Laboratorio di Apprendimento Automatico

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What is clustering?

- Clustering: the process of grouping a set of objects into groups of similar objects
  - The commonest form of *unsupervised learning*
    - Unsupervised learning = learning from raw data, as opposed to supervised data where a classification of examples is given
  - A common and important task that finds many applications
  - Not only Example Clustering (e.g. feature)

The Clustering Problem

Given:

- A set of examples $D=\{d_1,..d_n\}$
- A similarity measure (or distance metric)
- A partitioning criterion
- A desired number of clusters K

Compute:

- An assignment function $\gamma : D \rightarrow \{1,..,K\}$ s.t.
  - None of the clusters is empty
  - Satisfies the partitioning criterion w.r.t. the similarity measure

Issues for clustering

- Representation for clustering
  - Representation
    - Vector space? Normalization?
  - Need a notion of similarity/distance
- How many clusters?
  - Fixed a priori?
  - Completely data driven?
    - Avoid "trivial" clusters - too large or small
      - In an application, if a cluster's too large, then for navigation purposes you've wasted an extra user click without whittling down the set of documents much.

Objective Functions

- Often, the goal of a clustering algorithm is to optimize an objective function
- In this cases, clustering is a search (optimization) problem
- K^N / $K!$ different clustering available
- Most partitioning algorithms start from a guess and then refine the partition
- Many local minima in the objective function implies that different starting point may lead to very different (and unoptimal) final partitions

What Is A Good Clustering?

- Internal criterion: A good clustering will produce high quality clusters in which:
  - the intra-class (that is, intra-cluster) similarity is high
  - the inter-class similarity is low
  - The measured quality of a clustering depends on both the document representation and the similarity measure used

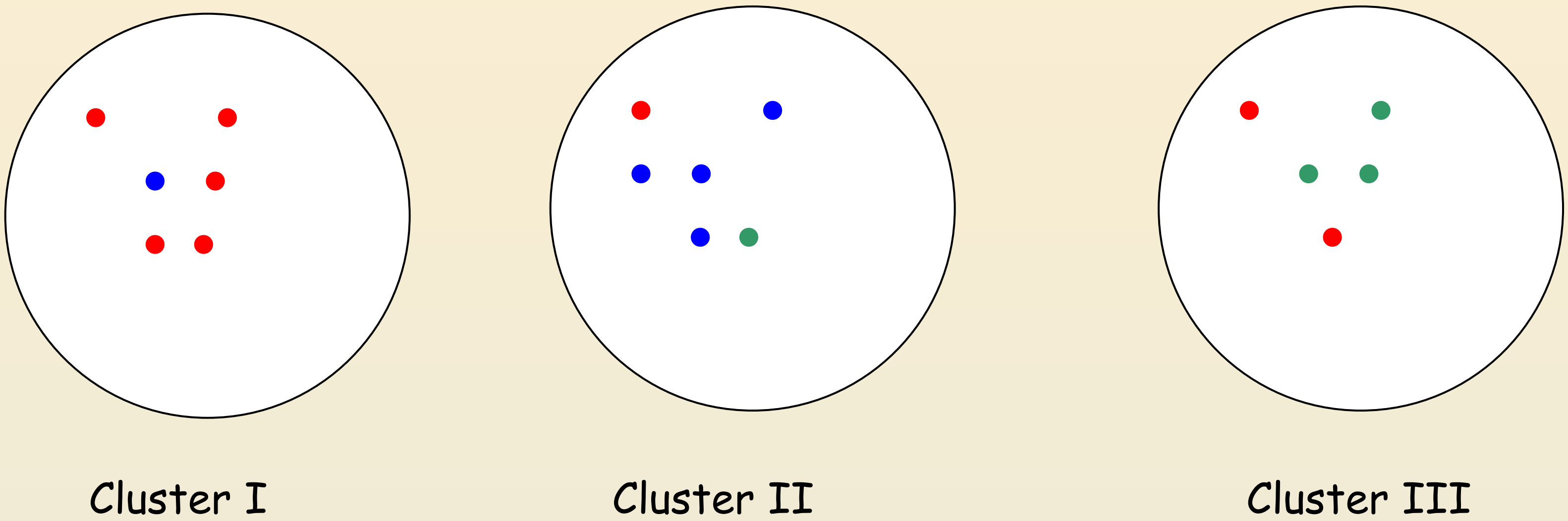
External criteria for clustering quality

- Quality measured by its ability to discover some or all of the hidden patterns or latent classes in gold standard data
- Assesses a clustering with respect to ground truth
- Assume documents with C gold standard classes, while our clustering algorithms produce K clusters, $\omega_1,\ldots,\omega_k$ with n_i members.

