Lezione del 16 Ottobre

- programma dettagliato svolto fino a oggi
  , 1ª e 2ª settimana
- Moodle — esercizi 1a e 2a settim.
  — link CONNECT del libro

Funzioni trigonometriche e loro inverse

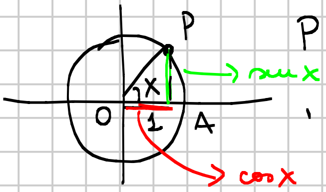
$f(x) = \operatorname{sen} x$ $\qquad$ x = misura in radianti di un angolo

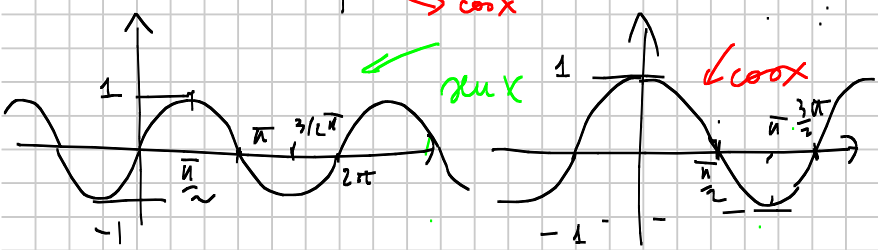
$f(x) = \cos x$

$x \in [0, 2\pi)$

sono definite $\forall x \in \mathbb{R}$

$P = (\cos x, \operatorname{sen} x)$





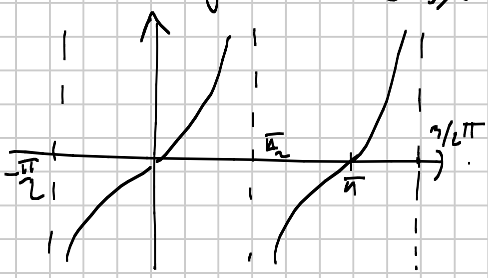
- $\operatorname{Im} \operatorname{sen} x = \operatorname{Im} \cos x = [-1, 1]$
- $\operatorname{sen} x$ e $\cos x$ sono f. periodiche di periodo 2π $\quad (f(x+P) = f(x))$

  4π
- $|\operatorname{sen} x| \leq 1, \quad |\cos x| \leq 1$
- $\operatorname{sen}^2 x + \cos^2 x = 1$
- $\operatorname{sen} x$ dispari $\qquad \operatorname{sen}(-x) = -\operatorname{sen} x$

  $\cos x$ pari $\qquad \cos(-x) = \cos x$
- $\cos(\alpha + \beta) =$ $\qquad$ p. 32-32 riguardare

  $\operatorname{sen} 2x = 2 \operatorname{sen} x \cos x$

def. $\operatorname{Tg} x := \dfrac{\operatorname{sen} x}{\cos x}$ $\qquad x \neq \dfrac{\pi}{2} + k\pi, \quad k \in \mathbb{Z}$



periodica di periodo π

$$\operatorname{tg} x : X = \{x \in \mathbb{R},\ x \neq \tfrac{\pi}{2} + k\pi\} \longrightarrow \mathbb{R}$$

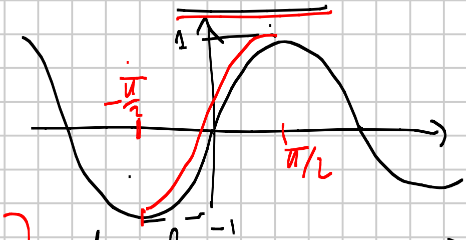
dominio — Immagine

$\operatorname{tg}(-x) = -\operatorname{tg} x$ <u>dispari</u>

$$\operatorname{cotg} x := \frac{\cos x}{\operatorname{sen} x} = \frac{1}{\operatorname{tg} x} \qquad x \neq k\pi,\ k \in \mathbb{Z}$$

Funzioni trigonometriche <u>inverse</u>.

