
        "Local Tangent Space Alignment of the digits (time %.2fs)" %
        (time() - t0))

# MDS embedding of the digits dataset
print "Computing MDS embedding"
c = manifold.MDS(n_components=2, n_init=1, max_iter=100)
t0 = time()
X m ds = c.fit_transform(euclidean_distances(X))
print "Done. Stress: %.2f" % c.stress_
plot_embedding(X_mds,
    "MDS embedding of the digits (time %.2fs)" %
    (time() - t0))

plt.show()

Figure 2.108: Multi-dimensional scaling

Multi-dimensional scaling

An illustration of the metric and non-metric MDS on generated noisy data.

The reconstructed points using the metric MDS and non metric MDS are slightly shifted to avoid overlapping.

Python source code: plot_mds.py
import numpy as np
from matplotlib import pyplot as plt
from matplotlib.collections import LineCollection
from sklearn import manifold
from sklearn.metrics import euclidean_distances
from sklearn.decomposition import PCA

n_samples = 20
seed = np.random.RandomState(seed=3)
X_true = seed.randint(0, 20, 2 * n_samples).astype(np.float)
X_true = X_true.reshape((n_samples, 2))
# Center the data
X_true -= X_true.mean()
similarities = euclidean_distances(X_true)

# Add noise to the similarities
noise = np.random.rand(n_samples, n_samples)
noise = noise + noise.T
noise[np.arange(noise.shape[0]), np.arange(noise.shape[0])] = 0
similarities += noise

mds = manifold.MDS(n_components=2, max_iter=3000,
                    eps=1e-9, random_state=seed,
                    n_jobs=1)
pos = mds.fit(similarities).embedding_

nmds = manifold.MDS(n_components=2, metric=False,
                     max_iter=3000,
                     eps=1e-9, random_state=seed, n_jobs=1)
npos = mds.fit_transform(similarities)

# Rotate the data
clf = PCA(n_components=2)
X_true = clf.fit_transform(X_true)
pos = clf.fit_transform(pos)
npos = clf.fit_transform(pos)

fig = plt.figure(1)
ax = plt.axes([0., 0., 1., 1.])
plt.scatter(X_true[:, 0], X_true[:, 1], c='r', s=20)
plt.scatter(pos[:, 0] + 0.2, pos[:, 1] + 0.2, s=20, c='g')
plt.scatter(npos[:, 0] - 0.2, npos[:, 1] - 0.2, s=20, c='b')
plt.legend(('True position', 'MDS', 'NMDS'), loc='best')
similarities = similarities.max() / similarities * 100
similarities[np.isinf(similarities)] = 0

# Plot the edges
start_idx, end_idx = np.where(pos)
# a sequence of (*line0*, *line1*, *line2*), where::
# linen = (x0, y0), (x1, y1), ... (xm, ym)
segments = [[pos[i, :], pos[j, :]]
    for i in range(len(pos)) for j in range(len(pos))]
values = np.abs(similarities)
lc = LineCollection(segments,
    zorder=0, cmap=plt.cm.hot_r,
    norm=plt.Normalize(0, values.max()))
lc.set_array(similarities.flatten())
lc.set_linewidths(0.5 * np.ones(len(segments)))
ax.add_collection(lc)

plt.show()

Figure 2.109: Swiss Roll reduction with LLE

**Swiss Roll reduction with LLE**

An illustration of Swiss Roll reduction with locally linear embedding
Script output:
Computing LLE embedding
Done. Reconstruction error: 9.68564e-08

Python source code: plot_swissroll.py

```python
# Author: Fabian Pedregosa -- <fabian.pedregosa@inria.fr>
# License: BSD, (C) INRIA 2011

print __doc__

import pylab as pl

# This import is needed to modify the way figure behaves
from mplot_toolkits.mplot3d import Axes3D

# Locally linear embedding of the swiss roll

from sklearn import manifold, datasets
X, color = datasets.samples_generator.make_swiss_roll(n_samples=1500)

print "Computing LLE embedding"
X_r, err = manifold.locally_linear_embedding(X, n_neighbors=12,
                                           n_components=2)
```

2.1. Examples
```python
print "Done. Reconstruction error: \$g\$ \% err

# Plot result

fig = pl.figure()

try:
    # compatibility matplotlib < 1.0
    ax = fig.add_subplot(211, projection='3d')
    ax.scatter(X[:, 0], X[:, 1], X[:, 2], c=color, cmap=pl.cm.Spectral)
except:
    ax = fig.add_subplot(211)
    ax.scatter(X[:, 0], X[:, 2], c=color, cmap=pl.cm.Spectral)

ax.set_title("Original data")
ax.scatter(X_r[:, 0], X_r[:, 1], c=color, cmap=pl.cm.Spectral)
pl.axis('tight')
pl.xticks([]), pl.yticks([])
pl.title('Projected data')
pl.show()
```

### 2.1.12 Gaussian Mixture Models

Examples concerning the `sklearn.mixture` package.

**Figure 2.110: Gaussian Mixture Model Ellipsoids**

**Gaussian Mixture Model Ellipsoids**

Plot the confidence ellipsoids of a mixture of two gaussians with EM and variational dirichlet process.

Both models have access to five components with which to fit the data. Note that the EM model will necessarily use all five components while the DP model will effectively only use as many as are needed for a good fit. This is a property of the Dirichlet Process prior. Here we can see that the EM model splits some components arbitrarily, because it is trying to fit too many components, while the Dirichlet Process model adapts it number of state automatically.

This example doesn’t show it, as we’re in a low-dimensional space, but another advantage of the dirichlet process model is that it can fit full covariance matrices effectively even when there are less examples per cluster than there are dimensions in the data, due to regularization properties of the inference algorithm.
import itertools
import numpy as np
from scipy import linalg
import pylab as pl
import matplotlib as mpl
from sklearn import mixture

# Number of samples per component
n_samples = 500

# Generate random sample, two components
np.random.seed(0)
C = np.array([[0., -0.1], [1.7, .4]])
X = np.r_[np.dot(np.random.randn(n_samples, 2), C),
          .7 * np.random.randn(n_samples, 2) + np.array([-6, 3])]

# Fit a mixture of gaussians with EM using five components
gmm = mixture.GMM(n_components=5, covariance_type='full')
gmm.fit(X)

dpgmm = mixture.DPGMM(n_components=5, covariance_type='full')
dpgmm.fit(X)

color_iter = itertools.cycle(['r', 'g', 'b', 'c', 'm'])

for i, (clf, title) in enumerate([(gmm, 'GMM'),
                                  (dpgmm, 'Dirichlet Process GMM')):
    splot = pl.subplot(2, 1, 1 + i)
    Y_ = clf.predict(X)
    for i, (mean, covar, color) in enumerate(zip(clf.means_, clf._get_covars(), color_iter)):
        v, w = linalg.eigh(covar)
        u = w[0] / linalg.norm(w[0])
        # as the DP will not use every component it has access to
        # unless it needs it, we shouldn’t plot the redundant
        # components.
        if not np.any(Y_ == i):
            continue
        pl.scatter(X[Y_ == i, 0], X[Y_ == i, 1], .8, color=color)
        # Plot an ellipse to show the Gaussian component
        angle = np.arctan(u[1] / u[0])
        angle = 180 * angle / np.pi
        ell = mpl.patches.Ellipse(mean, v[0], v[1], 180 + angle, color=color)
        ell.set_clip_box(splot.bbox)
        ell.set_alpha(0.5)
        splot.add_artist(ell)

pl.xlim(-10, 10)
pl.ylim(-3, 6)
pl.xticks(())
pl.yticks(())
pl.title(title)
pl.show()

Figure 2.111: GMM classification

GMM classification

Demonstration of Gaussian mixture models for classification.

Plots predicted labels on both training and held out test data using a variety of GMM classifiers on the iris dataset.

Compared GMMs with spherical, diagonal, full, and tied covariance matrices in increasing order of performance. Although one would expect full covariance to perform best in general, it is prone to overfitting on small datasets and does not generalize well to held out test data.

On the plots, train data is shown as dots, while test data is shown as crosses. The iris dataset is four-dimensional. Only
the first two dimensions are shown here, and thus some points are separated in other dimensions.

**Python source code:** plot_gmm_classifier.py

```python
print __doc__

# Author: Ron Weiss <ronweiss@gmail.com>, Gael Varoquaux
# License: BSD Style.

# $Id$

import pylab as pl
import matplotlib as mpl
import numpy as np

from sklearn import datasets
from sklearn.cross_validation import StratifiedKFold
from sklearn.mixture import GMM
```

2.1. Examples
```python
def make_ellipses(gmm, ax):
    for n, color in enumerate('rgb'):
        v, w = np.linalg.eigh(gmm._get_covars()[n][:2, :2])
        u = w[0] / np.linalg.norm(w[0])
        angle = 180 * angle / np.pi  # convert to degrees
        v *= 9
        ell = mpl.patches.Ellipse(gmm.means_[n, :2], v[0], v[1],
                                   180 + angle, color=color)
        ell.set_clip_box(ax.bbox)
        ell.set_alpha(0.5)
        ax.add_artist(ell)

iris = datasets.load_iris()

# Break up the dataset into non-overlapping training (75%) and testing
# (25%) sets.
skf = StratifiedKFold(iris.target, k=4)
# Only take the first fold.
train_index, test_index = skf.__iter__().next()

X_train = iris.data[train_index]
y_train = iris.target[train_index]
X_test = iris.data[test_index]
y_test = iris.target[test_index]
n_classes = len(np.unique(y_train))

# Try GMMs using different types of covariances.
classifiers = dict((covar_type, GMM(n_components=n_classes,
                                    covariance_type=covar_type, init_params='wc', n_iter=20))
                   for covar_type in ['spherical', 'diag', 'tied', 'full'])

n_classifiers = len(classifiers)

pl.figure(figsize=(3 * n_classifiers / 2, 6))
pl.subplots_adjust(bottom=.01, top=0.95, hspace=.15, wspace=.05,
                   left=.01, right=.99)

for index, (name, classifier) in enumerate(classifiers.items()):
    # Since we have class labels for the training data, we can
    # initialize the GMM parameters in a supervised manner.
    classifier.means_ = np.array([X_train[y_train == i].mean(axis=0)
                                 for i in xrange(n_classes)])

    # Train the other parameters using the EM algorithm.
    classifier.fit(X_train)

    h = pl.subplot(2, n_classifiers / 2, index + 1)
    make_ellipses(classifier, h)

    for n, color in enumerate('rgb'):
        data = iris.data[iris.target == n]
        pl.scatter(data[:, 0], data[:, 1], 0.8, color=color,
                    label=iris.target_names[n])
    # Plot the test data with crosses
```

---

Chapter 2. Example Gallery
for n, color in enumerate('rgb'):
    data = X_test[y_test == n]
    pl.plot(data[:, 0], data[:, 1], 'x', color=color)

y_train_pred = classifier.predict(X_train)
train_accuracy = np.mean(y_train_pred.ravel() == y_train.ravel()) * 100
pl.text(0.05, 0.9, 'Train accuracy: %.1f' % train_accuracy,
    transform=h.transAxes)

y_test_pred = classifier.predict(X_test)
test_accuracy = np.mean(y_test_pred.ravel() == y_test.ravel()) * 100
pl.text(0.05, 0.8, 'Test accuracy: %.1f' % test_accuracy,
    transform=h.transAxes)

pl.xticks(()
pl.yticks(()
pl.title(name)
pl.legend(loc='lower right', prop=dict(size=12))

pl.show()
import numpy as np
import pylab as pl
from sklearn import mixture

n_samples = 300

# generate random sample, two components
np.random.seed(0)
C = np.array([[0., -0.7], [3.5, .7]])
X_train = np.r_[np.dot(np.random.randn(n_samples, 2), C),
                np.random.randn(n_samples, 2) + np.array([20, 20])]

clf = mixture.GMM(n_components=2, covariance_type='full')
clf.fit(X_train)

x = np.linspace(-20.0, 30.0)
y = np.linspace(-20.0, 40.0)

XX = np.c_[X, Y]
Z = np.log(-clf.eval(XX)[0])
Z = Z.reshape(X.shape)

CS = pl.contour(X, Y, Z)
CB = pl.colorbar(CS, shrink=0.8, extend='both')
```python
pl.scatter(X_train[:, 0], X_train[:, 1], .8)
pl.axis('tight')
pl.show()
```

Figure 2.113: Gaussian Mixture Model Selection

**Gaussian Mixture Model Selection**

This example shows that model selection can be performed with Gaussian Mixture Models using information-theoretic criteria (BIC). Model selection concerns both the covariance type and the number of components in the model. In that case, AIC also provides the right result (not shown to save time), but BIC is better suited if the problem is to identify the right model. Unlike Bayesian procedures, such inferences are prior-free.

In that case, the model with 2 components and full covariance (which corresponds to the true generative model) is selected.
import itertools
import numpy as np
from scipy import linalg
import pylab as pl
import matplotlib as mpl
from sklearn import mixture

# Number of samples per component
n_samples = 500

# Generate random sample, two components
np.random.seed(0)
C = np.array([[0., -0.1], [1.7, .4]])
X = np.r_[np.dot(np.random.randn(n_samples, 2), C),
         .7 * np.random.randn(n_samples, 2) + np.array([-6, 3])]

lowest_bic = np.infty
bic = []
n_components_range = range(1, 7)
cv_types = ['spherical', 'tied', 'diag', 'full']
for cv_type in cv_types:
    for n_components in n_components_range:
        # Fit a mixture of gaussians with EM
        gmm = mixture.GMM(n_components=n_components, covariance_type=cv_type)
        gmm.fit(X)
        bic.append(gmm.bic(X))
        if bic[-1] < lowest_bic:
            lowest_bic = bic[-1]
            best_gmm = gmm
bic = np.array(bic)
color_iter = itertools.cycle(['k', 'r', 'g', 'b', 'c', 'm', 'y'])
clf = best_gmm
bars = []

# Plot the BIC scores
spl = pl.subplot(2, 1, 1)
for i, (cv_type, color) in enumerate(zip(cv_types, color_iter)):
    xpos = np.array(n_components_range) + .2 * (i - 2)
    bars.append(pl.bar(xpos, bic[i * len(n_components_range):
    (i + 1) * len(n_components_range)],
    width=.2, color=color))
pl.xticks(n_components_range)
pl.ylim([bic.min() * 1.01 - .01 * bic.max(), bic.max()])
pl.title('BIC score per model')
xpos = np.mod(bic.argmin(), len(n_components_range)) + .65 +
.2 * np.floor(bic.argmin() / len(n_components_range))
pl.text(xpos, bic.min() * 0.97 + .03 * bic.max(), '*', fontsize=14)
pl.set_xlabel('Number of components')
pl.legend([b[0] for b in bars], cv_types)

# Plot the winner
splot = pl.subplot(2, 1, 2)
Y_ = clf.predict(X)
for i, (mean, covar, color) in enumerate(zip(clf.means_, clf.covars_,
    color_iter)):
    v, w = linalg.eigh(covar)
    if not np.any(Y_ == i):
        continue
    pl.scatter(X[Y_ == i, 0], X[Y_ == i, 1], .8, color=color)
    # Plot an ellipse to show the Gaussian component
    angle = np.arctan2(w[0][1], w[0][0]) # convert to degrees
    v *= 4
    ell = mpl.patches.Ellipse(mean, v[0], v[1], 180 + angle, color=color)
    ell.set_clip_box(splot.bbox)
    ell.set_alpha(.5)
    splot.add_artist(ell)
pl.xlim(-10, 10)
pl.ylim(-3, 6)
pl.xticks(())
pl.yticks(())
pl.title('Selected GMM: full model, 2 components')
pl.subplots_adjust(hspace=.35, bottom=.02)
pl.show()
This example highlights the advantages of the Dirichlet Process: complexity control and dealing with sparse data. The dataset is formed by 100 points loosely spaced following a noisy sine curve. The fit by the GMM class, using the expectation-maximization algorithm to fit a mixture of 10 gaussian components, finds too-small components and very little structure. The fits by the dirichlet process, however, show that the model can either learn a global structure for the data (small alpha) or easily interpolate to finding relevant local structure (large alpha), never falling into the problems shown by the GMM class.

Python source code: plot_gmm_sin.py

```python
import itertools
import numpy as np
```
from scipy import linalg
import numpy as np
import matplotlib as mpl

from sklearn import mixture

# Number of samples per component
n_samples = 100

# Generate random sample following a sine curve
np.random.seed(0)
X = np.zeros((n_samples, 2))
step = 4 * np.pi / n_samples
for i in range(X.shape[0]):
    x = i * step - 6
    X[i, 0] = x + np.random.normal(0, 0.1)
    X[i, 1] = 3 * (np.sin(x) + np.random.normal(0, 0.2))

color_iter = itertools.cycle(['r', 'g', 'b', 'c', 'm'])

for i, (clf, title) in enumerate([(mixture.GMM(n_components=10, covariance_type='full', n_iter=100), "Expectation-maximization"),
                                  (mixture.DPGMM(n_components=10, covariance_type='full', alpha=0.01, n_iter=100), "Dirichlet Process, alpha=0.01"),
                                  (mixture.DPGMM(n_components=10, covariance_type='diag', alpha=100., n_iter=100), "Dirichlet Process, alpha=100.")):
    clf.fit(X)
splot = pl.subplot(3, 1, 1 + i)
Y_ = clf.predict(X)
for i, (mean, covar, color) in enumerate(zip(clf.means_, clf._get_covars(), color_iter)):
    v, w = linalg.eigh(covar)
    u = w[0] / linalg.norm(w[0])
    # as the DP will not use every component it has access to
    # unless it needs it, we shouldn't plot the redundant
    # components.
    if not np.any(Y_ == i):
        continue
    pl.scatter(X[Y_ == i, 0], X[Y_ == i, 1], .8, color=color)

    # Plot an ellipse to show the Gaussian component
    angle = np.arctan(u[1] / u[0])
    angle = 180 * angle / np.pi # convert to degrees
    ell = mpl.patches.Ellipse(mean, v[0], v[1], 180 + angle, color=color)
    ell.set_clip_box(splot.bbox)
    ell.set_alpha(0.5)
    splot.add_artist(ell)

pl.xlim(-6, 4 * np.pi - 6)
pl.ylim(-5, 5)

2.1. Examples
2.1.13 Nearest Neighbors

Examples concerning the `sklearn.neighbors` package.

**Figure 2.115: Nearest Neighbors Classification**

**Nearest Neighbors Classification**

Sample usage of Nearest Neighbors classification. It will plot the decision boundaries for each class.
Python source code: plot_classification.py

```python
print __doc__

import numpy as np
import pylab as pl
from matplotlib.colors import ListedColormap
from sklearn import neighbors, datasets

n_neighbors = 15

# import some data to play with
iris = datasets.load_iris()
X = iris.data[:, :2]  # we only take the first two features. We could
# avoid this ugly slicing by using a two-dim dataset
y = iris.target

h = .02  # step size in the mesh

# Create color maps
cmap_light = ListedColormap(['#FFAAAA', '#AAFFAA', '#AAAAFF'])
cmap_bold = ListedColormap(['#FF0000', '#00FF00', '#0000FF'])

for weights in ['uniform', 'distance']:
    # we create an instance of Neighbours Classifier and fit the data.
    clf = neighbors.KNeighborsClassifier(n_neighbors, weights=weights)
    clf.fit(X, y)

    # Plot the decision boundary. For that, we will assign a color to each
    # point in the mesh [x_min, m_max]x[y_min, y_max].
    x_min, x_max = X[:, 0].min() - 1, X[:, 0].max() + 1
    y_min, y_max = X[:, 1].min() - 1, X[:, 1].max() + 1
    xx, yy = np.meshgrid(np.arange(x_min, x_max, h),
                         np.arange(y_min, y_max, h))
    Z = clf.predict(np.c_[xx.ravel(), yy.ravel()])
    Z = Z.reshape(xx.shape)

    # Put the result into a color plot
    pl.figure()
    pl.pcolormesh(xx, yy, Z, cmap=cmap_light)
    pl.scatter(X[:, 0], X[:, 1], c=y, cmap=cmap_bold)
    pl.title("3-Class classification (k = %i, weights = %s)"
             % (n_neighbors, weights))
    pl.axis('tight')

pl.show()

Nearest Centroid Classification

Sample usage of Nearest Centroid classification. It will plot the decision boundaries for each class.
```

2.1. Examples 933
Figure 2.116: Nearest Centroid Classification

Script output:
None 0.813333333333
0.1 0.826666666667

Python source code: plot_nearest_centroid.py

```python
print __doc__

import numpy as np
import matplotlib as mpl
from matplotlib.colors import ListedColormap
from sklearn import datasets
from sklearn.neighbors import NearestCentroid

n_neighbors = 15

# import some data to play with
```
```
iris = datasets.load_iris()
X = iris.data[:, :2]  # we only take the first two features. We could
                        # avoid this ugly slicing by using a two-dim dataset
y = iris.target

h = .02  # step size in the mesh

# Create color maps
cmap_light = ListedColormap(['#FFAAAA', '#A0FFA0', '#AAAAFF'])
cmap_bold = ListedColormap(['#FF0000', '#00FF00', '#0000FF'])

for shrinkage in [None, 0.1]:
    # we create an instance of Neighbours Classifier and fit the data.
    clf = NearestCentroid(shrink_threshold=shrinkage)
    clf.fit(X, y)
    y_pred = clf.predict(X)
    # Plot the decision boundary. For that, we will asign a color to each
    # point in the mesh [x_min, m_max]x[y_min, y_max].
    x_min, x_max = X[:, 0].min() - 1, X[:, 0].max() + 1
    y_min, y_max = X[:, 1].min() - 1, X[:, 1].max() + 1
    xx, yy = np.meshgrid(np.arange(x_min, x_max, h),
                         np.arange(y_min, y_max, h))
    Z = clf.predict(np.c_[xx.ravel(), yy.ravel()])
    Z = Z.reshape(xx.shape)
    plt.figure()
    plt.pcolormesh(xx, yy, Z, cmap=cmap_light)
    # Plot also the training points
    plt.scatter(X[:, 0], X[:, 1], c=y, cmap=cmap_bold)
    plt.title("3-Class classification (shrink_threshold=\$r\$)"
              % shrinkage)
    plt.axis('tight')
plt.show()
```

Figure 2.117: Nearest Neighbors regression

Nearest Neighbors regression

Demonstrate the resolution of a regression problem using a k-Nearest Neighbor and the interpolation of the target using both barycenter and constant weights.
# Author: Alexandre Gramfort <alexandre.gramfort@inria.fr>
# Fabian Pedregosa <fabian.pedregosa@inria.fr>
#
# License: BSD, (C) INRIA

### Generate sample data

```python
import numpy as np
import matplotlib.pyplot as plt
from sklearn import neighbors

np.random.seed(0)
X = np.sort(5 * np.random.rand(40, 1), axis=0)
T = np.linspace(0, 5, 500)[..., np.newaxis]
y = np.sin(X).ravel
y[::5] += 1.0 * (0.5 - np.random.rand(8))
```

### Fit regression model

```python
```
n_neighbors = 5

for i, weights in enumerate(['uniform', 'distance']):
    knn = neighbors.KNeighborsRegressor(n_neighbors, weights=weights)
    y_ = knn.fit(X, y).predict(T)

    pl.subplot(2, 1, i + 1)
    pl.scatter(X, y, c='k', label='data')
    pl.plot(T, y_, c='g', label='prediction')
    pl.axis('tight')
    pl.legend()
    pl.title('KNeighborsRegressor (k = %i, weights = %s)' % (n_neighbors, weights))

pl.show()

2.1.14 Semi Supervised Classification

Examples concerning the sklearn.semi_supervised package.

Figure 2.118: Label Propagation digits: Demonstrating performance

Label Propagation digits: Demonstrating performance

This example demonstrates the power of semisupervised learning by training a Label Spreading model to classify handwritten digits with sets of very few labels.

The handwritten digit dataset has 1797 total points. The model will be trained using all points, but only 30 will be labeled. Results in the form of a confusion matrix and a series of metrics over each class will be very good.

