Lezione del 15 Ottobre

- f. iniettive, suriettive, biettive
- Composizione di funzioni
- f. inverse

$f : X \to Y$ iniettiva

$f : X \to \mathrm{Im} f$ biettiva

$\forall y \in \mathrm{Im} f \ \exists ! \, x \in X : \ y = f(x)$

suriettiva   iniettiva

$$f^{-1} : \mathrm{Im} f \to X$$
$$y \to x \qquad x = f^{-1}(y) \iff y = f(x)$$

Def f è invertibile in X se $\exists$ $f^{-1} : \mathrm{Im} f \to X$

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oss. $f \circ f^{-1}(y) = y \qquad y \in \mathrm{Im} f$

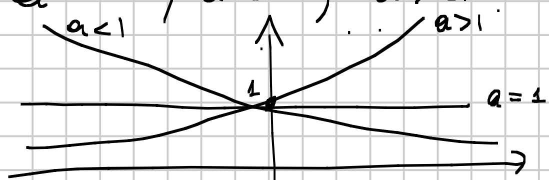
$f^{-1} \circ f(x) = x \qquad x \in X$

Funzione esponenziale

$f(x) = a^x, \quad a \in \mathbb{R}, \quad a > 0$

$f(0) = a^0 = 1$

$(0, 1)$



$a = 1 \qquad f(x) = 1$ non è iniettiva

$\nexists f^{-1}$

$a > 1$
$a < 1$ $\qquad f(x) : \mathbb{R} \to (0, +\infty)$

si può dim. che $\forall y \in (0, +\infty) \ \exists ! \, x \in \mathbb{R}$ t.c. $a^x = y$

$$f^{-1} : (0, +\infty) \to \mathbb{R}$$
$$y \to x \ : \ a^x = y$$

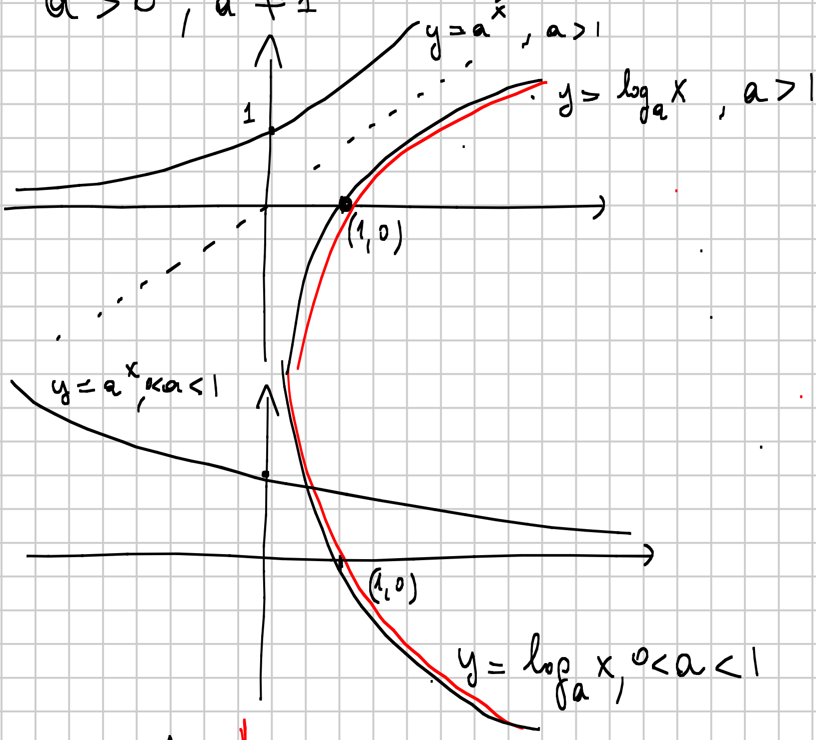
$f^{-1}(y) = \log_a y = x \iff a^x = y$ logaritmo in base a di y

