Definizione $X \subseteq \mathbb{R}$, $x_0 \in \mathbb{R}$

x_0 è pto accumulazione dx per X
se $\forall \delta > 0$ $(x_0, x_0+\delta) \cap X \neq \emptyset$

sx

$(x_0-\delta, x_0) \cap X \neq \emptyset$

Definizione $f: X \to \mathbb{R}$, x_0 di acc. dx per X

Diremo che il limite destro per x che tende a x_0 di $f(x)$ è uguale a $\ell \in \mathbb{R}^*$

$$\lim_{x \to x_0^+} f(x) = \ell$$

Se $\forall\, \mathcal{U}$ intorno di ℓ $\exists \delta > 0$:

$\forall x \in X$ $x_0 < x < x_0 + \delta$ si ha $f(x) \in \mathcal{U}$

sinistra

$$\lim_{x \to x_0^-} f(x) = \ell$$

$\forall x \in X$ $x_0 - \delta < x < x_0$ si ha $f(x) \in \mathcal{U}$

Teorema

$f : X \subseteq \mathbb{R} \to \mathbb{R}$ x_0 di acc. per X

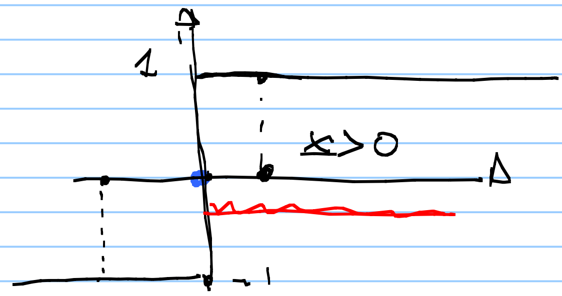
$\lim_{x \to x_0} f(x) = \ell \iff \lim_{x \to x_0^+} f(x) = \ell$

e

$\lim_{x \to x_0^-} f(x) = \ell$

Esempio

$f(x) = \frac{x}{|x|}$



$\lim_{x \to x_0^+} f(x) = 1$

$\cancel{=}$

$\lim_{x \to x_0^-} f(x) = -1$

$\nexists \lim_{x \to x_0} \frac{x}{|x|}$

Limiti di successioni

$\{x_n\}_{n \in \mathbb{N}}$ $f : \mathbb{N} \subseteq \mathbb{R} \to \mathbb{R}$

$+\infty$ è l'unico pto di acc. per $\mathbb{N}$

$$\boxed{\lim_{n \to \infty} x_n = \ell}$$ $\ell \in \mathbb{R}^*$

Teorema (ponte)

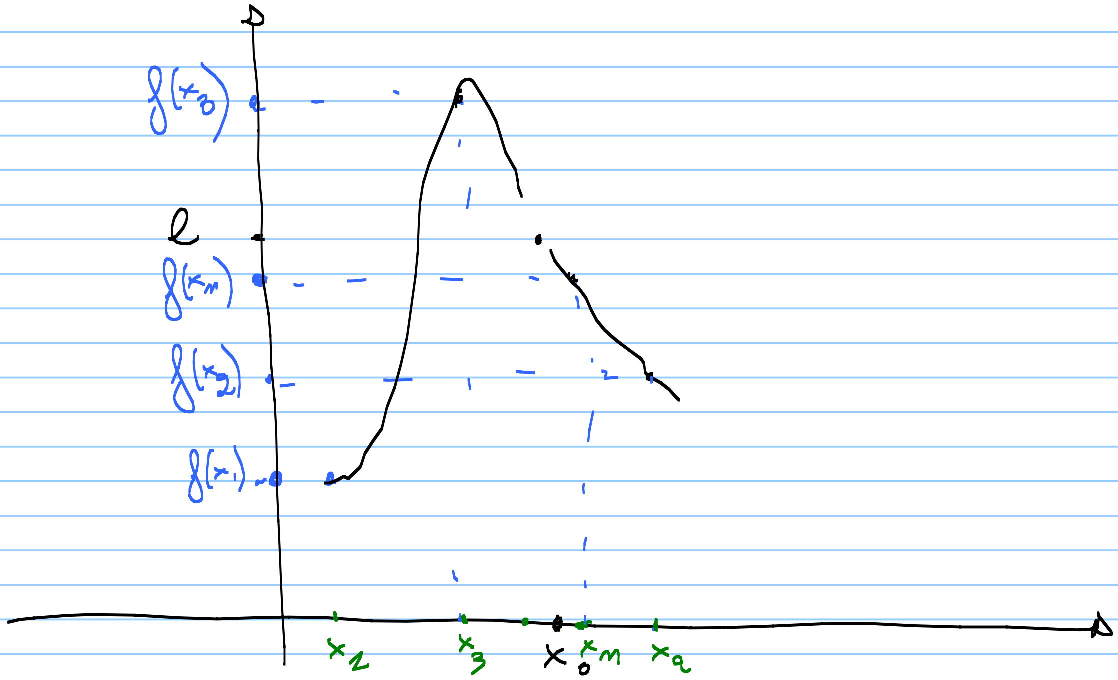
$f : X \to \mathbb{R}$, x_0 di acc. per X