At the end, the top 10 most uncertain predictions will be shown.
Learning with small amount of labeled data

<table>
<thead>
<tr>
<th>predict</th>
<th>true</th>
<th>Label Spreading model: 30 labeled &amp; 300 unlabeled points (330 total)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>3</td>
<td>precision recall f1-score support</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>0.58 0.54 0.56 28</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>0.96 0.93 0.95 29</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>0.00 0.00 0.00 28</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>0.91 0.80 0.85 25</td>
</tr>
<tr>
<td>5</td>
<td>0</td>
<td>0.96 0.79 0.87 33</td>
</tr>
<tr>
<td>6</td>
<td>0</td>
<td>0.97 0.97 0.97 36</td>
</tr>
<tr>
<td>7</td>
<td>0</td>
<td>0.89 1.00 0.94 34</td>
</tr>
<tr>
<td>8</td>
<td>0</td>
<td>0.48 0.83 0.61 29</td>
</tr>
<tr>
<td>9</td>
<td>0</td>
<td>0.54 0.77 0.64 35</td>
</tr>
<tr>
<td>avg / total</td>
<td>0.73</td>
<td>0.77 0.74 300</td>
</tr>
</tbody>
</table>

Confusion matrix

```
[[23  0  0  0  0  0  0  0  0]
 [ 0 15  1  0  1  0 11  0  0]
 [ 0  0 27  0  0  2  0  0  0]
 [ 0  0 20  0  0  0  0  0  0]
 [ 0  0  0 26  0  0  1  6  0]
 [ 0  0  0  0 35  0  0  0  0]
 [ 0  0  0  0  0 34  0  0  0]
 [ 0  0  0  0  0  0 24  0  0]
 [ 0  0  0  0  0  0  0 27 37]]
```
import numpy as np
import pylab as pl

from scipy import stats
from sklearn import datasets
from sklearn.semi_supervised import label_propagation
from sklearn.metrics import metrics

digits = datasets.load_digits()
X = digits.data[:330]
y = digits.target[:330]
images = digits.images[:330]

n_total_samples = len(y)
indices = np.arange(n_total_samples)
unlabeled_set = indices[n_labeled_points:]

y_train = np.copy(y)
y_train[unlabeled_set] = -1

gamma = 0.25
lp_model = label_propagation.LabelSpreading(gamma=gamma, max_iters=5)
lp_model.fit(X, y_train)
predicted_labels = lp_model.transduction_[unlabeled_set]
true_labels = y[unlabeled_set]

print "Label Spreading model: %d labeled & %d unlabeled points (%d total)" %
      (n_labeled_points, n_total_samples - n_labeled_points, n_total_samples)
print metrics.classification_report(true_labels, predicted_labels)
print "Confusion matrix"
print cm

# calculate uncertainty values for each transduced distribution
pred_entropies = stats.distributions.entropy(lp_model.label_distributions_.T)
# pick the top 10 most uncertain labels
uncertainty_index = np.argsort(pred_entropies)[-10:]

f = plt.figure(figsize=(7, 5))
for index, image_index in enumerate(uncertainty_index):
    image = images[image_index]
    sub = f.add_subplot(2, 5, index + 1)
    sub.imshow(image, cmap=pl.cm.gray_r)
    pl.xticks([])
    pl.yticks([])
    sub.set_title('predict: %i
true: %i' % (lp_model.transduction_[image_index], y[image_index]))

f.suptitle('Learning with small amount of labeled data')
plt.show()
Active learning with Label Propagation.
Rows show 5 most uncertain labels to learn with the next model.

```
model 1
fit with predict: 1  predict: 1  predict: 1  predict: 9  predict: 8
  10 labels true: 4    true: 8    true: 8    true: 3

model 2
fit with predict: 8  predict: 1  predict: 8  predict: 8  predict: 3
  15 labels true: 3    true: 4    true: 3    true: 3

model 3
fit with predict: 9  predict: 7  predict: 2  predict: 2
  20 labels true: 9    true: 1    true: 5    true: 7

model 4
fit with predict: 1  predict: 6  predict: 9  predict: 9
  25 labels true: 1    true: 6    true: 5    true: 5

model 5
fit with predict: 4  predict: 8  predict: 3  predict: 8  predict: 8
  30 labels true: 4    true: 8    true: 3    true: 8
```

Script output:

```
Iteration 0
Label Spreading model: 10 labeled & 320 unlabeled (330 total)
precision    recall    f1-score    support
 0         0.00    0.00    0.00          24
 1         0.49    0.90    0.63          29
 2         0.88    0.97    0.92          31
 3         0.00    0.00    0.00          28
 4         0.00    0.00    0.00          27
 5         0.89    0.49    0.63          35
 6         0.86    0.95    0.90          40
 7         0.75    0.92    0.83          36
 8         0.54    0.79    0.64          33
 9         0.41    0.86    0.56          37

avg / total    0.52    0.63    0.55         320

Confusion matrix
[[26  1  0  0  1  0  1]
 [ 1 30  0  0  0  0  0]
 [ 0  0 17  6  0  2 10]
 [ 2  0  0 38  0  0  0]
 [ 0  3  0  0 33  0  0]
 [ 7  0  0  0  0 26  0]]
```
Iteration 1

Label Spreading model: 15 labeled & 315 unlabeled (330 total)

<table>
<thead>
<tr>
<th></th>
<th>precision</th>
<th>recall</th>
<th>f1-score</th>
<th>support</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>23</td>
</tr>
<tr>
<td>1</td>
<td>0.61</td>
<td>0.59</td>
<td>0.60</td>
<td>29</td>
</tr>
<tr>
<td>2</td>
<td>0.91</td>
<td>0.97</td>
<td>0.94</td>
<td>31</td>
</tr>
<tr>
<td>3</td>
<td>1.00</td>
<td>0.56</td>
<td>0.71</td>
<td>27</td>
</tr>
<tr>
<td>4</td>
<td>0.79</td>
<td>0.88</td>
<td>0.84</td>
<td>26</td>
</tr>
<tr>
<td>5</td>
<td>0.89</td>
<td>0.46</td>
<td>0.60</td>
<td>35</td>
</tr>
<tr>
<td>6</td>
<td>0.86</td>
<td>0.95</td>
<td>0.90</td>
<td>40</td>
</tr>
<tr>
<td>7</td>
<td>0.97</td>
<td>0.92</td>
<td>0.94</td>
<td>36</td>
</tr>
<tr>
<td>8</td>
<td>0.54</td>
<td>0.84</td>
<td>0.66</td>
<td>31</td>
</tr>
<tr>
<td>9</td>
<td>0.70</td>
<td>0.81</td>
<td>0.75</td>
<td>37</td>
</tr>
</tbody>
</table>

avg / total 0.82 0.80 0.79 315

Confusion matrix

```
[[23  0  0  0  0  0  0  0  0  0]
 [ 0 17  1  0  2  0  0  1  7  1]
 [ 0  1 30  0  0  0  0  0  0  0]
 [ 0  0  0 15  0  0  0  0  10  2]
 [ 0  3  0  0 23  0  0  0  0  0]
 [ 0  0  0  0  1 16  6  0  2 10]
 [ 0  2  0  0  0  0 38  0  0  0]
 [ 0  0  2  0  1 0  0 33  0  0]
 [ 0  5  0  0  0  0  0  0 26  0]
 [ 0  0  0  0  2 2  0  0 30  0]]
```

Iteration 2

Label Spreading model: 20 labeled & 310 unlabeled (330 total)

<table>
<thead>
<tr>
<th></th>
<th>precision</th>
<th>recall</th>
<th>f1-score</th>
<th>support</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>23</td>
</tr>
<tr>
<td>1</td>
<td>0.68</td>
<td>0.59</td>
<td>0.63</td>
<td>29</td>
</tr>
<tr>
<td>2</td>
<td>0.91</td>
<td>0.97</td>
<td>0.94</td>
<td>31</td>
</tr>
<tr>
<td>3</td>
<td>0.96</td>
<td>1.00</td>
<td>0.98</td>
<td>23</td>
</tr>
<tr>
<td>4</td>
<td>0.81</td>
<td>1.00</td>
<td>0.89</td>
<td>25</td>
</tr>
<tr>
<td>5</td>
<td>0.89</td>
<td>0.46</td>
<td>0.60</td>
<td>35</td>
</tr>
<tr>
<td>6</td>
<td>0.86</td>
<td>0.95</td>
<td>0.90</td>
<td>40</td>
</tr>
<tr>
<td>7</td>
<td>0.97</td>
<td>0.92</td>
<td>0.94</td>
<td>36</td>
</tr>
<tr>
<td>8</td>
<td>0.68</td>
<td>0.84</td>
<td>0.75</td>
<td>31</td>
</tr>
<tr>
<td>9</td>
<td>0.75</td>
<td>0.81</td>
<td>0.78</td>
<td>37</td>
</tr>
</tbody>
</table>

avg / total 0.85 0.84 0.83 310

Confusion matrix

```
[[23  0  0  0  0  0  0  0  0  0]
 [ 0 17  1  0  2  0  0  1  7  1]
 [ 0  1 30  0  0  0  0  0  0  0]
 [ 0  0  0 23  0  0  0  0  0  0]
 [ 0  0  0  0 25  0  0  0  0  0]
 [ 0  0  0  0  1 16  6  0  2 10]
 [ 0  2  0  0  0  0 38  0  0  0]
 [ 0  2  0  0  1 0  0 33  0  0]
 [ 0  5  0  0  0  0  0  0 26  0]
 [ 0  0  0  0  2 2  0  0 30  0]]
```

Iteration 3
Label Spreading model: 25 labeled & 305 unlabeled (330 total)

<table>
<thead>
<tr>
<th></th>
<th>precision</th>
<th>recall</th>
<th>f1-score</th>
<th>support</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>23</td>
</tr>
<tr>
<td>1</td>
<td>0.70</td>
<td>0.85</td>
<td>0.77</td>
<td>27</td>
</tr>
<tr>
<td>2</td>
<td>1.00</td>
<td>0.90</td>
<td>0.95</td>
<td>31</td>
</tr>
<tr>
<td>3</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>23</td>
</tr>
<tr>
<td>4</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>25</td>
</tr>
<tr>
<td>5</td>
<td>0.96</td>
<td>0.74</td>
<td>0.83</td>
<td>34</td>
</tr>
<tr>
<td>6</td>
<td>1.00</td>
<td>0.95</td>
<td>0.97</td>
<td>40</td>
</tr>
<tr>
<td>7</td>
<td>0.90</td>
<td>1.00</td>
<td>0.95</td>
<td>35</td>
</tr>
<tr>
<td>8</td>
<td>0.83</td>
<td>0.81</td>
<td>0.82</td>
<td>31</td>
</tr>
<tr>
<td>9</td>
<td>0.75</td>
<td>0.83</td>
<td>0.79</td>
<td>36</td>
</tr>
</tbody>
</table>

avg / total 0.91 0.90 0.90 305

Confusion matrix

```python
[[23 0 0 0 0 0 0 0 0 0
  [ 0 23 0 0 0 0 0 0 4 0
  [ 0 1 28 0 0 0 0 2 0 0
  [ 0 0 0 23 0 0 0 0 0 0
  [ 0 0 0 0 25 0 0 0 0 0
  [ 0 0 0 0 0 25 0 0 9 0
  [ 0 2 0 0 0 0 38 0 0 0
  [ 0 0 0 0 0 0 35 0 0 0
  [ 0 5 0 0 0 0 0 25 1 0
  [ 0 2 0 0 0 1 0 2 130]]
```

Iteration 4

Label Spreading model: 30 labeled & 300 unlabeled (330 total)

<table>
<thead>
<tr>
<th></th>
<th>precision</th>
<th>recall</th>
<th>f1-score</th>
<th>support</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>23</td>
</tr>
<tr>
<td>1</td>
<td>0.77</td>
<td>0.88</td>
<td>0.82</td>
<td>26</td>
</tr>
<tr>
<td>2</td>
<td>1.00</td>
<td>0.90</td>
<td>0.95</td>
<td>31</td>
</tr>
<tr>
<td>3</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>23</td>
</tr>
<tr>
<td>4</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>25</td>
</tr>
<tr>
<td>5</td>
<td>0.94</td>
<td>0.97</td>
<td>0.95</td>
<td>32</td>
</tr>
<tr>
<td>6</td>
<td>1.00</td>
<td>0.97</td>
<td>0.99</td>
<td>39</td>
</tr>
<tr>
<td>7</td>
<td>0.90</td>
<td>1.00</td>
<td>0.95</td>
<td>35</td>
</tr>
<tr>
<td>8</td>
<td>0.89</td>
<td>0.81</td>
<td>0.85</td>
<td>31</td>
</tr>
<tr>
<td>9</td>
<td>0.94</td>
<td>0.89</td>
<td>0.91</td>
<td>35</td>
</tr>
</tbody>
</table>

avg / total 0.94 0.94 0.94 300

Confusion matrix

```python
[[23 0 0 0 0 0 0 0 0 0
  [ 0 23 0 0 0 0 0 0 3 0
  [ 0 1 28 0 0 0 0 2 0 0
  [ 0 0 0 23 0 0 0 0 0 0
  [ 0 0 0 0 25 0 0 0 0 0
  [ 0 0 0 0 31 0 0 0 1 0
  [ 0 1 0 0 0 0 38 0 0 0
  [ 0 0 0 0 0 0 35 0 0 0
  [ 0 5 0 0 0 0 0 25 1 0
  [ 0 0 0 0 0 0 2 0 2 130]]
```

Python source code: plot_label_propagation_digits_active_learning.py

2.1. Examples 943
import numpy as np
import pylab as pl
from scipy import stats
from sklearn import datasets
from sklearn.semi_supervised import label_propagation
from sklearn.metrics import classification_report, confusion_matrix

digits = datasets.load_digits()
rng = np.random.RandomState(0)
indices = np.arange(len(digits.data))
rng.shuffle(indices)

X = digits.data[indices[:330]]
y = digits.target[indices[:330]]
images = digits.images[indices[:330]]

n_total_samples = len(y)
n_labeled_points = 10

unlabeled_indices = np.arange(n_total_samples)[n_labeled_points:]

f = pl.figure()
for i in range(5):
    y_train = np.copy(y)
    y_train[unlabeled_indices] = -1

    lp_model = label_propagation.LabelSpreading(gamma=0.25, max_iters=5)
    lp_model.fit(X, y_train)

    predicted_labels = lp_model.transduction_[unlabeled_indices]
    true_labels = y[unlabeled_indices]

    cm = confusion_matrix(true_labels, predicted_labels,
                           labels=lp_model.classes_)

    print('Iteration %i ' + 70 * '_') % i
    print("Label Spreading model: \$d labeled & \$d unlabeled (\$d total)" %
          (n_labeled_points, n_total_samples - n_labeled_points, n_total_samples))
    print(classification_report(true_labels, predicted_labels))
    print "Confusion matrix"
    print cm

    # compute the entropies of transduced label distributions
    pred_entropies = stats.distributions.entropy(
        lp_model.label_distributions_.T)

    # select five digit examples that the classifier is most uncertain about
    uncertainty_index = np.argsort(pred_entropies)[-5:]

    # keep track of indicies that we get labels for
delete_indices = np.array([])

f.text(.05, (1 - (i + 1) * .183),
    "model $\hat{y}$ fit with $\hat{y}$ labels" % ((i + 1), i * 5 + 10), size=10)
for index, image_index in enumerate(uncertainty_index):
    image = images[image_index]
    sub = f.add_subplot(5, 5, index + 1 + (5 * i))
    sub.imshow(image, cmap=pl.cm.gray_r)
    sub.set_title('predict: %i
true: %i' % (lp_model.transduction_[image_index], y[image_index]), size=10)
    sub.axis('off')

# labeling 5 points, remote from labeled set
delete_index, = np.where(unlabeled_indices == image_index)
delete_indices = np.concatenate((delete_indices, delete_index))

unlabeled_indices = np.delete(unlabeled_indices, delete_indices)
n_labeled_points += 5

f.suptitle("Active learning with Label Propagation.
Rows show 5 most "
"uncertain labels to learn with the next model.")
pl.subplots_adjust(0.12, 0.03, 0.9, 0.8, 0.2, 0.45)
pl.show()
# Authors: Clay Woolam <clay@woolam.org>
# Andreas Mueller <amueller@ais.uni-bonn.de>
# Licence: BSD

```python
import numpy as np
import pylab as pl
from sklearn.semi_supervised import label_propagation
from sklearn.datasets import make_circles

# generate ring with inner box
n_samples = 200
X, y = make_circles(n_samples=n_samples, shuffle=False)
outer, inner = 0, 1
labels = -np.ones(n_samples)
labels[0] = outer
labels[-1] = inner

# Learn with LabelSpreading
label_spread = label_propagation.LabelSpreading(kernel='knn', alpha=1.0)
label_spread.fit(X, labels)

# Plot output labels
output_labels = label_spread.transduction_
pl.figure(figsize=(8.5, 4))
pl.subplot(1, 2, 1)
plot_outer_labeled, = pl.plot(X[labels == outer, 0],
                             X[labels == outer, 1], 'rs')
plot_unlabeled, = pl.plot(X[labels == -1, 0], X[labels == -1, 1], 'g.')
plot_inner_labeled, = pl.plot(X[labels == inner, 0],
                             X[labels == inner, 1], 'bs')
pl.legend((plot_outer_labeled, plot_inner_labeled, plot_unlabeled),
          ('Outer Labeled', 'Inner Labeled', 'Unlabeled'), 'upper left',
```

---

**Python source code:** plot_label_propagation_structure.py

```python
print __doc__

# Authors: Clay Woolam <clay@woolam.org>
#       Andreas Mueller <amueller@ais.uni-bonn.de>
# Licence: BSD

import numpy as np
import pylab as pl
from sklearn.semi_supervised import label_propagation
from sklearn.datasets import make_circles

# generate ring with inner box
n_samples = 200
X, y = make_circles(n_samples=n_samples, shuffle=False)
outer, inner = 0, 1
labels = -np.ones(n_samples)
labels[0] = outer
labels[-1] = inner

# Learn with LabelSpreading
label_spread = label_propagation.LabelSpreading(kernel='knn', alpha=1.0)
label_spread.fit(X, labels)

# Plot output labels
output_labels = label_spread.transduction_
pl.figure(figsize=(8.5, 4))
pl.subplot(1, 2, 1)
plot_outer_labeled, = pl.plot(X[labels == outer, 0],
                             X[labels == outer, 1], 'rs')
plot_unlabeled, = pl.plot(X[labels == -1, 0], X[labels == -1, 1], 'g.')
plot_inner_labeled, = pl.plot(X[labels == inner, 0],
                             X[labels == inner, 1], 'bs')
pl.legend((plot_outer_labeled, plot_inner_labeled, plot_unlabeled),
          ('Outer Labeled', 'Inner Labeled', 'Unlabeled'), 'upper left',
```
numpoints=1, shadow=False)
pl.title("Raw data (2 classes=red and blue")

pl.subplot(1, 2, 2)
output_label_array = np.asarray(output_labels)
outer_numbers = np.where(output_label_array == outer)[0]
inner_numbers = np.where(output_label_array == inner)[0]
plot_outer = pl.plot(X[outer_numbers, 0], X[outer_numbers, 1], 'rs')
plot_inner = pl.plot(X[inner_numbers, 0], X[inner_numbers, 1], 'bs')
pl.legend((plot_outer, plot_inner), ('Outer Learned', 'Inner Learned'),
    'upper left', numpoints=1, shadow=False)
pl.title("Labels learned with Label Spreading (KNN")

pl.subplots_adjust(left=0.07, bottom=0.07, right=0.93, top=0.92)
pl.show()

Figure 2.121: Decision boundary of label propagation versus SVM on the Iris dataset

Decision boundary of label propagation versus SVM on the Iris dataset

Comparison for decision boundary generated on iris dataset between Label Propagation and SVM.
This demonstrates Label Propagation learning a good boundary even with a small amount of labeled data.
Python source code: plot_label_propagation_versus_svm_iris.py

```python
print __doc__

# Authors: Clay Woolam <clay@woolam.org>
# Licence: BSD

import numpy as np
import pylab as pl
from sklearn import datasets
from sklearn import svm
from sklearn.semi_supervised import label_propagation

rng = np.random.RandomState(0)

iris = datasets.load_iris()

X = iris.data[:, :2]
y = iris.target

# step size in the mesh
h = .02

y_30 = np.copy(y)
y_30[rng.rand(len(y)) < 0.3] = -1
y_50 = np.copy(y)
```

Unlabeled points are colored white
y_50[rng.rand(len(y)) < 0.5] = -1
# we create an instance of SVM and fit out data. We do not scale our
# data since we want to plot the support vectors
ls30 = (label_propagation.LabelSpreading().fit(X, y_30), y_30)
ls50 = (label_propagation.LabelSpreading().fit(X, y_50), y_50)
ls100 = (label_propagation.LabelSpreading().fit(X, y), y)
rbf_svc = (svm.SVC(kernel='rbf').fit(X, y), y)

# create a mesh to plot in
x_min, x_max = X[:, 0].min() - 1, X[:, 0].max() + 1
y_min, y_max = X[:, 1].min() - 1, X[:, 1].max() + 1
xx, yy = np.meshgrid(np.arange(x_min, x_max, h),
                     np.arange(y_min, y_max, h))

titles = ['Label Spreading 30% data',
          'Label Spreading 50% data',
          'Label Spreading 100% data',
          'SVC with rbf kernel']

color_map = {-1: (1, 1, 1), 0: (0, 0, .9), 1: (1, 0, 0), 2: (.8, .6, 0)}

for i, (clf, y_train) in enumerate((ls30, ls50, ls100, rbf_svc)):
    # Plot the decision boundary. For that, we will assign a color to each
    # point in the mesh [x_min, m_max]x[y_min, y_max].
    pl.subplot(2, 2, i + 1)
    Z = clf.predict(np.c_[xx.ravel(), yy.ravel()])
    Z = Z.reshape(xx.shape)
    pl.contourf(xx, yy, Z, cmap=pl.cm.Paired)
    pl.axis('off')

    # Put the result into a color plot
    Z = Z.reshape(xx.shape)
    pl.contourf(xx, yy, Z, cmap=pl.cm.Paired)
    pl.axis('off')

    # Plot also the training points
    colors = [color_map[y] for y in y_train]
    pl.scatter(X[:, 0], X[:, 1], c=colors, cmap=pl.cm.Paired)
    pl.title(titles[i])

pl.text(.90, 0, "Unlabeled points are colored white")
pl.show()
Figure 2.122: SVM with custom kernel

3-Class classification using Support Vector Machine with custom kernel

Python source code: plot_custom_kernel.py

```python
print __doc__

import numpy as np
import pylab as pl
from sklearn import svm, datasets

# import some data to play with
iris = datasets.load_iris()
X = iris.data[:, :2]  # we only take the first two features. We could
                     # avoid this ugly slicing by using a two-dim dataset
Y = iris.target
```
```python
def my_kernel(x, y):
    """
    We create a custom kernel:
    \[
    \begin{pmatrix}
    2 & 0 \\
    0 & 1
    \end{pmatrix}
    \]
    \( k(x, y) = x \begin{pmatrix} 1 \\ 0 \end{pmatrix} y^\top \)
    """
    M = np.array([[2, 0], [0, 1.0]])
    return np.dot(np.dot(x, M), y.T)

h = .02  # step size in the mesh

# we create an instance of SVM and fit out data.
clf = svm.SVC(kernel=my_kernel)
clf.fit(X, Y)

# Plot the decision boundary. For that, we will assign a color to each
# point in the mesh [x_min, m_max]x[y_min, y_max].
x_min, x_max = X[:, 0].min() - 1, X[:, 0].max() + 1
y_min, y_max = X[:, 1].min() - 1, X[:, 1].max() + 1
xx, yy = np.meshgrid(np.arange(x_min, x_max, h), np.arange(y_min, y_max, h))
Z = clf.predict(np.c_[xx.ravel(), yy.ravel()])

# Put the result into a color plot
Z = Z.reshape(xx.shape)
plt.pcolormesh(xx, yy, Z, cmap=plt.cm.Paired)

# Plot also the training points
plt.scatter(X[:, 0], X[:, 1], c=Y, cmap=plt.cm.Paired)
plt.title('3-Class classification using Support Vector Machine with custom'
          ' kernel')
plt.axis('tight')
plt.show()
```

**Figure 2.123: **Plot different SVM classifiers in the iris dataset

**Plot different SVM classifiers in the iris dataset**

Comparison of different linear SVM classifiers on the iris dataset. It will plot the decision surface for four different SVM classifiers.
Python source code: plot_iris.py

```python
from sklearn import svm, datasets
from sklearn.svm import SVC, LinearSVC
import numpy as np
import matplotlib.pyplot as plt

# import some data to play with
iris = datasets.load_iris()
X = iris.data[:, :2]  # we only take the first two features. We could
                      # avoid this ugly slicing by using a two-dim dataset
Y = iris.target

h = .02  # step size in the mesh

# we create an instance of SVM and fit out data. We do not scale our
# data since we want to plot the support vectors
C = 1.0  # SVM regularization parameter
svc = SVC(kernel='linear', C=C).fit(X, Y)
svc1 = SVC(kernel='rbf', gamma=0.7, C=C).fit(X, Y)
svc2 = SVC(kernel='poly', degree=3, C=C).fit(X, Y)
svc3 = SVC(kernel='sigmoid', degree=1, C=C).fit(X, Y)

# create a mesh to plot in
x_min, x_max = X[:, 0].min() - 1, X[:, 0].max() + 1
y_min, y_max = X[:, 1].min() - 1, X[:, 1].max() + 1
xx, yy = np.meshgrid(np.linspace(x_min, x_max, h),
                     np.linspace(y_min, y_max, h))

# Plot the decision boundry.
plt.contourf(xx, yy, svc2.score(xx, yy),
             cmap=plt.cm.Paired, alpha=0.8)

plt.scatter(X[:, 0], X[:, 1], c=Y, cmap=plt.cm.Paired, edgecolors='k',
            s=20, label='training samples')
plt.axis('tight')
plt.show()
```
y_min, y_max = X[:, 1].min() - 1, X[:, 1].max() + 1
xx, yy = np.meshgrid(np.arange(x_min, x_max, h),
                     np.arange(y_min, y_max, h))

# title for the plots
titles = ['SVC with linear kernel',
          'SVC with RBF kernel',
          'SVC with polynomial (degree 3) kernel',
          'LinearSVC (linear kernel)']

for i, clf in enumerate((svc, rbf_svc, poly_svc, lin_svc)):
    # Plot the decision boundary. For that, we will assign a color to each
    # point in the mesh [x_min, m_max]x[y_min, y_max].
    pl.subplot(2, 2, i + 1)
    Z = clf.predict(np.c_[xx.ravel(), yy.ravel()])
    Z = Z.reshape(xx.shape)
    pl.contourf(xx, yy, Z, cmap=pl.cm.Paired)
    pl.axis('off')

    # Plot also the training points
    pl.scatter(X[:, 0], X[:, 1], c=Y, cmap=pl.cm.Paired)

    pl.title(titles[i])

pl.show()

Figure 2.124: One-class SVM with non-linear kernel (RBF)

One-class SVM with non-linear kernel (RBF)

One-class SVM is an unsupervised algorithm that learns a decision function for novelty detection: classifying new data as similar or different to the training set.
```python
from sklearn import svm

xx, yy = np.meshgrid(np.linspace(-5, 5, 500), np.linspace(-5, 5, 500))

# Generate train data
X = 0.3 * np.random.randn(100, 2)
X_train = np.r_[X + 2, X - 2]

# Generate some regular novel observations
X = 0.3 * np.random.randn(20, 2)
X_test = np.r_[X + 2, X - 2]

# Generate some abnormal novel observations
X_outliers = np.random.uniform(low=-4, high=4, size=(20, 2))

# fit the model
clf = svm.OneClassSVM(nu=0.1, kernel="rbf", gamma=0.1)
clf.fit(X_train)
y_pred_train = clf.predict(X_train)
y_pred_test = clf.predict(X_test)
y_pred_outliers = clf.predict(X_outliers)
n_error_train = y_pred_train[y_pred_train == -1].size
```

Python source code: plot_oneclass.py
n_error_test = y_pred_test[y_pred_test == -1].size
n_error_outliers = y_pred_outliers[y_pred_outliers == 1].size

# plot the line, the points, and the nearest vectors to the plane
Z = clf.decision_function(np.c_[xx.ravel(), yy.ravel()])
Z = Z.reshape(xx.shape)
pl.title("Novelty Detection")
pl.contourf(xx, yy, Z, levels=np.linspace(Z.min(), 0, 7), cmap=pl.cm.Blues_r)
a = pl.contour(xx, yy, Z, levels=[0], linewidths=2, colors='red')
pl.contourf(xx, yy, Z, levels=[0, Z.max()], colors='orange')
b1 = pl.scatter(X_train[:, 0], X_train[:, 1], c='white')
b2 = pl.scatter(X_test[:, 0], X_test[:, 1], c='green')
c = pl.scatter(X_outliers[:, 0], X_outliers[:, 1], c='red')
pl.axis('tight')
pl.xlim((-5, 5))
pl.ylim((-5, 5))
pl.legend([a.collections[0], b1, b2, c],
          ['learned frontier', "training observations", "new regular observations", "new abnormal observations"],
          loc="upper left",
          prop=matplotlib.font_manager.FontProperties(size=11))
pl.xlabel("error train: %d/200 ; errors novel regular: %d/20 ; " % (n_error_train, n_error_test, n_error_outliers))
pl.ylabel("errors novel abnormal: %d/20" % (n_error_outliers))
pl.show()

Figure 2.125: RBF SVM parameters

RBF SVM parameters

This example illustrates the effect of the parameters gamma and C of the rbf kernel SVM.

Intuitively, the gamma parameter defines how far the influence of a single training example reaches, with low values meaning ‘far’ and high values meaning ‘close’. The C parameter trades off misclassification of training examples against simplicity of the decision surface. A low C makes the decision surface smooth, while a high C aims at classifying all training examples correctly.

