2) $f(x) = \log\left(\frac{x+4}{(x+1)^2}\right)$ (13/12/12)

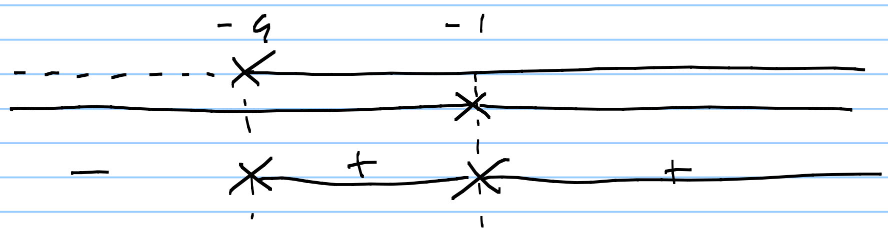
1) dom f

$\rightarrow \frac{x+4}{(x+1)^2} > 0$

$\rightarrow x+1 \neq 0$

$N \geq 0 \qquad x+4 > 0 \qquad x > -4$

$D > 0 \qquad (x+1)^2 > 0 \qquad x \neq -1$



$\text{dom} f = \left\{ x \in \mathbb{R} : \frac{x+4}{(x+1)^2} > 0 \text{ e } x \neq -1 \right\} =$

$= \{ x \in \mathbb{R} : x > -4 , x \neq -1 \} =$

$= (-4, -1) \cup (-1, +\infty)$

2) $A = \{$ pti di accumulazione del dom $f\}$

$\forall x_0 \in A, \ \lim_{x \to x_0} f(x)$

$A = [-4, -1] \cup [-1, +\infty) \cup \{+\infty\}$

$= [-4, +\infty) \cup \{+\infty\}$

$\lim_{x \to x_0} f(x) = ? \qquad \forall x_0 \in A$

$x_0 \in A \cap (\operatorname{dom} f) = (-4,-1) \cup (-1,+\infty)$

$\underbrace{x_0 \in A \setminus (\operatorname{dom} f)} = \underbrace{\{-4\} \cup \{-1\} \cup \{+\infty\}}$

$\lim_{x \to x_0} f(x) = f(x_0) \quad \forall x_0 \in \operatorname{dom} f (\cap A)$

$$\lim_{x \to -4^+} \log\left(\frac{x+4}{(x+1)^2}\right) = -\infty$$

$$\lim_{x \to -1} \log\underbrace{\left(\frac{x+4}{(x+1)^2}\right)}_{\downarrow \, +\infty} = +\infty$$

$$\lim_{x \to +\infty} \log\left(\frac{x+4}{\underbrace{(x+1)^2}}\right) = -\infty$$

$\to 0$

3) determinare eventuali asintoti

verticali: $x=-4$, $x=-1$ sono asintoti verticali

orizzontali: non ce ne sono

obliqui: $\lim_{x \to +\infty} \dfrac{\log\left(\frac{x+4}{(x+1)^2}\right)}{x} =$

$$= \lim_{x \to +\infty} \underbrace{\frac{1}{x}}_{\downarrow \, 0} \underbrace{\log\left(\underbrace{\frac{x+4}{(x+1)^2}}_{\downarrow \, 0}\right)}_{\to -\infty} =$$

$$= \lim_{x \to 0^+} \underbrace{\frac{(x+1)^2}{x(x+4)}}_{1} \; \frac{x+4}{(x+1)^2} \log\left(\frac{x+4}{(x+1)^2}\right) = 0$$

$\frac{x+4}{(x+1)^2} = t \; ; \; t \to 0 \;$ se $x \to +\infty$

$$\lim_{t \to 0} t \log t = 0$$

non ci sono asintoti obliqui!

4) Determinare l'insieme B dei punti nei quali f è derivabile e calcolare $f'(x)$, $\forall x \in B$.

Per il teorema sull'algebra delle derivate e il teorema sulla composizione di funzioni derivabili

f è derivabile in x, $\forall x \in \operatorname{dom} f$

$\operatorname{dom} f = B = (-4,-1) \cup (-1,+\infty)$

$$f'(x) = \frac{\cancel{(x+1)^{2}}}{x+4} \cdot \frac{(x+1)^{\cancel{2}} - 2(x+4)\cancel{(x+1)}}{(x+1)^{\cancel{4}\,\cancel{3}}} =$$

$$= \frac{(x+1) - 2x - 8}{(x+4)(x+1)}$$

$$= \frac{-x-7}{(x+4)(x+1)}$$

Calcolare $\displaystyle\lim_{x \to -1^{\pm}} f'(x) = \lim_{x \to -1^{\pm}} \frac{\overbrace{-x-7}^{<0,\ <0}}{\underbrace{(x+4)}_{>0,\ >0}\underbrace{(x+1)}_{>0,\ <0}} = \mp\infty$

5) Determinare gli intervalli di monotonia.