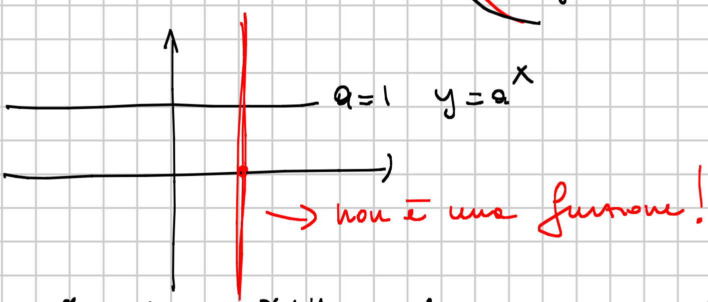
si può def. se $a > 1$, $a < 1$ (no se $a = 1$)

poi la chiameremo $f(x) = \log_a x = y \iff a^y = x$

$g(x) = \log_a x \qquad \text{dom } g = (0, +\infty)$

$a > 0, \ a \neq 1 \qquad x > 0$





$a^x \cdot a^y = a^{x+y}$

$\dfrac{a^x}{a^y} = a^{x-y}$

$\log_a xy = \log_a x + \log_a y$

$\log_a \dfrac{x}{y} = \log_a x - \log_a y$

$\log_a x^\alpha = \alpha \log_a x$

$\log_a x = \dfrac{\log_b x}{\log_b a}$

$\log_a 1 = 0 \quad \forall a > 0, \ a \neq 1$

dom $\log_a x = (0, +\infty)$

Im $\log_a x = \mathbb{R}$

strett. crescente se $a > 1$

" decrescente se $a < 1$

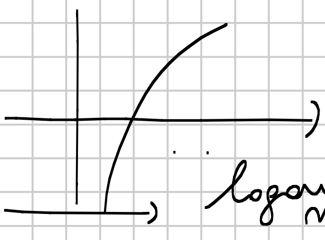
$a = e$ numero di Nepero

$e = 2,71 \ldots \qquad e > 1$

$\log_e x =$

$= \log x$

$= \ln x$



logaritmo naturale

$e^x \iff a^x$

FUNZIONI IPERBOLICHE e^x

$$f(x) = \operatorname{senh} x = \frac{e^x - e^{-x}}{2} \qquad \text{seno iperbolico} \quad x \in \mathbb{R}$$

$$\cosh x = \frac{e^x + e^{-x}}{2} \qquad \text{coseno iperbolico} \quad x \in \mathbb{R}$$

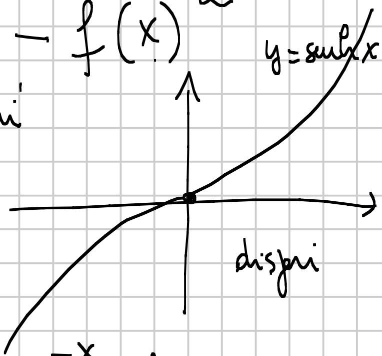
$f(x) = \operatorname{senh} x$ ha simmetrie?

$$f(-x) = \frac{e^{-x} - e^{x}}{2} = -\frac{e^x - e^{-x}}{2} = -f(x)$$

$f(x) = \operatorname{senh} x$ è dispari

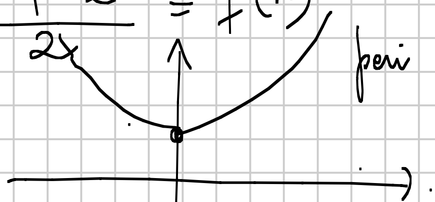
$\operatorname{dom} f = \mathbb{R}$

$\operatorname{Im} f = \mathbb{R}$



$f(x) = \cosh x \doteq \frac{e^x + e^{-x}}{2}$ ha simmetrie?

$$f(-x) = \frac{e^{-x} + e^{x}}{2} = f(x)$$



<u>OSS.</u> $f(x) = \operatorname{senh} x$ è strett. monotone in $\mathbb{R}$

quindi $\exists\, f^{-1} : \mathbb{R} \to \mathbb{R}$

$$\underbrace{x = \operatorname{sett\,senh} y}_{f^{-1}(y)} \iff \boxed{y = \operatorname{senh} x}$$

si può scrivere esplicitamente sett senh y

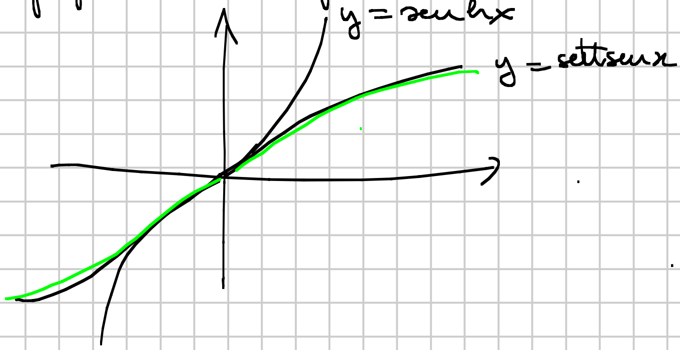
$$y = \frac{e^x - e^{-x}}{2} \Rightarrow x = ?$$