Two plots are generated. The first is a visualization of the decision function for a variety of parameter values, and the second is a heatmap of the classifier’s cross-validation accuracy as a function of C and gamma.
Script output:

('The best classifier is: ', SVC(C=1000000.0, cache_size=200, class_weight=None, coef0=0.0, degree=3, gamma=0.0001, kernel='rbf', probability=False, shrinking=True, tol=0.001, verbose=False))

Python source code: plot_rbf_parameters.py

```python
import numpy as np
import matplotlib.pyplot as plt

from sklearn.svm import SVC
from sklearn.preprocessing import StandardScaler
from sklearn.datasets import load_iris
from sklearn.cross_validation import StratifiedKFold
from sklearn.grid_search import GridSearchCV

# Load and prepare data set
# dataset for grid search
iris = load_iris()
X = iris.data
Y = iris.target

# dataset for decision function visualization
X_2d = X[:, :2]
```

Chapter 2. Example Gallery
X_2d = X_2d[Y > 0]
Y_2d = Y[Y > 0]
Y_2d -= 1

# It is usually a good idea to scale the data for SVM training.
# We are cheating a bit in this example in scaling all of the data,
# instead of fitting the transformation on the training set and
# just applying it on the test set.

scaler = Scaler()
X = scaler.fit_transform(X)
X_2d = scaler.fit_transform(X_2d)

# Train classifier

# For an initial search, a logarithmic grid with basis
# 10 is often helpful. Using a basis of 2, a finer
# tuning can be achieved but at a much higher cost.

C_range = 10.0 ** np.arange(-2, 9)
gamma_range = 10.0 ** np.arange(-5, 4)
param_grid = dict(gamma=gamma_range, C=C_range)
grid = GridSearchCV(SVC(), param_grid=param_grid, cv=StratifiedKFold(y=Y, k=3))
grid.fit(X, Y)
print("The best classifier is: ", grid.best_estimator_)

# Now we need to fit a classifier for all parameters in the 2d version
# (we use a smaller set of parameters here because it takes a while to train)
C_2d_range = [1, 1e2, 1e4]
gamma_2d_range = [1e-1, 1, 1e1]
classifiers = []
for C in C_2d_range:
    for gamma in gamma_2d_range:
        clf = SVC(C=C, gamma=gamma)
        clf.fit(X_2d, Y_2d)
        classifiers.append((C, gamma, clf))

# draw visualization of parameter effects
pl.figure(figsize=(8, 6))
xx, yy = np.meshgrid(np.linspace(-5, 5, 200), np.linspace(-5, 5, 200))
for (k, (C, gamma, clf)) in enumerate(classifiers):
    Z = clf.decision_function(np.c_[xx.ravel(), yy.ravel()])
    Z = Z.reshape(xx.shape)
    pl.subplot(len(C_2d_range), len(gamma_2d_range), k + 1)
    pl.title("gamma 10^{%d}, C 10^{%d}" % (np.log10(gamma), np.log10(C)), size='medium')
    # visualize parameter’s effect on decision function

2.1. Examples
SVM: Maximum margin separating hyperplane

Plot the maximum margin separating hyperplane within a two-class separable dataset using a Support Vector Machines classifier with linear kernel.
scikit-learn user guide, Release 0.12-git

Python source code: plot_separating_hyperplane.py

```python
print __doc__

import numpy as np
import pylab as pl
from sklearn import svm

# we create 40 separable points
np.random.seed(0)
X = np.r_[np.random.randn(20, 2) - [2, 2], np.random.randn(20, 2) + [2, 2]]
Y = [0] * 20 + [1] * 20

# fit the model
clf = svm.SVC(kernel='linear')
clf.fit(X, Y)

# get the separating hyperplane
w = clf.coef_[0]
a = -w[0] / w[1]
xx = np.linspace(-5, 5)
yy = a * xx - (clf.intercept_[0]) / w[1]

# plot the parallels to the separating hyperplane that pass through the
# support vectors
b = clf.support_vectors_[0]
```

2.1. Examples
yy_down = a * xx + (b[1] - a * b[0])
b = clf.support_vectors_[-1]
yy_up = a * xx + (b[1] - a * b[0])

# plot the line, the points, and the nearest vectors to the plane
pl.plot(xx, yy, 'k-')
pl.plot(xx, yy_down, 'k--')
pl.plot(xx, yy_up, 'k--')

pl.scatter(clf.support_vectors_[:, 0], clf.support_vectors_[:, 1], s=80, facecolors='none')
pl.scatter(X[:, 0], X[:, 1], c=Y, cmap=pl.cm.Paired)

pl.axis('tight')
pl.show()

Figure 2.127: SVM: Separating hyperplane for unbalanced classes

SVM: Separating hyperplane for unbalanced classes

Find the optimal separating hyperplane using an SVC for classes that are unbalanced.

We first find the separating plane with a plain SVC and then plot (dashed) the separating hyperplane with automatically correction for unbalanced classes.
```python
import numpy as np
import pylab as pl
from sklearn import svm

# we create 40 separable points
rng = np.random.RandomState(0)
n_samples_1 = 1000
n_samples_2 = 100
X = np.r_[1.5 * rng.randn(n_samples_1, 2),
          0.5 * rng.randn(n_samples_2, 2) + [2, 2]]
y = [0] * (n_samples_1) + [1] * (n_samples_2)

# fit the model and get the separating hyperplane
clf = svm.SVC(kernel='linear', C=1.0)
clf.fit(X, y)

w = clf.coef_[0]
a = -w[0] / w[1]
xx = np.linspace(-5, 5)
yy = a * xx - clf.intercept_[0] / w[1]
```

Python source code: plot_separating_hyperplane_unbalanced.py
# get the separating hyperplane using weighted classes
wclf = svm.SVC(kernel='linear', class_weight={1: 10})
wclf.fit(X, y)

ww = wclf.coef_[0]
wa = -ww[0] / ww[1]
wy = wa * xx - wclf.intercept_[0] / ww[1]

# plot separating hyperplanes and samples
h0 = pl.plot(xx, yy, 'k-', label='no weights')
h1 = pl.plot(xx, wy, 'k--', label='with weights')
pl.scatter(X[:, 0], X[:, 1], c=y, cmap=pl.cm.Paired)
pl.legend()
pl.axis('tight')
pl.show()

Figure 2.128: SVM-Anova: SVM with univariate feature selection

**SVM-Anova: SVM with univariate feature selection**

This example shows how to perform univariate feature before running a SVC (support vector classifier) to improve the classification scores.
import numpy as np
import matplotlib.pyplot as plt
from sklearn import svm, datasets, feature_selection, cross_validation
from sklearn.pipeline import Pipeline

def main():
    # Import some data to play with
    digits = datasets.load_digits()
    y = digits.target
    # Throw away data, to be in the curse of dimension settings
    y = y[:200]
    X = digits.data[:200]
    n_samples = len(y)
    X = X.reshape((n_samples, -1))
    # add 200 non-informative features
    X = np.hstack((X, 2 * np.random.random((n_samples, 200))))

    # Create a feature-selection transform and an instance of SVM that we
    # combine together to have an full-blown estimator
    transform = feature_selection.SelectPercentile(feature_selection.f_classif)

    # Create a full-blown estimator
    pipe = Pipeline([('select', transform), ('svm', svm.SVC())])

    # Evaluate using cross-validation
    scores = cross_validation.cross_val_score(pipe, X, y, cv=5)

    # Fit and get the prediction rate
    pipe = pipe.fit(X, y)
    y_pred = pipe.predict(X)
    scores = pipe.score(X, y)

    # Plot
    plt.figure()
    plt.plot([0, 100], [0, 1], 'k-')
    plt.plot([0, 100], scores, 'o-')
    plt.xlabel('Percentile')
    plt.ylabel('Prediction rate')
    plt.xlim(0, 100)
    plt.ylim(0, 1)
    plt.title('Performance of the SVM-Anova varying the percentile of features selected')

    plt.show()

if __name__ == '__main__':
    main()
clf = Pipeline([('anova', transform), ('svc', svm.SVC(C=1.0))])

# Plot the cross-validation score as a function of percentile of features
score_means = list()
score_stds = list()
percentiles = (1, 3, 6, 10, 15, 20, 30, 40, 60, 80, 100)

for percentile in percentiles:
    clf.set_params(anova__percentile=percentile)
    # Compute cross-validation score using all CPUs
    this_scores = cross_validation.cross_val_score(clf, X, y, n_jobs=1)
    score_means.append(this_scores.mean())
    score_stds.append(this_scores.std())

pl.errorbar(percentiles, score_means, np.array(score_stds))

pl.title('Performance of the SVM-Anova varying the percentile of features selected')
pl.xlabel('Percentile')
pl.ylabel('Prediction rate')
pl.axis('tight')
pl.show()

Figure 2.129: SVM-SVC (Support Vector Classification)

SVM-SVC (Support Vector Classification)

The classification application of the SVM is used below. The Iris dataset has been used for this example.
The decision boundaries, are shown with all the points in the training-set.
```python
from sklearn import svm, datasets

# import some data to play with
iris = datasets.load_iris()
X = iris.data[:, :2]  # we only take the first two features.
Y = iris.target

h = .02  # step size in the mesh
clf = svm.SVC(C=1.0, kernel='linear')
clf.fit(X, Y)

xx, yy = np.meshgrid(np.arange(x_min, x_max, h), np.arange(y_min, y_max, h))
Z = clf.predict(np.c_[xx.ravel(), yy.ravel()])
Z = Z.reshape(xx.shape)

plt.figure(1, figsize=(4, 3))
plt.pcolormesh(xx, yy, Z, cmap=pl.cm.Paired)
plt.show()
```

---

2.1. Examples
SVM-Kernels

Three different types of SVM-Kernels are displayed below. The polynomial and RBF are especially useful when the data-points are not linearly separable.

Python source code: plot_svm_kernels.py
print __doc__

# Code source: Gael Varoquaux
# License: BSD

import numpy as np
import pylab as pl
from sklearn import svm

# Our dataset and targets
X = np.c_[
    (.4, -.7),
    (-1.5, -1),
    (-1.4, -.9),
    (-1.3, -1.2),
    (-1.1, -.2),
    (-1.2, -.4),
    (-.5, 1.2),
    (-1.5, 2.1),
    (1, 1),
    # --
    (1.3, .8),
    (1.2, .5),
    (.2, -2),
    (.5, -2.4),
    (.2, -2.3),
    (0, -2.7),
    (1.3, 2.1),
].T
Y = [0] * 8 + [1] * 8

# figure number
fignum = 1

# fit the model
for kernel in ('linear', 'poly', 'rbf'):
    clf = svm.SVC(kernel=kernel, gamma=2)
    clf.fit(X, Y)

    # plot the line, the points, and the nearest vectors to the plane
    pl.figure(fignum, figsize=(4, 3))
    pl.clf()
    pl.scatter(clf.support_vectors_[:, 0], clf.support_vectors_[:, 1],
               s=80, facecolors='none', zorder=10)
    pl.scatter(X[:, 0], X[:, 1], c=Y, zorder=10, cmap=pl.cm.Paired)

    pl.axis('tight')
    x_min = -3
    x_max = 3
    y_min = -3
    y_max = 3
    XX, YY = np.mgrid[x_min:x_max:200j, y_min:y_max:200j]
    Z = clf.decision_function(np.c_[XX.ravel(), YY.ravel()])

    # Put the result into a color plot

2.1. Examples
Z = Z.reshape(XX.shape)
pl.figure(fignum, figsize=(4, 3))
pl.pcolormesh(XX, YY, Z > 0, cmap=pl.cm.Paired)
pl.contour(XX, YY, Z, colors=['k', 'k', 'k'],
           linestyles=['--', '-', '--'],
           levels=[-.5, 0, .5])
pl.xlim(x_min, x_max)
pl.ylim(y_min, y_max)
pl.xticks(())
pl.yticks(())
fignum = fignum + 1
pl.show()

Figure 2.131: SVM Margins Example

SVM Margins Example

The plots below illustrate the effect the parameter $C$ has on the separation line. A large value of $C$ basically tells our model that we do not have that much faith in our data's distribution, and will only consider points close to line of separation.

A small value of $C$ includes more/all the observations, allowing the margins to be calculated using all the data in the area.

Python source code: plot_svm_margin.py

# Code source: Gael Varoquaux
import numpy as np
import matplotlib.pyplot as plt
from sklearn import svm

# we create 40 separable points
np.random.seed(0)
X = np.r_[np.random.randn(20, 2) - [2, 2], np.random.randn(20, 2) + [2, 2]]
Y = [0] * 20 + [1] * 20

# figure number
fignum = 1

# fit the model
for name, penalty in (['unreg', 1], ['reg', 0.05]):
    clf = svm.LinearSVC(kernel='linear', C=penalty)
    clf.fit(X, Y)

    # get the separating hyperplane
    w = clf.coef_[0]
    a = -w[0] / w[1]
    xx = np.linspace(-5, 5)
    yy = a * xx - (clf.intercept_[0]) / w[1]

    # plot the parallels to the separating hyperplane that pass through the
    # support vectors
    margin = 1 / np.sqrt(np.sum(clf.coef_ ** 2))
    yy_down = yy + a * margin
    yy_up = yy - a * margin

    # plot the line, the points, and the nearest vectors to the plane
    plt.figure(fignum, figsize=(4, 3))
    plt.clf()
    plt.plot(xx, yy, 'k-')
    plt.plot(xx, yy_down, 'k--')
    plt.plot(xx, yy_up, 'k--')
    plt.scatter(clf.support_vectors_[:, 0], clf.support_vectors_[:, 1], s=80, facecolors='none', zorder=10)
    plt.scatter(X[:, 0], X[:, 1], c=Y, zorder=10, cmap=plt.cm.Paired)
    plt.axis('tight')
    x_min = -4.8
    x_max = 4.2
    y_min = -6
    y_max = 6

    XX, YY = np.mgrid[x_min:x_max:200j, y_min:y_max:200j]
    Z = clf.predict(np.c_[XX.ravel(), YY.ravel()])

    # Put the result into a color plot
    Z = Z.reshape(XX.shape)
    plt.figure(fignum, figsize=(4, 3))
    plt.pcolormesh(XX, YY, Z, cmap=plt.cm.Paired)
Non-linear SVM

Perform binary classification using non-linear SVC with RBF kernel. The target to predict is a XOR of the inputs. The color map illustrates the decision function learn by the SVC.
```python
import numpy as np
import pylab as pl
from sklearn import svm

xx, yy = np.meshgrid(np.linspace(-3, 3, 500),
                     np.linspace(-3, 3, 500))

np.random.seed(0)
X = np.random.randn(300, 2)
Y = np.logical_xor(X[:, 0] > 0, X[:, 1] > 0)

# fit the model
clf = svm.NuSVC()
clf.fit(X, Y)

# plot the decision function for each datapoint on the grid
Z = clf.decision_function(np.c_[xx.ravel(), yy.ravel()])
Z = Z.reshape(xx.shape)
pl.imshow(Z, interpolation='nearest',
          extent=(xx.min(), xx.max(), yy.min(), yy.max()),
          aspect='auto', origin='lower', cmap=pl.cm.PuOr_r)
contours = pl.contour(xx, yy, Z, levels=[0], lineweight=2)
```

**Python source code:** plot_svm_nonlinear.py
Support Vector Regression (SVR) using linear and non-linear kernels

Toy example of 1D regression using linear, polynominial and RBF kernels.

Python source code: plot_svm_regression.py
# Generate sample data
import numpy as np
X = np.sort(5 * np.random.rand(40, 1), axis=0)
y = np.sin(X).ravel()

# Add noise to targets
y[::5] += 3 * (0.5 - np.random.rand(8))

# Fit regression model
from sklearn.svm import SVR
svr_rbf = SVR(kernel='rbf', C=1e3, gamma=0.1)
svr_lin = SVR(kernel='linear', C=1e3)
svr_poly = SVR(kernel='poly', C=1e3, degree=2)
y_rbf = svr_rbf.fit(X, y).predict(X)
y_lin = svr_lin.fit(X, y).predict(X)
y_poly = svr_poly.fit(X, y).predict(X)

# look at the results
import pylab as pl
pl.scatter(X, y, c='k', label='data')
pl.hold('on')
pl.plot(X, y_rbf, c='g', label='RBF model')
pl.plot(X, y_lin, c='r', label='Linear model')
pl.plot(X, y_poly, c='b', label='Polynomial model')
pl.xlabel('data')
pl.ylabel('target')
pl.title('Support Vector Regression')
pl.legend()
pl.show()

Figure 2.134: SVM: Weighted samples

**SVM: Weighted samples**

Plot decision function of a weighted dataset, where the size of points is proportional to its weight.
Python source code: plot_weighted_samples.py

```python
import numpy as np
import pylab as pl
from sklearn import svm

# we create 20 points
np.random.seed(0)
X = np.r_[np.random.randn(10, 2) + [1, 1], np.random.randn(10, 2)]
Y = [1] * 10 + [-1] * 10
sample_weight = 100 * np.abs(np.random.randn(20))
# and assign a bigger weight to the last 10 samples
sample_weight[:10] *= 10

# fit the model
clf = svm.SVC()
clf.fit(X, Y, sample_weight=sample_weight)

# plot the decision function
xx, yy = np.meshgrid(np.linspace(-4, 5, 500), np.linspace(-4, 5, 500))
Z = clf.decision_function(np.c_[xx.ravel(), yy.ravel()])
Z = Z.reshape(xx.shape)
```

974 Chapter 2. Example Gallery
# plot the line, the points, and the nearest vectors to the plane
pl.contourf(xx, yy, Z, alpha=0.75, cmap=pl.cm.bone)
pl.scatter(X[:, 0], X[:, 1], c=Y, s=sample_weight, alpha=0.9, cmap=pl.cm.bone)

pl.axis('off')
pl.show()

## 2.1.16 Decision Trees

Examples concerning the `sklearn.tree` package.

![Plot the decision surface of a decision tree on the iris dataset](image)

**Figure 2.135: Plot the decision surface of a decision tree on the iris dataset**

**Plot the decision surface of a decision tree on the iris dataset**

Plot the decision surface of a decision tree trained on pairs of features of the iris dataset.

For each pair of iris features, the decision tree learns decision boundaries made of combinations of simple thresholding rules inferred from the training samples.
from sklearn.datasets import load_iris
from sklearn.tree import DecisionTreeClassifier

# Parameters
n_classes = 3
plot_colors = "bry"
plot_step = 0.02

# Load data
iris = load_iris()

for pairidx, pair in enumerate(
    [[0, 1], [0, 2], [0, 3], [1, 2], [1, 3], [2, 3]]):
    # We only take the two corresponding features
    X = iris.data[:, pair]
y = iris.target

    # Shuffle
    idx = np.arange(X.shape[0])
```python	np.random.seed(13)
np.random.shuffle(idx)
X = X[idx]
y = y[idx]

# Standardize
mean = X.mean(axis=0)
std = X.std(axis=0)
X = (X - mean) / std

# Train
clf = DecisionTreeClassifier().fit(X, y)

# Plot the decision boundary
pl.subplot(2, 3, pairidx + 1)
x_min, x_max = X[:, 0].min() - 1, X[:, 0].max() + 1
y_min, y_max = X[:, 1].min() - 1, X[:, 1].max() + 1
xx, yy = np.meshgrid(np.arange(x_min, x_max, plot_step),
                     np.arange(y_min, y_max, plot_step))
Z = clf.predict(np.c_[xx.ravel(), yy.ravel()])
Z = Z.reshape(xx.shape)
cs = pl.contourf(xx, yy, Z, cmap=pl.cm.Paired)
pl.xlabel(iris.feature_names[pair[0]])
pl.ylabel(iris.feature_names[pair[1]])
pl.axis("tight")

# Plot the training points
for i, color in zip(xrange(n_classes), plot_colors):
    idx = np.where(y == i)
    pl.scatter(X[idx, 0], X[idx, 1], c=color, label=iris.target_names[i],
               cmap=pl.cm.Paired)
pl.axis("tight")
pl.suptitle("Decision surface of a decision tree using paired features")
pl.legend()
pl.show()
```

Figure 2.136: Decision Tree Regression

**Decision Tree Regression**

1D regression with *decision trees*: the decision tree is used to fit a sine curve with addition noisy observation. As a result, it learns local linear regressions approximating the sine curve.
We can see that if the maximum depth of the tree (controlled by the `max_depth` parameter) is set too high, the decision trees learn too fine details of the training data and learn from the noise, i.e. they overfit.

**Python source code:** `plot_tree_regression.py`

```python
print __doc__

import numpy as np

# Create a random dataset
rng = np.random.RandomState(1)
X = np.sort(5 * rng.rand(80, 1), axis=0)
y = np.sin(X).ravel()
y[::5] += 3 * (0.5 - rng.rand(16))

# Fit regression model
from sklearn.tree import DecisionTreeRegressor

clf_1 = DecisionTreeRegressor(max_depth=2)
clf_2 = DecisionTreeRegressor(max_depth=5)
clf_1.fit(X, y)
clf_2.fit(X, y)

# Predict
X_test = np.arange(0.0, 5.0, 0.01)[:, np.newaxis]
y_1 = clf_1.predict(X_test)
```

![Decision Tree Regression](image)
y_2 = clf_2.predict(X_test)

# Plot the results
import pylab as pl

pl.figure()
pl.scatter(X, y, c="k", label="data")
pl.plot(X_test, y_1, c="g", label="max_depth=2", linewidth=2)
pl.plot(X_test, y_2, c="r", label="max_depth=5", linewidth=2)
pl.xlabel("data")
pl.ylabel("target")
pl.title("Decision Tree Regression")
pl.legend()
pl.show()
CHAPTER THREE

DEVELOPMENT

3.1 Contributing

This project is a community effort, and everyone is welcome to contribute.
The project is hosted on http://github.com/scikit-learn/scikit-learn

3.1.1 Submitting a bug report

In case you experience issues using this package, do not hesitate to submit a ticket to the Bug Tracker. You are also welcome to post feature requests or links to pull requests.

3.1.2 Retrieving the latest code

We use Git for version control and GitHub for hosting our main repository.
You can check out the latest sources with the command:

```
git clone git://github.com/scikit-learn/scikit-learn.git
```
or if you have write privileges:

```
git clone git@github.com:scikit-learn/scikit-learn.git
```

If you run the development version, it is cumbersome to reinstall the package each time you update the sources. It is thus preferred that you add the scikit-learn directory to your PYTHONPATH and build the extension in place:

```
python setup.py build_ext --inplace
```

On Unix-like systems, you can simply type make in the top-level folder to build in-place and launch all the tests. Have a look at the Makefile for additional utilities.

3.1.3 Contributing code

**Note:** To avoid duplicating work, it is highly advised that you contact the developers on the mailing list before starting work on a non-trivial feature.

https://lists.sourceforge.net/lists/listinfo/scikit-learn-general
How to contribute

The preferred way to contribute to scikit-learn is to fork the main repository on GitHub:

1. Create an account on GitHub if you do not already have one.
2. Fork the project repository: click on the ‘Fork’ button near the top of the page. This creates a copy of the code under your account on the GitHub server.
3. Clone this copy to your local disk:
   
   $ git clone git@github.com:YourLogin/scikit-learn.git

4. Create a branch to hold your changes:
   
   $ git checkout -b my-feature

   and start making changes. Never work in the master branch!
5. Work on this copy, on your computer, using Git to do the version control. When you’re done editing, do:
   
   $ git add modified_files
   $ git commit

   to record your changes in Git, then push them to GitHub with:

   $ git push -u origin my-feature

Finally, go to the web page of the your fork of the scikit-learn repo, and click ‘Pull request’ to send your changes to the maintainers for review. request. This will send an email to the committers, but might also send an email to the mailing list in order to get more visibility.

Note: In the above setup, your origin remote repository points to YourLogin/scikit-learn.git. If you wish to fetch/merge from the main repository instead of your forked one, you will need to add another remote to use instead of origin. If we choose the name upstream for it, the command will be:

   $ git remote add upstream https://github.com/scikit-learn/scikit-learn.git

(If any of the above seems like magic to you, then look up the Git documentation on the web.)

It is recommended to check that your contribution complies with the following rules before submitting a pull request:

- Follow the coding-guidelines (see below).
- When applicable, use the Validation tools and other code in the sklearn.utils submodule. A list of utility routines available for developers can be found in the Utilities for Developers page.
- All public methods should have informative docstrings with sample usage presented as doctests when appropriate.
- All other tests pass when everything is rebuilt from scratch. On Unix-like systems, check with (from the toplevel source folder):

   $ make

- When adding additional functionality, provide at least one example script in the examples/ folder. Have a look at other examples for reference. Examples should demonstrate why the new functionality is useful in practice and, if possible, compare it to other methods available in scikit-learn.
- At least one paragraph of narrative documentation with links to references in the literature (with PDF links when possible) and the example.
The documentation should also include expected time and space complexity of the algorithm and scalability, e.g., "this algorithm can scale to a large number of samples > 100000, but does not scale in dimensionality: n_features is expected to be lower than 100".

To build the documentation, see the documentation section below.

You can also check for common programming errors with the following tools:

- **Code with a good unittest coverage (at least 80%), check with:**
  
  ```
  $ pip install nose coverage
  $ nosetests --with-coverage path/to/tests_for_package
  ```

- **No pyflakes warnings, check with:**
  
  ```
  $ pip install pyflakes
  $ pyflakes path/to/module.py
  ```

- **No PEP8 warnings, check with:**
  
  ```
  $ pip install pep8
  $ pep8 path/to/module.py
  ```

- **AutoPEP8 can help you fix some of the easy redundant errors:**
  
  ```
  $ pip install autopep8
  $ autopep8 path/to/pep8.py
  ```

Bonus points for contributions that include a performance analysis with a benchmark script and profiling output (please report on the mailing list or on the GitHub wiki).

Also check out the *How to optimize for speed* guide for more details on profiling and Cython optimizations.

**Note:** The current state of the scikit-learn code base is not compliant with all of those guidelines, but we expect that enforcing those constraints on all new contributions will get the overall code base quality in the right direction.

### EasyFix Issues

A great way to start contributing to scikit-learn is to pick an item from the list of EasyFix issues in the issue tracker. Resolving these issues allow you to start contributing to the project without much prior knowledge. Your assistance in this area will be greatly appreciated by the more experienced developers as it helps free up their time to concentrate on other issues.

### Documentation

We are glad to accept any sort of documentation: function docstrings, reStructuredText documents (like this one), tutorials, etc. reStructuredText documents live in the source code repository under the doc/ directory.

You can edit the documentation using any text editor, and then generate the HTML output by typing `make html` from the doc/ directory. Alternatively, `make html-noplot` can be used to quickly generate the documentation without the example gallery. The resulting HTML files will be placed in _build/html/ and are viewable in a web browser. See the README file in the doc/ directory for more information.

For building the documentation, you will need **sphinx** and **matplotlib**.

When you are writing documentation, it is important to keep a good compromise between mathematical and algorithmic details, and give intuition to the reader on what the algorithm does. It is best to always start with a small paragraph
with a hand-waiving explanation of what the method does to the data and a figure (coming from an example) illustrating it.

**Warning:** Sphinx version
While we do our best to have the documentation build under as many version of Sphinx as possible, the different versions tend to behave slightly differently. To get the best results, you should use version 1.0.

**Developers web site**

More information can be found on the developer's wiki.