$$f'(x) = -\frac{(x+7)}{(x+4)(x+1)} > 0$$

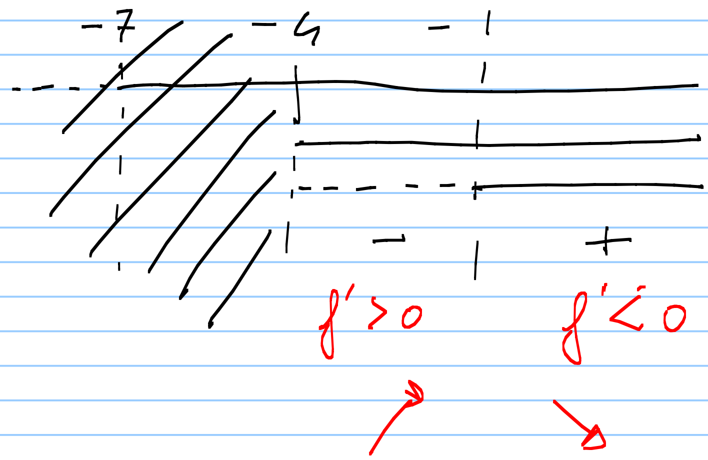
$\Updownarrow$

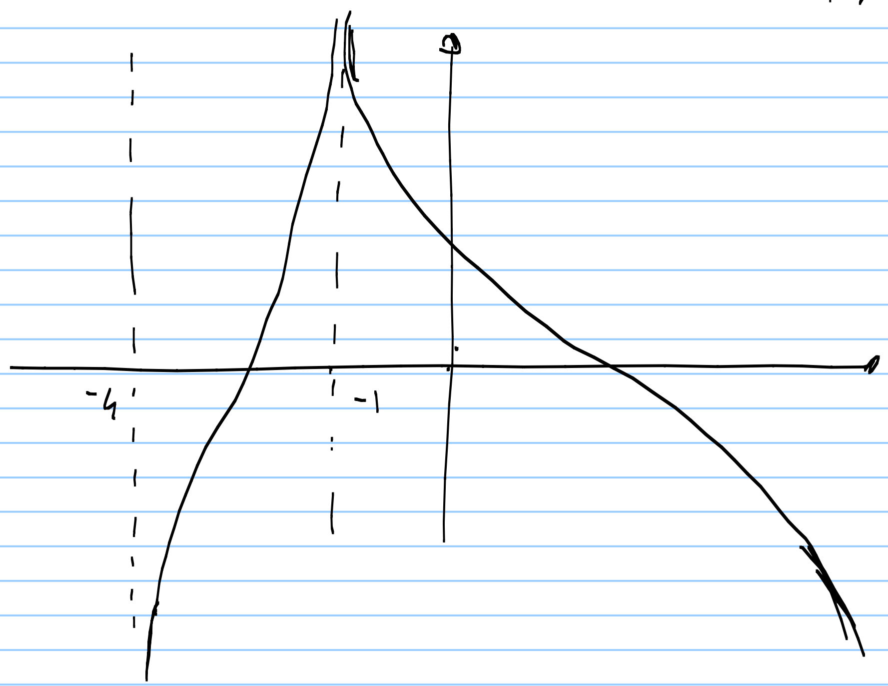
$$\frac{(x+7)}{(x+4)(x+1)} < 0$$

$(x+7) > 0 \iff x > -7$

$(x+4) > 0 \iff x > -4$

$(x+1) > 0 \iff x > -1$





f è strettamente crescente $(-4, -1)$

f è strettamente decrescente $(-1, +\infty)$

6) Determinare eventuali punti di max/min rel e/o assoluto.

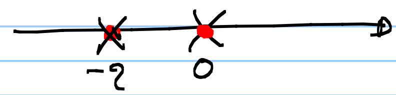
non ci sono max / min rel né assoluti.

3) $f(x) = \dfrac{(x+2)}{x}\, e^{-\frac{1}{(x+2)}}$ (19/12/11)

1) $\operatorname{dom} f = \{x \in \mathbb{R},\ x \neq 0,\ x+2 \neq 0\}$

$= \{x \in \mathbb{R} : x \neq 0,\ x \neq -2\}$

$= (-\infty, -2) \cup (-2, 0) \cup (0, +\infty)$



2) $A = \{\text{pti di acc. di } \operatorname{dom} f\} =$

$= \{-\infty\} \cup (-\infty, -2] \cup [-2, 0] \cup [0, +\infty) \cup \{+\infty\}$