$$\boxed{\lim_{x \to x_0} f(x) = \ell}$$

$\Updownarrow$

$\forall \{x_n\}_{n \in \mathbb{N}} \subseteq X : \lim_{n \to \infty} x_n = x_0$

si h_2 $\lim\limits_{n \to \infty} f(x_n) = \ell$



$\lim\limits_{x \to +\infty} \sin x$

$x_n = \frac{\pi}{2} + 2n\pi \qquad \lim\limits_{n \to \infty} x_n = +\infty$

$\lim\limits_{n \to +\infty} \sin(x_n) = \lim\limits_{n \to \infty} 1 = 1$

$y_n = n\pi \qquad \lim\limits_{n \to \infty} y_n = +\infty$

$\lim\limits_{n \to \infty} \sin y_n = \lim\limits_{n \to \infty} 0 = 0$

$\exists\ x_n, y_n \qquad \lim\limits_{n \to \infty} x_n = \lim\limits_{n \to \infty} y_n = +\infty$

t.c. $\lim\limits_{n \to \infty} f(x_n) \neq \lim\limits_{n \to +\infty} f(y_n)$

Teorema (unicità del limite)

$f: X \to \mathbb{R}$ $\quad x_0$ di acc. per X e se

$\lim_{x \to x_0} f(x) = l_1$ e $\lim_{x \to x_0} f(x) = l_2$

$\Rightarrow l_1 = l_2$

Teorema (della permanenza del segno)

$f: X \subseteq \mathbb{R} \to \mathbb{R}$ $\quad x_0$ di acc. per X

Se $\lim_{x \to x_0} f(x) = l \in \mathbb{R}^*$, $\quad l > 0$

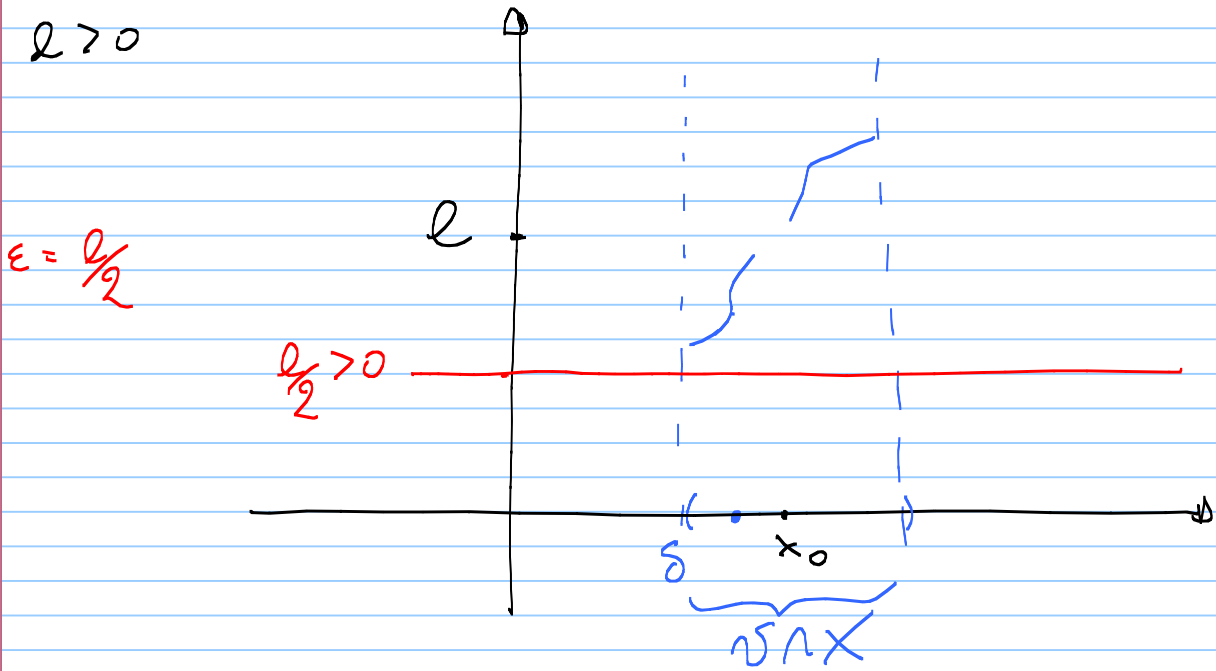
Allora $\exists\, V$ intorno di x_0 t.c. $f(x) > 0$