### 3.1.4 Other ways to contribute

Code is not the only way to contribute to scikit-learn. For instance, documentation is also a very important part of the project and often doesn’t get as much attention as it deserves. If you find a typo in the documentation, or have made improvements, do not hesitate to send an email to the mailing list or submit a GitHub pull request. Full documentation can be found under the doc/ directory.

It also helps us if you spread the word: reference the project from your blog and articles, link to it from your website, or simply say “I use it”:

### 3.1.5 Coding guidelines

The following are some guidelines on how new code should be written. Of course, there are special cases and there will be exceptions to these rules. However, following these rules when submitting new code makes the review easier so new code can be integrated in less time.

Uniformly formatted code makes it easier to share code ownership. The scikit-learn project tries to closely follow the official Python guidelines detailed in PEP8 that detail how code should be formatted and indented. Please read it and follow it.

In addition, we add the following guidelines:

- Use underscores to separate words in non class names: `n_samples` rather than `nsamples`.
- Avoid multiple statements on one line. Prefer a line return after a control flow statement (`if/for`).
- Use relative imports for references inside scikit-learn.
- **Please don’t use `import *` in any case.** It is considered harmful by the official Python recommendations. It makes the code harder to read as the origin of symbols is no longer explicitly referenced, but most important, it prevents using a static analysis tool like `pyflakes` to automatically find bugs in scikit-learn.
- Use the numpy docstring standard in all your docstrings.

A good example of code that we like can be found here.

### Input validation

The module `sklearn.utils` contains various functions for doing input validation and conversion. Sometimes, `np.asarray` suffices for validation; do not use `np.asanyarray` or `np.atleast_2d`, since those let NumPy’s `np.matrix` through, which has a different API (e.g., `*` means dot product on `np.matrix`, but Hadamard product on `np.ndarray`).
In other cases, be sure to call `safe_asarray`, `atleast2d_or_csr`, `as_float_array` or `array2d` on any array-like argument passed to a scikit-learn API function. The exact function to use depends mainly on whether `scipy.sparse` matrices must be accepted.

For more information, refer to the `Utilities for Developers` page.

### Random Numbers

If your code depends on a random number generator, do not use `numpy.random.random()` or similar routines. To ensure repeatability in error checking, the routine should accept a keyword `random_state` and use this to construct a `numpy.random.RandomState` object. See `sklearn.utils.check_random_state` in `Utilities for Developers`.

Here’s a simple example of code using some of the above guidelines:

```python
from sklearn.utils import array2d, check_random_state

def choose_random_sample(X, random_state=0):
    
    Choose a random point from X

    Parameters
    ----------
    X : array-like, shape = (n_samples, n_features)
        array representing the data
    random_state : RandomState or an int seed (0 by default)
        A random number generator instance to define the state of the
        random permutations generator.

    Returns
    -------
    x : numpy array, shape = (n_features,)
        A random point selected from X

    X = array2d(X)
    random_state = check_random_state(random_state)
    i = random_state.randint(X.shape[0])
    return X[i]
```

### 3.1.6 APIs of scikit-learn objects

To have a uniform API, we try to have a common basic API for all the objects. In addition, to avoid the proliferation of framework code, we try to adopt simple conventions and limit to a minimum the number of methods an object must implement.

#### Different objects

The main objects in scikit-learn are (one class can implement multiple interfaces):

- **Estimator** The base object, implements:
  
  ```python
  estimator = obj.fit(data)
  ```

- **Predictor** For supervised learning, or some unsupervised problems, implements:
prediction = obj.predict(data)

**Transformer** For filtering or modifying the data, in a supervised or unsupervised way, implements:

```
new_data = obj.transform(data)
```

When fitting and transforming can be performed much more efficiently together than separately, implements:

```
new_data = obj.fit_transform(data)
```

**Model** A model that can give a goodness of fit or a likelihood of unseen data, implements (higher is better):

```
score = obj.score(data)
```

### Estimators

The API has one predominant object: the estimator. A estimator is an object that fits a model based on some training data and is capable of inferring some properties on new data. It can be, for instance, a classifier or a regressor. All estimators implement the fit method:

```
estimator.fit(X, y)
```

All built-in estimators also have a set_params method, which sets data-independent parameters (overriding previous parameter values passed to __init__). This method is not required for an object to be an estimator.

All estimators should inherit from `sklearn.base.BaseEstimator`.

### Instantiation

This concerns the creation of an object. The object’s __init__ method might accept constants as arguments that determine the estimator’s behavior (like the C constant in SVMs). It should not, however, take the actual training data as an argument, as this is left to the fit() method:

```
clf2 = SVC(C=2.3)
clf3 = SVC([[1, 2], [2, 3]], [-1, 1]) # WRONG!
```

The arguments accepted by __init__ should all be keyword arguments with a default value. In other words, a user should be able to instantiate an estimator without passing any arguments to it. The arguments should all correspond to hyperparameters describing the model or the optimisation problem the estimator tries to solve.

In addition, **every keyword argument accepted by ‘__init__‘ should correspond to an attribute on the instance.** Scikit-learn relies on this to find the relevant attributes to set on an estimator when doing model selection.

To summarize, a __init__ should look like:

```
def __init__(self, param1=1, param2=2):
    self.param1 = param1
    self.param2 = param2
```

There should be no logic, and the parameters should not be changed. The corresponding logic should be put where the parameters are used. The following is wrong:

```
def __init__(self, param1=1, param2=2, param3=3):
    # WRONG: parameters should not be modified
    if param1 > 1:
```

```
param2 += 1
self.param1 = param1
# WRONG: the object's attributes should have exactly the name of
# the argument in the constructor
self.param3 = param2

Scikit-learn relies on this mechanism to introspect objects to set their parameters by cross-validation.

Fitting

The next thing you will probably want to do is to estimate some parameters in the model. This is implemented in the
fit() method.

The fit() method takes the training data as arguments, which can be one array in the case of unsupervised learning,
or two arrays in the case of supervised learning.

Note that the model is fitted using X and y, but the object holds no reference to X and y. There are, however, some
exceptions to this, as in the case of precomputed kernels where this data must be stored for use by the predict method.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>X</td>
<td>array-like, with shape = [N, D], where N is the number of samples and D is the number of features.</td>
</tr>
<tr>
<td>y</td>
<td>array, with shape = [N], where N is the number of samples.</td>
</tr>
<tr>
<td>kwargs</td>
<td>optional data-dependent parameters.</td>
</tr>
</tbody>
</table>

X.shape[0] should be the same as y.shape[0]. If this requisite is not met, an exception of type ValueError
should be raised.

y might be ignored in the case of unsupervised learning. However, to make it possible to use the estimator as part of a
pipeline that can mix both supervised and unsupervised transformers, even unsupervised estimators are kindly asked
to accept a y=None keyword argument in the second position that is just ignored by the estimator.

The method should return the object (self). This pattern is useful to be able to implement quick one liners in an
IPython session such as:

```
y_predicted = SVC(C=100).fit(X_train, y_train).predict(X_test)
```

Depending on the nature of the algorithm, fit can sometimes also accept additional keywords arguments. However,
any parameter that can have a value assigned prior to having access to the data should be an __init__ keyword
argument. fit parameters should be restricted to directly data dependent variables. For instance a Gram matrix
or an affinity matrix which are precomputed from the data matrix X are data dependent. A tolerance stopping criterion
tol is not directly data dependent (although the optimal value according to some scoring function probably is).

Any attribute that ends with _ is expected to be overridden when you call fit a second time without taking any
previous value into account: fit should be idempotent.

Optional Arguments

In iterative algorithms, the number of iterations should be specified by an integer called n_iter.

Unresolved API issues

Some things are must still be decided:

- what should happen when predict is called before fit()?
- which exception should be raised when the shape of arrays do not match in fit()?
Working notes

For unresolved issues, TODOs, and remarks on ongoing work, developers are advised to maintain notes on the GitHub wiki.

Specific models

In linear models, coefficients are stored in an array called \texttt{coef\_}, and the independent term is stored in \texttt{intercept\_}.

3.2 How to optimize for speed

The following gives some practical guidelines to help you write efficient code for the scikit-learn project.

\textbf{Note:} While it is always useful to profile your code so as to check performance assumptions, it is also highly recommended to review the literature to ensure that the implemented algorithm is the state of the art for the task before investing into costly implementation optimization.

Times and times, hours of efforts invested in optimizing complicated implementation details have been rendered irrelevant by the late discovery of simple algorithmic tricks, or by using another algorithm altogether that is better suited to the problem.

The section \textit{A sample algorithmic trick: warm restarts for cross validation} gives an example of such a trick.

3.2.1 Python, Cython or C/C++?

In general, the scikit-learn project emphasizes the readability of the source code to make it easy for the project users to dive into the source code so as to understand how the algorithm behaves on their data but also for ease of maintainability (by the developers).

When implementing a new algorithm is thus recommended to start implementing it in Python using Numpy and Scipy by taking care of avoiding looping code using the vectorized idioms of those libraries. In practice this means trying to replace any nested for loops by calls to equivalent Numpy array methods. The goal is to avoid the CPU wasting time in the Python interpreter rather than crunching numbers to fit your statistical model.

Sometimes however an algorithm cannot be expressed efficiently in simple vectorized Numpy code. In this case, the recommended strategy is the following:

1. \textbf{Profile} the Python implementation to find the main bottleneck and isolate it in a dedicated module level function. This function will be reimplemented as a compiled extension module.
2. If there exists a well maintained BSD or MIT C/C++ implementation of the same algorithm that is not too big, you can write a Cython wrapper for it and include a copy of the source code of the library in the scikit-learn source tree: this strategy is used for the classes \texttt{svm.LinearSVC}, \texttt{svm.SVC} and \texttt{linear_model.LogisticRegression} (wrappers for liblinear and libsvm).
3. Otherwise, write an optimized version of your Python function using Cython directly. This strategy is used for the \texttt{linear_model.ElasticNet} and \texttt{linear_model.SGDClassifier} classes for instance.
4. Move the Python version of the function in the tests and use it to check that the results of the compiled extension are consistent with the gold standard, easy to debug Python version.
5. Once the code is optimized (not simple bottleneck spottable by profiling), check whether it is possible to have coarse grained parallelism that is amenable to multi-processing by using the \texttt{joblib.Parallel} class.
When using Cython, include the generated C source code alongside with the Cython source code. The goal is to make it possible to install the scikit on any machine with Python, Numpy, Scipy and C/C++ compiler.

### 3.2.2 Profiling Python code

In order to profile Python code we recommend to write a script that loads and prepare you data and then use the IPython integrated profiler for interactively exploring the relevant part for the code.

Suppose we want to profile the Non Negative Matrix Factorization module of the scikit. Let us setup a new IPython session and load the digits dataset as in the Recognizing hand-written digits example:

```python
In [1]: from sklearn.decomposition import NMF
In [2]: from sklearn.datasets import load_digits
In [3]: X = load_digits().data
```

Before starting the profiling session and engaging in tentative optimization iterations, it is important to measure the total execution time of the function we want to optimize without any kind of profiler overhead and save it somewhere for later reference:

```python
In [4]: %timeit NMF(n_components=16, tol=1e-2).fit(X)
1 loops, best of 3: 1.7 s per loop
```

To have a look at the overall performance profile using the `%prun` magic command:

```python
In [5]: %prun -l nmf.py NMF(n_components=16, tol=1e-2).fit(X)
```

Here is the beginning of the output of the same command without the `-l nmf.py` filter:

```plaintext
ncalls  tottime  percall  cumtime  percall filename:lineno(function)
     1      0.000      0.000     1.681      1.681 nmf.py:461(fit)
   673      0.001      0.000     0.001      0.000 nmf.py:23(_neg)
   36      0.001      0.000     0.010      0.000 nmf.py:36(_sparseness)
    1      0.000      0.000     1.681      1.681 nmf.py:461(fit)

The `tottime` columns is the most interesting: it gives to total time spent executing the code of a given function ignoring the time spent in executing the sub-functions. The real total time (local code + sub-function calls) is given by the `cumtime` column.

Note the use of the `-l nmf.py` that restricts the output to lines that contains the “nmf.py” string. This is useful to have a quick look at the hotspot of the nmf Python module itself ignoring anything else.

Here is the beginning of the output of the same command without the `-l nmf.py` filter:

```python
In [5] %prun NMF(n_components=16, tol=1e-2).fit(X)
```

```plaintext
ncalls  tottime  percall  cumtime  percall filename:lineno(function)
   2833      0.653      0.000     0.653      0.000 {numpy.core._dotblas.dot}
```
The above results show that the execution is largely dominated by dot products operations (delegated to blas). Hence there is probably no huge gain to expect by rewriting this code in Cython or C/C++: in this case out of the 1.7s total execution time, almost 0.7s are spent in compiled code we can consider optimal. By rewriting the rest of the Python code and assuming we could achieve a 1000% boost on this portion (which is highly unlikely given the shallowness of the Python loops), we would not gain more than a 2.4x speed-up globally.

Hence major improvements can only be achieved by **algorithmic improvements** in this particular example (e.g. trying to find operation that are both costly and useless to avoid computing them rather than trying to optimize their implementation).

It is however still interesting to check what’s happening inside the `_nls_subproblem` function which is the hotspot if we only consider Python code: it takes around 100% of the cumulated time of the module. In order to better understand the profile of this specific function, let us install `line-prof` and wire it to IPython:

```
$ pip install line-profiler
```

**Under IPython <= 0.10**, edit `~/.ipython/ipy_user_conf.py` and ensure the following lines are present:

```python
import IPython.ipapi
ip = IPython.ipapi.get()

Towards the end of the file, define the `%lprun` magic:

```python
import line_profiler
ip.expose_magic('lprun', line_profiler.magic_lprun)
```

**Under IPython 0.11+**, first create a configuration profile:

```
$ ipython profile create
```

Then create a file named `~/.ipython/extensions/line_profiler_ext.py` with the following content:

```python
import line_profiler
def load_ipython_extension(ip):
    ip.define_magic('lprun', line_profiler.magic_lprun)
```

Then register it in `~/.ipython/profile_default/ipython_config.py`:

```python
c.TerminalIPythonApp.extensions = [‘line_profiler_ext’,]
c.InteractiveShellApp.extensions = [‘line_profiler_ext’,]
```
This will register the `%lprun` magic command in the IPython terminal application and the other frontends such as qtconsole and notebook.

Now restart IPython and let us use this new toy:

```
In [1]: from sklearn.datasets import load_digits

In [2]: from sklearn.decomposition.nmf import _nls_subproblem, NMF

In [3]: X = load_digits().data

In [4]: %lprun -f _nls_subproblem NMF(n_components=16, tol=1e-2).fit(X)
```

File: sklearn/decomposition/nmf.py  
Function: _nls_subproblem at line 137  
Total time: 1.73153 s

<table>
<thead>
<tr>
<th>Line #</th>
<th>Hits</th>
<th>Time</th>
<th>Per Hit</th>
<th>% Time</th>
<th>Line Contents</th>
</tr>
</thead>
<tbody>
<tr>
<td>137</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td><strong>def _nls_subproblem(V, W, H_init, tol, max_iter):</strong></td>
</tr>
<tr>
<td>138</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td><em><strong><strong>Non-negative least square solver</strong></strong></em></td>
</tr>
<tr>
<td>139</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td><strong>&quot;&quot;&quot;</strong></td>
</tr>
<tr>
<td>140</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td><strong>if (H_init &lt; 0).any():</strong></td>
</tr>
<tr>
<td>141</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td><strong>raise ValueError(&quot;Negative values in H_init passed to NLS solver.&quot;);</strong></td>
</tr>
<tr>
<td>142</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td><strong>H = H_init</strong></td>
</tr>
<tr>
<td>143</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td><strong>WtV = np.dot(W.T, V)</strong></td>
</tr>
<tr>
<td>144</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td><strong>WtW = np.dot(W.T, W)</strong></td>
</tr>
<tr>
<td>145</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td><strong># values justified in the paper</strong></td>
</tr>
<tr>
<td>146</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td><strong>alpha = 1</strong></td>
</tr>
<tr>
<td>147</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td><strong>beta = 0.1</strong></td>
</tr>
<tr>
<td>148</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td><strong>for n_iter in xrange(1, max_iter + 1):</strong></td>
</tr>
<tr>
<td>149</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td><strong>grad = np.dot(WtW, H) - WtV</strong></td>
</tr>
<tr>
<td>150</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td><strong>proj_gradient = norm(grad[np.logical_or(grad &lt; 0, H &gt; 0)])</strong></td>
</tr>
<tr>
<td>151</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td><strong>if proj_gradient &lt; tol:</strong></td>
</tr>
<tr>
<td>152</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td><strong>break</strong></td>
</tr>
<tr>
<td>153</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td><strong>#</strong></td>
</tr>
<tr>
<td>154</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td><strong>Hn = np.where(Hn &gt; 0, Hn, 0)</strong></td>
</tr>
<tr>
<td>155</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td><strong>Hn = _pos(Hn)</strong></td>
</tr>
<tr>
<td>156</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td><strong>d = Hn - H</strong></td>
</tr>
<tr>
<td>157</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td><strong>gradd = np.sum(grad * d)</strong></td>
</tr>
<tr>
<td>158</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td><strong>dQd = np.sum(np.dot(WtW, d) * d)</strong></td>
</tr>
</tbody>
</table>

By looking at the top values of the **Time** column it is really easy to pin-point the most expensive expressions that would deserve additional care.

### 3.2.3 Performance tips for the Cython developer

If profiling of the Python code reveals that the Python interpreter overhead is larger by one order of magnitude or more than the cost of the actual numerical computation (e.g. for loops over vector components, nested evaluation of conditional expression, scalar arithmetics...), it is probably adequate to extract the hotspot portion of the code as a
standalone function in a .pyx file, add static type declarations and then use Cython to generate a C program suitable to be compiled as a Python extension module.

The official documentation available at http://docs.cython.org/ contains a tutorial and reference guide for developing such a module. In the following we will just highlight a couple of tricks that we found important in practice on the existing cython codebase in the scikit-learn project.

TODO: html report, type declarations, bound checks, division by zero checks, memory alignement, direct blas calls...


3.2.4 Profiling compiled extensions

When working with compiled extensions (written in C/C++ with a wrapper or directly as Cython extension), the default Python profiler is useless: we need a dedicated tool to instrospect what’s happening inside the compiled extension itself.

In order to profile compiled Python extensions one could use gprof after having recompiled the project with gcc -pg and using the python-dbg variant of the interpreter on debian / ubuntu: however this approach requires to also have numpy and scipy recompiled with -pg which is rather complicated to get working.

Fortunately there exist two alternative profilers that don’t require you to recompile everything.

Using google-perftools

TODO

- https://github.com/fabianp/yep
- http://fseoane.net/blog/2011/a-profiler-for-python-extensions/

Note: google-perftools provides a nice ‘line by line’ report mode that can be triggered with the --lines option. However this does not seem to work correctly at the time of writing. This issue can be tracked on the project issue tracker.

Using valgrind / callgrind / kcachegrind

TODO

3.2.5 Multi-core parallelism using joblib.Parallel

TODO: give a simple teaser example here.

Checkout the official joblib documentation:

- http://packages.python.org/joblib/

3.2.6 A sample algorithmic trick: warm restarts for cross validation

TODO: demonstrate the warm restart tricks for cross validation of linear regression with Coordinate Descent.
3.3 Utilities for Developers

Scikit-learn contains a number of utilities to help with development. These are located in `sklearn.utils`, and include tools in a number of categories. All the following functions and classes are in the module `sklearn.utils`.

**Warning:** These utilities are meant to be used internally within the scikit-learn package. They are not guaranteed to be stable between versions of scikit-learn. Backports, in particular, will be removed as the scikit-learn dependencies evolve.

### 3.3.1 Validation Tools

These are tools used to check and validate input. When you write a function which accepts arrays, matrices, or sparse matrices as arguments, the following should be used when applicable.

- **assert_all_finite**: Throw an error if array contains NaNs or Infs.
- **safe_asarray**: Convert input to array or sparse matrix. Equivalent to `np.asarray`, but sparse matrices are passed through.
- **as_float_array**: convert input to an array of floats. If a sparse matrix is passed, a sparse matrix will be returned.
- **array2d**: equivalent to `np.atleast_2d`, but the order and dtype of the input are maintained.
- **atleast2d_or_csr**: equivalent to `array2d`, but if a sparse matrix is passed, will convert to csr format. Also calls `assert_all_finite`.
- **check_arrays**: check that all input arrays have consistent first dimensions. This will work for an arbitrary number of arrays.
- **warn_if_not_float**: Warn if input is not a floating-point value. the input `X` is assumed to have `X.dtype`.

If your code relies on a random number generator, it should never use functions like `numpy.random.random` or `numpy.random.normal`. This approach can lead to repeatability issues in unit tests. Instead, a `numpy.random.RandomState` object should be used, which is built from a `random_state` argument passed to the class or function. The function `check_random_state`, below, can then be used to create a random number generator object.

- **check_random_state**: create a `np.random.RandomState` object from a parameter `random_state`.
  - If `random_state` is `None` or `np.random`, then a randomly-initialized `RandomState` object is returned.
  - If `random_state` is an integer, then it is used to seed a new `RandomState` object.
  - If `random_state` is a `RandomState` object, then it is passed through.

For example:

```python
>>> from sklearn.utils import check_random_state
>>> random_state = 0
>>> random_state = check_random_state(random_state)
>>> random_state.rand(4)
array([[ 0.5488135 , 0.71518937, 0.60276338, 0.54488318]])
```
3.3.2 Efficient Linear Algebra & Array Operations

- `extmath.randomized_range_finder`: construct an orthonormal matrix whose range approximates the range of the input. This is used in `extmath.randomized_svd`, below.
- `extmath.randomized_svd`: compute the k-truncated randomized SVD. This algorithm finds the exact truncated singular values decomposition using randomization to speed up the computations. It is particularly fast on large matrices on which you wish to extract only a small number of components.
- `arrayfuncs.cholesky_delete`: (used in `sklearn.linear_model.least_angle.lars_path`) Remove an item from a cholesky factorization.
- `arrayfuncs.min_pos`: (used in `sklearn.linear_model.least_angle`) Find the minimum of the positive values within an array.
- `extmath.norm`: computes Euclidean (L2) vector norm by directly calling the BLAS nrm2 function. This is more stable than `scipy.linalg.norm`. See Fabian’s blog post for a discussion.
- `extmath.fast_logdet`: efficiently compute the log of the determinant of a matrix.
- `extmath.density`: efficiently compute the density of a sparse vector
- `extmath.safe_sparse_dot`: dot product which will correctly handle `scipy.sparse` inputs. If the inputs are dense, it is equivalent to `numpy.dot`.
- `extmath.logsumexp`: compute the sum of X assuming X is in the log domain. This is equivalent to calling `np.log(np.sum(np.exp(X)))`, but is robust to overflow/underflow errors. Note that there is similar functionality in `np.logaddexp.reduce`, but because of the pairwise nature of this routine, it is slower for large arrays. Scipy has a similar routine in `scipy.misc.logsumexp` (In scipy versions < 0.10, this is found in `scipy.maxentropy.logsumexp`), but the scipy version does not accept an axis keyword.
- `extmath.weighted_mode`: an extension of `scipy.stats.mode` which allows each item to have a real-valued weight.
- `resample`: Resample arrays or sparse matrices in a consistent way. used in `shuffle`, below.
- `shuffle`: Shuffle arrays or sparse matrices in a consistent way. Used in `sklearn.cluster.k_means`.

3.3.3 Efficient Routines for Sparse Matrices

The `sklearn.utils.sparsefuncs` cython module hosts compiled extensions to efficiently process `scipy.sparse` data.

- `sparsefuncs.mean_variance_axis0`: compute the means and variances along axis 0 of a CSR matrix. Used for normalizing the tolerance stopping criterion in `sklearn.cluster.k_means_.KMeans`.
- `sparsefuncs.inplace_csr_row_normalize_l1` and `sparsefuncs.inplace_csr_row_normalize_l2`: can be used to normalize individual sparse samples to unit l1 or l2 norm as done in `sklearn.preprocessing.Normalizer`.
- `sparsefuncs.inplace_csr_column_scale`: can be used to multiply the columns of a CSR matrix by a constant scale (one scale per column). Used for scaling features to unit standard deviation in `sklearn.preprocessing.Scaler`.

3.3.4 Graph Routines

- `graph.single_source_shortest_path_length`: (not currently used in scikit-learn) Return the shortest path from a single source to all connected nodes on a graph. Code is adapted from networkx.
If this is ever needed again, it would be far faster to use a single iteration of Dijkstra’s algorithm from `graph_shortest_path`.

- `graph.graph_laplacian`: (used in sklearn.cluster.spectral.spectral_embedding) Return the Laplacian of a given graph. There is specialized code for both dense and sparse connectivity matrices.

- `graph_shortest_path.graph_shortest_path`: (used in :class:sklearn.manifold.Isomap) Return the shortest path between all pairs of connected points on a directed or undirected graph. Both the Floyd-Warshall algorithm and Dijkstra’s algorithm are available. The algorithm is most efficient when the connectivity matrix is a `scipy.sparse.csr_matrix`.

### 3.3.5 Backports

- `fixes.Counter` (partial backport of collections.Counter from Python 2.7) Used in sklearn.feature_extraction.text.

- `fixes.unique` (backport of np.unique from numpy 1.4). Find the unique entries in an array. In numpy versions < 1.4, np.unique is less flexible. Used in sklearn.cross_validation.

- `fixes.copysign` (backport of np.copysign from numpy 1.4). Change the sign of x1 to that of x2, element-wise.

- `fixes.in1d` (backport of np.in1d from numpy 1.4). Test whether each element of an array is in a second array. Used in sklearn.datasets.twenty_newsgroups and sklearn.feature_extraction.image.

- `fixes.savemat` (backport of scipy.io.savemat from scipy 0.7.2). Save an array in MATLAB-format. In earlier versions, the keyword `oned_as` is not available.

- `fixes.count_nonzero` (backport of np.count_nonzero from numpy 1.6). Count the nonzero elements of a matrix. Used in tests of sklearn.linear_model.

- `arrayfuncs.solve_triangular` (Back-ported from scipy v0.9) Used in sklearn.linear_model.omp, independent back-ports in sklearn.mixture.gmm and sklearn.gaussian_process.

- `sparsetools.cs_graph_components` (backported from scipy.sparse.cs_graph_components in scipy 0.9). Used in sklearn.cluster.hierarchical, as well as in tests for sklearn.feature_extraction.

### ARPACK

- `arpack.eigs` (backported from scipy.sparse.linalg.eigs in scipy 0.10) Sparse non-symmetric eigenvalue decomposition using the Arnoldi method. A limited version of eigs is available in earlier scipy versions.

- `arpack.eigsh` (backported from scipy.sparse.linalg.eigsh in scipy 0.10) Sparse non-symmetric eigenvalue decomposition using the Arnoldi method. A limited version of eigsh is available in earlier scipy versions.

- `arpack.svds` (backported from scipy.sparse.linalg.svds in scipy 0.10) Sparse non-symmetric eigenvalue decomposition using the Arnoldi method. A limited version of svds is available in earlier scipy versions.
Benchmarks

- `bench.total_seconds` (back-ported from `timedelta.total_seconds` in Python 2.7). Used in `benchmarks/bench_glm.py`.

3.3.6 Testing Functions

- `testing.assert_in`, `testing.assert_not_in`: Assertions for container membership. Designed for forward compatibility with Nose 1.0.
- `mock_urllib2`: Object which mocks the urllib2 module to fake requests of mldata. Used in tests of `sklearn.datasets`.

3.3.7 Helper Functions

- `gen_even_slices`: generator to create n-packs of slices going up to n. Used in `sklearn.decomposition.dict_learning` and `sklearn.cluster.k_means`.
- `arraybuilder.ArrayBuilder`: Helper class to incrementally build a 1-d numpy.ndarray. Currently used in `sklearn.datasets._svmlight_format.pyx`.
- `safe_mask`: Helper function to convert a mask to the format expected by the numpy array or scipy sparse matrix on which to use it (sparse matrices support integer indices only while numpy arrays support both boolean masks and integer indices).

3.3.8 Hash Functions

- `murmurhash3_32` provides a python wrapper for the `MurmurHash3_x86_32` C++ non cryptographic hash function. This hash function is suitable for implementing lookup tables, Bloom filters, Count Min Sketch, feature hashing and implicitly defined sparse random projections:

  ```python
  >>> from sklearn.utils import murmurhash3_32
  >>> murmurhash3_32("some feature", seed=0)
  -384616559
  >>> murmurhash3_32("some feature", seed=0, positive=True)
  3910350737L
  ```

  The `sklearn.utils.murmurhash` module can also be “cimported” from other cython modules so as to benefit from the high performance of MurmurHash while skipping the overhead of the Python interpreter.

3.3.9 Warnings and Exceptions

- `deprecated`: Decorator to mark a function or class as deprecated.
- `ConvergenceWarning`: Custom warning to catch convergence problems. Used in `sklearn.covariance.graph_lasso`.

996 Chapter 3. Development
3.4 Developers’ Tips for Debugging

3.4.1 Memory errors: debugging Cython with valgrind

While python/numpy’s built-in memory management is relatively robust, it can lead to performance penalties for some routines. For this reason, much of the high-performance code in scikit-learn is written in cython. This performance gain comes with a tradeoff, however: it is very easy for memory bugs to crop up in cython code, especially in situations where that code relies heavily on pointer arithmetic.

Memory errors can manifest themselves a number of ways. The easiest ones to debug are often segmentation faults and related glibc errors. Uninitialized variables can lead to unexpected behavior that is difficult to track down. A very useful tool when debugging these sorts of errors is valgrind.

Valgrind is a command-line tool that can trace memory errors in a variety of code. Follow these steps:

1. Install valgrind on your system.
2. Download the python valgrind suppression file: valgrind-python.supp.
3. Follow the directions in the README.valgrind file to customize your python suppressions. If you don’t, you will have spurious output coming related to the python interpreter instead of your own code.
4. Run valgrind as follows:

   $> valgrind -v --suppressions=valgrind-python.supp python my_test_script.py

The result will be a list of all the memory-related errors, which reference lines in the C-code generated by cython from your .pyx file. If you examine the referenced lines in the .c file, you will see comments which indicate the corresponding location in your .pyx source file. Hopefully the output will give you clues as to the source of your memory error.

For more information on valgrind and the array of options it has, see the tutorials and documentation on the valgrind web site.

3.5 About us

This is a community effort, and as such many people have contributed to it over the years.

3.5.1 History

This project was started in 2007 as a Google Summer of Code project by David Cournapeau. Later that year, Matthieu Brucher started work on this project as part of his thesis.

In 2010 Fabian Pedregosa, Gael Varoquaux, Alexandre Gramfort and Vincent Michel of INRIA took leadership of the project and made the first public release, February the 1st 2010. Since then, several releases have appeared following a ~3 month cycle, and a striving international community has been leading the development.

3.5.2 People

- David Cournapeau
- Fred Mailhot
- David Cooke
- David Huard
- Dave Morrill
3.5.3 Citing scikit-learn

If you use scikit-learn in scientific publication, we would appreciate citations to the following paper:


Bibtex entry:

```latex
@article{scikit-learn,
  title={{Scikit-learn: Machine Learning in Python }},
  journal={Journal of Machine Learning Research},
  volume={12},
  pages={2825--2830},
  year={2011}
}
```
3.5.4 Funding

INRIA actively supports this project. It has provided funding for Fabian Pedregosa to work on this project full time in the period 2010-2012. It also hosts coding sprints and other events.

Google sponsored David Cournapeau with a Summer of Code Scholarship in the summer of 2007 and Vlad Niculae in 2011. If you would like to participate in the next Google Summer of code program, please see this page.

The NeuroDebian project providing Debian packaging and contributions is supported by Dr. James V. Haxby (Dartmouth College).

3.6 Support

There are several ways to get in touch with the developers.

3.6.1 Mailing List

The main mailing list is scikit-learn-general. There is also a commit list scikit-learn-commits, where updates to the main repository get notified.

3.6.2 Bug tracker

If you think you’ve encountered a bug, please report it to the issue tracker:
https://github.com/scikit-learn/scikit-learn/issues

3.6.3 IRC

Some developers like to hang out on channel #scikit-learn on irc.freenode.net.

If you do not have an IRC client or are behind a firewall this web client works fine: http://webchat.freenode.net

3.6.4 Documentation resources

This documentation is relative to 0.12-git. Documentation for other versions can be found here:

- Development version
- 0.10
- 0.9
- 0.8
- 0.7
- 0.6
- 0.5

Printable pdf documentation for all versions can be found here.
3.7 0.12

3.7.1 Changelog

- Added `preprocessing.LabelBinarizer`, a simple utility class to normalize labels or transform non-numerical labels, by Mathieu Blondel.
- Added the epsilon-insensitive loss and the ability to make probabilistic predictions with the modified huber loss in *Stochastic Gradient Descent*, by Mathieu Blondel.
- Added *Multi-dimensional Scaling (MDS)*, by Nelle Varoquaux
- SVMlight file format loader now detects compressed (gzip/bzip2) files and decompresses them on the fly.

3.7.2 API changes summary

- In `hmm` objects, like `hmm.GaussianHMM`, `hmm.MultinomialHMM`, etc., all parameters must be passed to the object when initialising it and not through `fit`. Now `fit` will only accept the data as an input parameter.

3.8 0.11

3.8.1 Changelog

Highlights

- Gradient boosted regression trees (*Gradient Tree Boosting*) for classification and regression by Peter Prettenhofer and Scott White.
- Simple dict-based feature loader with support for categorical variables (`feature_extraction.DictVectorizer`) by Lars Buitinck.
- Added Matthews correlation coefficient (`metrics.matthews_corrcoef`) and added macro and micro average options to `metrics.precision_score`, `metrics.recall_score` and `metrics.f1_score` by Satrajit Ghosh.
- *Out of Bag Estimates* of generalization error for *Ensemble methods* by Andreas Müller.
- Randomized sparse models: Randomized sparse linear models for feature selection, by Alexandre Gramfort and Gael Varoquaux
- *Label Propagation* for semi-supervised learning, by Clay Woolam. Note the semi-supervised API is still work in progress, and may change.
- Added BIC/AIC model selection to classical *Gaussian mixture models* and unified the API with the remainder of scikit-learn, by Bertrand Thirion
- Added `sklearn.cross_validation.StratifiedShuffleSplit`, which is a `sklearn.cross_validation.ShuffleSplit` with balanced splits, by Yannick Schwartz.
- `sklearn.neighbors.NearestCentroid` classifier added, along with a `shrink_threshold` parameter, which implements *shrunken centroid classification*, by Robert Layton.
Other changes

- Merged dense and sparse implementations of Stochastic Gradient Descent module and exposed utility extension types for sequential datasets seq_dataset and weight vectors weight_vector by Peter Prettenhofer.

- Added partial_fit (support for online/minibatch learning) and warm_start to the Stochastic Gradient Descent module by Mathieu Blondel.

- Dense and sparse implementations of Support Vector Machines classes and linear_model.LogisticRegression merged by Lars Buitinck.

- Regressors can now be used as base estimator in the Multiclass and multilabel algorithms module by Mathieu Blondel.

- Added n_jobs option to metrics.pairwise.pairwise_distances and metrics.pairwise.pairwise_kernels for parallel computation, by Mathieu Blondel.

- K-means can now be run in parallel, using the n_jobs argument to either K-means or KMeans, by Robert Layton.

- Improved Cross-Validation: evaluating estimator performance and Grid Search: setting estimator parameters documentation and introduced the new cross_validation.train_test_split helper function by Olivier Grisel

- svm.SVC members coef_ and intercept_ changed sign for consistency with decision_function; for kernel==linear, coef_ was fixed in the the one-vs-one case, by Andreas Müller.

- Performance improvements to efficient leave-one-out cross-validated Ridge regression, esp. for the n_samples > n_features case, in linear_model.RidgeCV, by Reuben Fletcher-Costin.

- Refactoring and simplification of the Text feature extraction API and fixed a bug that caused possible negative IDF, by Olivier Grisel.

- Beam pruning option in _BaseHMM module has been removed since it is difficult to cythonize. If you are interested in contributing a cython version, you can use the python version in the git history as a reference.

- Classes in Nearest Neighbors now support arbitrary Minkowski metric for nearest neighbors searches. The metric can be specified by argument p.

3.8.2 API changes summary

- covariance.EllipticEnvelop is now deprecated - Please use covariance.EllipticEnvelope instead.

- NeighborsClassifier and NeighborsRegressor are gone in the module Nearest Neighbors. Use the classes KNeighborsClassifier, RadiusNeighborsClassifier, KNeighborsRegressor and/or RadiusNeighborsRegressor instead.

- Sparse classes in the Stochastic Gradient Descent module are now deprecated.

- In mixture.GMM, mixture.DPGMM and mixture.VBGMM, parameters must be passed to an object when initialising it and not through fit. Now fit will only accept the data as an input parameter.

- methods rvs and decode in GMM module are now deprecated. sample and score or predict should be used instead.

- attribute _scores and _pvalues in univariate feature selection objects are now deprecated. scores_ or pvalues_ should be used instead.

- In LogisticRegression, LinearSVC, SVC and NuSVC, the class_weight parameter is now an initialization parameter, not a parameter to fit. This makes grid searches over this parameter possible.

- LFW data is now always shape (n_samples, n_features) to be consistent with the Olivetti faces dataset. Use images and pairs attribute to access the natural images shapes instead.
• In `svm.LinearSVC`, the meaning of the `multi_class` parameter changed. Options now are 'ovr' and 'cramer_singer', with 'ovr' being the default. This does not change the default behavior but hopefully is less confusing.

• Class `feature_selection.textVectorizer` is deprecated and replaced by `feature_selection.text.TfidfVectorizer`.

• The preprocessor / analyzer nested structure for text feature extraction has been removed. All those features are now directly passed as flat constructor arguments to `feature_selection.text.TfidfVectorizer` and `feature_selection.text.CountVectorizer`, in particular the following parameters are now used:
  – `analyzer` can be 'word' or 'char' to switch the default analysis scheme, or use a specific python callable (as previously).
  – `tokenizer` and `preprocessor` have been introduced to make it still possible to customize those steps with the new API.
  – `input` explicitly control how to interpret the sequence passed to `fit` and `predict`: filenames, file objects or direct (byte or unicode) strings.
  – charset decoding is explicit and strict by default.
  – the `vocabulary`, fitted or not is now stored in the `vocabulary_` attribute to be consistent with the project conventions.

• Class `feature_selection.text.TfidfVectorizer` now derives directly from `feature_selection.text.CountVectorizer` to make grid search trivial.

• Methods `rvs` in `_BaseHMM` module are now deprecated. `sample` should be used instead.

• Beam pruning option in `_BaseHMM` module is removed since it is difficult to be Cythonized. If you are interested, you can look in the history codes by git.

• The SVMlight format loader now supports files with both zero-based and one-based column indices, since both occur “in the wild”.

• Arguments in class `ShuffleSplit` are now consistent with `StratifiedShuffleSplit`. Arguments `test_fraction` and `train_fraction` are deprecated and renamed to `test_size` and `train_size` and can accept both float and int.

• Arguments in class `Bootstrap` are now consistent with `StratifiedShuffleSplit`. Arguments `n_test` and `n_train` are deprecated and renamed to `test_size` and `train_size` and can accept both float and int.

• Argument `p` added to classes in `Nearest Neighbors` to specify an arbitrary Minkowski metric for nearest neighbors searches.

### 3.8.3 People

- 282 Andreas Müller
- 239 Peter Prettenhofer
- 198 Gael Varoquaux
- 129 Olivier Grisel
- 114 Mathieu Blondel
- 103 Clay Woolam
- 96 Lars Buitinck
3.9.1 Changelog

- Python 2.5 compatibility was dropped; the minimum Python version needed to use scikit-learn is now 2.6.
- Sparse inverse covariance estimation using the graph Lasso, with associated cross-validated estimator, by Gael Varoquaux.
- New Tree module by Brian Holt, Peter Prettenhofer, Satrajit Ghosh and Gilles Louppe. The module comes with complete documentation and examples.
- Fixed a bug in the RFE module by Gilles Louppe (issue #378).
- Fixed a memory leak in in Support Vector Machines module by Brian Holt (issue #367).
- Faster tests by Fabian Pedregosa and others.
- Silhouette Coefficient cluster analysis evaluation metric added as sklearn.metrics.silhouette_score by Robert Layton.
- Fixed a bug in K-means in the handling of the n_init parameter: the clustering algorithm used to be run n_init times but the last solution was retained instead of the best solution by Olivier Grisel.
- Minor refactoring in Stochastic Gradient Descent module; consolidated dense and sparse predict methods; Enhanced test time performance by converting model parameters to fortran-style arrays after fitting (only multi-class).
- Adjusted Mutual Information metric added as sklearn.metrics.adjusted_mutual_info_score by Robert Layton.
- Models like SVC/SVR/LinearSVC/LogisticRegression from libsvm/liblinear now support scaling of C regularization parameter by the number of samples by Alexandre Gramfort.
- New Ensemble Methods module by Gilles Louppe and Brian Holt. The module comes with the random forest algorithm and the extra-trees method, along with documentation and examples.
- Novelty and Outlier Detection: outlier and novelty detection, by Virgile Fritsch.
- Kernel Approximation: a transform implementing kernel approximation for fast SGD on non-linear kernels by Andreas Müller.
- Fixed a bug due to atom swapping in Orthogonal Matching Pursuit (OMP) by Vlad Niculae.
- Sparse coding with a precomputed dictionary by Vlad Niculae.
- Mini Batch K-Means performance improvements by Olivier Grisel.
- K-means support for sparse matrices by Mathieu Blondel.
- Improved documentation for developers and for the sklearn.utils module, by Jake VanderPlas.
• Vectorized 20newsgroups dataset loader (sklearn.datasets.fetch_20newsgroups_vectorized) by Mathieu Blondel.

• Multiclass and multilabel algorithms by Lars Buitinck.

• Utilities for fast computation of mean and variance for sparse matrices by Mathieu Blondel.

• Make sklearn.preprocessing.scale and sklearn.preprocessing.Scaler work on sparse matrices by Olivier Grisel

• Feature importances using decision trees and/or forest of trees, by Gilles Louppe.

• Parallel implementation of forests of randomized trees by Gilles Louppe.

• sklearn.cross_validation.ShuffleSplit can subsample the train sets as well as the test sets by Olivier Grisel.

• Errors in the build of the documentation fixed by Andreas Müller.

3.9.2 API changes summary

Here are the code migration instructions when upgrading from scikit-learn version 0.9:

• Some estimators that may overwrite their inputs to save memory previously had overwrite_ parameters; these have been replaced with copy_ parameters with exactly the opposite meaning. This particularly affects some of the estimators in linear_model. The default behavior is still to copy everything passed in.

• The SVMlight dataset loader sklearn.datasets.load_svmlight_file no longer supports loading two files at once; use load_svmlight_files instead. Also, the (unused) buffer_mb parameter is gone.

• Sparse estimators in the Stochastic Gradient Descent module use dense parameter vector coef_ instead of sparse_coef_. This significantly improves test time performance.

• The Covariance estimation module now has a robust estimator of covariance, the Minimum Covariance Determinant estimator.

• Cluster evaluation metrics in metrics.cluster have been refactored but the changes are backwards compatible. They have been moved to the metrics.cluster.supervised, along with metrics.cluster.unsupervised which contains the Silhouette Coefficient.

• The permutation_test_score function now behaves the same way as cross_val_score (i.e. uses the mean score across the folds.)

• Cross Validation generators now use integer indices (indices=True) by default instead of boolean masks. This make it more intuitive to use with sparse matrix data.

• The functions used for sparse coding, sparse_encode and sparse_encode_parallel have been combined into sklearn.decomposition.sparse_encode, and the shapes of the arrays have been transposed for consistency with the matrix factorization setting, as opposed to the regression setting.

• Fixed an off-by-one error in the SVMlight/LibSVM file format handling; files generated using sklearn.datasets.dump_svmlight_file should be re-generated. (They should continue to work, but accidentally had one extra column of zeros prepended.)

• BaseDictionaryLearning class replaced by SparseCodingMixin.

• sklearn.utils.extmath.fast_svd has been renamed sklearn.utils.extmath.randomized_svd and the default oversampling is now fixed to 10 additional random vectors instead of doubling the number of components to extract. The new behavior follows the reference paper.
3.9.3 People

The following people contributed to scikit-learn since last release:

- 246 Andreas Müller
- 242 Olivier Grisel
- 220 Gilles Louppe
- 183 Brian Holt
- 166 Gael Varoquaux
- 144 Lars Buitinck
- 73 Vlad Niculae
- 65 Peter Prettenhofer
- 64 Fabian Pedregosa
- 60 Robert Layton
- 55 Mathieu Blondel
- 52 Jake Vanderplas
- 44 Noel Dawe
- 38 Alexandre Gramfort
- 24 Virgile Fritsch
- 23 Satrajit Ghosh
- 3 Jan Hendrik Metzen
- 3 Kenneth C. Arnold
- 3 Shiqiao Du
- 3 Tim Sheerman-Chase
- 3 Yaroslav Halchenko
- 2 Bala Subrahmaniam Varanasi
- 2 DraXus
- 2 Michael Eickenberg
- 1 Bogdan Trach
- 1 Félix-Antoine Fortin
- 1 Juan Manuel Caicedo Carvajal
- 1 Nelle Varoquaux
- 1 Nicolas Pinto
- 1 Tiziano Zito
- 1 Xinfan Meng
scikit-learn 0.9 was released on September 2011, three months after the 0.8 release and includes the new modules Manifold learning, The Dirichlet Process as well as several new algorithms and documentation improvements. This release also includes the dictionary-learning work developed by Vlad Niculae as part of the Google Summer of Code program.

### 3.10.1 Changelog

- New Manifold learning module by Jake Vanderplas and Fabian Pedregosa.
- New Dirichlet Process Gaussian Mixture Model by Alexandre Passos
- Nearest Neighbors module refactoring by Jake Vanderplas: general refactoring, support for sparse matrices in input, speed and documentation improvements. See the next section for a full list of API changes.
- Improvements on the Feature selection module by Gilles Louppe: refactoring of the RFE classes, documentation rewrite, increased efficiency and minor API changes.
- Sparse Principal Components Analysis (SparsePCA and MiniBatchSparsePCA) by Vlad Niculae, Gael Varoquaux and Alexandre Gramfort
- Printing an estimator now behaves independently of architectures and Python version thanks to Jean Kossaifi.
- Loader for libsvm/svmlight format by Mathieu Blondel and Lars Buitinck
- Documentation improvements: thumbnails in example gallery by Fabian Pedregosa.
- Important bugfixes in Support Vector Machines module (segfaults, bad performance) by Fabian Pedregosa.
- Added Multinomial Naive Bayes and Bernoulli Naive Bayes by Lars Buitinck
- Text feature extraction optimizations by Lars Buitinck
- Chi-Square feature selection (feature_selection.univariate_selection.chi2) by Lars Buitinck.
- Sample generators module refactoring by Gilles Louppe
- Multiclass and multilabel algorithms by Mathieu Blondel
- Ball tree rewrite by Jake Vanderplas
- Implementation of DBSCAN algorithm by Robert Layton
- Kmeans predict and transform by Robert Layton
- Preprocessing module refactoring by Olivier Grisel
- Faster mean shift by Conrad Lee
• New *Bootstrapping cross-validation, Random permutations cross-validation a.k.a. Shuffle & Split* and various other improvements in cross validation schemes by Olivier Grisel and Gael Varoquaux

• Adjusted Rand index and V-Measure clustering evaluation metrics by Olivier Grisel

• Added Orthogonal Matching Pursuit by Vlad Niculae

• Added 2D-patch extractor utilities in the *Feature extraction* module by Vlad Niculae

• Implementation of `linear_model.LassoLarsCV` (cross-validated Lasso solver using the Lars algorithm) and `linear_model.LassoLarsIC` (BIC/AIC model selection in Lars) by Gael Varoquaux and Alexandre Gramfort

• Scalability improvements to `metrics.roc_curve` by Olivier Hervieu

• Distance helper functions `metrics.pairwise.pairwise_distances` and `metrics.pairwise.pairwise_kernels` by Robert Layton

• Mini-Batch K-Means by Nelle Varoquaux and Peter Prettenhofer.

• *Downloading datasets from the mldata.org repository* utilities by Pietro Berkes.

• *The Olivetti faces dataset* by David Warde-Farley.

### 3.10.2 API changes summary

Here are the code migration instructions when upgrading from scikit-learn version 0.8:

• The `scikits.learn` package was renamed `sklearn`. There is still a `scikits.learn` package alias for backward compatibility.

  Third-party projects with a dependency on scikit-learn 0.9+ should upgrade their codebase. For instance under Linux / MacOSX just run (make a backup first!):

  ```
  find -name "*.py" | xargs sed -i 's/scikits.learn
  b/bscikits.learn\b/sklearn/g'
  ```

• Estimators no longer accept model parameters as fit arguments: instead all parameters must be only be passed as constructor arguments or using the now public `set_params` method inherited from `base.BaseEstimator`.

  Some estimators can still accept keyword arguments on the *fit* but this is restricted to data-dependent values (e.g. a Gram matrix or an affinity matrix that are precomputed from the *X* data matrix.

• The `cross_val` package has been renamed to `cross_validation` although there is also a `cross_val` package alias in place for backward compatibility.

  Third-party projects with a dependency on scikit-learn 0.9+ should upgrade their codebase. For instance under Linux / MacOSX just run (make a backup first!):