$= \mathbb{R} \cup \{-\infty\} \cup \{+\infty\}$

$\lim_{x \to x_0} f(x) \qquad \forall x_0 \in A$

•) $x_0 \in A \cap (\operatorname{dom} f) = \operatorname{dom} f = (-\infty, -2) \cup (-2, 0) \cup (0, +\infty)$

f è continua in $x_0 \Rightarrow \lim_{x \to x_0} f(x) = f(x_0)$

•) $x_0 \in A \setminus (\operatorname{dom} f) = \{+\infty\} \cup \{-\infty\} \cup \{-2\} \cup \{0\}$

$$\lim_{\substack{x \to +\infty \\ -\infty}} \underbrace{\frac{(x+2)}{x}}_{\to 1} \underbrace{e^{\overbrace{-\frac{1}{(x+2)}}^{\to 0}}}_{\to 1} = 1$$

$$\lim_{x \to -2^+} \underbrace{\frac{(x+2)}{x}}_{\to 0} \underbrace{e^{-\frac{1}{(x+2)}}}_{\to 0} = 0$$

$$\lim_{x \to -2^-} \underbrace{\frac{(x+2)}{x}}_{\to 0} \cdot \underbrace{e^{\overbrace{-\frac{1}{(x+2)}}^{t}}}_{\to +\infty} =$$

$t \to +\infty$

$$\lim_{t \to +\infty} \frac{e^t}{t} = +\infty$$

$$\frac{\cancel{x+2}}{x}\left(-\frac{1}{\cancel{(x+2)}}\right)\frac{e^{-\frac{1}{(x+2)}}}{-\frac{1}{(x+2)}} = -\underbrace{\boxed{\frac{1}{x}}}_{\to\frac{1}{2}}\ \underbrace{\boxed{\frac{e^{-\frac{1}{(x+2)}}}{-\frac{1}{(x+2)}}}}_{\to+\infty}$$

$$= +\infty$$

$$\lim_{x\to 0^{\pm}} \overbrace{\boxed{\frac{x+2}{x}}}^{\to 0}\ \overbrace{\boxed{e^{-\frac{1}{x+2}}}}^{\to 0} = \begin{matrix} +\infty \\ -\infty \end{matrix}$$

3) Asintoti

orizzontali: $y=1$ sia a $+\infty$ che a $-\infty$

verticali: $x=0$, $x=-2$

obliqui non ce ne sono (ci sono gli asintoti orizzontali)

4) Determinare $B = \{$pti dove f è derivabile$\}$ e calcolare $f'(x)$, $\forall x \in B$.

$$f(x) = \frac{x+2}{x}\, e^{-\frac{1}{x+2}}$$

per il teo sull'algebra delle derivate e sulla composizione di funzioni derivabili $B = \operatorname{dom} f$ e

$$f'(x) = \left[\frac{\cancel{x} - (\cancel{x}+2)}{x^2}\right] e^{-\frac{1}{x+2}}$$

$$+ \left[\frac{\cancel{x+2}}{x}\right] e^{-\frac{1}{x+2}} \left(\frac{1}{(x+2)^{\cancel{2}}}\right)$$

$$= e^{-\frac{1}{x+2}} \left[-\frac{2}{x^2} + \frac{1}{x(x+2)} \right]$$

$$= e^{-\frac{1}{x+2}} \frac{-2x-4+x}{x^2(x+2)} =$$

$$= - e^{-\frac{1}{x+2}} \frac{4+x}{x^2(x+2)}$$

5) Intervalli di monotonia

$$f'(x) = - e^{-\frac{1}{x+2}} \frac{4+x}{x^2(x+2)} > 0$$

$$\Updownarrow$$

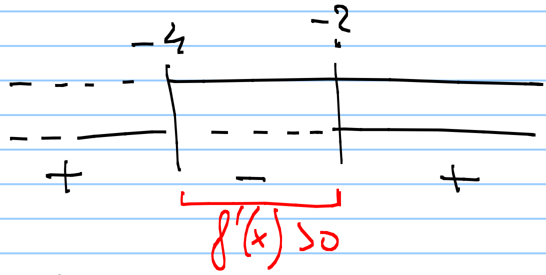
$$\underbrace{e^{-\frac{1}{x+2}}}_{>0} \frac{4+x}{\underbrace{x^2}_{>0}(x+2)} < 0$$