- $f(x) = \operatorname{sen} x$



<u>non è</u> <u>iniettiva</u>

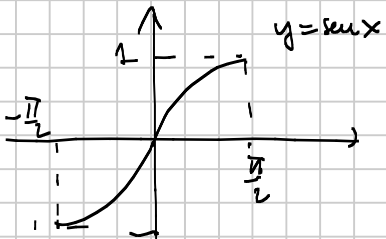
se $x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ la funzione $\operatorname{sen} x$ è strett. monotone e quindi invertibile in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$f(x) = \operatorname{sen} x \qquad f : \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \longrightarrow [-1, 1]$

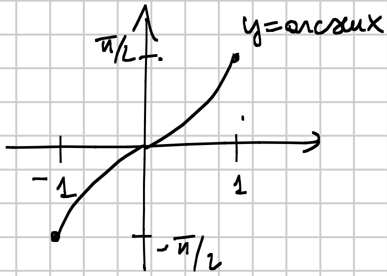
$\exists\ f^{-1}(y) =: \operatorname{arcsen} y : [-1, 1] \longrightarrow \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

arcoseno

$$\boxed{\begin{array}{lcl} \operatorname{arcsen} y = x & \Leftrightarrow & \operatorname{sen} x = y \\ y \in [-1,1] & & \forall x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \end{array}}$$

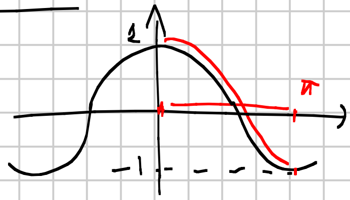


dispari



dispari

- $f(x) = \cos x$



se $x \in [0, \pi]$

la funzione $\cos x$ è strett. monotone e quindi invertibile

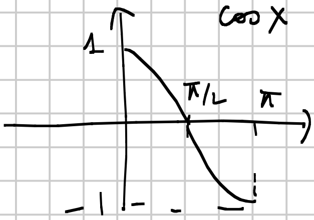
$\cos x : [0, \pi] \longrightarrow [-1, 1]$

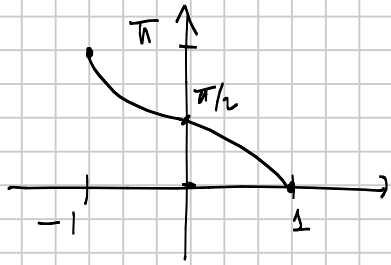
arcocoseno

$f^{-1}(y) =: \arccos y : [-1, 1] \longrightarrow [0, \pi]$

$x = \arccos y \iff y = \cos x$

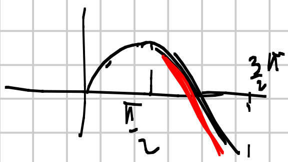
$y \in [-1,1] \to x \in [0, \pi]$





oss. Avrei potuto def. la funzione inversa del sen x o del cos x anche in un altro intervallo nel quale sen x o cos x sono strett. monotone

per es. $x \in \left[\frac{\pi}{2}, \frac{3}{2}\pi\right]$ per sen x



anche in questo intervallo sen x è invertibile ma la f^{-1} non è arcsen y

se si parla di $y = \operatorname{arcsen} x$ significa $x \in [-1,1]$, $y \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$$f \circ f^{-1}(y) = y$$

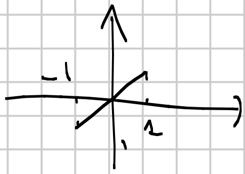
$$f^{-1} \circ f(x) = x$$

$\operatorname{sen}(\operatorname{arcsen} x) = x \iff x \in [-1,1]$

$\operatorname{arcsen}(\operatorname{sen} x) = x \iff x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$x \longrightarrow x$

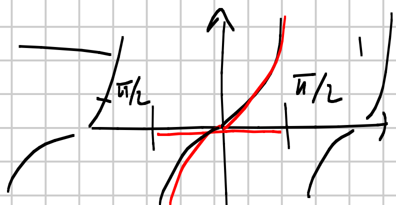
$y = x$



$\operatorname{arcsen}(\operatorname{sen} x) \neq x \quad x \in \mathbb{R}$

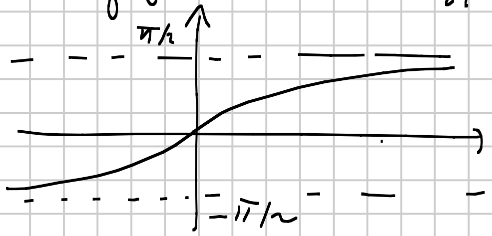
se $x \notin \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