  ```
  find -name "*.py" | xargs sed -i 's/cross_val\b/cross_validation/g'
  ```

• The `score_func` argument of the `sklearn.cross_validation.cross_val_score` function is now expected to accept `y_test` and `y_predicted` as only arguments for classification and regression tasks or `X_test` for unsupervised estimators.

• *gamma* parameter for support vector machine algorithms is set to 1 / *n_features* by default, instead of 1 / *n_samples*.

• The `sklearn.hmm` has been marked as orphaned: it will be removed from scikit-learn in version 0.11 unless someone steps up to contribute documentation, examples and fix lurking numerical stability issues.
• `sklearn.neighbors` has been made into a submodule. The two previously available estimators, `NeighborsClassifier` and `NeighborsRegressor` have been marked as deprecated. Their functionality has been divided among five new classes: `NearestNeighbors` for unsupervised neighbors searches, `KNeighborsClassifier` & `RadiusNeighborsClassifier` for supervised classification problems, and `KNeighborsRegressor` & `RadiusNeighborsRegressor` for supervised regression problems.

• `sklearn.ball_tree.BallTree` has been moved to `sklearn.neighbors.BallTree`. Using the former will generate a warning.

• `sklearn.linear_model.LARS()` and related classes (LassoLARS, LassoLARSCV, etc.) have been renamed to `sklearn.linear_model.Lars()`.

• All distance metrics and kernels in `sklearn.metrics.pairwise` now have a `Y` parameter, which by default is `None`. If not given, the result is the distance (or kernel similarity) between each sample in `Y`. If given, the result is the pairwise distance (or kernel similarity) between samples in `X` to `Y`.

• `sklearn.metrics.pairwise.l1_distance` is now called `manhattan_distance`, and by default returns the pairwise distance. For the component wise distance, set the parameter `sum_over_features` to `False`.

Backward compatibility package aliases and other deprecated classes and functions will be removed in version 0.11.

### 3.10.3 People

38 people contributed to this release.

- 387 Vlad Niculae
- 320 Olivier Grisel
- 192 Lars Buitinck
- 179 Gael Varoquaux
- 168 Fabian Pedregosa (INRIA, Parietal Team)
- 127 Jake Vanderplas
- 120 Mathieu Blondel
- 85 Alexandre Passos
- 67 Alexandre Gramfort
- 57 Peter Prettenhofer
- 56 Gilles Louppe
- 42 Robert Layton
- 38 Nelle Varoquaux
- 32 Jean Kossaifi
- 30 Conrad Lee
- 22 Pietro Berkes
- 18 andy
- 17 David Warde-Farley
- 12 Brian Holt
- 11 Robert
scikit-learn 0.8 was released on May 2011, one month after the first “international” scikit-learn coding sprint and is marked by the inclusion of important modules: Hierarchical clustering, Partial Least Squares, Non-negative matrix factorization (NMF or NNMF), initial support for Python 3 and by important enhacements and bug fixes.

3.11.1 Changelog

Several new modules were introduced during this release:

- New Hierarchical clustering module by Vincent Michel, Bertrand Thirion, Alexandre Gramfort and Gael Varoquaux.
- Kernel PCA implementation by Mathieu Blondel
- The Labeled Faces in the Wild face recognition dataset by Olivier Grisel.
- New Partial Least Squares module by Edouard Duchesnay.
- Non-negative matrix factorization (NMF or NNMF) module Vlad Niculae
- Implementation of the Oracle Approximating Shrinkage algorithm by Virgile Fritsch in the Covariance estimation module.

Some other modules benefited from significant improvements or cleanups.

- Initial support for Python 3: builds and imports cleanly, some modules are usable while others have failing tests by Fabian Pedregosa.
- decomposition.PCA is now usable from the Pipeline object by Olivier Grisel.
- Guide How to optimize for speed by Olivier Grisel.
• Fixes for memory leaks in libsvm bindings, 64-bit safer BallTree by Lars Buitinck.
• bug and style fixing in K-means algorithm by Jan Schlüter.
• Add attribute covered to Gaussian Mixture Models by Vincent Schut.
• Implement transform, predict_log_proba in LDA by Mathieu Blondel.
• Refactoring in the Support Vector Machines module and bug fixes by Fabian Pedregosa, Gael Varoquaux and Amit Aides.
• Refactored SGD module (removed code duplication, better variable naming), added interface for sample weight by Peter Prettenhofer.
• Wrapped BallTree with Cython by Thouis (Ray) Jones.
• Added functions by Paolo Losi.
• Typos, doc style, etc. by Yaroslav Halchenko, Gael Varoquaux, Olivier Grisel, Yann Malet, Nicolas Pinto, Lars Buitinck and Fabian Pedregosa.

3.11.2 People

People that made this release possible preceded by number of commits:

• 159 Olivier Grisel
• 96 Gael Varoquaux
• 96 Vlad Niculae
• 94 Fabian Pedregosa
• 36 Alexandre Gramfort
• 32 Paolo Losi
• 31 Edouard Duchesnay
• 30 Mathieu Blondel
• 25 Peter Prettenhofer
• 22 Nicolas Pinto
• 11 Virgile Fritsch
• 7 Lars Buitinck
• 6 Vincent Michel
• 5 Bertrand Thirion
• 4 Thouis (Ray) Jones
• 4 Vincent Schut
• 3 Jan Schlüter
• 2 Julien Miotte
• 2 Matthieu Perrot
• 2 Yann Malet
• 2 Yaroslav Halchenko
• 1 Amit Aides
scikit-learn 0.7 was released in March 2011, roughly three months after the 0.6 release. This release is marked by the speed improvements in existing algorithms like k-Nearest Neighbors and K-Means algorithm and by the inclusion of an efficient algorithm for computing the Ridge Generalized Cross Validation solution. Unlike the preceding release, no new modules were added to this release.

3.12.1 Changelog

- Performance improvements for Gaussian Mixture Model sampling [Jan Schlüter].
- Better handling of collinearity and early stopping in `linear_model.lars_path` [Alexandre Gramfort and Fabian Pedregosa].
- Fixes for liblinear ordering of labels and sign of coefficients [Dan Yamins, Paolo Losi, Mathieu Blondel and Fabian Pedregosa].
- Performance improvements for Nearest Neighbors algorithm in high-dimensional spaces [Fabian Pedregosa].
- Performance improvements for `cluster.KMeans` [Gael Varoquaux and James Bergstra].
- Sanity checks for SVM-based classes [Mathieu Blondel].
- Refactoring of `neighbors.NeighborsClassifier` and `neighbors.kneighbors_graph` : added different algorithms for the k-Nearest Neighbor Search and implemented a more stable algorithm for finding barycenter weights. Also added some developer documentation for this module, see notes_neighbors for more information [Fabian Pedregosa].
- Documentation improvements: Added `pca.RandomizedPCA` and `linear_model.LogisticRegression` to the class reference. Also added references of matrices used for clustering and other fixes [Gael Varoquaux, Fabian Pedregosa, Mathieu Blondel, Olivier Grisel, Virgile Fritsch, Emmanuelle Gouillart].
- Binded decision_function in classes that make use of liblinear, dense and sparse variants, like `svm.LinearSVC` or `linear_model.LogisticRegression` [Fabian Pedregosa].
- Performance and API improvements to `metrics.euclidean_distances` and to `pca.RandomizedPCA` [James Bergstra].
- Fix compilation issues under NetBSD [Kamel Ibn Hassen Derouiche].
- Allow input sequences of different lengths in `hmm.GaussianHMM` [Ron Weiss].
- Fix bug in affinity propagation caused by incorrect indexing [Xinfan Meng].

3.12.2 People

People that made this release possible preceded by number of commits:

- 85 Fabian Pedregosa
3.13 0.6

scikit-learn 0.6 was released on December 2010. It is marked by the inclusion of several new modules and a general renaming of old ones. It is also marked by the inclusion of new examples, including applications to real-world datasets.

3.13.1 Changelog

- New stochastic gradient descent module by Peter Prettenhofer. The module comes with complete documentation and examples.
- Improved svm module: memory consumption has been reduced by 50%, heuristic to automatically set class weights, possibility to assign weights to samples (see SVM: Weighted samples for an example).
- New Gaussian Processes module by Vincent Dubourg. This module also has great documentation and some very neat examples. See Gaussian Processes regression: basic introductory example or Gaussian Processes classification example: exploiting the probabilistic output for a taste of what can be done.
- It is now possible to use liblinear’s Multi-class SVC (option multi_class in svm.LinearSVC)
- New features and performance improvements of text feature extraction.
- Improved sparse matrix support, both in main classes (grid_search.GridSearchCV) as in modules sklearn.svm.sparse and sklearn.linear_model.sparse.
- Lots of cool new examples and a new section that uses real-world datasets was created. These include: Faces recognition example using eigenfaces and SVMs, Species distribution modeling, Libsvm GUI, Wikipedia principal eigenvector and others.
- Faster Least Angle Regression algorithm. It is now 2x faster than the R version on worst case and up to 10x times faster on some cases.
Faster coordinate descent algorithm. In particular, the full path version of lasso (\texttt{linear\_model.lasso\_path}) is more than 200x times faster than before.

It is now possible to get probability estimates from a \texttt{linear\_model.LogisticRegression} model.

module renaming: the glm module has been renamed to linear\_model, the gmm module has been included into the more general mixture model and the sgd module has been included in linear\_model.

Lots of bug fixes and documentation improvements.

### 3.13.2 People

People that made this release possible proceeded by number of commits:

- 207 Olivier Grisel
- 167 Fabian Pedregosa
- 97 Peter Prettenhofer
- 68 Alexandre Gramfort
- 59 Mathieu Blondel
- 55 Gael Varoquaux
- 33 Vincent Dubourg
- 21 Ron Weiss
- 9 Bertrand Thirion
- 3 Alexandre Passos
- 3 Anne-Laure Fouque
- 2 Ronan Amicel
- 1 Christian Osendorfer

### 3.14 0.5

#### 3.14.1 Changelog

#### 3.14.2 New classes

- Support for sparse matrices in some classifiers of modules \texttt{svm} and \texttt{linear\_model}(see \texttt{svm.sparse.SVC}, \texttt{svm.sparse.SVR}, \texttt{svm.sparse.LinearSVC}, \texttt{linear\_model.sparse.Lasso}, \texttt{linear\_model.sparse.ElasticNet})
- New \texttt{pipeline.Pipeline} object to compose different estimators.
- Recursive Feature Elimination routines in module \textit{Feature selection}.
- Addition of various classes capable of cross validation in the linear\_model module (\texttt{linear\_model.LassoCV}, \texttt{linear\_model.ElasticNetCV}, etc.).
- New, more efficient LARS algorithm implementation. The Lasso variant of the algorithm is also implemented. See \texttt{linear\_model.lars\_path}, \texttt{linear\_model.Lars} and \texttt{linear\_model.LassoLars}.
• New Hidden Markov Models module (see classes `hmm.GaussianHMM`, `hmm.MultinomialHMM`, `hmm.GMMHMM`)
• New module feature_extraction (see `class reference`)
• New FastICA algorithm in module sklearn.fastica

3.14.3 Documentation

• Improved documentation for many modules, now separating narrative documentation from the class reference. As an example, see documentation for the SVM module and the complete class reference.

3.14.4 Fixes

• API changes: adhere variable names to PEP-8, give more meaningful names.
• Fixes for svm module to run on a shared memory context (multiprocessing).
• It is again possible to generate latex (and thus PDF) from the sphinx docs.

3.14.5 Examples

• new examples using some of the mlcomp datasets: Classification of text documents: using a MLComp dataset, Classification of text documents using sparse features
• Many more examples. See here the full list of examples.

3.14.6 External dependencies

• Joblib is now a dependency of this package, although it is shipped with (sklearn.externals.joblib).

3.14.7 Removed modules

• Module ann (Artificial Neural Networks) has been removed from the distribution. Users wanting this sort of algorithms should take a look into pybrain.

3.14.8 Misc

• New sphinx theme for the web page.

3.14.9 Authors

The following is a list of authors for this release, preceeded by number of commits:
• 262 Fabian Pedregosa
• 240 Gael Varoquaux
• 149 Alexandre Gramfort
• 116 Olivier Grisel
• 40 Vincent Michel
3.15.0.4

3.15.1 Changelog

Major changes in this release include:

- Coordinate Descent algorithm (Lasso, ElasticNet) refactoring & speed improvements (roughly 100x times faster).
- Coordinate Descent Refactoring (and bug fixing) for consistency with R’s package GLMNET.
- New metrics module.
- New GMM module contributed by Ron Weiss.
- Implementation of the LARS algorithm (without Lasso variant for now).
- feature_selection module redesign.
- Migration to GIT as content management system.
- Removal of obsolete attrselect module.
- Rename of private compiled extensions (aded underscore).
- Removal of legacy unmaintained code.
- Documentation improvements (both docstring and rst).
- Improvement of the build system to (optionally) link with MKL. Also, provide a lite BLAS implementation in case no system-wide BLAS is found.
- Lots of new examples.
- Many, many bug fixes ...

3.15.2 Authors

The committer list for this release is the following (preceded by number of commits):

- 143 Fabian Pedregosa
- 35 Alexandre Gramfort
- 34 Olivier Grisel
• 11 Gael Varoquaux
• 5 Yaroslav Halchenko
• 2 Vincent Michel
• 1 Chris Filo Gorgolewski

3.16 Presentations and Tutorials on Scikit-Learn

For written tutorials, see the Tutorial section of the documentation.

3.16.1 Videos

• Introduction to scikit-learn by Gael Varoquaux at ICML 2010
  A three minute video from a very early stage of the scikit, explaining the basic idea and approach we are following.

• Introduction to statistical learning with scikit learn by Gael Varoquaux at SciPy 2011
  An extensive tutorial, consisting of four sessions of one hour. The tutorial covers basics of machine learning, many algorithms and how to apply them using scikit-learn. The material corresponding is now in the scikit-learn documentation section A tutorial on statistical-learning for scientific data processing.

• Statistical Learning for Text Classification with scikit-learn and NLTK (and slides) by Olivier Grisel at PyCon 2011
  Thirty minute introduction to text classification. Explains how to use NLTK and scikit-learn to solve real-world text classification tasks and compares against cloud-based solutions.

• Introduction to Interactive Predictive Analytics in Python with scikit-learn by Olivier Grisel at PyCon 2012
  3-hours long introduction to prediction tasks using the scikit-learn.

• scikit-learn - Machine Learning in Python by Jake Vanderplas at the 2012 PyData workshop at Google
  Interactive demonstration of some scikit-learn features. 75 minutes.
BIBLIOGRAPHY


http://www-ist.massey.ac.nz/smarsland/Code/10/lle.py


[MRT] A randomized algorithm for the decomposition of matrices Per-Gunnar Martinsson, Vladimir Rokhlin and Mark Tygert


http://www2.imm.dtu.dk/~hbn/dace/dace.pdf

http://www.jstor.org/pss/1269548


http://citeseerx.ist.psu.edu/viewdoc/summary?doi=10.1.1.70.382


[R42] Wikipedia entry for the Adjusted Mutual Information

http://www.springerlink.com/content/x64124718341j1j0/


PYTHON MODULE INDEX

S
sklearn.cluster, 227
sklearn.covariance, 245
sklearn.cross_validation, 269
sklearn.datasets, 281
sklearn.decomposition, 303
sklearn.ensemble, 339
sklearn.feature_extraction, 361
sklearn.feature_extraction.image, 364
sklearn.feature_extraction.text, 367
sklearn.feature_selection, 375
sklearn.gaussian_process, 388
sklearn.grid_search, 396
sklearn.hmm, 399
sklearn.kernel_approximation, 412
sklearn.lda, 422
sklearn.linear_model, 424
sklearn.linear_model.sparse, 497
sklearn.manifold, 509
sklearn.metrics, 517
sklearn.metrics.cluster, 526
sklearn.metrics.pairwise, 535
sklearn.mixture, 541
sklearn.multiclass, 553
sklearn.naive_bayes, 559
sklearn.neighbors, 566
sklearn.pipeline, 601
sklearn.pls, 592
sklearn.preprocessing, 603
sklearn.qda, 615
sklearn.semi_supervised, 417
sklearn.svm, 617
sklearn.tree, 644
sklearn.utils, 656
S
sklearn.cluster, 227
sklearn.covariance, 245
sklearn.cross_validation, 269
sklearn.datasets, 281
sklearn.decomposition, 303
sklearn.ensemble, 339
sklearn.feature_extraction, 361
sklearn.feature_extraction.image, 364
sklearn.feature_extraction.text, 367
sklearn.feature_selection, 375
sklearn.gaussian_process, 388
sklearn.grid_search, 396
sklearn.hmm, 399
sklearn.kernel_approximation, 412
sklearn.lda, 422
sklearn.linear_model, 424
sklearn.linear_model.sparse, 497
sklearn.manifold, 509
sklearn.metrics, 517
sklearn.metrics.cluster, 526
sklearn.metrics.pairwise, 535
sklearn.mixture, 541
sklearn.multiclass, 553
sklearn.naive_bayes, 559
sklearn.neighbors, 566
sklearn.pipeline, 601
sklearn.pls, 592
sklearn.preprocessing, 603
sklearn.qda, 615
sklearn.semi_supervised, 417
sklearn.svm, 617
sklearn.tree, 644
sklearn.utils, 656
__init__() (sklearn.neighbors.RadiusNeighborsRegressor method), 583
__init__() (sklearn.pipeline.Pipeline method), 603
__init__() (sklearn.pls.CCA method), 599
__init__() (sklearn.pls.PLSCanonical method), 597
__init__() (sklearn.pls.PLSRRegression method), 594
__init__() (sklearn.pls.PLSSVD method), 601
__init__() (sklearn.preprocessing.Binarizer method), 608
__init__() (sklearn.preprocessing.KernelCenterer method), 612
__init__() (sklearn.preprocessing.LabelBinarizer method), 611
__init__() (sklearn.preprocessing.Normalizer method), 606
__init__() (sklearn.preprocessing.Scaler method), 605
__init__() (sklearn.qda.QDA method), 616
__init__() (sklearn.semi_supervised.LabelPropagation method), 418
__init__() (sklearn.semi_supervised.LabelSpreading method), 420
__init__() (sklearn.svm.LinearSVC method), 624
__init__() (sklearn.svm.NuSVC method), 627
__init__() (sklearn.svm.NuSVR method), 635
__init__() (sklearn.svm.OneClassSVM method), 638
__init__() (sklearn.svm.SVC method), 620
__init__() (sklearn.svm.SVR method), 631
__init__() (sklearn.tree.DecisionTreeClassifier method), 645
__init__() (sklearn.tree.DecisionTreeRegressor method), 649
__init__() (sklearn.tree.ExtraTreeClassifier method), 651
__init__() (sklearn.tree.ExtraTreeRegressor method), 654
absolute_exponential() (in module sklearn.gaussian_process.correlation_models), 392
AdditiveChi2Sampler (class in sklearn.kernel_approximation), 414
adjusted_mutual_info_score() (in module sklearn.metrics), 527
adjusted_rand_score() (in module sklearn.metrics), 528
affinity_propagation() (in module sklearn.cluster), 242
AffinityPropagation (class in sklearn.cluster), 227
aic() (sklearn.mixture.DPGMM method), 547
aic() (sklearn.mixture.GMM method), 543
aic() (sklearn.mixture.VBGMM method), 551
algorithm (sklearn.hmm.GaussianHMM attribute), 402
algorithm (sklearn.hmm.GMMHMM attribute), 409
algorithm (sklearn.hmm.MultinomialHMM attribute), 406
ARDRegression (class in sklearn.linear_model), 483
arg_max_reduced_likelihood_function() (sklearn.gaussian_process.GaussianProcess method), 390
auc() (in module sklearn.metrics), 519
BallTree (class in sklearn.neighbors), 586
BayesianRidge (class in sklearn.linear_model), 480
BernoulliNB (class in sklearn.naive_bayes), 564
best_estimator (sklearn.grid_search.GridSearchCV attribute), 398
best_score (sklearn.grid_search.GridSearchCV attribute), 398
bic() (sklearn.mixture.DPGMM method), 547
bic() (sklearn.mixture.GMM method), 543
bic() (sklearn.mixture.VBGMM method), 551
binarize() (in module sklearn.preprocessing), 615
Binarizer (class in sklearn.preprocessing), 607
Bootstrap (class in sklearn.cross_validation), 269
build_analyzer() (sklearn.feature_extraction.text.CountVectorizer method), 370
build_analyzer() (sklearn.feature_extraction.text.TfidfVectorizer method), 373
build_preprocessor() (sklearn.feature_extraction.text.CountVectorizer method), 370
build_preprocessor() (sklearn.feature_extraction.text.TfidfVectorizer method), 374
build_tokenizer() (sklearn.feature_extraction.text.CountVectorizer method), 370
build_tokenizer() (sklearn.feature_extraction.text.TfidfVectorizer method), 374
CCA (class in sklearn.pls), 598
correct_covariance() (sklearn.covariance.EllipticEnvelope method), 249
correct_covariance() (sklearn.covariance.MinCovDet method), 259
correct_covariance() (sklearn.covariance.MinCovDet method), 259
classification_report() (in module sklearn.metrics), 524
classification_report() (in module sklearn.metrics), 529
class_prior (sklearn.naive_bayes.GaussianNB attribute), 386
classes (sklearn.linear_model.Perceptron attribute), 469
classes (sklearn.linear_model.SGDClassifier attribute), 473
classes (sklearn.linear_model.sparse.SGDClassifier attribute), 501
classification_report() (in module sklearn.metrics), 524
classification_report() (in module sklearn.metrics), 529
classification_report() (in module sklearn.metrics), 518
classification_report() (sklearn.svm.GaussianProcessRegressionModel), 395
classification_report() (sklearn.svm.GaussianProcessRegressionModel), 395
confidence_matrix() (in module sklearn.metrics), 518
covariance_type (sklearn.hmm.GaussianHMM attribute), 402
covariance_type (sklearn.hmm.GaussianHMM attribute), 409
Index
covars_ (sklearn.hmm.GaussianHMM attribute), 402
cross_val_score() (in module sklearn.cross_validation), 279
cross_validation() (in module sklearn.svm.libsvm), 643
cubic() (in module sklearn.gaussian_process.correlation_models), 394

DBSCAN (class in sklearn.cluster), 228
dbscan() (in module sklearn.cluster), 243
decision_function() (sklearn.svm.libsvm), 642
decision_function() (sklearn.covariance.EllipticEnvelope method), 249
decision_function() (sklearn.lda.LDA method), 423
decision_function() (sklearn.linear_model.ARDRegression method), 484
decision_function() (sklearn.linear_model.BayesianRidge method), 481
decision_function() (sklearn.linear_model.ElasticNet method), 444
decision_function() (sklearn.linear_model.ElasticNetCV method), 176, 447
decision_function() (sklearn.linear_model.Lars method), 450
decision_function() (sklearn.linear_model.LarsCV method), 167, 455
decision_function() (sklearn.linear_model.Lasso method), 437
decision_function() (sklearn.linear_model.LassoCV method), 172, 440
decision_function() (sklearn.linear_model.LassoLars method), 452
decision_function() (sklearn.linear_model.LassoLarsCV method), 170, 457
decision_function() (sklearn.linear_model.LassoLarsIC method), 180, 460
decision_function() (sklearn.linear_model.LinearRegression method), 426
decision_function() (sklearn.linear_model.LogisticRegression method), 462, 507
decision_function() (sklearn.linear_model.OrthogonalMatchingPursuit method), 466
decision_function() (sklearn.linear_model.Perceptron method), 469
decision_function() (sklearn.linear_model.Ridge method), 428
decision_function() (sklearn.linear_model.RidgeCV method), 163, 435
decision_function() (sklearn.linear_model.SGDClassifier method), 473
decision_function() (sklearn.linear_model.SGDRegressor method), 478
decision_function() (sklearn.linear_model.sparse.ElasticNet method), 499
decision_function() (sklearn.linear_model.sparse.ElasticNetCV method), 447
decision_function() (sklearn.linear_model.sparse.SGDClassifier method), 501
decision_function() (sklearn.linear_model.sparse.SGDRegressor method), 504
decision_function() (sklearn.pipeline.Pipeline method), 603
decision_function() (sklearn.qda.QDA method), 616
decision_function() (sklearn.svm.LinearSVC method), 624
decision_function() (sklearn.svm.NuSVC method), 627
decision_function() (sklearn.svm.NuSVR method), 635
decision_function() (sklearn.svm.OneClassSVM method), 638
decision_function() (sklearn.svm.SVC method), 620
decision_function() (sklearn.svm.SVR method), 631
DecisionTreeClassifier (class in sklearn.tree), 644
DecisionTreeRegressor (class in sklearn.tree), 647
decode() (sklearn.feature_extraction.text.CountVectorizer method), 370
decode() (sklearn.feature_extraction.text.TfidfVectorizer method), 374
dict_learning() (in module sklearn.decomposition), 336
dict_learning_online() (in module sklearn.decomposition), 337
DictionaryLearning (class in sklearn.decomposition), 328
DictVectorizer (class in sklearn.feature_extraction), 361
distance_metrics() (in module sklearn.metrics.pairwise), 538
DPGMM (class in sklearn.mixture), 545
ElasticNet (class in sklearn.linear_model), 442
ElasticNetCV (class in sklearn.linear_model), 174, 445
EllipticEnvelope (class in sklearn.covariance), 248
empirical_covariance() (sklearn.hmm.MultinomialHMM attribute), 406
error_norm() (sklearn.covariance.EllipticEnvelope method), 249
error_norm() (sklearn.covariance.EmpiricalCovariance method), 246
EmpiricalCovariance (class in sklearn.covariance), 246

1030 Index
fit() (sklearn.kernel_approximation.AdditiveChi2Sampler method), 414
fit() (sklearn.kernel_approximation.RBFSampler method), 413
fit() (sklearn.kernel_approximation.SkewedChi2Sampler method), 416
fit() (sklearn lda LDA method), 423
fit() (sklearn.linear_model.ARDRegression method), 485
fit() (sklearn.linear_model.BayesianRidge method), 482
fit() (sklearn.linear_model.ElasticNet method), 444
fit() (sklearn.linear_model.ElasticNetCV method), 176, 447
fit() (sklearn.linear_model.Lars method), 450
fit() (sklearn.linear_model.LarsCV method), 167, 455