$$\Updownarrow$$

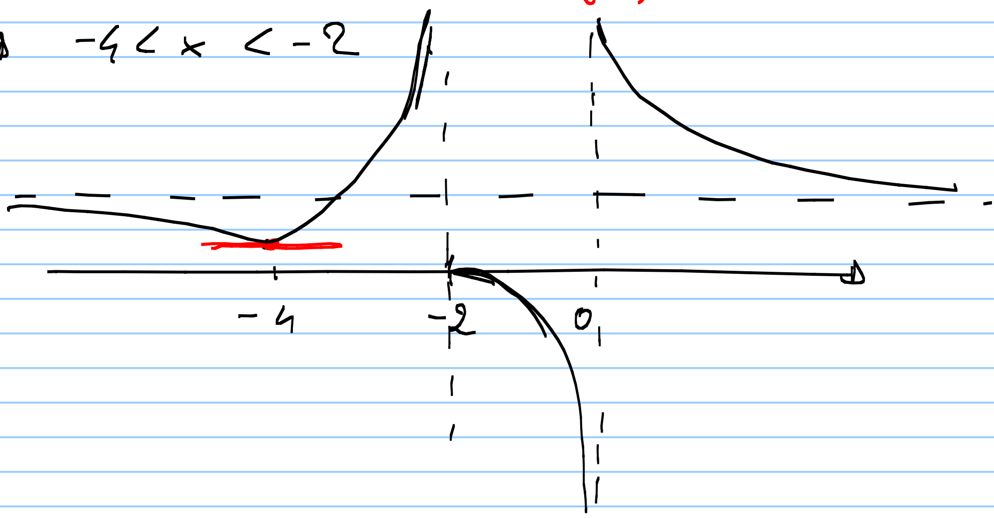
$$\frac{4+x}{(x+2)} < 0$$

$(4+x) > 0 \iff x > -4$

$(x+2) > 0 \iff x > -2$



$$f'(x) > 0 \iff -4 < x < -2$$



f è decrescente in $(-\infty, -4)$
in $(-2, 0)$
in $(0, +\infty)$

f è crescente in $(-4, -2)$

6) max / min.
$f'(x) = 0 \Rightarrow x = -4$

$x = -4$ è di min relativo
non assoluto $\left(\lim_{x \to 0^-} f(x) = -\infty \right)$

$$\lim_{x \to -2^+} f'(x) = \lim_{x \to -2^+} - \underbrace{e^{-\frac{1}{x+2}}}_{\substack{\downarrow \\ 0}} \underbrace{\frac{4+x}{x^2(x+2)}}_{\substack{\downarrow \\ +\infty}} =$$

$$= \lim_{x \to -2^+} - \frac{\frac{4+x}{x^2(x+2)}}{e^{\frac{1}{x+2}}} =$$

$$= \lim_{x \to -2^+} - \underbrace{\frac{4+x}{x^2}}_{\substack{\downarrow \\ \frac{1}{2}}} \frac{\frac{1}{(x+2)}}{e^{\frac{1}{(x+2)}}} = 0$$

$t = \frac{1}{x+2}$ $\quad x \to -2^-$ $\quad t \to +\infty$

$$\lim_{t \to +\infty} \frac{t}{e^t} = 0$$

Derivate successive (di ordine superiore)

$f: I \subseteq \mathbb{R} \longrightarrow \mathbb{R}$ derivabile in I

$f'(x) \quad \forall x \in I$

$f': I \subseteq \mathbb{R} \longrightarrow \mathbb{R}$ se è derivabile in I

$f''(x) \quad \forall x \in I$ $\quad \boxed{f''(x) = (f')'(x)}$ derivata seconda di f

$$f''(x) = \frac{d^2}{dx^2} f \Big|_x = \frac{d^2}{dx^2} f(x)$$

$f''(x): I \subseteq \mathbb{R} \longrightarrow \mathbb{R}$ se è derivabile in I

$\boxed{f'''(x) = (f'')'(x)}$ derivata terza di f

$$f'''(x) = \frac{d^3}{dx^3} f \Big|_x = \frac{d^3}{dx^3} f(x)$$

$\vdots$

$\boxed{f^{(n)}(x) = \left(f^{(n-1)}\right)'(x)}$ derivata n-esima di f

$$f^{(n)}(x) = \frac{d^n}{dx^n} f \Big|_x = \frac{d^n}{dx^n} f(x)$$

$\vdots$

Esempi: $\quad f(x) = x^n \qquad n \in \mathbb{N} \qquad (n \geq 1)$

$f'(x) = n\, x^{n-1}$ $\qquad$ se $n = 1$ $\quad f'(x) = 1$

$$f''(x) = \begin{cases} 0 & \text{se } n = 1 \\ n(n-1)\, x^{(n-2)} \end{cases}$$

se $n = 2$ $\quad f''(x) = 2$

se $n > 2$ $\quad f''(x) = n(n-1)\, x^{n-2}$

$$f'''(x) = n(n-1)(n-2)\, x^{n-3}$$

$\vdots$

se $n > k-1$

$$f^{(k)}(x) = n(n-1)(n-2)\dots(n-k+1)\, x^{n-k}$$

$$= \frac{n(n-1)(n-2)\dots(n-k+1)(n-k)(n-k-1)\dots 3\cdot 2\cdot 1}{(n-k)!}\, x^{n-k}$$