$\forall x \in V \cap X$

$l < 0$

$f(x) < 0$

$l > 0$



Teorema (algebra dei limiti)

$f, g : X \subseteq \mathbb{R} \to \mathbb{R}$ $\qquad$ x_0 di acc. per X

Se $\lim_{x \to x_0} f(x) = l_1 \in \mathbb{R}$ $\quad$ e $\quad$ $\lim_{x \to x_0} g(x) = l_2 \in \mathbb{R}$

1) $\lim_{x \to x_0} c\, f(x) = c\, l_1 \quad , \forall c \in \mathbb{R}$

2) $\lim_{x \to x_0} \left[f(x) + g(x) \right] = \left[\lim_{x \to x_0} f(x) \right] + \left[\lim_{x \to x_0} g(x) \right] = l_1 + l_2$

3) $\lim_{x \to x_0} \left[f(x)\, g(x) \right] = \left[\lim_{x \to x_0} f(x) \right] \left[\lim_{x \to x_0} g(x) \right] = l_1\, l_2$

4) $l_2 \neq 0 \implies \lim_{x \to x_0} \frac{1}{g(x)} = \frac{1}{l_2}$

5) $l_2 \neq 0 \implies \lim_{x \to x_0} \frac{f(x)}{g(x)} = \frac{l_1}{l_2}$

Teorema (del confronto / dei due carabinieri / sandwich)

$f, g, h : X \subseteq \mathbb{R} \to \mathbb{R}$ $\qquad$ x_0 di acc. per X

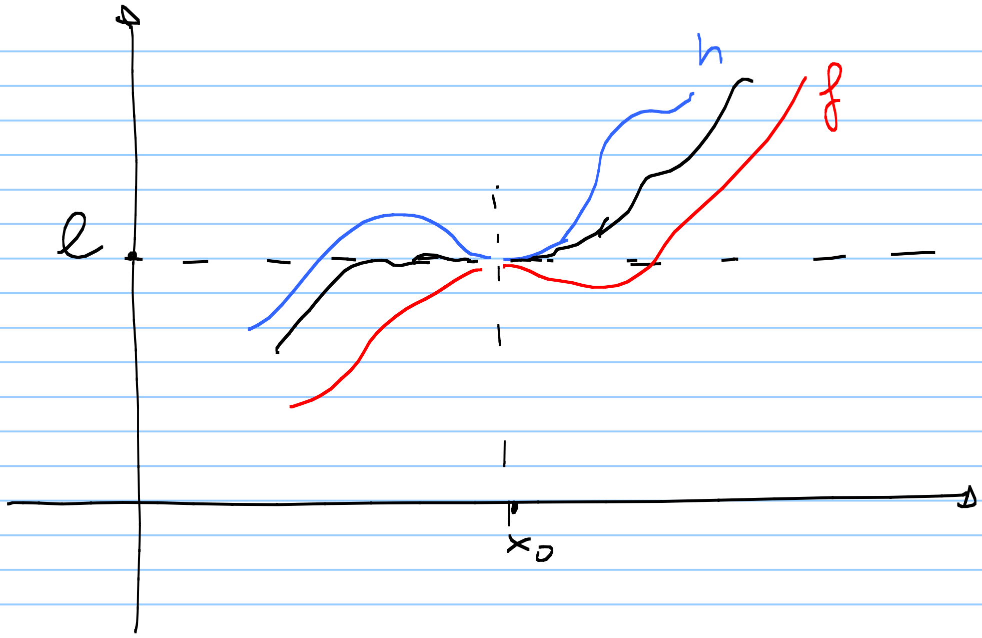
$\lim_{x \to x_0} f(x) = \lim_{x \to x} h(x) = l$

