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Pricing the Zero Coupon Bond Under Jump-Diffusion Model with Machine Learning

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Abstract

A data-driven approach called CaML (Calibration Machine Learning) is proposed to calibrate financial asset price models using Machine Learning methodologies. From the theoretical point of view, determining optimal values of the model parameters is formulated as training hidden neurons within a machine learning framework, based on available financial zero-coupon bond prices. For this purpose, we consider a dynamic system using discretization on stochastic variables. To train unknown parameters of the model, we consider the loss function in the form of the difference between the real and computed value of derivatives. Finally, we investigated the performance of this method by estimating the market price of risk of Zero Coupon bonds.

Keywords: financial model calibration, machine learning, deep neural networks, zero coupon bond, jump-diffusion model, approximation method.

Introduction

The bond market, also known as fixed income securities, is a market in which fixed income instruments are traded. The most important and common fixed income securities are bonds. A bond is a type of financial contract that the issuer of these bonds promises to pay bondholders stream of coupon over a given time period and the eventual return the principal at maturity date. A debt security that does not pay any interest coupon payments is called Zero Coupon Bond(ZCB). These bonds are very important to economy and financial market. In this paper, we attempt to present a model for these type of bonds and then solve it by using the Radial Basis Function (RBF)method. Many Scientists and researchers study about these bonds in markets, universities and across the world (the many references cited). Since in the modelling discussion, some parameters appear that can't be observed easily such as the most important one ,the market price of risk, in this paper we intend to approximate this parameter using machine learning. In this field, Poggi in 2017 and Horva in 2019 have articles about neural networks in finance which we want to use machine learning and fast calibration method for approximating the risk market price parameter.

Modeling

Bond modelling bonds on two variables, time and interest rate which in terms of interest rate having various models. If the interest rate is constant, then the formula for calculating interest rates is straightforward but if it is a deterministic variable then the model of bond would be an Ordinary Differential Equation (ODE). However, the model that has accurate description of market, is determined by stochastic interest rate.

In this paper we assume that the stochastic interest rate has the following model:

$$dr = k(\theta - r)dt + \sigma\sqrt{r}dW + JdN(\rho t),$$

Where W is a standard Brownian motion, $N(\rho t)$ is a Poission driving process with an intensity function ρ, and J is the jump size.

Now, let $P = P(r,t)$ be the current price of bond in time t with interest rate r Considering the Ito lemma, the price change of this bond is as follow:

$$dP = d_g + dP_J$$

If we apply the portfolio strategy of Black-Scholes and borrow an independent portfolio of two bonds with maturity date T1 and T2 then we reach the following Partial differential equation (PDE):

$$\frac{\partial P}{\partial t} + \frac{1}{2}\sigma^2 r \frac{\partial^2 P}{\partial r^2} + k(\theta - r)\frac{\partial P}{\partial r} - \lambda \int_{-\infty}^{\infty} (P(r+z,t,T) - P(r,t)d = 0$$

Where λ is a market price of risk P is the present value. Whenever the only unknown variable in the problem is p, we call it Direct Finance Problem (DFP) and if P as well as λ are unknown variables, the problem is called Inverse Finance Problem (IFP).

Solving the DFP by using RBF

The obtained PDE in prior section with initial and boundary condition is called initial boundary value problem. This problem can be solved numerically and many researchers have worked on it. In this paper, with the assumption that knowing all parameters, the problem is solved by RBF method.

Machin Learning

In this study, we try to reach a discrete dynamical system to predict future market price of risk value. In fact, using data, which are time series, a discrete dynamical system is trained and used to predict the next time step by previous data. To begin, we must pay attention to the data pattern, which is a time series and is arranged in the following order:

Database

$$\Omega = \{(\boldsymbol{x}_1, t_1), P^1), (\boldsymbol{x}_2, t_2), P^2), \ldots, (\boldsymbol{x}_M, t_M), P^M)\}$$

Without losing the generality of problem, we consider the input value of this learning problem model as a stochastic process:

$$d\boldsymbol{x} = \mu(t, \boldsymbol{x})dt + \sigma(t, \boldsymbol{x})dW$$

Where $\boldsymbol{x} = (r, k, \theta, \sigma, \lambda; T)$, and $\boldsymbol{\mu}$ and $\boldsymbol{\sigma}$ are drift and volatility respectively. Without losing the whole problem, we limit the range of changes of $\mathbf{x}$ as $\mathbf{x}_{min} \leq \mathbf{x} \leq \mathbf{x}_{max}$ and discretize it to M points, and estimate the P_i value for the discretized points using dataset the P_i value for the discretized points using dataset and the RBF so the new dataset is obtained as bellow set:

$$\left\{ P^1 = \begin{bmatrix} P(\boldsymbol{x}_1, t_1) \\ \vdots \\ (\boldsymbol{x}_M, t_1) \end{bmatrix}, P^2 = \begin{bmatrix} P(\boldsymbol{x}_1, t_2) \\ \vdots \\ (\boldsymbol{x}_M, t_2) \end{bmatrix}, \ldots, P^M = \begin{bmatrix} P(\boldsymbol{x}_1, t_M) \\ \vdots \\ (\boldsymbol{x}_M, t_M) \end{bmatrix} \right\}$$

Considering the Ito lemma and other concepts such as risk-free portfolio and Feynman-Kac formula , the following statement can be reached:

$$P^{i+1} = \psi P^i$$

where ψ is derivatives matrix that is obtained from PDE.

Training

In this section, we attempt to estimate λ parameter. To do this, we arrange the all parameters in a single vector as follows:

$$\varphi = (r, t, k, \theta, \sigma, \lambda)$$

By using RBF solution, we write the error norm for observable values in market and obtained values in the following way:

$$E_i = \|P_{RBF}(r_i, t_i, \varphi) - P_{market}(r_i, t_i)\|$$

$$L(\varphi) = \sum_{i=1}^{i=M} w_i {E_i}^2 + \alpha H(\varphi)$$

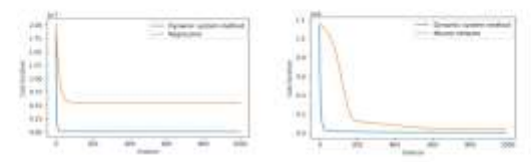
Where $\|.\|$ is L_2 norm and M is the number of simulations. So we have:

$$\varphi^* = arg \min_{\varphi} L(\varphi)$$

Where α is regulator parameter. Therefore, calibration as an optimization problem is considered only as a set of educational data. For example, the point between input and output of network are obtained with 80% of simulated data and with the remaining 20 %, we teach network in order to return corresponding output.

Implementation

To investigate the performance of this method, we illustrate a numerical example and the accuracy of this method respect to linear regression and neural network is as follow:



(a) Comparison of the cost function between regression and dynamic system (b) Comparison of the cost function between neural network and dynamic system

Figure 1: Comparison of the cost function

In addition, the result for testing data is as bellow:

Table 1: Test data error

	Min error	Mean error	Max error
Linear regression	0.2178	30123.72812	422787.8738
Neural network	210.3775	21501.6408	210575.5093
Dynamic system	1.4752e-03	0.9380	3.9026

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