$$= \frac{n!}{(n-k)!}\, x^{n-k}$$

2) $f(x) = \sin x$

$f'(x) = \cos x$

$f''(x) = -\sin x \quad = f^{(6)}(x) = f^{(10)}(x)$

$f'''(x) = -\cos x \quad = f^{(7)}(x)$

$f^{IV}(x) = \sin x \quad = f^{(8)}(x)$

$f^{V}(x) = \cos x \quad = f^{(9)}(x)$

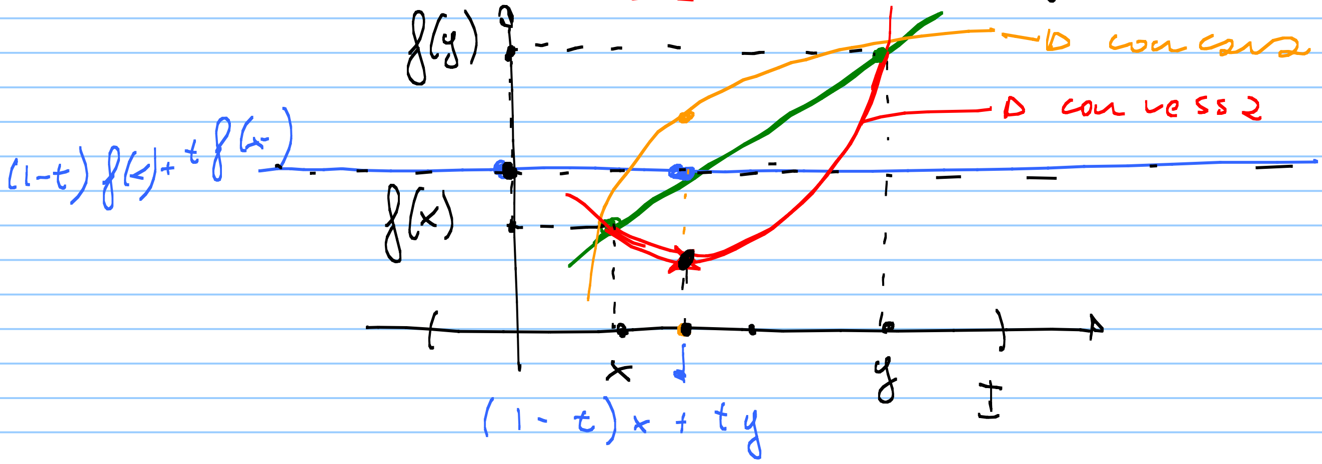
Funzioni convesse e funzioni concave

$f: I \subseteq \mathbb{R} \to \mathbb{R}$ diremo che f è

convessa (concava) se $\forall\, x, y \in I$ e $\forall\, t \in [0,1]$

si ha

$$f((1-t)x + ty) \le \boxed{(1-t) f(x) + t f(y)} \quad (\ge)$$



f si dice strettamente convessa (concava) se $\forall x, y \in I$
$\forall\, t \in (0,1)$ si ha

$$f((1-t)x + ty) < (1-t) f(x) + t f(y) \quad (>)$$

Teorema

$f : I \to \mathbb{R}$ $\quad I$ chiuso

se f è convessa (concava) $\Rightarrow$ f è continua in I

Teorema

$f : I \to \mathbb{R}$ convessa (concava)

$\forall x, y, z \quad x < y < z$

$$\frac{f(y) - f(x)}{y - x} \leq (\geq) \frac{f(z) - f(y)}{z - y}$$

Teorema $f : I \subseteq \mathbb{R} \to \mathbb{R}$ $\quad I$ chiuso

è convessa

Allora f è derivabile in $\mathring{I} \setminus (X)$, X insieme al più numerabile, e $f'(x)$ è monotona crescente (decrescente)

Teorema $f : I \subseteq \mathbb{R} \to \mathbb{R}$ aperto

$f', f'' : I \to \mathbb{R}$. Allora

f è convessa $\Leftrightarrow$ $f''(x) \geq 0 \quad \forall x \in I$

f è concava $\Leftrightarrow$ $f''(x) \leq 0 \quad \forall x \in I$

Se $f''(x) > 0 \quad \forall x \in I \Rightarrow f$ è strett. convessa

Se $f''(x) < 0 \quad \forall x \in I \Rightarrow f$ è strett. concava.

Studio di funzione

$$f(x) = \arcsin\left(x^2 - 4|x| + 3\right)$$

Osservazione

$\boxed{f \text{ è pari} \quad f(-x) = f(x)}$

1) $\text{dom} f = \{x \in \mathbb{R} : -1 \le x^2 - 4|x| + 3 \le 1\}$

$\forall x \in \mathbb{R}$

$x^2 - 4|x| + 3 \ge -1$

•) $x \ge 0$ $\quad x^2 - 4x + 3 \ge -1 \quad x^2 - 4x + 4 \ge 0$

$(x-2)^2 \ge 0 \quad \forall x \in \mathbb{R}$

•) $x < 0$ $\quad x^2 + 4x + 3 \ge -1 \quad x^2 + 4x + 4 \ge 0$

$(x+2)^2 \ge 0 \quad \forall x \in \mathbb{R}$

$x^2 - 4|x| + 3 \le 1$

•) $x \ge 0$ $\quad x^2 - 4x + 2 \le 0$

$x_{1,2} = 2 \pm \sqrt{2}$

$0 < 2 - \sqrt{2} \le x \le 2 + \sqrt{2}$

•) $x < 0$ $\quad x^2 + 4x + 2 \le 0$

$x_{1,2} = -2 \pm \sqrt{2}$

$-2 - \sqrt{2} \le x \le -2 + \sqrt{2} < 0$

$$\text{dom} f = [-2-\sqrt{2}, -2+\sqrt{2}] \cup [2-\sqrt{2}, 2+\sqrt{2}]$$

2) $A = \{\text{punti di acc. di dom} f\} = \text{dom} f$

$\lim_{x \to x_0} f(x) = f(x_0) \quad \forall x_0 \in A$

perché f è continua in dom f

$$f(x) = \arcsin\left(x^2 - 4|x| + 3\right)$$

3) no asintoti

4) $B = \{ x \in \mathrm{dom}\, f : f$ è derivabile in $x \}$

$|x|$ è derivabile se $x \neq 0$

non è derivabile se $x = 0$

$\arcsin(x) : [-1, 1] \to \mathbb{R}$ continua

è derivabile in $(-1, 1)$

- se $x = 0$ $(|x| !!)$??
- se $x^2 - 4|x| + 3 = \pm 1$??