- $f(x) = \operatorname{tg} x$



$f(x) = \operatorname{tg} x : \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \longrightarrow \mathbb{R}$ è strett. monotona $\Rightarrow$ invertibile

$f^{-1}(y) = \operatorname{arctg} y : \mathbb{R} \to \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$



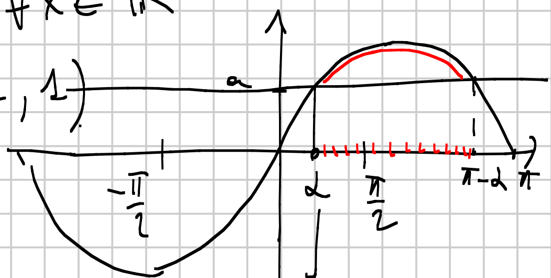
strett crescente e dispari

Disequazioni Trigonometriche

1) $\operatorname{sen} x > a$, $a \in \mathbb{R}$

$\begin{cases} \operatorname{sen} x > 1/2 \\ \operatorname{sen} x > 5 \\ \operatorname{sen} x > -18 \end{cases}$

- $a \geq 1$ $\nexists x$ soluz. diseq.
- $a < -1$ $\forall x \in \mathbb{R}$
- $a \in [-1, 1)$



$\operatorname{sen} \alpha = a$

$\alpha \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ $\quad$ $\alpha = \arcsin a$

$x \in (\alpha, \pi - \alpha) \cup (\alpha + 2\pi, \pi - \alpha + 2\pi) \cup$ - - -

$x \in (\alpha + 2k\pi, \pi - \alpha + 2k\pi)$

es. $\operatorname{sen} x > \frac{1}{10}$ $\quad$ $\operatorname{sen} \alpha = \frac{1}{10}$

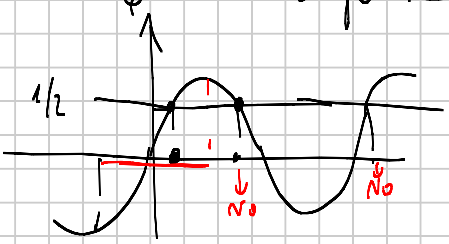
$\alpha = \arcsin \frac{1}{10}$

$\operatorname{sen} x > \frac{1}{2}$ $\quad$ $\alpha \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

t.c. $\operatorname{sen} \alpha = \frac{1}{2}$

$\alpha = \frac{\pi}{6}$ $\quad$ qui si può calcolare esplicitamente

$\arcsin \frac{1}{2} = \frac{\pi}{6}$



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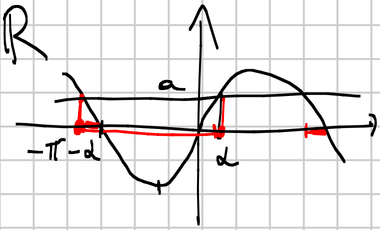
2) $\operatorname{sen} x < a$

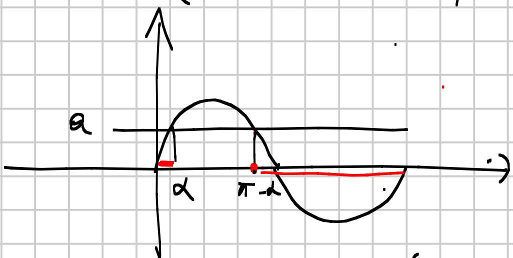
- $a \geq 1$ $\quad \forall x \in \mathbb{R}$
- $a \leq -1$ $\quad \nexists x$
- $a \in (-1, 1)$



$\operatorname{sen} \alpha = a \iff \alpha = \operatorname{arcsen} a$
$\alpha \in [-\pi/2, \pi/2]$