Se $\exists\, V$ di x_0 t.c. $\forall x \in (V \setminus \{x_0\}) \cap X$

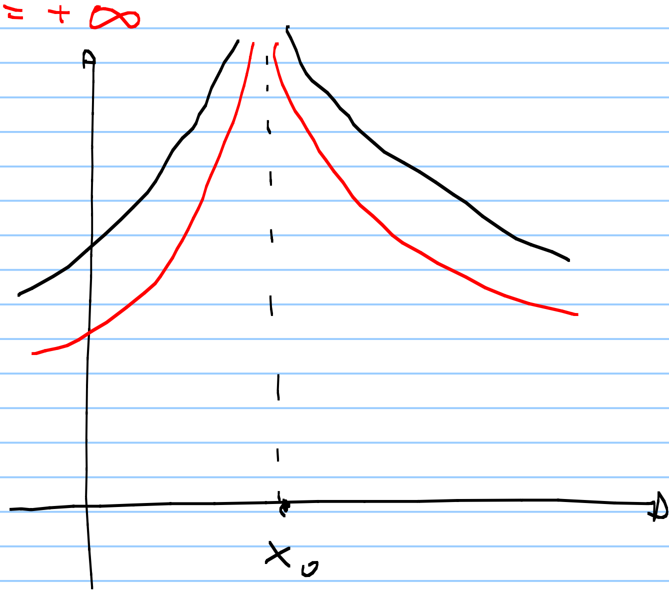
si ha $\quad f(x) \leq g(x) \leq h(x)$

Allora

$\lim_{x \to x_0} g(x) = l$



Caso speciale $\quad l = +\infty$



Se $\lim_{x \to x_0} f(x) = +\infty$ e $\underbrace{g(x) \geq f(x)}$

$\Rightarrow \lim_{x \to x_0} g(x) = +\infty$

Analogamente

$l = -\infty$ e $f(x) \geq g(x)$

Se $\lim_{x \to x_0} f(x) = -\infty$

$\Rightarrow \lim_{x \to x_0} g(x) = -\infty$

Esempi 1) $\lim_{x \to x_0} x = x_0$ | $\lim_{x \to x_0} |x| = |x_0|$

$\forall \varepsilon > 0$ $\quad |f(x) - \ell| = |x - x_0| < \varepsilon$

$\exists \delta = \varepsilon$ $\quad |x - x_0| < \delta = \varepsilon$

2) $\lim_{x \to 0} x \cos\left(\frac{1}{x}\right) =$

$$\left|\cos\left(\frac{1}{x}\right)\right| \leq 1$$

$$\left|x \cos \frac{1}{x}\right| = |x| \left|\cos\left(\frac{1}{x}\right)\right| \leq |x|$$

$$\underbrace{-|x|}_{f(x)} \leq x \cos\left(\frac{1}{x}\right) \leq \underbrace{|x|}_{h(x)}$$

$$\lim_{x \to 0} h(x) = \lim_{x \to 0} |x| = 0$$

$$\lim_{x \to 0} -|x| = -\lim_{x \to 0} |x| = 0$$

per il teo del confronto $\Rightarrow \lim_{x \to 0} x \cos\left(\frac{1}{x}\right) = 0$

Esempio 3) $\lim_{x \to 1} \dfrac{x^3 + x^2}{5x^5 + 2x^2} = \dfrac{2}{7}$

4) $\lim_{x \to 0} \dfrac{x^3 + x^2}{5x^5 + 2x^2}$ (*) $=$

se $x \neq 0$ $\quad \dfrac{x^3 + x^2}{5x^5 + 2x^2} = \dfrac{x + 1}{5x^3 + 2}$

$$8 = \lim_{x \to 0} \frac{x+1}{5x^3+2} = \frac{1}{2}$$

Aritmetizzazione parziale di $\mathbb{R}^*$

Siano $f, g : X \to \mathbb{R}$, x_0 di accumulazione per X

i) Se $\lim_{x \to x_0} f(x) = +\infty$ $(-\infty)$

e g è limitata inferiormente (superiormente)

Allora $\lim_{x \to x_0} [\underbrace{f(x) + g(x)}_{\geq f(x) + M}] = +\infty$ $(-\infty)$

ii) Se $\lim_{x \to x_0} f(x) = +\infty$ e $g(x) > m > 0$ $(< m < 0)$

Allora $\lim_{x \to x_0} f(x) g(x) = +\infty$ $(-\infty)$

Se $\lim_{x \to x_0} f(x) = -\infty$ e $g(x) > m > 0$ $(< m < 0)$

Allora $\lim_{x \to x_0} f(x) g(x) = -\infty$ $(+\infty)$

iii) Se $\lim_{x \to x_0} f(x) = 0$ → (f è infinitesima in x_0)

e $g(x)$ è limitata

Allora $\lim_{x \to x_0} f(x) g(x) = 0$

iv) Se $\lim_{x \to x_0} f(x) = 0$ e $f(x) > 0$ (< 0)

Allora $\lim_{x \to x_0} \frac{1}{f(x)} = +\infty$ $(-\infty)$

v) Se $\lim_{x \to x_0} f(x) = +\infty$ $(-\infty)$

Allora $\lim_{x \to x_0} \frac{1}{f(x)} = 0$

Osservazione: $\left[\lim_{x \to x_0} g(x) = \ell \quad \ell \in \mathbb{R} \text{ o } \ell = +\infty \right.$