risolvo questa eq. in x come se y fosse costante

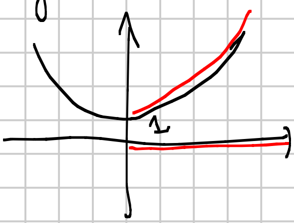
non si fa il conto e si ottiene

$$\boxed{x = \log\left(y + \sqrt{1+y^2}\right) =: \operatorname{sett\,senh} y}$$

Grafico di $y = \text{sett senh}\, x$



funzione inversa di $f(x) = \cosh x$



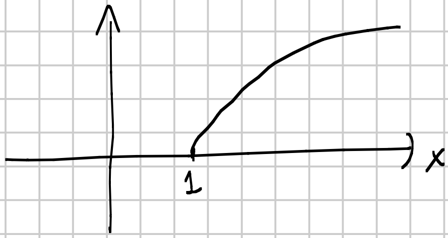
$f : \mathbb{R} \longrightarrow \underbrace{[1, +\infty)}_{\text{Im} f}$

non è iniettiva

$\cosh x : [0, +\infty) \longrightarrow [1, +\infty)$ è biettiva

è invertibile

$\text{sett}\cosh y = f^{-1}(y) \; : \; [1, +\infty) \to [0, +\infty)$



$y = \text{sett}\cosh x$

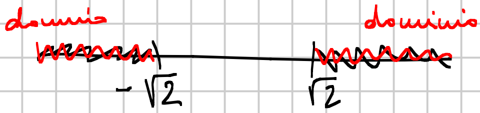
è def. $\forall x \geq 1$

es. $f(x) = \text{sett}\cosh(x^2 - 1)$

qual'è il dominio?

$x^2 - 1 \geq 1 \Rightarrow x^2 \geq 2 \iff$

$x \geq \sqrt{2}$ o $x \leq -\sqrt{2}$



C'è una forma esplicita per scrivere

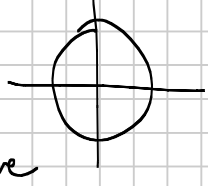
$y = \text{sett}\cosh x \doteq \log\left(x + \sqrt{x^2 - 1}\right)$

$x \geq 1$

relazione tra senh x e cosh x

$$\boxed{(\cosh x)^2 - (\text{senh}\, x)^2 = 1}$$

$(\cos x)^2 + (\text{sen}\, x)^2 = 1$

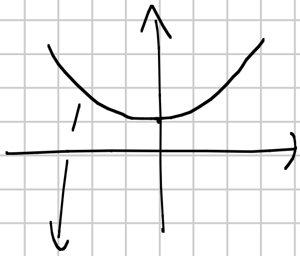


$x_1^2 - x_2^2 = 1$ iperboliche

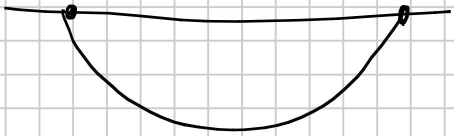
↓ eq iperbole

Dal punto di vista "fisico"

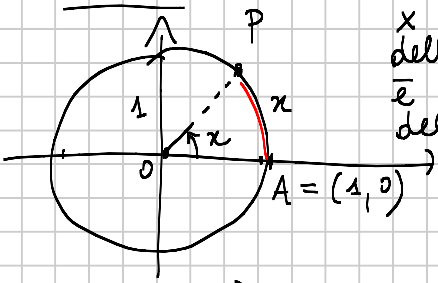
$$y = \cosh x = \frac{e^{x} + e^{-x}}{2}$$



catenaria.