$x \in (-\pi - \alpha, \alpha)$ e i multipli di 2π

$x \in (-\pi - \alpha + 2k\pi, \alpha + 2k\pi)_{k \in \mathbb{Z}}$



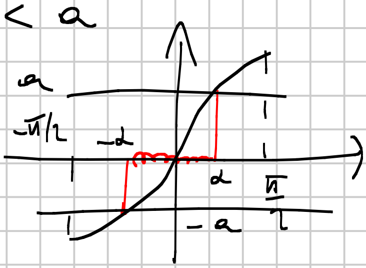
$\operatorname{sen} x < a$

$x \in (0, \alpha) \cup (\pi - \alpha, 2\pi)$
+ multipli di 2π

3) $|\operatorname{sen} x| < a$

$-a < \operatorname{sen} x < a$

$$\begin{cases} \operatorname{sen} x < a \\ \operatorname{sen} x > -a \end{cases}$$



$\alpha = \operatorname{arcsen} a$

es. $f(x) = \operatorname{arcsen}\left(\dfrac{x-5}{|x+2|}\right)$

1) trovare il dominio
2) studiare segno
3) trovare x : $f(x) < \dfrac{\pi}{6}$

$$\left|\frac{x-5}{|x+2|}\right| \leq 1$$

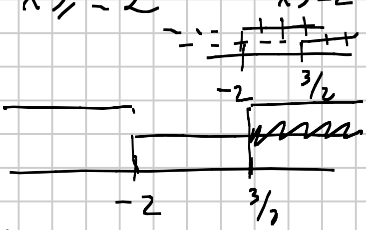
$$\frac{|x-5|}{|x+2|} \leq 1 \qquad \boxed{\left|\frac{x-5}{x+2}\right| \leq 1}$$

$$\begin{cases} \dfrac{x-5}{x+2} \le 1 & \Rightarrow \dfrac{x-5}{x+2} - 1 \le 0 \\ \dfrac{x-5}{x+2} \ge -1 & \Rightarrow \dfrac{x-5}{x+2} + 1 \ge 0 \end{cases}$$

$$\begin{cases} \dfrac{\cancel{x}-5-\cancel{x}-2}{x+2} \le 0 \\ \dfrac{x-5+x+2}{x+2} \ge 0 \end{cases} \qquad \begin{cases} \dfrac{-7}{x+2} \le 0 \\ \dfrac{2x-3}{x+2} \ge 0 \end{cases}$$

$x \ge 3/2$
$x > -2$

$$\begin{cases} x+2 \ge 0 \\ x < -2, \quad x \ge \frac{3}{2} \end{cases} \qquad x \ge -2$$

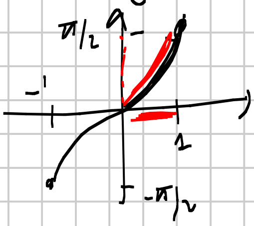


$$\begin{cases} x \ge -2 \\ x < -2, \quad x \ge 3/2 \end{cases} \Rightarrow x \ge 3/2$$

$D = [\frac{3}{2}, +\infty)$

?

2) segno di $\arcsin\left(\dfrac{x-5}{|x+2|}\right) \ge 0$



$\arcsin y \ge 0 \iff y \ge 0$

$\arcsin\left(\dfrac{x-5}{|x+2|}\right) \ge 0 \iff$

$x \ge 5 \iff \boxed{\dfrac{x-5}{\underbrace{|x+2|}_{>0}} \ge 0}$

$f(x) \ge 0 \iff x \ge 5$

3) x : $\arcsin\left(\dfrac{x-5}{|x+2|}\right) < \dfrac{\pi}{6}$

?

? $\sin\left(\arcsin\left(\dfrac{x-5}{|x+2|}\right)\right) < \sin\dfrac{\pi}{6}$

? $x_1 < x_2 \Rightarrow f(x_1) < f(x_2)$?

$\sin(\ )$ è crescente in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

sì è vero!

$\dfrac{x-5}{|x+2|} < \dfrac{1}{2}$

R.: $[\frac{3}{2}, 12)$