se $x \neq 0$ e $x^2 - 4|x| + 3 \neq \pm 1$ possiamo applicare i teoremi sull'algebra delle derivate e sulla composizione di funzioni derivabile

$$f'(x) = \begin{cases} \dfrac{1}{\sqrt{1 + (x^2 - 4|x| + 3)^2}} (2x - 4) & \text{se } x > 0 \text{ e } x^2 - 4|x| + 3 \neq \pm 1 \\[2ex] \dfrac{1}{\sqrt{1 + (x^2 - 4|x| + 3)^2}} (2x + 4) & \text{se } x < 0 \text{ e } x^2 - 4|x| + 3 \neq \pm 1 \end{cases}$$

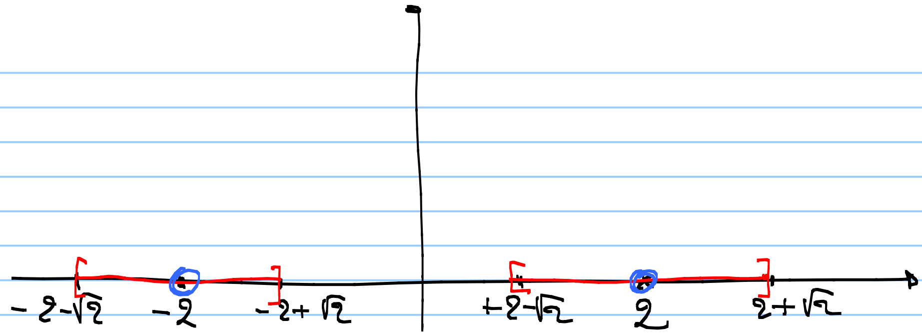
$x = 0$, $\underline{x^2 - 4|x| + 3 = \pm 1}$

$\notin \mathrm{dom}\, f$!

$x^2 - 4|x| + 3 = -1 \iff x = 2$ o $x = -2$

$x^2 - 4|x| + 3 = 1 \iff x = -2 - \sqrt{2}$ o $x = -2 + \sqrt{2}$

o $x = 2 - \sqrt{2}$ o $x = 2 + \sqrt{2}$



$x=2$ $$\lim_{x \to 2} \frac{\arcsin(x^2-4x+3)+\frac{\pi}{2}}{(x-2)}$$

$$x^2-4x+3 = \sin t$$
$$(x-2)^2 = \sin t + 1$$
$$(x-2) = \pm\sqrt{\sin t + 1}$$
$$x = 2 \pm \sqrt{\sin t + 1}$$

$$\lim_{x \to 2^{\pm}} \frac{\arcsin(x^2-4x+3)+\frac{\pi}{2}}{(x-2)} =$$ $x = 2-\sqrt{\sin t + 1}$

$$= \lim_{t \to -\frac{\pi}{2}^+} \frac{\arcsin(\sin t)+\frac{\pi}{2}}{-\sqrt{\sin t + 1}} =$$

$$= \lim_{t \to -\frac{\pi}{2}^+} - \frac{t+\frac{\pi}{2}}{\sqrt{\sin t + 1}} =$$

$t+\frac{\pi}{2} = u \qquad u \to 0$

$t = u-\frac{\pi}{2} = -\left(\frac{\pi}{2}-u\right)$

$$= -\lim_{u \to 0^+} \frac{u}{\sqrt{\sin\left(-\left(\frac{\pi}{2}-u\right)\right)+1}} =$$

$$= \lim_{u \to 0^+} \frac{u}{\sqrt{-\sin\left(\frac{\pi}{2}-u\right)+1}} =$$

$$= \lim_{u \to 0^+} \frac{u}{\sqrt{1-\cos u}} = \lim_{u \to 0^+} \sqrt{\frac{u^2}{1-\cos u}} = \pm\sqrt{2}$$

f non è derivabile in $x=2$. (e nemmeno in $x=-2$)

$f(x) = \arcsin\left(x^2 - 4|x| + 3\right) \qquad (x > 0 \,!!)$

$$f'(x) = \frac{1}{\sqrt{1-(x^2-4x+3)^2}} \cdot (2x-4) \qquad \left(\forall x \in \left(2-\sqrt{2},\, 2+\sqrt{2}\right),\ x \neq 2\right)$$