External Evaluation of Cluster Quality

- Simple measure: purity, the ratio between the dominant class in the cluster π_i and the size of cluster ω_i
- Others are entropy of classes in clusters (or mutual information between classes and clusters)

Purity example



Cluster I: Purity = 1/6 (max(5, 1, 0)) = 5/6

Cluster II: Purity = 1/6 (max(1, 4, 1)) = 4/6

Cluster III: Purity = 1/5 (max(2, 0, 3)) = 3/5

Rand Index

Number of points	Same Cluster in clustering	Different Clusters in clustering
Same class in ground truth	A (tp)	C (fn)
Different classes in ground truth	B (fp)	D (tn)

Rand index: symmetric version

$$RI = \frac{A + D}{A + B + C + D}$$

Compare with standard Precision and Recall.

$$P = \frac{A}{A + B}$$

$$R = \frac{A}{A + C}$$

Rand Index example: 0.68

Number of points	Same Cluster in clustering	Different Clusters in clustering
Same class in ground truth	20	24
Different classes in ground truth	20	72

Clustering Algorithms

- Partitional algorithms
  - Usually start with a random (partial) partitioning
  - Refine it iteratively
    - *K* means clustering
    - Model based clustering
- Hierarchical algorithms
  - Bottom-up, agglomerative
  - Top-down, divisive

Partitioning Algorithms

- Partitioning method: Construct a partition of *n* documents into a set of *K* clusters

- Given: a set of documents and the number *K*
- Find: a partition of *K* clusters that optimizes the chosen partitioning criterion
  - Globally optimal: exhaustively enumerate all partitions
  - Effective heuristic methods: *K*-means and *K*-medoids algorithms

K-Means

- Assumes documents are real-valued vectors.
- Clusters based on *centroids* (aka the *center of gravity* or mean) of points in a cluster, *c*:

$$\vec{\mu}(\mathrm{c}) = \frac{1}{|c|} \sum_{\vec{x} \in c} \vec{x}$$

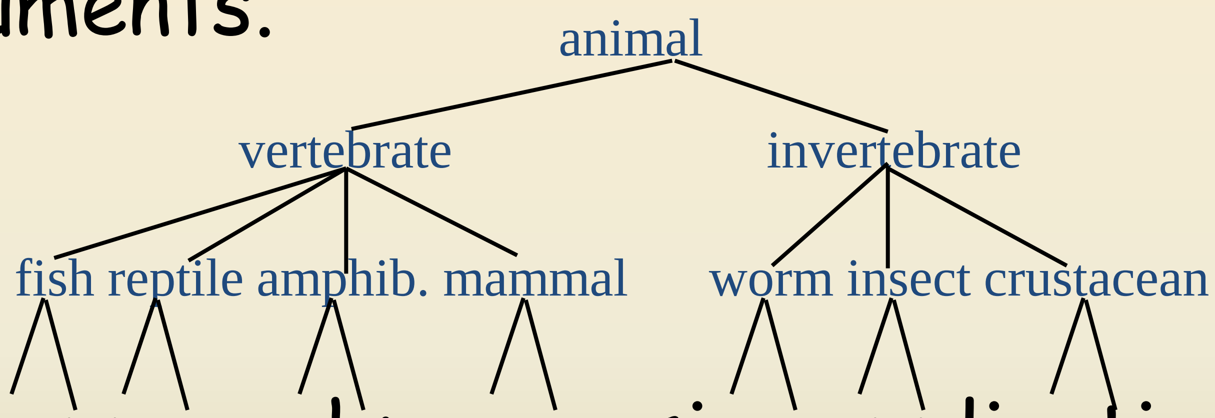
- Reassignment of instances to clusters is based on distance to the current cluster centroids.
  - (Or one can equivalently phrase it in terms of similarities)

How Many Clusters?

- Number of clusters *K* is given
  - Partition *n* docs into predetermined number of clusters
- Finding the "right" number of clusters is part of the problem
  - Given a set of examples, partition into an "appropriate" number of subsets.
  - E.g., for query results - ideal value of *K* not known up front - though UI may impose limits.