$\left| \Rightarrow g(x) \text{ è inf. limitata} \right.$

Quindi corollario al caso i)

se $\lim_{x \to x_0} f(x) = +\infty$ e $\lim_{x \to x_0} g(x) = \ell$

$\Rightarrow \lim_{x \to x_0} f(x) + g(x) = +\infty$

Riassumendo: i) $+\infty + \ell = +\infty \quad \ell \in \mathbb{R}$

$+\infty + \infty = +\infty$

$-\infty + \ell = -\infty \quad \ell \in \mathbb{R}$

$-\infty + (-\infty) = -\infty$

ii) $(+\infty)(\ell) = +\infty$ se $\ell > 0$

$(+\infty)(+\infty) = +\infty$

$(+\infty)(\ell) = -\infty$ se $\ell < 0$

$(+\infty)(-\infty) = -\infty$

$(-\infty)(\ell) = -\infty$ se $\ell > 0$

$(-\infty)(+\infty) = -\infty$

$(-\infty)(\ell) = +\infty$ se $\ell < 0$

$(-\infty)(-\infty) = +\infty$

iv) $\frac{1}{0^+} = +\infty$

$\frac{1}{0^-} = -\infty$

(Il risultato ha lo stesso segno del denominatore)

v) $\frac{1}{\pm\infty} = 0$

Forme indeterminate

- $(+\infty) - (+\infty)$
- $(-\infty) + (+\infty)$
- $(-\infty) - (-\infty)$
- $+\infty \cdot 0$
- $-\infty \cdot 0$
- $\frac{0}{0}$
- $\frac{-\infty}{+\infty}$ $\quad \frac{+\infty}{-\infty}$ $\quad \frac{+\infty}{+\infty}$ $\quad \frac{-\infty}{-\infty}$

Esercizi

1) $\lim_{x \to +\infty} x = +\infty$

$\lim_{x \to -\infty} x = -\infty$

$\lim_{x \to \pm\infty} |x| = +\infty$

def. di limite!

2) $\lim_{x \to +\infty} \underbrace{x^2}_{\underbrace{x \cdot x}_{\to +\infty}} + \underbrace{\cos(3x)}_{\geq -1} = +\infty$

3) $\lim_{x \to 0} \frac{3x^2 + x^3}{x^4 + 6x^7} = \lim_{x \to 0} \frac{3 + x}{x^2 + 6x^5} = +\infty$

infatti $x^2 + 6x^5 > 0$ $\quad x^2(1 + 6x^3) > 0$ in un intorno di 0 !!

4) $\lim_{x \to -\infty} \boxed{\left(\underbrace{\frac{x^3 + 2x + 1}{x - 1}}_{\frac{-\infty}{-\infty}}\right)} \underbrace{\left|\left[\cos\left(\frac{x}{2}\right)\right] + 2\right|}_{\geq 1} = +\infty$

$\to +\infty$

$\frac{1}{x^2} - \frac{1}{x^3} = \frac{x-1}{x^3} > 0$

"vicino a $-\infty$"

$$\frac{1 + \frac{2}{x^2} + \frac{1}{x}}{\frac{1}{x^2} - \frac{1}{x^3}} \to +\infty$$

$\cos(\cdot) \geq -1$

$|\cos(\cdot) + 2| \geq 1$

5) $\lim_{x \to +\infty} \overbrace{\underbrace{\left(\frac{x^3 + 2x + 1}{x - 1}\right)}_{\to +\infty} \underbrace{\left|\left[\cos\left(\frac{x}{2}\right)\right] + 1\right|}_{\geq 0}}^{f(x)}$ $\nexists$

$\cos\left(\frac{x}{2}\right) = -1$ $\quad \frac{x}{2} = \pi + 2k\pi$ $\quad x = 2\pi + 4k\pi$

6) $\lim_{x \to 0} \frac{1}{x}$ $\nexists$

$\left.\begin{array}{l} \lim_{x \to 0^+} \frac{1}{x} = +\infty \\ \lim_{x \to 0^-} \frac{1}{x} = -\infty \end{array}\right] \neq$

7) $\lim_{x \to 0} \frac{1}{x^2} = +\infty$

$\boxed{\text{Teorema}}$ (Esistenza del limite per funzioni monotone)

$f: X \to \mathbb{R}$, x_0 punto di accumulazione sinistro per X

f monotona crescente (decrescente)

Allora $\lim_{x \to x_0^-} f(x) = \sup \{ f(x), x \in X \cap (-\infty, x_0) \}$

$$= \sup_{X \cap (-\infty, x_0)} f$$

(inf)

$f: X \to \mathbb{R}$ x_0 punto di accumulazione destro per X

f monotona crescente (decrescente)

