Computing fixed-point of decreasing operators by relaxed iterations

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In this work we solve numerically fixed-point systems $\mathbf{u} = A(\mathbf{u})$, where $A : \mathbb{R}^m_+ \to \mathbb{R}^m_+$ is a continuous decreasing (antitone) mapping, with no distinct and comparable coupled fixed-points (i.e. if $\mathbf{u}, \mathbf{v} \in \mathbb{R}^m_+$ and $\mathbf{u} \preceq \mathbf{v}$, $\mathbf{u} = A(\mathbf{v}), \mathbf{v} = A(\mathbf{u})$, then $\mathbf{u} = \mathbf{v}$). Global convergence of Picard, updated Picard, Jacobi, and Gauss-Seidel (under)relaxed iterations is proved, in the general framework of decreasing operators in ordered Banach spaces. Relaxed iterations are applicable for instance, to discrete Hammerstein equations of the form $u_i = A_i(\mathbf{u}) = \sum_{j=1}^m b_{ij} f_j(u_j)$, $i = 1, ..., m$, where $b_{ij} \geq 0$, and the $f_j : \mathbb{R}_+ \to \mathbb{R}_+$ are suitable continuous and decreasing functions. Such methods are compared with Newton-like solvers on the decreasing form of the discrete Chandrasekhar H-equation. Two-grid Picard and Picard-Broyden solvers are also tested on such equation.

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