fit() (sklearn.linear_model.Lasso method), 437
fit() (sklearn.linear_model.LassoCV method), 172, 440
fit() (sklearn.linear_model.LassoLars method), 452
fit() (sklearn.linear_model.LassoLarsCV method), 170, 457
fit() (sklearn.linear_model.LassoLarsIC method), 180, 460
fit() (sklearn.linear_model.LinearRegression method), 426
fit() (sklearn.linear_model.LogisticRegression method), 463, 507
fit() (sklearn.linear_model.OrthogonalMatchingPursuit method), 466
fit() (sklearn.linear_model.Perceptron method), 469
fit() (sklearn.linear_model.RandomizedLasso method), 488
fit() (sklearn.linear_model.RandomizedLogisticRegression method), 491
fit() (sklearn.linear_model.Ridge method), 428
fit() (sklearn.linear_model.RidgeClassifier method), 430
fit() (sklearn.linear_model.RidgeClassifierCV method), 165, 432
fit() (sklearn.linear_model.RidgeCV method), 163, 435
fit() (sklearn.linear_model.SGDClassifier method), 474
fit() (sklearn.linear_model.SGDRegressor method), 478
fit() (sklearn.linear_model.sparse.ElasticNet method), 500
fit() (sklearn.linear_model.sparse.Lasso method), 498
fit() (sklearn.linear_model.sparse.SGDClassifier method), 501
fit() (sklearn.linear_model.sparse.SGDRegressor method), 504
fit() (sklearn.manifold.Isomap method), 513
fit() (sklearn.manifold.LocallyLinearEmbedding method), 511
fit() (sklearn.manifold.MDS method), 515
fit() (sklearn.mixture.DPGMM method), 548
fit() (sklearn.mixture.GMM method), 544
fit() (sklearn.mixture.VBGMM method), 552
fit() (sklearn.multiclass.OneVsOneClassifier method), 555
fit() (sklearn.multiclass.OneVsRestClassifier method), 554
fit() (sklearn.multiclass.OutputCodeClassifier method), 557
fit() (sklearn naive bayes.BernoulliNB method), 565
fit() (sklearn naive bayes.GaussianNB method), 560
fit() (sklearn naive bayes.MultinomialNB method), 562
fit() (sklearn.neighbors.KNeighborsClassifier method), 572
fit() (sklearn.neighbors.KNeighborsRegressor method), 580
fit() (sklearn.neighbors.NearestCentroid method), 590
fit() (sklearn.neighbors.NearestNeighbors method), 568
fit() (sklearn.neighbors.RadiusNeighborsClassifier method), 576
fit() (sklearn.neighbors.RadiusNeighborsRegressor method), 583
fit() (sklearn.pipeline.Pipeline method), 603
fit() (sklearn.preprocessing.Binarizer method), 608
fit() (sklearn.preprocessing.KernelCenterer method), 612
fit() (sklearn.preprocessing.LabelBinarizer method), 611
fit() (sklearn.preprocessing.Normalizer method), 606
fit() (sklearn.preprocessing.Scaler method), 605
fit() (sklearn.qda.QDA method), 616
fit() (sklearn.semi_supervised.LabelPropagation method), 418
fit() (sklearn.semi_supervised.LabelSpreading method), 420
fit() (sklearn.svm.LinearSVC method), 624
fit() (sklearn.svm.NuSVC method), 627
fit() (sklearn.svm.NuSVR method), 635
fit() (sklearn.svm.OneClassSVM method), 638
fit() (sklearn.svm.SVC method), 620
fit() (sklearn.svm.SVR method), 631
fit() (sklearn.tree.DecisionTreeClassifier method), 645
fit() (sklearn.tree.DecisionTreeRegressor method), 649
fit() (sklearn.tree.ExtraTreeClassifier method), 651
fit() (sklearn.tree.ExtraTreeRegressor method), 654
fit_ecoc() (in module sklearn.multiclass), 558
fit_ovo() (in module sklearn.multiclass), 558
fit_ovr() (in module sklearn.multiclass), 558
fit_predict() (sklearn.cluster.KMeans method), 231
fit_predict() (sklearn.cluster.MINIBATCHKMEANS method), 234
fit_stage() (sklearn.ensemble.GradientBoostingClassifier method), 198, 357
fit_stage() (sklearn.ensemble.GradientBoostingRegressor method), 201, 360
fit_transform() (sklearn.decomposition.DictionaryLearning method), 330
fit_transform() (sklearn.decomposition.kernelPCA method), 316
<table>
<thead>
<tr>
<th>Method</th>
<th>Page(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>fit_transform() (sklearn.decomposition.MiniBatchDictionaryLearning)</td>
<td>333</td>
</tr>
<tr>
<td>fit_transform() (sklearn.decomposition.MiniBatchSparsePCA)</td>
<td>324</td>
</tr>
<tr>
<td>fit_transform() (sklearn.decomposition.NMF)</td>
<td>320</td>
</tr>
<tr>
<td>fit_transform() (sklearn.decomposition.PCA)</td>
<td>305</td>
</tr>
<tr>
<td>fit_transform() (sklearn.decomposition.ProbabilisticPCA)</td>
<td>308</td>
</tr>
<tr>
<td>fit_transform() (sklearn.decomposition.ProjectedGradientNMF)</td>
<td>311</td>
</tr>
<tr>
<td>fit_transform() (sklearn.decomposition.RandomizedPCA)</td>
<td>313</td>
</tr>
<tr>
<td>fit_transform() (sklearn.decomposition.SparseCoder)</td>
<td>327</td>
</tr>
<tr>
<td>fit_transform() (sklearn.decomposition.SparsePCA)</td>
<td>322</td>
</tr>
<tr>
<td>fit_transform() (sklearn.ensemble.ExtraTreesClassifier)</td>
<td>191, 350</td>
</tr>
<tr>
<td>fit_transform() (sklearn.ensemble.ExtraTreesRegressor)</td>
<td>195, 354</td>
</tr>
<tr>
<td>fit_transform() (sklearn.ensemble.RandomForestClassifier)</td>
<td>184, 342</td>
</tr>
<tr>
<td>fit_transform() (sklearn.ensemble.RandomForestRegressor)</td>
<td>187, 346</td>
</tr>
<tr>
<td>fit_transform() (sklearn.feature_extraction.DictVectorizer)</td>
<td>362</td>
</tr>
<tr>
<td>fit_transform() (sklearn.feature_extraction.text.CountVectorizer)</td>
<td>370</td>
</tr>
<tr>
<td>fit_transform() (sklearn.feature_extraction.text.TfidfVectorizer)</td>
<td>374</td>
</tr>
<tr>
<td>fit_transform() (sklearn.feature_extraction.text.TfidfVectorizer)</td>
<td>372</td>
</tr>
<tr>
<td>fit_transform() (sklearn.feature_selection.SelectFdr)</td>
<td>379</td>
</tr>
<tr>
<td>fit_transform() (sklearn.feature_selection.SelectFpr)</td>
<td>378</td>
</tr>
<tr>
<td>fit_transform() (sklearn.feature_selection.SelectFwe)</td>
<td>381</td>
</tr>
<tr>
<td>fit_transform() (sklearn.feature_selection.SelectKBest)</td>
<td>377</td>
</tr>
<tr>
<td>fit_transform() (sklearn.feature_selection.SelectPercentile)</td>
<td>375</td>
</tr>
<tr>
<td>fit_transform() (sklearn.kernel_approximation.AdditiveChi2Sampler)</td>
<td>415</td>
</tr>
<tr>
<td>fit_transform() (sklearn.kernel_approximation.RBFSampler)</td>
<td>413</td>
</tr>
<tr>
<td>fit_transform() (sklearn.kernel_approximation.SkewedChi2Sampler)</td>
<td>416</td>
</tr>
<tr>
<td>fit_transform() (sklearn.llda.LDA)</td>
<td>423</td>
</tr>
<tr>
<td>fit_transform() (sklearn.linear_model.LogisticRegression)</td>
<td>463, 508</td>
</tr>
<tr>
<td>fit_transform() (sklearn.linear_model.Perceptron)</td>
<td>469</td>
</tr>
<tr>
<td>fit_transform() (sklearn.linear_model.RandomizedLasso)</td>
<td>488</td>
</tr>
<tr>
<td>fit_transform() (sklearn.linear_model.RandomizedLogisticRegression)</td>
<td>491</td>
</tr>
<tr>
<td>fit_transform() (sklearn.linear_model.SGDClassifier)</td>
<td>474</td>
</tr>
<tr>
<td>fit_transform() (sklearn.linear_model.SGDRegressor)</td>
<td>478</td>
</tr>
<tr>
<td>fit_transform() (sklearn.linear_model.sparse.SGDClassifier)</td>
<td>501</td>
</tr>
<tr>
<td>fit_transform() (sklearn.linear_model.sparse.SGDRegressor)</td>
<td>504</td>
</tr>
<tr>
<td>fit_transform() (sklearn.manifold.Isomap)</td>
<td>513</td>
</tr>
<tr>
<td>fit_transform() (sklearn.manifold.LocallyLinearEmbedding)</td>
<td>511</td>
</tr>
<tr>
<td>fit_transform() (sklearn.manifold.MDS)</td>
<td>515</td>
</tr>
<tr>
<td>fit_transform() (sklearn.pipeline.Pipeline)</td>
<td>603</td>
</tr>
<tr>
<td>fit_transform() (sklearn.preprocessing.Binarizer)</td>
<td>608</td>
</tr>
<tr>
<td>fit_transform() (sklearn.preprocessing.KernelCenterer)</td>
<td>613</td>
</tr>
<tr>
<td>fit_transform() (sklearn.preprocessing.LabelBinarizer)</td>
<td>611</td>
</tr>
<tr>
<td>fit_transform() (sklearn.preprocessing.Normalizer)</td>
<td>606</td>
</tr>
<tr>
<td>fit_transform() (sklearn.preprocessing.Scaler)</td>
<td>605</td>
</tr>
<tr>
<td>fit_transform() (sklearn.svm.LinearSVC)</td>
<td>624</td>
</tr>
<tr>
<td>fit_transform() (sklearn.tree.DecisionTreeClassifier)</td>
<td>646</td>
</tr>
<tr>
<td>fit_transform() (sklearn.tree.DecisionTreeRegressor)</td>
<td>649</td>
</tr>
<tr>
<td>fit_transform() (sklearn.tree.ExtraTreeClassifier)</td>
<td>652</td>
</tr>
<tr>
<td>fit_transform() (sklearn.tree.ExtraTreeRegressor)</td>
<td>654</td>
</tr>
<tr>
<td>GaussianHMM (class in sklearn.hmm)</td>
<td>400</td>
</tr>
<tr>
<td>GaussianNB (class in sklearn.naive_bayes)</td>
<td>559</td>
</tr>
<tr>
<td>GaussianProcess (class in sklearn.gaussian_process)</td>
<td>388</td>
</tr>
<tr>
<td>generalized_exponential() (in module sklearn.gaussian_process.correlation_models)</td>
<td>393</td>
</tr>
<tr>
<td>get_feature_names() (sklearn.feature_extraction.DictVectorizer)</td>
<td>362</td>
</tr>
<tr>
<td>get_feature_names() (sklearn.feature_extraction.text.CountVectorizer)</td>
<td>370</td>
</tr>
<tr>
<td>get_feature_names() (sklearn.feature_extraction.text.TfidfVectorizer)</td>
<td>374</td>
</tr>
<tr>
<td>get_mixing_matrix() (sklearn.decomposition.FastICA)</td>
<td>317</td>
</tr>
</tbody>
</table>
get_params() (sklearn.cluster.AffinityPropagation method), 228
get_params() (sklearn.cluster.DBSCAN method), 229
get_params() (sklearn.cluster.KMeans method), 231
get_params() (sklearn.cluster.MeanShift method), 236
get_params() (sklearn.cluster.MiniBatchKMeans method), 234
get_params() (sklearn.cluster.SpectralClustering method), 238
get_params() (sklearn.cluster.Ward method), 239
get_params() (sklearn.covariance.EllipticEnvelope method), 250
get_params() (sklearn.covariance.EmpiricalCovariance method), 247
get_params() (sklearn.covariance.GraphLasso method), 252
get_params() (sklearn.covariance.GraphLassoCV method), 255
get_params() (sklearn.covariance.LedoitWolf method), 257
get_params() (sklearn.covariance.MinCovDet method), 260
get_params() (sklearn.covariance.OAS method), 263
get_params() (sklearn.covariance.ShrunkCovariance method), 265
get_params() (sklearn.decomposition.DictionaryLearning method), 330
get_params() (sklearn.decomposition.FastICA method), 318
get_params() (sklearn.decomposition.KernelPCA method), 316
get_params() (sklearn.decomposition.MiniBatchDictionaryLearning method), 333
get_params() (sklearn.decomposition.MiniBatchSparsePCA method), 325
get_params() (sklearn.decomposition.NMF method), 320
get_params() (sklearn.decomposition.PCA method), 305
get_params() (sklearn.decomposition.ProbabilisticPCA method), 308
get_params() (sklearn.decomposition.ProjectedGradientNMF method), 311
get_params() (sklearn.decomposition.RandomizedPCA method), 313
get_params() (sklearn.decomposition.SparseCoder method), 327
get_params() (sklearn.decomposition.SparsePCA method), 322
get_params() (sklearn.ensemble.ExtraTreesClassifier method), 192, 350
get_params() (sklearn.ensemble.ExtraTreesRegressor method), 196, 354
get_params() (sklearn.ensemble.GradientBoostingClassifier method), 198, 357
get_params() (sklearn.ensemble.GradientBoostingRegressor method), 201, 360
get_params() (sklearn.ensemble.RandomForestClassifier method), 184, 342
get_params() (sklearn.ensemble.RandomForestRegressor method), 188, 346
get_params() (sklearn.feature_extraction.DictVectorizer method), 363
get_params() (sklearn.feature_extraction.image.PatchExtractor method), 367
get_params() (sklearn.feature_extraction.text.CountVectorizer method), 370
get_params() (sklearn.feature_extraction.text.TfidfTransformer method), 372
get_params() (sklearn.feature_extraction.text.TfidfVectorizer method), 374
get_params() (sklearn.feature_selection.RFE method), 383
get_params() (sklearn.feature_selection.RFECV method), 385
get_params() (sklearn.feature_selection.SelectFdr method), 378
get_params() (sklearn.feature_selection.SelectFpr method), 381
get_params() (sklearn.feature_selection.SelectKBest method), 377
get_params() (sklearn.feature_selection.SelectPercentile method), 376
get_params() (sklearn.gaussian_process.GaussianProcess method), 391
get_params() (sklearn.grid_search.GridSearchCV method), 398
get_params() (sklearn.hmm.GaussianHMM method), 403
get_params() (sklearn.hmm.GMMHMM method), 411
get_params() (sklearn.hmm.MultinomialHMM method), 407
get_params() (sklearn.kernel_approximation.AdditiveChi2Sampler method), 415
get_params() (sklearn.kernel_approximation.RBFSampler method), 413
get_params() (sklearn.kernel_approximation.SkewedChi2Sampler method), 416
get_params() (sklearn.lda.LDA method), 423
get_params() (sklearn.linear_model.ARDRegression method), 485
get_params() (sklearn.linear_model.BayesianRidge method), 482
get_params() (sklearn.linear_model.ElasticNet method), 444
get_params() (sklearn.linear_model.ElasticNetCV method), 176, 447
get_params() (sklearn.linear_model.Lars method), 450
get_params() (sklearn.linear_model.LarsCV method), 167, 455
get_params() (sklearn.linear_model.Lasso method), 438
get_params() (sklearn.linear_model.LassoCV method), 172, 440
get_params() (sklearn.linear_model.LassoLars method), 453
get_params() (sklearn.linear_model.LassoLarsCV method), 170, 458
get_params() (sklearn.linear_model.LassoLarsIC method), 180, 460
get_params() (sklearn.linear_model.LinearRegression method), 426
get_params() (sklearn.linear_model.LogisticRegression method), 463, 508
get_params() (sklearn.linear_model.OrthogonalMatchingPursuit method), 466
get_params() (sklearn.linear_model.Perceptron method), 470
get_params() (sklearn.linear_model.RandomizedLasso method), 488
get_params() (sklearn.linear_model.RandomizedLogisticRegression method), 491
get_params() (sklearn.linear_model.Ridge method), 429
get_params() (sklearn.linear_model.RidgeClassifier method), 431
get_params() (sklearn.linear_model.RidgeClassifierCV method), 165, 433
get_params() (sklearn.linear_model.RidgeCV method), 163, 435
get_params() (sklearn.linear_model.SGDClassifier method), 474
get_params() (sklearn.linear_model.SGDRegressor method), 479
get_params() (sklearn.linear_model.sparse.ElasticNet method), 500
get_params() (sklearn.linear_model.sparse.Lasso method), 498
get_params() (sklearn.linear_model.sparse.SGDClassifier method), 502
get_params() (sklearn.linear_model.sparse.SGDRegressor method), 504
get_params() (sklearn.manifold.Isomap method), 513
get_params() (sklearn.manifold.LocallyLinearEmbedding method), 511
get_params() (sklearn.manifold.MDS method), 516
get_params() (sklearn.mixture.DPGMM method), 548
get_params() (sklearn.mixture.GMM method), 544
get_params() (sklearn.mixture.VBGMM method), 552
get_params() (sklearn.multiclass.OneVsOneClassifier method), 555
get_params() (sklearn.multiclass.OneVsRestClassifier method), 554
get_params() (sklearn.multiclass.OutputCodeClassifier method), 557
get_params() (sklearn.naive_bayes.BernoulliNB method), 565
get_params() (sklearn.naive_bayes.GaussianNB method), 560
get_params() (sklearn.naive_bayes.MultinomialNB method), 563
get_params() (sklearn.neighbors.KNeighborsClassifier method), 573
get_params() (sklearn.neighbors.KNeighborsRegressor method), 580
get_params() (sklearn.neighbors.NearestCentroid method), 590
get_params() (sklearn.neighbors.NearestNeighbors method), 568
get_params() (sklearn.neighbors.RadiusNeighborsClassifier method), 576
get_params() (sklearn.neighbors.RadiusNeighborsRegressor method), 584
get_params() (sklearn.pls.CCA method), 599
get_params() (sklearn.pls.PLSRegression method), 594
get_params() (sklearn.pls.PLSVDD method), 601
get_params() (sklearn.preprocessing.Binarizer method), 608
get_params() (sklearn.preprocessing.KernelCenterer method), 613
get_params() (sklearn.preprocessing.LabelBinarizer method), 611
get_params() (sklearn.preprocessing.Normalizer method), 607
get_params() (sklearn.preprocessing.Scaler method), 605
get_params() (sklearn.qda.QDA method), 616
get_params() (sklearn.semi_supervised.LabelPropagation method), 419
get_params() (sklearn.semi_supervised.LabelSpreading method), 421
get_params() (sklearn.svm.LinearSVC method), 624
get_params() (sklearn.svm.NuSVC method), 628
get_params() (sklearn.svm.NuSVR method), 636
get_params() (sklearn.svm.OneClassSVM method), 639
get_params() (sklearn.svm.SVC method), 620
get_params() (sklearn.svm.SVR method), 632
get_params() (sklearn.tree.DecisionTreeClassifier method), 646
get_params() (sklearn.tree.DecisionTreeRegressor method), 650
get_params() (sklearn.tree.ExtraTreeClassifier method), 652
get_params() (sklearn.tree.ExtraTreeRegressor method), 655
get_stop_words() (sklearn.feature_extraction.text.CountVectorizer method), 370
LassoLarsCV (class in sklearn.linear_model), 168, 456
LassoLarsIC (class in sklearn.linear_model), 178, 458
LDA (class in sklearn.lda), 422
LeaveOneLabelOut (class in sklearn.cross_validation), 271
LeaveOneOut (class in sklearn.cross_validation), 272
LeavePOut (class in sklearn.cross_validation), 273
LassoLarsIC (class in sklearn.linear_model), 458
LedoitWolf (class in sklearn.covariance), 255
linear() (in module sklearn.gaussian_process.correlation_models), 395
linear() (in module sklearn.gaussian_process.regression_models), 395
linear_kernel() (in module sklearn.metrics.pairwise), 537
LinearRegression (class in sklearn.linear_model), 425
LinearSVC (class in sklearn.svm), 622
load_20newsgroups() (in module sklearn.datasets), 282
load_boston() (in module sklearn.datasets), 283
load_diabetes() (in module sklearn.datasets), 283
load_digits() (in module sklearn.datasets), 284
load_files() (in module sklearn.datasets), 284
load_iris() (in module sklearn.datasets), 285
load_digits() (in module sklearn.datasets), 286
load_digits() (in module sklearn.datasets), 287
load_digits() (in module sklearn.datasets), 288
load_digits() (in module sklearn.datasets), 289
load_digits() (in module sklearn.datasets), 290
locally_linear_embedding() (in module sklearn.manifold), 516
LocallyLinearEmbedding (class in sklearn.manifold), 509
LogisticRegression (class in sklearn.linear_model), 461, 506
lower_bound() (sklearn.mixture.DPGMM method), 548
lower_bound() (sklearn.mixture.VBGMM method), 552
mahalanobis() (sklearn.covariance.EllipticEnvelope method), 250
mahalanobis() (sklearn.covariance.EmpiricalCovariance method), 247
mahalanobis() (sklearn.covariance.GraphLasso method), 252
mahalanobis() (sklearn.covariance.GraphLassoCV method), 255
mahalanobis() (sklearn.covariance.LedoitWolf method), 257
mahalanobis() (sklearn.covariance.MinCovDet method), 260
mahalanobis() (sklearn.covariance.OAS method), 263
mahalanobis() (sklearn.covariance.ShrunkCovariance method), 265
make_blobs() (in module sklearn.datasets), 291
make_classification() (in module sklearn.datasets), 292
make_classification() (in module sklearn.datasets), 294
make_classification() (in module sklearn.datasets), 296
make_classification() (in module sklearn.datasets), 298
make_classification() (in module sklearn.datasets), 300
make_classification() (in module sklearn.datasets), 302
make_classification() (in module sklearn.datasets), 304
make_classification() (in module sklearn.datasets), 306
make_classification() (in module sklearn.datasets), 308
make_classification() (in module sklearn.datasets), 310
make_classification() (in module sklearn.datasets), 312
make_classification() (in module sklearn.datasets), 314
make_classification() (in module sklearn.datasets), 316
make_classification() (in module sklearn.datasets), 318
make_classification() (in module sklearn.datasets), 320
make_classification() (in module sklearn.datasets), 322
make_classification() (in module sklearn.datasets), 324
make_classification() (in module sklearn.datasets), 326
make_classification() (in module sklearn.datasets), 328
make_classification() (in module sklearn.datasets), 330
make_classification() (in module sklearn.datasets), 332
make_classification() (in module sklearn.datasets), 334
make_classification() (in module sklearn.datasets), 336
make_classification() (in module sklearn.datasets), 338
make_classification() (in module sklearn.datasets), 340
make_classification() (in module sklearn.datasets), 342
make_classification() (in module sklearn.datasets), 344
make_classification() (in module sklearn.datasets), 346
make_classification() (in module sklearn.datasets), 348
make_classification() (in module sklearn.datasets), 350
make_classification() (in module sklearn.datasets), 352
make_classification() (in module sklearn.datasets), 354
make_classification() (in module sklearn.datasets), 356
make_classification() (in module sklearn.datasets), 358
make_classification() (in module sklearn.datasets), 360
make_classification() (in module sklearn.datasets), 362
make_classification() (in module sklearn.datasets), 364
make_classification() (in module sklearn.datasets), 366
make_classification() (in module sklearn.datasets), 368
make_classification() (in module sklearn.datasets), 370
make_classification() (in module sklearn.datasets), 372
make_classification() (in module sklearn.datasets), 374
make_classification() (in module sklearn.datasets), 376
make_classification() (in module sklearn.datasets), 378
make_classification() (in module sklearn.datasets), 380
make_classification() (in module sklearn.datasets), 382
make_classification() (in module sklearn.datasets), 384
make_classification() (in module sklearn.datasets), 386
make_classification() (in module sklearn.datasets), 388
make_classification() (in module sklearn.datasets), 390
make_classification() (in module sklearn.datasets), 392
make_classification() (in module sklearn.datasets), 394
mean_shift() (in module sklearn.cluster), 243
mean_squared_error() (in module sklearn.metrics), 526
mean_shift() (in module sklearn.cluster), 245
MultinomialHMM (class in sklearn.hmm), 404
MultinomialNB (class in sklearn.naive_bayes), 561
mutual_info_score() (in module sklearn.metrics), 532
NearestCentroid (class in sklearn.neighbors), 589
NearestNeighbors (class in sklearn.neighbors), 566
NMF (class in sklearn.decomposition), 318
normalized_mutual_info_score() (in module sklearn.metrics), 532
Normalization (class in sklearn.preprocessing), 606
NuSVC (class in sklearn.svm), 625
OneClassSVM (class in sklearn.svm), 637
OneVsOneClassifier (class in sklearn.multiclass), 555
OneVsRestClassifier (class in sklearn.multiclass), 553
OAS (class in sklearn.covariance), 261
oas() (in module sklearn.covariance), 267
OneClassSVM (class in sklearn.svm), 637
OneVsOneClassifier (class in sklearn.multiclass), 555
OneVsRestClassifier (class in sklearn.multiclass), 553
orthogonal_mp() (in module sklearn.linear_model), 494
orthogonal_mp_gram() (in module sklearn.linear_model), 495
OrthogonalMatchingPursuit (class in sklearn.linear_model), 465
OutputCodeClassifier (class in sklearn.multiclass), 556
pairwise_distances() (in module sklearn.metrics.pairwise), 539
pairwise_kernels() (in module sklearn.metrics.pairwise), 540
partial_fit() (sklearn.cluster.MiniBatchKMeans method), 234
partial_fit() (sklearn.decomposition.MiniBatchDictionaryLearning method), 333
partial_fit() (sklearn.linear_model.Perceptron method), 470
partial_fit() (sklearn.linear_model.SGDClassifier method), 474
partial_fit() (sklearn.linear_model.SGDRegressor method), 479
partial_fit() (sklearn.linear_model.sparse.SGDClassifier method), 502
partial_fit() (sklearn.linear_model.sparse.SGDRegressor method), 504
PatchExtractor (class in sklearn.feature_extraction.image), 366
path() (sklearn.linear_model.ElasticNetCV static method), 176, 447
path() (sklearn.linear_model.LassoCV static method), 174, 440
PCA (class in sklearn.decomposition), 304
Perceptron (class in sklearn.linear_model), 467
permutation_test_score() (in module sklearn.cross_validation), 280
Pipeline (class in sklearn.pipeline), 602
PLSCanonical (class in sklearn.pls), 595
PLSRegression (class in sklearn.pls), 593
PLSSVD (class in sklearn.pls), 600
polynomial_kernel() (in module sklearn.metrics.pairwise), 538
precision_recall_curve() (in module sklearn.metrics), 524
precision_recall_fscore_support() (in module sklearn.metrics), 523
precision_score() (in module sklearn.metrics), 519
predict() (in module sklearn.svm.libsvm), 642
predict() (sklearn.cluster.KMeans method), 232
predict() (sklearn.cluster.MiniBatchKMeans method), 234
predict() (sklearn.covariance.EllipticEnvelope method), 250
predict() (sklearn.ensemble.ExtraTreesClassifier method), 192, 350
predict() (sklearn.ensemble.ExtraTreesRegressor method), 196, 354
predict() (sklearn.ensemble.GradientBoostingClassifier method), 199, 357
predict() (sklearn.ensemble.GradientBoostingRegressor method), 202, 360
predict() (sklearn.ensemble.RandomForestClassifier method), 184, 342
predict() (sklearn.ensemble.RandomForestRegressor method), 188, 346
predict() (sklearn.feature_selection.RFE method), 383
predict() (sklearn.feature_selection.RFECV method), 386
predict() (sklearn.gaussian_process.GaussianProcess method), 391
predict() (sklearn.hmm.GaussianHMM method), 403
predict() (sklearn.hmm.GMMHMM method), 411
predict() (sklearn.hmm.MultinomialHMM method), 407
predict() (sklearn lda.LDA method), 424
predict() (sklearn.linear_model.ARDRegression method), 485
predict() (sklearn.linear_model.BayesianRidge method), 482
predict() (sklearn.linear_model.ElasticNet method), 444
predict() (sklearn.linear_model.ElasticNetCV method), 178, 448
predict() (sklearn.linear_model.Lars method), 450
predict() (sklearn.linear_model.LarsCV method), 168, 455
predict() (sklearn.linear_model.Lasso method), 438
predict() (sklearn.linear_model.LassoCV method), 174, 442