Allora $\lim_{x \to x_0^+} f(x) = \inf \{ f(x) : x \in X \cap (x_0, +\infty) \}$

$$= \inf_{X \cap (x_0, +\infty)} f$$

(sup)

Limiti delle funzioni elementari

- $$\lim_{x \to x_0} x^\alpha = x_0^\alpha \qquad \forall \alpha \in \mathbb{R} \qquad x_0 \in \mathbb{R}^+$$

- $$\lim_{x \to 0^+} x^\alpha = \begin{cases} 0 & \text{se } \alpha > 0 \\ 1 & \text{se } \alpha = 0 \\ +\infty & \text{se } \alpha < 0 \end{cases}$$

- $$\lim_{x \to +\infty} x^\alpha = \begin{cases} +\infty & \text{se } \alpha > 0 \\ 1 & \text{se } \alpha = 0 \\ 0 & \text{se } \alpha < 0 \end{cases}$$

- $$\lim_{x \to x_0} a^x = a^{x_0} \qquad \forall x_0 \in \mathbb{R}$$

- $$\lim_{x \to +\infty} a^x = \begin{cases} +\infty & a > 1 \\ 0 & 0 < a < 1 \end{cases}$$

- $$\lim_{x \to -\infty} a^x = \begin{cases} 0 & a > 1 \\ +\infty & 0 < a < 1 \end{cases}$$

- $$\lim_{x \to x_0} \log_a x = \log_a x_0 \qquad \forall x_0 > 0$$

- $$\lim_{x \to +\infty} \log_a x = \begin{cases} +\infty & a > 1 \\ -\infty & 0 < a < 1 \end{cases}$$

- $$\lim_{x \to 0^+} \log_a x = \begin{cases} -\infty & a > 1 \\ +\infty & 0 < a < 1 \end{cases}$$

Inoltre si può dimostrare che

- $\lim_{x \to x_0} \sin x = \sin x_0 \quad \forall x_0 \in \mathbb{R}$

- $\lim_{x \to x_0} \cos x = \cos x_0 \quad \forall x_0 \in \mathbb{R}$

- $\lim_{x \to +\infty \, (-\infty)} \sin x \quad \nexists$

  $\lim_{x \to +\infty \, (-\infty)} \cos x \quad \nexists$

- $\lim_{x \to x_0} tg\, x = tg\, x_0 \quad \forall x_0 \in \mathbb{R} \setminus \{\frac{\pi}{2} + k\pi, \, k \in \mathbb{Z}\}$

- $\lim_{x \to +\infty \, (-\infty)} tg\, x \quad \nexists$

- $\lim_{x \to (\frac{\pi}{2} + k\pi)^{\pm}} tg\, x = \mp \infty$

dom $tg = \mathbb{R} \setminus \{\frac{\pi}{2} + k\pi\}$

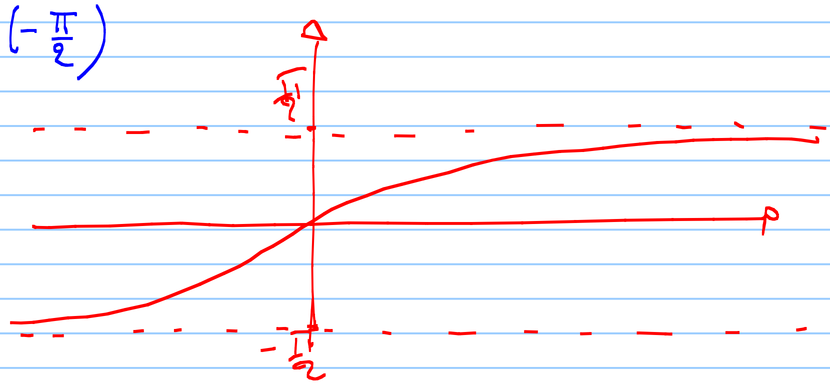
$\frac{\pi}{2} + k\pi$ è di acc per dom tg

- $\lim_{x \to x_0} \arcsin x = \arcsin x_0 \quad \forall x_0 \in [-1, 1]$

- $\lim_{x \to x_0} \arccos x = \arccos x_0 \quad \forall x_0 \in [-1, 1]$

- $\lim_{x \to x_0} \arctan x = \arctan x_0 \quad \forall x_0 \in \mathbb{R}$

- $\lim_{x \to +\infty \, (-\infty)} \arctan x = \frac{\pi}{2} \; \left(-\frac{\pi}{2}\right)$



Teorema (limite della funzione composta)