$f'(x) > 0 \iff x > 2$

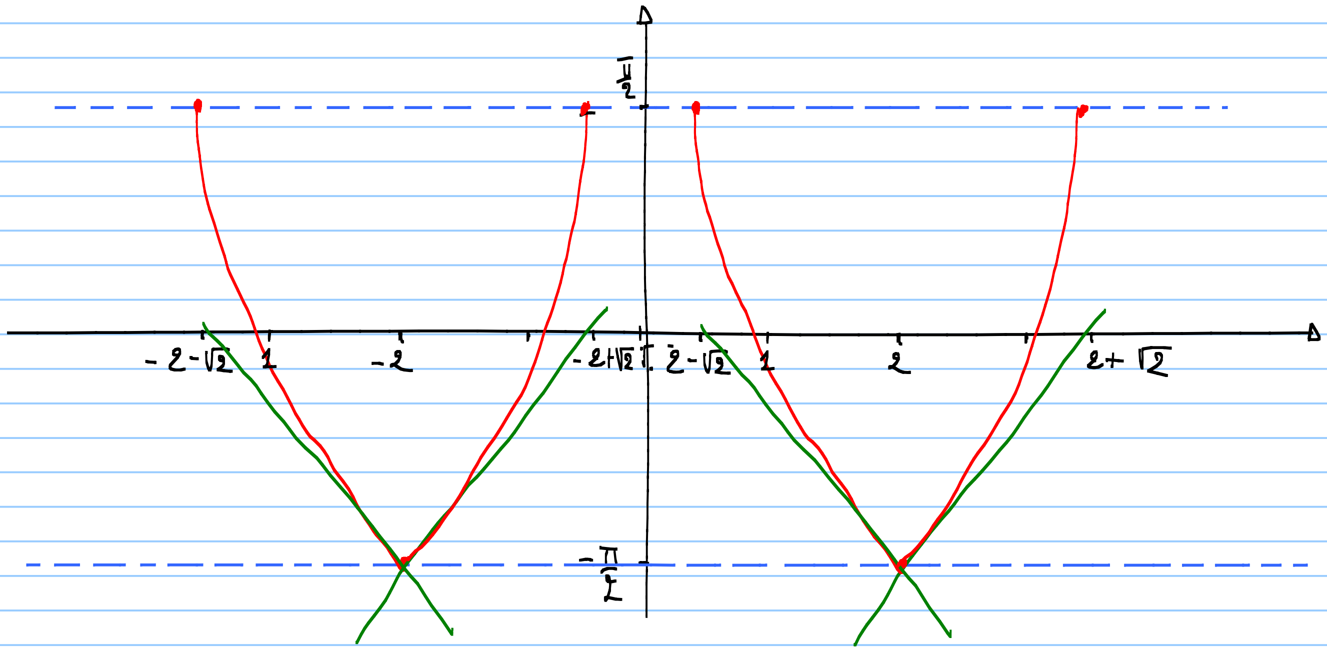
f è monotona strettamente crescente in $[2, 2+\sqrt{2}]$

" " decrescente in $[2-\sqrt{2}, 2]$

$$f''(x) = \frac{1}{\cancel{2}} \frac{1}{\left[1-(x^2+4x+3)^2\right]^{\frac{3}{2}}} \left[\cancel{2}(x^2-4x+3)(2x-4)^2\right] +$$

$$+ \frac{2}{\sqrt{1-(x^2-4x+3)^2}}$$

$$= \frac{2(x-2)^4}{\left(1-(x^2-4x+3)^2\right)^{\frac{3}{2}}} > 0 \qquad \forall x \in \left(2-\sqrt{2},\, 2+\sqrt{2}\right),\ x \neq 2$$



Teorema (di de l'Hopital)

$f, g: I \subseteq \mathbb{R} \to \mathbb{R}$ I intervallo aperto $I = (a, b)$

derivabili in I

t.c. $\lim_{x \to a^+} f(x) = \lim_{x \to a^+} g(x) = 0$ $(+\infty)$ $(-\infty)$

Se $\exists \lim_{x \to a^+} \frac{f'(x)}{g'(x)} = \ell \ (\in \mathbb{R}^*)$

Allora $\lim_{x \to a^+} \frac{f(x)}{g(x)} = \ell$

Osservazione ($\lim_{x \to b^-}$ ....)

dim: $f, g : (a, b)$, $\lim_{x \to a^+} f(x) = \lim_{x \to a^+} g(x) = 0$

$\bar{f}, \bar{g} : [a, b) \to \mathbb{R}$

$$\bar{f}(x) = \begin{cases} f(x) & x \in (a, b) \\ 0 & x = a \end{cases}$$

$$\bar{g}(x) = \begin{cases} g(x) & x \in (a, b) \\ 0 & x = a \end{cases}$$

$\bar{f}, \bar{g}$ sono continue in $[a, b)$

$\bar{f}, \bar{g}$ sono derivabili in (a, b)

$$\lim_{x \to a^+} \frac{f(x)}{g(x)} = \lim_{x \to a^+} \frac{\bar{f}(x)}{\bar{g}(x)}$$

$$\frac{\bar{f}(x)}{\bar{g}(x)} = \frac{\bar{f}(x) - 0}{\bar{g}(x) - 0} = \frac{\bar{f}(x) - \bar{f}(a)}{\bar{g}(x) - \bar{g}(a)}$$