Funzioni trigonometriche e loro inverse

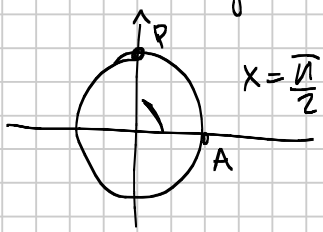


x si dice ampiezza dell'angolo $A\hat{O}P$ se x è la lunghezza dell'arco AP

x = misura in RADIANTI dell'angolo $A\hat{O}P$

$x \in [0, 2\pi)$

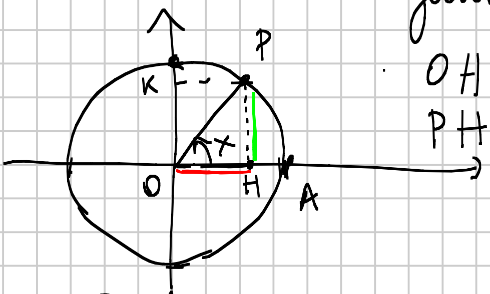
x = radiante



$x = \frac{\pi}{2}$

$\forall x \in [0, 2\pi)$

def. $(\cos x, \operatorname{sen} x)$ = le coordinate del punto P



$OH = \cos x$

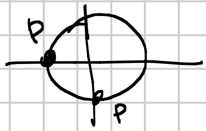
$PH = \operatorname{sen} x$

es. $P = A$ $\quad x = 0$ $\quad A = (\overset{\cos 0}{1}, \overset{\operatorname{sen} 0}{0})$

$\cos 0 = 1$

$\operatorname{sen} 0 = 0$

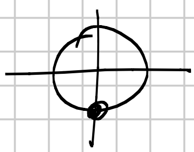
$x = \frac{\pi}{2}$ $\quad P = (\overset{\cos\frac{\pi}{2}}{0}, \overset{\operatorname{sen}\frac{\pi}{2}}{1})$



$\cos \frac{\pi}{2} = 0$ $\quad \operatorname{sen} \frac{\pi}{2} = 1$

$x = \pi$ $\quad P = (-1, 0)$

$\cos \pi = -1$ $\quad \operatorname{sen} \pi = 0$



$x = \frac{3}{2}\pi$

$P = (0, -1)$

$\cos \frac{3}{2}\pi = 0$, $\quad \operatorname{sen} \frac{3}{2}\pi = -1$

$X = 2\pi \qquad P = (1, 0)$

$\cos 2\pi = 1 = \cos 0$

$\operatorname{sen} 2\pi = 0 = \operatorname{sen} 0$

$\cos x$, $\operatorname{sen} x$ le abbiamo definite per $x \in [0, 2\pi)$

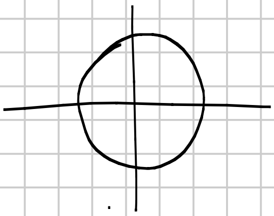
ma si possono definire $\cos x$ e $\operatorname{sen} x$ $\forall x \in \mathbb{R}$ estendendole in maniera periodica. Periodo 2π

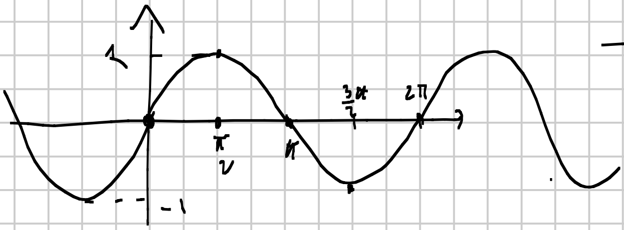
$x \in \mathbb{R}$

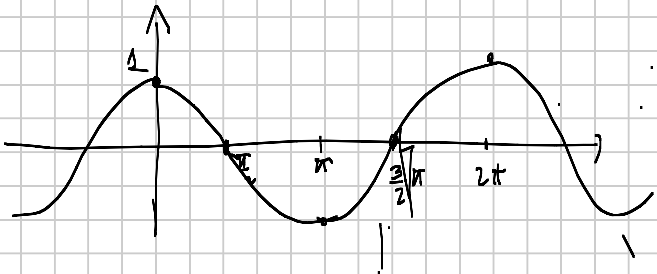
$$\cos(x + 2k\pi) = \cos x$$
$$\operatorname{sen}(x + 2k\pi) = \operatorname{sen} x \quad k \in \mathbb{Z}$$
$$\forall x \in [0, 2\pi)$$

$\operatorname{sen} x : \mathbb{R} \longrightarrow [-1, 1]$

$\cos x : \mathbb{R} \longrightarrow [-1, 1]$







$$\operatorname{tg} x := \frac{\operatorname{sen} x}{\cos x} \qquad \cos x \neq 0$$

$$\operatorname{tg} x : \mathbb{R} \setminus \left\{ x = \frac{\pi}{2} + k\pi,\ k \in \mathbb{Z} \right\}$$