$f: X \subseteq \mathbb{R} \to \mathbb{R}$ $\qquad g: Y \subseteq \mathbb{R} \to \mathbb{R}$, $f(X) \subseteq Y$,

x_0 acc. di X

Se $\lim_{x \to x_0} f(x) = \ell$ e $f(x) \neq \ell$ in un intorno di x_0

ℓ acc. per Y

$\lim_{y \to \ell} g(y) = k$

Allora $\lim_{x \to x_0} (g \circ f)(x) = \lim_{x \to x_0} g(f(x)) = \lim_{y \to \ell} g(y) = k$

Esempi 1) $\lim_{x \to 0} \cos\left(\frac{x^2 - 4x^5}{x - 1}\right)$

$f(x) = \frac{x^2 - 4x^5}{x - 1}$ $\qquad g(y) = \cos y$

$(g \circ f)(x)$

$\lim_{x \to 0} f(x) = \lim_{x \to 0} \frac{x^2 - 4x^5}{x - 1} = \lim_{x \to 0} \frac{x - 4x^4}{1 - \frac{1}{x}} = \boxed{0}$

$\lim_{x \to 0} \cos\Big(\underbrace{\frac{x^2 - 4x^5}{x - 1}}_{y \to \boxed{0}}\Big) = \lim_{y \to 0} \cos y = 1$

2) $\lim_{x \to \frac{\pi}{2}} \log_4 \frac{1}{1 - \sin^4 x}$

$\sin x \to 1$

$t = (\sin x)^4 = (y)^4 \xrightarrow[y \to 1]{} 1$ $\qquad y = \sin x$

$v = 1 - (\sin x)^4 = \underbrace{1 - t}_{\text{se } t \to 1} \to 0$ $\qquad t = y^4 = (\sin x)^4$

$$u = \frac{1}{1-t} = \frac{1}{\underbrace{1-(\sin x)^4}_{v \to 0,\ v > 0}} \to +\infty \qquad 1-(\sin x)^4 > 0$$

in un intorno di $\frac{\pi}{2}$!

$$\lim_{x \to \frac{\pi}{2}} \log_4 \underbrace{\frac{1}{1-(\sin x)^4}}_{+\infty} = \lim_{z \to +\infty} \log_4 z = +\infty$$

Definizione 1) $f: X \subset \mathbb{R} \to \mathbb{R}$ $\quad x_0$ di acc. per X

Diremo che f è un infinitesimo in x_0 se

$$\lim_{x \to x_0} f(x) = 0$$

2) Se $f, g: X \subset \mathbb{R} \to \mathbb{R}$ $\quad x_0$ di acc per X

f, g infinitesime in x_0. Diremo che

f è un infinitesimo di ordine

superiore a g se

$$\lim_{x \to x_0} \frac{f(x)}{g(x)} = 0 \qquad \text{e} \qquad f(x) = o\big(g(x)\big)$$

"o piccolo" di Landau

3) f, g

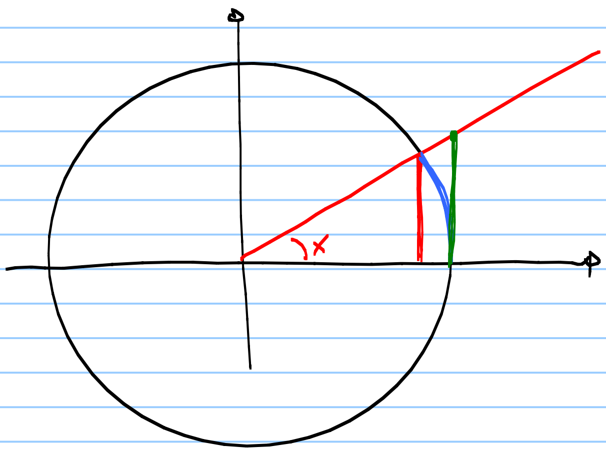
infinitesime in x_0. Diremo che

f e g sono infinitesime dello stesso

ordine $\quad \lim_{x \to x_0} \left| \frac{f(x)}{g(x)} \right| = \ell \in \mathbb{R} \qquad \underline{\ell > 0}$

Primi Limiti notevoli

- $\lim_{x \to 0} \frac{\sin x}{x} = 1$
- $\lim_{x \to 0} \frac{1-\cos x}{x^2} = \frac{1}{2}$ $\qquad \left(\begin{array}{ll} 1-\cos x \simeq \frac{1}{2}x^2 & \text{vicino a } 0 \\ \cos x \simeq 1 - \frac{1}{2}x^2 & \text{"} \end{array} \right)$
- $\lim_{x \to 0} \frac{\operatorname{tg} x}{x} =$
- $\lim_{x \to 0} \frac{\arcsin x}{x} =$
- $\lim_{x \to 0} \frac{\operatorname{arctan} x}{x} =$