$[a, x] \subset [a, b)$

$\bar{f}, \bar{g}$ sono continue in $[a, x]$, derivabili in (a, x)

teo. di Cauchy:

$$\frac{\bar{f}(x) - \bar{f}(a)}{\bar{g}(x) - \bar{g}(a)} = \frac{\bar{f}'(c)}{\bar{g}'(c)}$$

$\boxed{c \in (a, x)}$

$a < c < x$

$\lim_{x \to a} c = a$

$$\lim_{x \to a^+} \frac{\bar{f}(x) - \bar{f}(a)}{\bar{g}(x) - \bar{g}(a)} =$$

$$\lim_{c \to a^+} \frac{\bar{f}'(c)}{\bar{g}'(c)} = \lim_{c \to a^+} \frac{f'(c)}{g'(c)}$$

Osservazione $\lim_{x \to 0} \frac{e^x - 1}{x} \overset{(H)}{=}$

$$\lim_{x \to 0} \frac{e^x}{1} = 1$$

$$\lim_{x \to 0} \frac{e^{\frac{x^3}{x^4 + x}} - \cos x}{(\sin x)(\operatorname{tg} x)} \overset{(H)}{=}$$

$$= \lim_{x \to 0} \frac{e^{\frac{x^3}{x^4+x}} \cdot \frac{3x^2(x^4+x) - x^3(4x^3+1)}{(x^4+x)^2} + \sin x}{(\cos x)(\operatorname{tg} x) + \sin x\left(\frac{1}{\cos^2 x}\right)}$$

Esempi

2) $$\lim_{x \to 0} x^{\frac{3}{2}} \log(\sin x) =$$

3) $$\lim_{x \to 0} \frac{e^x - \left(1 + x + \frac{1}{2} x^2\right)}{x^2}$$

4) $$f(x) = \begin{cases} \dfrac{e^x - 1}{x^a} & \text{se } x > 0 \\ 1 + bx + x^2 & \text{se } x \leq 0 \end{cases}$$

$a > 0, \quad b \in \mathbb{R}$

5) $$f(x) = \begin{cases} \dfrac{1 - \cos x^a}{x^2} & \text{se } x > 0 \\ x^2 + b & \text{se } x \leq 0 \end{cases}$$

$$\lim_{x \to 0} \frac{1 - \cos x^a}{x^{2a}} \cdot x^{2a-2} = \begin{cases} 0 & a > 1 \\ \frac{1}{2} & a = 1 \\ +\infty & a < 1 \end{cases}$$

$a = 1$ e $b = \frac{1}{2}$

$a > 1$ e $b = 0$

6) $$\lim_{x \to 0} x \sin\left(\frac{1}{x}\right) = \lim_{x \to 0} \frac{\sin \frac{1}{x}}{\frac{1}{x}} \overset{(H)}{=}$$

Esempi

1) $$\lim_{x \to 0} \frac{x - \sin x}{x^3} =$$

$$\frac{x}{x^3} - \frac{\sin x}{x^3} = \underbrace{\frac{1}{x^2}}_{\to +\infty} - \underbrace{\underbrace{\frac{1}{x^2}}_{\to +\infty} \underbrace{\frac{\sin x}{x}}_{\to 1}}_{+\infty}$$

$+\infty - \infty$ f. i.

mi fermo!

$$\lim_{x \to 0} \frac{x - \sin x}{x^3} \overset{(H)}{=} \lim_{x \to 0} \underbrace{\frac{1 - \cos x}{3x^2}}_{\to \frac{1}{2}} = \frac{1}{6}$$

2) $$\lim_{x \to 0^+} \underbrace{x^{\frac{3}{2}}}_{\to 0} \underbrace{\log(\sin x)}_{\to -\infty} = \lim_{x \to 0^+} \frac{\overbrace{\log(\sin x)}^{\to \infty}}{\underbrace{\frac{1}{x^{\frac{3}{2}}}}_{\to +\infty}}$$

$$\frac{1}{x^{\frac{3}{2}}} = x^{-\frac{3}{2}}$$

$$\overset{(H)}{=} \lim_{x \to 0^+} \frac{\frac{1}{\sin x} \cdot \cos x}{-\frac{3}{2} x^{-\frac{5}{2}}} =$$

$$= \lim_{x \to 0^+} -\frac{2}{3} x^{\frac{5}{2}} \frac{\cos x}{\sin x} =$$

$$= \lim_{x \to 0^+} -\frac{2}{3} \underbrace{x^{\frac{3}{2}}}_{\to 0} \cdot \underbrace{\frac{x}{\sin x}}_{\to 1} \cdot \underbrace{\cos x}_{\to 1} = 0$$