Hierarchical Clustering

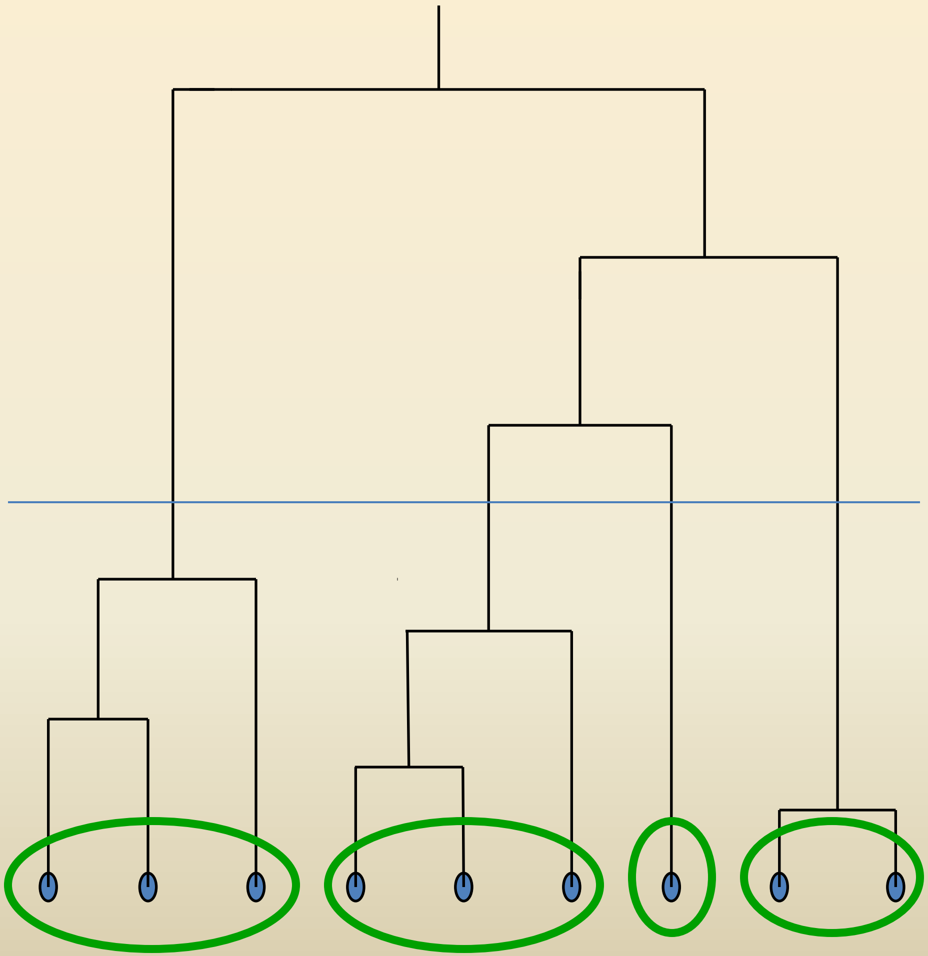
- Build a tree-based hierarchical taxonomy (*dendrogram*) from a set of documents.



- One approach: recursive application of a partitional clustering algorithm

Dendrogram: Hierarchical Clustering

Clustering obtained by cutting the dendrogram at a desired level: each connected component forms a cluster.



The dendrogram

- The y-axis of the dendogram represents the combination similarities, i.e. the similarities of the clusters merged by a the horizontal lines for a particular y
- Assumption: The merge operation is monotonic, i.e. if $s_1,..,s_{k-1}$ are successive combination similarities, then

  $s_1 > s_2 > \ldots > s_{k-1}$ must hold

Hierarchical Agglomerative Clustering (HAC)

- Starts with each example in a separate cluster
  - then repeatedly joins the closest pair of clusters, until there is only one cluster.
- The history of merging forms a binary tree or hierarchy.

Closest pair of clusters

- Many variants to defining closest pair of clusters
- Single-link
  - Similarity of the *most* cosine-similar (single-link)
- Complete-link
  - Similarity of the "furthest" points, the *least* cosine-similar
- Centroid
  - Clusters whose centroids (centers of gravity) are the most cosine-similar
- Average-link
  - Average cosine between pairs of elements

Summarizing

Single-link	Max sim of any two points	$O(N^2)$	Chaining effect
Complete-link	Min sim of any two points	$O(N^2 \log N)$	Sensitive to outliers
Centroid	Similarity of centroids	$O(N^2 \log N)$	Non monotonic
Group-average	Avg sim of any two points	$O(N^2 \log N)$	OK