$x > 0$

$\sin x < x < \operatorname{tg} x$

$1 < \frac{x}{\sin x} < \frac{1}{\cos x}$

$\underbrace{\cos x}_{f(x)} < \frac{\sin x}{x} < \underbrace{1}_{h(x)}$

$\lim_{x \to 0} f(x) = \lim_{x \to 0} h = 1$

$\Rightarrow \boxed{\lim_{x \to 0^+} \frac{\sin x}{x} = 1}$

$x < 0 \qquad -x > 0 \qquad \frac{\sin x}{x} = \frac{\sin(-(-x))}{-(-x)} =$

$= \frac{\cancel{+}\sin(-x)}{\cancel{+}(-x)} = \frac{\sin(-x)}{(-x)} \qquad y = (-x)$

$\lim_{x \to 0^-} \frac{\sin x}{x} = \lim_{x \to 0^-} \frac{\sin(-x)}{(-x)} =$

$-x = y \qquad x \to 0^- \implies y \to 0^+$

$$\Rightarrow \lim_{y \to 0^+} \frac{\sin y}{y} = 1$$

$$\lim_{x \to 0} \frac{1-\cos x}{x^2}$$

$\cos x = \cos^2 \frac{x}{2} - \sin^2 \frac{x}{2}$

$= 1 - 2 \sin^2 \frac{x}{2}$

$$= \lim_{x \to 0} \frac{2 \sin^2\left(\frac{x}{2}\right)}{x^2} =$$

$$= \lim_{x \to 0} \frac{2 \sin^2\left(\frac{x}{2}\right)}{4\left(\frac{x}{2}\right)^2} = \lim_{x \to 0} \frac{1}{2} \left[\frac{\sin\left(\frac{x}{2}\right)}{\frac{x}{2}}\right]^2 =$$

$$= \frac{1}{2} \lim_{x \to 0} \left[\frac{\sin\left(\frac{x}{2}\right)}{\frac{x}{2}}\right]^2 =$$

$\frac{x}{2} = t \qquad$ se $x \to 0 \implies t \to 0$

$$= \frac{1}{2} \lim_{t \to 0} \left[\frac{\sin t}{t}\right]^2 = \frac{1}{2}$$

$$\lim_{x \to 0} \frac{\operatorname{tg} x}{x} = \lim_{x \to 0} \underbrace{\frac{1}{\cos x}}_{\downarrow \atop 1} \cdot \underbrace{\frac{\sin x}{x}}_{\downarrow \atop 1} = 1$$

$$\lim_{x \to 0} \frac{\arcsin x}{x} =$$

$y = \sin x$

$y \to 0$ se $x \to 0$

$$= \lim_{y \to 0} \frac{y}{\sin y} = 1$$

$$\lim_{x \to 0} \frac{\operatorname{arctg} x}{x} =$$

$y = \operatorname{tg} x$

se $x \to 0 \Rightarrow y \to 0$

$$= \lim_{y \to 0} \frac{y}{\operatorname{tg} y} = 1$$

Esercizi

1) $$\lim_{x \to 0} \frac{\sin(x^2)}{\sqrt{3x^4 + x^5 \cos x}} = \lim_{x \to 0} \underbrace{\boxed{\frac{\sin(x^2)}{x^2}}}_{\to 1} \cdot \frac{x^2}{\sqrt{3x^4 + x^5 \cos x}} =$$

$$= \lim_{x \to 0} \frac{x^2}{\sqrt{3x^4 + x^5 \cos x}} =$$

$$= \lim_{x \to 0} \frac{\cancel{x^2}}{\cancel{x^2}\sqrt{3 + x \cos x}} = \frac{1}{\sqrt{3}}$$

2) $$\lim_{x \to +\infty} x\left(\frac{\pi}{2} - \arccos \underbrace{\frac{1}{x}}_{>0}\right) =$$

facciamo la sostituzione $t = \frac{1}{x}$, osservando che

se $x \to +\infty \Rightarrow t \to 0$ (e $t > 0$)

$$= \lim_{t \to 0^+} \frac{\left(\frac{\pi}{2} - \arccos t\right)}{t}$$

con la sostituzione

$\arccos t = y$ $\quad$ ($t = \cos y$)

se $t \to 0^+ \Rightarrow y \to \frac{\pi}{2}$

$$= \lim_{y \to \frac{\pi}{2}} \frac{\frac{\pi}{2} - y}{\cos y} = \lim_{y \to \frac{\pi}{2}} \frac{\frac{\pi}{2} - y}{\sin\left(\frac{\pi}{2} - y\right)}$$

e con la sostituzione

$\frac{\pi}{2} - y = u$

si ha che se $y \to \frac{\pi}{2} \Rightarrow u \to 0$

$$= \lim_{u \to 0} \frac{u}{\sin u} = 1$$