predict() (sklearn.linear_model.LassoLars method), 453
predict() (sklearn.linear_model.LassoLarsCV method), 170, 458
predict() (sklearn.linear_model.LassoLarsIC method), 180, 460
predict() (sklearn.linear_model.LinearRegression method), 426
predict() (sklearn.linear_model.LogisticRegression method), 463, 508
predict() (sklearn.linear_model.OrthogonalMatchingPursuit method), 466
predict() (sklearn.linear_model.Perceptron method), 470
predict() (sklearn.linear_model.Ridge method), 429
predict() (sklearn.linear_model.RidgeClassifier method), 431
predict() (sklearn.linear_model.RidgeClassifierCV method), 165, 433
predict() (sklearn.linear_model.RidgeCV method), 163, 435
predict() (sklearn.linear_model.SGDClassifier method), 475
predict() (sklearn.linear_model.SGDRegressor method), 479
<table>
<thead>
<tr>
<th>Class/Method</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>predict()</code> (sklearn.linear_model.sparse.ElasticNet method)</td>
<td>500</td>
</tr>
<tr>
<td><code>predict()</code> (sklearn.linear_model.sparse.Lasso method)</td>
<td>498</td>
</tr>
<tr>
<td><code>predict()</code> (sklearn.linear_model.sparse.SGDClassifier method)</td>
<td>502</td>
</tr>
<tr>
<td><code>predict()</code> (sklearn.linear_model.sparse.SGDRegressor method)</td>
<td>505</td>
</tr>
<tr>
<td><code>predict()</code> (sklearn.mixture.DPGMM method)</td>
<td>548</td>
</tr>
<tr>
<td><code>predict()</code> (sklearn.mixture.GMM method)</td>
<td>544</td>
</tr>
<tr>
<td><code>predict()</code> (sklearn.mixture.VBGMM method)</td>
<td>552</td>
</tr>
<tr>
<td><code>predict()</code> (sklearn.multiclass.OneVsOneClassifier method)</td>
<td>556</td>
</tr>
<tr>
<td><code>predict()</code> (sklearn.multiclass.OneVsRestClassifier method)</td>
<td>554</td>
</tr>
<tr>
<td><code>predict()</code> (sklearn.multiclass.OutputCodeClassifier method)</td>
<td>557</td>
</tr>
<tr>
<td><code>predict()</code> (sklearn.naive_bayes.BernoulliNB method)</td>
<td>565</td>
</tr>
<tr>
<td><code>predict()</code> (sklearn.naive_bayes.GaussianNB method)</td>
<td>560</td>
</tr>
<tr>
<td><code>predict()</code> (sklearn.naive_bayes.MultinomialNB method)</td>
<td>563</td>
</tr>
<tr>
<td><code>predict()</code> (sklearn.neighbors.KNeighborsClassifier method)</td>
<td>574</td>
</tr>
<tr>
<td><code>predict()</code> (sklearn.neighbors.KNeighborsRegressor method)</td>
<td>581</td>
</tr>
<tr>
<td><code>predict()</code> (sklearn.neighbors.NearestCentroid method)</td>
<td>590</td>
</tr>
<tr>
<td><code>predict()</code> (sklearn.neighbors.RadiusNeighborsClassifier method)</td>
<td>576</td>
</tr>
<tr>
<td><code>predict()</code> (sklearn.neighbors.RadiusNeighborsRegressor method)</td>
<td>584</td>
</tr>
<tr>
<td><code>predict()</code> (sklearn.pipeline.Pipeline method)</td>
<td>603</td>
</tr>
<tr>
<td><code>predict()</code> (sklearn.pls.CCA method)</td>
<td>600</td>
</tr>
<tr>
<td><code>predict()</code> (sklearn.pls.PLSCanonical method)</td>
<td>597</td>
</tr>
<tr>
<td><code>predict()</code> (sklearn.pls.PLSRegression method)</td>
<td>594</td>
</tr>
<tr>
<td><code>predict()</code> (sklearn.qda.QDA method)</td>
<td>617</td>
</tr>
<tr>
<td><code>predict()</code> (sklearn.svm.LinearSVC method)</td>
<td>625</td>
</tr>
<tr>
<td><code>predict()</code> (sklearn.svm.NuSVC method)</td>
<td>628</td>
</tr>
<tr>
<td><code>predict()</code> (sklearn.svm.NuSVR method)</td>
<td>636</td>
</tr>
<tr>
<td><code>predict()</code> (sklearn.svm.OneClassSVM method)</td>
<td>639</td>
</tr>
<tr>
<td><code>predict()</code> (sklearn.svm.SVC method)</td>
<td>621</td>
</tr>
<tr>
<td><code>predict()</code> (sklearn.svm.SVR method)</td>
<td>632</td>
</tr>
<tr>
<td><code>predict()</code> (sklearn.tree.DecisionTreeClassifier method)</td>
<td>646</td>
</tr>
<tr>
<td><code>predict()</code> (sklearn.tree.DecisionTreeRegressor method)</td>
<td>650</td>
</tr>
<tr>
<td><code>predict()</code> (sklearn.tree.ExtraTreeClassifier method)</td>
<td>652</td>
</tr>
<tr>
<td><code>predict()</code> (sklearn.tree.ExtraTreeRegressor method)</td>
<td>655</td>
</tr>
<tr>
<td><code>predict_ecoc()</code> (in module sklearn.multiclass)</td>
<td>559</td>
</tr>
<tr>
<td><code>predict_log_proba()</code> (sklearn.ensemble.ExtraTreesClassifier method)</td>
<td>192, 350</td>
</tr>
<tr>
<td><code>predict_log_proba()</code> (sklearn.ensemble.RandomForestClassifier method)</td>
<td>184, 343</td>
</tr>
<tr>
<td><code>predict_log_proba()</code> (sklearn.linear_model.LogisticRegression method)</td>
<td>463, 508</td>
</tr>
<tr>
<td><code>predict_log_proba()</code> (sklearn.naive_bayes.BernoulliNB method)</td>
<td>565</td>
</tr>
<tr>
<td><code>predict_log_proba()</code> (sklearn.naive_bayes.GaussianNB method)</td>
<td>560</td>
</tr>
<tr>
<td><code>predict_log_proba()</code> (sklearn.naive_bayes.MultinomialNB method)</td>
<td>563</td>
</tr>
<tr>
<td><code>predict_log_proba()</code> (sklearn.qda.QDA method)</td>
<td>617</td>
</tr>
<tr>
<td><code>predict_log_proba()</code> (sklearn.svm.NuSVC method)</td>
<td>628</td>
</tr>
<tr>
<td><code>predict_log_proba()</code> (sklearn.svm.NuSVR method)</td>
<td>636</td>
</tr>
<tr>
<td><code>predict_log_proba()</code> (sklearn.svm.OneClassSVM method)</td>
<td>639</td>
</tr>
<tr>
<td><code>predict_log_proba()</code> (sklearn.svm.SVC method)</td>
<td>621</td>
</tr>
<tr>
<td><code>predict_log_proba()</code> (sklearn.svm.SVR method)</td>
<td>632</td>
</tr>
<tr>
<td><code>predict_log_proba()</code> (sklearn.tree.DecisionTreeClassifier method)</td>
<td>646</td>
</tr>
<tr>
<td><code>predict_log_proba()</code> (sklearn.tree.ExtraTreeClassifier method)</td>
<td>652</td>
</tr>
<tr>
<td><code>predict_oov()</code> (in module sklearn.multiclass)</td>
<td>558</td>
</tr>
<tr>
<td><code>predict_ovr()</code> (in module sklearn.multiclass)</td>
<td>558</td>
</tr>
<tr>
<td><code>predict_proba()</code> (sklearn.ensemble.ExtraTreesClassifier method)</td>
<td>192, 350</td>
</tr>
<tr>
<td><code>predict_proba()</code> (sklearn.ensemble.GradientBoostingClassifier method)</td>
<td>199, 357</td>
</tr>
<tr>
<td><code>predict_proba()</code> (sklearn.ensemble.RandomForestClassifier method)</td>
<td>184, 343</td>
</tr>
<tr>
<td><code>predict_proba()</code> (sklearn.hmm.GaussianHMM method)</td>
<td>403</td>
</tr>
<tr>
<td><code>predict_proba()</code> (sklearn.hmm.GMMHMM method)</td>
<td>411</td>
</tr>
<tr>
<td><code>predict_proba()</code> (sklearn.hmm.MultinomialHMM method)</td>
<td>407</td>
</tr>
<tr>
<td><code>predict_proba()</code> (sklearn.la.lda.LDA method)</td>
<td>424</td>
</tr>
<tr>
<td><code>predict_proba()</code> (sklearn.linear_model.LogisticRegression method)</td>
<td>464, 508</td>
</tr>
<tr>
<td><code>predict_proba()</code> (sklearn.linear_model.Perceptron method)</td>
<td>470</td>
</tr>
<tr>
<td><code>predict_proba()</code> (sklearn.linear_model.SGDClassifier method)</td>
<td>475</td>
</tr>
<tr>
<td><code>predict_proba()</code> (sklearn.linear_model.sparse.SGDClassifier method)</td>
<td>502</td>
</tr>
<tr>
<td><code>predict_proba()</code> (sklearn.mixture.DPGMM method)</td>
<td>549</td>
</tr>
<tr>
<td><code>predict_proba()</code> (sklearn.mixture.GMM method)</td>
<td>544</td>
</tr>
<tr>
<td><code>predict_proba()</code> (sklearn.mixture.VBGMM method)</td>
<td>552</td>
</tr>
<tr>
<td><code>predict_proba()</code> (sklearn.naive_bayes.BernoulliNB method)</td>
<td>555</td>
</tr>
<tr>
<td><code>predict_proba()</code> (sklearn.naive_bayes.GaussianNB method)</td>
<td>561</td>
</tr>
</tbody>
</table>
predict_proba() (sklearn.naive_bayes.MultinomialNB method), 563
predict_proba() (sklearn.pipeline.Pipeline method), 603
predict_proba() (sklearn.qda.QDA method), 617
predict_proba() (sklearn.semi_supervised.LabelPropagation method), 419
predict_proba() (sklearn.semi_supervised.LabelSpreading method), 421
predict_proba() (sklearn.svm.NuSVC method), 628
predict_proba() (sklearn.svm.NuSVR method), 636
predict_proba() (sklearn.svm.OneClassSVM method), 639
predict_proba() (sklearn.svm.SVC method), 621
predict_proba() (sklearn.svm.SVR method), 632
predict_proba() (sklearn.tree.DecisionTreeClassifier method), 647
predict_proba() (sklearn.tree.ExtraTreeClassifier method), 652
ProbabilisticPCA (class in sklearn.decomposition), 306
ProjectedGradientNMF (class in sklearn.decomposition), 309
pure_nugget() (in module sklearn.gaussian_process.correlation_models), 394
QDA (class in sklearn.qda), 615
quadratic() (in module sklearn.gaussian_process.regression_models), 396
query() (sklearn.neighbors.BallTree method), 587
query_radius() (sklearn.neighbors.BallTree method), 588
r2_score() (in module sklearn.metrics), 526
radius_neighbors() (sklearn.neighbors.NearestNeighbors method), 569
radius_neighbors() (sklearn.neighbors.NearestNeighbors method), 577
radius_neighbors() (sklearn.neighbors.RadiusNeighborsClassifier method), 577
radius_neighbors() (sklearn.neighbors.RadiusNeighborsRegressor method), 584
radius_neighbors_graph() (in module sklearn.neighbors), 592
radius_neighbors_graph() (sklearn.neighbors.NearestNeighbors method), 570
radius_neighbors_graph() (sklearn.neighbors.RadiusNeighborsClassifier method), 577
radius_neighbors_graph() (sklearn.neighbors.RadiusNeighborsRegressor method), 585
RadiusNeighborsClassifier (class in sklearn.neighbors), 575
RadiusNeighborsRegressor (class in sklearn.neighbors), 582
RandomForestClassifier (class in sklearn.ensemble), 181, 340
RandomForestRegressor (class in sklearn.ensemble), 185, 344
RandomizedLasso (class in sklearn.linear_model), 486
RandomizedLogisticRegression (class in sklearn.linear_model), 489
RandomizedPCA (class in sklearn.decomposition), 311
rbf_kernel() (in module sklearn.metrics.pairwise), 538
RBFSampler (class in sklearn.kernel_approximation), 412
recall_score() (in module sklearn.metrics), 520
reconstruct_from_patches_2d() (in module sklearn.feature_extraction.image), 366
reconstruction_error() (sklearn.manifold.Isomap method), 513
reduced_likelihood_function() (sklearn.gaussian_process.GaussianProcess method), 391
reduced_likelihood_function_value (sklearn.gaussian_process.GaussianProcess attribute), 392
resample() (in module sklearn.utils), 657
restrict() (sklearn.feature_extraction.DictVectorizer method), 363
reweight_covariance() (sklearn.covariance.EllipticEnvelope method), 250
reweight_covariance() (sklearn.covariance.MinCovDet method), 260
RFE (class in sklearn.feature_selection), 382
RFECV (class in sklearn.feature_selection), 384
Ridge (class in sklearn.linear_model), 427
RidgeClassifier (class in sklearn.linear_model), 429
RidgeClassifierCV (class in sklearn.linear_model), 164, 431
RidgeCV (class in sklearn.linear_model), 161, 433
eoc_curve() (in module sklearn.metrics), 518
rvs() (sklearn.hmm.GaussianHMM method), 403
rvs() (sklearn.hmm.GMMHMM method), 411
rvs() (sklearn.hmm.MultinomialHMM method), 407
rvs() (sklearn.mixture.DPGMM method), 549
rvs() (sklearn.mixture.GMM method), 545
rvs() (sklearn.mixture.VBGMM method), 552
sample() (sklearn.hmm.GaussianHMM method), 403
sample() (sklearn.hmm.GMMHMM method), 411
sample() (sklearn.hmm.MultinomialHMM method), 407
sample() (sklearn.mixture.DPGMM method), 549
sample() (sklearn.mixture.GMM method), 545
sample() (sklearn.mixture.VBGMM method), 552
scale() (in module sklearn.preprocessing), 613
Scaler (class in sklearn.preprocessing), 604
score() (sklearn.cluster.KMeans method), 232
score() (sklearn.cluster.MiniBatchKMeans method), 235
Index 1041
scikit-learn user guide, Release 0.12-git

score() (sklearn.tree.DecisionTreeRegressor method), 650
score() (sklearn.tree.ExtraTreeClassifier method), 653
score() (sklearn.tree.ExtraTreeRegressor method), 655
SelectFdr (class in sklearn.feature_selection), 379
SelectFpr (class in sklearn.feature_selection), 378
SelectFwe (class in sklearn.feature_selection), 380
SelectKBest (class in sklearn.feature_selection), 376
SelectPercentile (class in sklearn.feature_selection), 375
set_params() (sklearn.cluster.AffinityPropagation method), 228
set_params() (sklearn.cluster.DBSCAN method), 230
set_params() (sklearn.cluster.KMeans method), 232
set_params() (sklearn.cluster.MeanShift method), 236
set_params() (sklearn.cluster.MinibatchKMeans method), 235
set_params() (sklearn.cluster.SpectralClustering method), 238
set_params() (sklearn.cluster.Ward method), 239
set_params() (sklearn.covariance.EllipticEnvelope method), 251
set_params() (sklearn.covariance.EmpiricalCovariance method), 247
set_params() (sklearn.covariance.GraphLasso method), 253
set_params() (sklearn.covariance.GraphLassoCV method), 255
set_params() (sklearn.covariance.LedoitWolf method), 258
set_params() (sklearn.covariance.MinCovDet method), 261
set_params() (sklearn.covariance.OAS method), 263
set_params() (sklearn.covariance.ShrunkCovariance method), 266
set_params() (sklearn.decomposition.DictionaryLearning method), 330
set_params() (sklearn.decomposition.FastICA method), 318
set_params() (sklearn.decomposition.KernelPCA method), 316
set_params() (sklearn.decomposition.MinibatchDictionaryLearning method), 334
set_params() (sklearn.decomposition.MinibatchSparsePCA method), 325
set_params() (sklearn.decomposition.NMF method), 320
set_params() (sklearn.decomposition.PCA method), 306
set_params() (sklearn.decomposition.ProbabilisticPCA method), 308
set_params() (sklearn.decomposition.ProjectedGradientNMF method), 311
set_params() (sklearn.decomposition.RandomizedPCA method), 314
set_params() (sklearn.decomposition.SparseCoder method), 327
set_params() (sklearn.decomposition.SparsePCA method), 323
set_params() (sklearn.ensemble.ExtraTreesClassifier method), 192, 351
set_params() (sklearn.ensemble.ExtraTreesRegressor method), 196, 354
set_params() (sklearn.ensemble.GradientBoostingClassifier method), 199, 357
set_params() (sklearn.ensemble.GradientBoostingRegressor method), 202, 360
set_params() (sklearn.ensemble.RandomForestClassifier method), 185, 343
set_params() (sklearn.ensemble.RandomForestRegressor method), 188, 347
set_params() (sklearn.feature_extraction.DictVectorizer method), 363
set_params() (sklearn.feature_extraction.image.PatchExtractor method), 367
set_params() (sklearn.feature_extraction.text.CountVectorizer method), 371
set_params() (sklearn.feature_extraction.text.TfidfTransformer method), 372
set_params() (sklearn.feature_extraction.text.TfidfVectorizer method), 374
set_params() (sklearn.feature_selection.RFE method), 383
set_params() (sklearn.feature_selection.RFECV method), 386
set_params() (sklearn.feature_selection.SelectFdr method), 380
set_params() (sklearn.feature_selection.SelectFpr method), 379
set_params() (sklearn.feature_selection.SelectFwe method), 381
set_params() (sklearn.feature_selection.SelectKBest method), 377
set_params() (sklearn.feature_selection.SelectPercentile method), 376
set_params() (sklearn.gaussian_process.GaussianProcess method), 392
set_params() (sklearn.grid_search.GridSearchCV method), 399
set_params() (sklearn.hmm.GaussianHMM method), 404
set_params() (sklearn.hmm.GMMHMM method), 412
set_params() (sklearn.hmm.MultinomialHMM method), 408
set_params() (sklearn.kernel_approximation.AdditiveChi2Sampler method), 415
set_params() (sklearn.kernel_approximation.RBFSampler method), 413
set_params() (sklearn.kernel_approximation.SkewedChi2Sampler method), 417
set_params() (sklearn.lda.LDA method), 424
set_params() (sklearn.linear_model.ARDRegression method), 485
set_params() (sklearn.linear_model.BayesianRidge method), 482
set_params() (sklearn.linear_model.ElasticNet method), 445
set_params() (sklearn.linear_model.ElasticNetCV method), 178, 448
set_params() (sklearn.linear_model.Lars method), 451
set_params() (sklearn.linear_model.Lasso method), 438
set_params() (sklearn.linear_model.LassoLars method), 453
set_params() (sklearn.linear_model.LassoIC method), 181, 461
set_params() (sklearn.linear_model.LinearRegression method), 427
set_params() (sklearn.linear_model.LogisticRegression method), 464, 509
set_params() (sklearn.linear_model.OrthogonalMatchingPursuit method), 467
set_params() (sklearn.linear_model.Perceptron method), 471
set_params() (sklearn.linear_model.RandomizedLasso method), 489
set_params() (sklearn.linear_model.Ridge method), 492
set_params() (sklearn.linear_model.Ridge method), 429
set_params() (sklearn.linear_model.RidgeClassifier method), 431
set_params() (sklearn.linear_model.RidgeClassifierCV method), 166, 433
set_params() (sklearn.linear_model.RidgeCV method), 164, 435
set_params() (sklearn.linear_model.SGDClassifier method), 475
set_params() (sklearn.linear_model.SGDRegressor method), 479
set_params() (sklearn.linear_model.sparse.ElasticNet method), 500
set_params() (sklearn.linear_model.sparse.Lasso method), 499
set_params() (sklearn.linear_model.sparse.SGDClassifier method), 503
set_params() (sklearn.linear_model.sparse.SGDRegressor method), 505
set_params() (sklearn.manifold.Isomap method), 514
set_params() (sklearn.manifold.LocallyLinearEmbedding method), 511
set_params() (sklearn.manifold.MDS method), 516
set_params() (sklearn.mixture.DPGMM method), 549
set_params() (sklearn.mixture.GMM method), 545
set_params() (sklearn.mixture.VBGMM method), 553
set_params() (sklearn.multiclass.OneVsOneClassifier method), 556
set_params() (sklearn.multiclass.OneVsRestClassifier method), 554
set_params() (sklearn.multiclass.OutputCodeClassifier method), 558
set_params() (sklearn.naive_bayes.BernoulliNB method), 566
set_params() (sklearn.naive_bayes.GaussianNB method), 561
set_params() (sklearn.naive_bayes.MultinomialNB method), 563
set_params() (sklearn.neighbors.KNeighborsClassifier method), 574
set_params() (sklearn.neighbors.KNeighborsRegressor method), 582
set_params() (sklearn.neighbors.NearestCentroid method), 591
set_params() (sklearn.neighbors.NearestNeighbors method), 571
set_params() (sklearn.neighbors.RadiusNeighborsClassifier method), 578
set_params() (sklearn.neighbors.RadiusNeighborsRegressor method), 585
set_params() (sklearn.pipeline.Pipeline method), 603
set_params() (sklearn.pls.CCA method), 600
set_params() (sklearn.pls.PLSCanonical method), 597
set_params() (sklearn.pls.PLSRegression method), 595
set_params() (sklearn.pls.PLSSVD method), 601
set_params() (sklearn.preprocessing.Binarizer method), 608
set_params() (sklearn.preprocessing.LabelBinarizer method), 613
set_params() (sklearn.preprocessing.LabelBinarizer method), 612
set_params() (sklearn.preprocessing.Normalizer method), 607
set_params() (sklearn.preprocessing.Scaler method), 605
set_params() (sklearn.qda.QDA method), 617
set_params() (sklearn.semi_supervised.LabelPropagation method), 419
set_params() (sklearn.semi_supervised.LabelSpreading method), 421
set_params() (sklearn.svm.LinearSVC method), 625
set_params() (sklearn.svm.NuSVC method), 629
set_params() (sklearn.svm.NuSVR method), 637
set_params() (sklearn.svm.OneClassSVM method), 640
set_params() (sklearn.svm.SVC method), 621
set_params() (sklearn.svm.SVR method), 633
<table>
<thead>
<tr>
<th>Method/Class</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>set_params() (sklearn.tree.DecisionTreeClassifier method)</td>
<td>647</td>
</tr>
<tr>
<td>set_params() (sklearn.tree.DecisionTreeRegressor method)</td>
<td>650</td>
</tr>
<tr>
<td>set_params() (sklearn.tree.ExtraTreeClassifier method)</td>
<td>653</td>
</tr>
<tr>
<td>set_params() (sklearn.tree.ExtraTreeRegressor method)</td>
<td>655</td>
</tr>
<tr>
<td>SGDClassifier (class in sklearn.linear_model)</td>
<td>471</td>
</tr>
<tr>
<td>SGDClassifier (class in sklearn.linear_model.sparse)</td>
<td>500</td>
</tr>
<tr>
<td>SGDRegressor (class in sklearn.linear_model)</td>
<td>476</td>
</tr>
<tr>
<td>SGDRegressor (class in sklearn.linear_model.sparse)</td>
<td>503</td>
</tr>
<tr>
<td>shrunk_covariance() (in module sklearn.covariance)</td>
<td>267</td>
</tr>
<tr>
<td>ShrunkCovariance (class in sklearn.covariance)</td>
<td>264</td>
</tr>
<tr>
<td>shuffle() (in module sklearn.utils)</td>
<td>658</td>
</tr>
<tr>
<td>ShuffleSplit (class in sklearn.cross_validation)</td>
<td>276</td>
</tr>
<tr>
<td>sigma (sklearn.naive_bayes.GaussianNB attribute)</td>
<td>561</td>
</tr>
<tr>
<td>silhouette_score() (in module sklearn.metrics)</td>
<td>533</td>
</tr>
<tr>
<td>SkewedChi2Sampler (class in sklearn.kernel_approximation)</td>
<td>415</td>
</tr>
<tr>
<td>sklearn.cluster (module)</td>
<td>227</td>
</tr>
<tr>
<td>sklearn.covariance (module)</td>
<td>245</td>
</tr>
<tr>
<td>sklearn.cross_validation (module)</td>
<td>269</td>
</tr>
<tr>
<td>sklearn.datasets (module)</td>
<td>281</td>
</tr>
<tr>
<td>sklearn.decomposition (module)</td>
<td>303</td>
</tr>
<tr>
<td>sklearn.ensemble (module)</td>
<td>339</td>
</tr>
<tr>
<td>sklearn.feature_extraction (module)</td>
<td>361</td>
</tr>
<tr>
<td>sklearn.feature_extraction.image (module)</td>
<td>364</td>
</tr>
<tr>
<td>sklearn.feature_extraction.text (module)</td>
<td>367</td>
</tr>
<tr>
<td>sklearn.feature_selection (module)</td>
<td>375</td>
</tr>
<tr>
<td>sklearn.gaussian_process (module)</td>
<td>388</td>
</tr>
<tr>
<td>sklearn.grid_search (module)</td>
<td>396</td>
</tr>
<tr>
<td>sklearn.hmm (module)</td>
<td>399</td>
</tr>
<tr>
<td>sklearn.kernel_approximation (module)</td>
<td>412</td>
</tr>
<tr>
<td>sklearn.lda (module)</td>
<td>422</td>
</tr>
<tr>
<td>sklearn.linear_model (module)</td>
<td>424</td>
</tr>
<tr>
<td>sklearn.linear_model.sparse (module)</td>
<td>497</td>
</tr>
<tr>
<td>sklearn.manifold (module)</td>
<td>509</td>
</tr>
<tr>
<td>sklearn.metrics (module)</td>
<td>517</td>
</tr>
<tr>
<td>sklearn.metrics.cluster (module)</td>
<td>526</td>
</tr>
<tr>
<td>sklearn.metrics.pairwise (module)</td>
<td>535</td>
</tr>
<tr>
<td>sklearn.mixture (module)</td>
<td>541</td>
</tr>
<tr>
<td>sklearn.multiclass (module)</td>
<td>553</td>
</tr>
<tr>
<td>sklearn.naive_bayes (module)</td>
<td>559</td>
</tr>
<tr>
<td>sklearn.neighbors (module)</td>
<td>566</td>
</tr>
<tr>
<td>sklearn.pipeline (module)</td>
<td>601</td>
</tr>
<tr>
<td>sklearn.pls (module)</td>
<td>592</td>
</tr>
<tr>
<td>sklearn.preprocessing (module)</td>
<td>603</td>
</tr>
<tr>
<td>sklearn.qda (module)</td>
<td>615</td>
</tr>
<tr>
<td>sklearn.semi_supervised (module)</td>
<td>417</td>
</tr>
<tr>
<td>sklearn.svm (module)</td>
<td>617</td>
</tr>
<tr>
<td>sklearn.tree (module)</td>
<td>644</td>
</tr>
<tr>
<td>sklearn.utils (module)</td>
<td>656</td>
</tr>
<tr>
<td>sparse_encode() (in module sklearn.decomposition)</td>
<td>338</td>
</tr>
<tr>
<td>SparseCoder (class in sklearn.decomposition)</td>
<td>326</td>
</tr>
<tr>
<td>SparsePCA (class in sklearn.decomposition)</td>
<td>321</td>
</tr>
<tr>
<td>spectral_clustering() (in module sklearn.cluster)</td>
<td>244</td>
</tr>
<tr>
<td>SpectralClustering (class in sklearn.cluster)</td>
<td>237</td>
</tr>
<tr>
<td>squared_exponential() (in module sklearn.gaussian_process.correlation_models)</td>
<td>393</td>
</tr>
<tr>
<td>staged_decision_function() (sklearn.ensemble.GradientBoostingClassifier method)</td>
<td>199, 357</td>
</tr>
<tr>
<td>staged_decision_function() (sklearn.ensemble.GradientBoostingRegressor method)</td>
<td>202, 360</td>
</tr>
<tr>
<td>staged_predict() (sklearn.ensemble.GradientBoostingRegressor method)</td>
<td>202, 361</td>
</tr>
<tr>
<td>startprob_ (sklearn.hmm.GaussianHMM attribute)</td>
<td>404</td>
</tr>
<tr>
<td>startprob_ (sklearn.hmm.GMMHMM attribute)</td>
<td>412</td>
</tr>
<tr>
<td>startprob_ (sklearn.hmm.MultinomialHMM attribute)</td>
<td>408</td>
</tr>
<tr>
<td>StratifiedKFold (class in sklearn.cross_validation)</td>
<td>275</td>
</tr>
<tr>
<td>StratifiedShuffleSplit (class in sklearn.cross_validation)</td>
<td>277</td>
</tr>
<tr>
<td>SVC (class in sklearn.svm)</td>
<td>618</td>
</tr>
<tr>
<td>SVR (class in sklearn.svm)</td>
<td>629</td>
</tr>
<tr>
<td>TfidfTransformer (class in sklearn.feature_extraction.text)</td>
<td>371</td>
</tr>
<tr>
<td>TfidfVectorizer (class in sklearn.feature_extraction.text)</td>
<td>373</td>
</tr>
<tr>
<td>theta (sklearn.gaussian_process.GaussianProcess attribute)</td>
<td>392</td>
</tr>
<tr>
<td>theta (sklearn.naive_bayes.GaussianNB attribute)</td>
<td>561</td>
</tr>
<tr>
<td>train_test_split() (in module sklearn.cross_validation)</td>
<td>278</td>
</tr>
<tr>
<td>transform() (sklearn.cluster.KMeans method)</td>
<td>232</td>
</tr>
<tr>
<td>transform() (sklearn.cluster.MinibatchKMeans method)</td>
<td>235</td>
</tr>
<tr>
<td>transform() (sklearn.decomposition.DictionaryLearning method)</td>
<td>330</td>
</tr>
<tr>
<td>transform() (sklearn.decomposition.FastICA method)</td>
<td>318</td>
</tr>
<tr>
<td>transform() (sklearn.decomposition.KernelPCA method)</td>
<td>316</td>
</tr>
<tr>
<td>transform() (sklearn.decomposition.MinibatchDictionaryLearning method)</td>
<td>334</td>
</tr>
<tr>
<td>transform() (sklearn.decomposition.MinibatchSparsePCA method)</td>
<td>325</td>
</tr>
<tr>
<td>transform() (sklearn.decomposition.NMF method)</td>
<td>320</td>
</tr>
<tr>
<td>transform() (sklearn.decomposition.PCA method)</td>
<td>306</td>
</tr>
<tr>
<td>transform() (sklearn.decomposition.ProbabilisticPCA method)</td>
<td>308</td>
</tr>
<tr>
<td>transform() (sklearn.decomposition.ProjectGradientNMF method)</td>
<td>311</td>
</tr>
</tbody>
</table>