• $$\int_2^3 x^3 e^{x^2} dx = \frac{1}{2}\int_2^3 x^2 \left(2x e^{x^2}\right) dx =$$

$$= \frac{1}{2}\left(\left[e^{x^2} x^2\right]_2^3 - \int_2^3 e^{x^2} 2x \, dx \right) =$$

$$= \frac{1}{2}\left(e^9 \cdot 9 - e^4 \cdot 4 - \left[e^{x^2}\right]_2^3 \right) =$$

$$= \frac{9}{2} e^9 - 2e^4 - \frac{e^9}{2} + \frac{e^4}{2}$$

$$= 4e^9 - \frac{3}{2} e^4$$

• $$\int \lg\left(\sqrt{x+1} + \sqrt{x-1}\right) dx =$$

$$= x \lg\left(\sqrt{x+1} + \sqrt{x-1}\right) - \int x \frac{1}{\sqrt{x+1} + \sqrt{x-1}} \left(\frac{1}{2\sqrt{1+x}} + \frac{1}{2\sqrt{x-1}} \right) dx$$

$$= x \lg\left(\sqrt{x+1} + \sqrt{x-1}\right) - \int \frac{x}{2\sqrt{x^2-1}} dx$$

$$= x \lg\left(\sqrt{x+1} + \sqrt{x-1}\right) - \frac{1}{2}\sqrt{x^2-1} + c$$

$$\int_0^1 \frac{e^t}{\sqrt{1+e^{2t}}}\,dt \qquad e^t = x \qquad t = \log x$$

$$dt = \frac{1}{x}\,dx$$

$$t = 0 \Rightarrow x = 1$$

$$t = 1 \Rightarrow x = e$$

$$\int_1^e \frac{x}{\sqrt{1+x^2}}\frac{1}{x}\,dx = \int_1^e \frac{dx}{\sqrt{1+x^2}}$$

$$x = \operatorname{senh} z \qquad dx = \cosh z\,dz$$

$$\int \frac{dx}{\sqrt{1+x^2}} = \int \frac{\cosh z\,dz}{\sqrt{1+\operatorname{sen}^2 z}} = \int \frac{\cosh z}{\sqrt{\cosh^2 z}}\,dz =$$

$$= \int dz = z = \operatorname{settsenh} x$$

quindi

$$\int_0^1 \frac{e^t}{\sqrt{1+e^{2t}}}\,dt = \int_1^e \frac{dx}{\sqrt{1+x^2}} = \Big[\operatorname{settsenh} x\Big]_1^e =$$

$$= \operatorname{settsenh} e - \operatorname{settsenh} 1 =$$

$$= \log\left(e+\sqrt{1+e^2}\right) - \log\left(1+\sqrt{1+1}\right) =$$

$$= \log\left(e+\sqrt{1+e^2}\right) - \log\left(1+\sqrt{2}\right)$$

$$= \log\left(\frac{e+\sqrt{1+e^2}}{1+\sqrt{2}}\right)$$

- $\int x \lg^2 x \, dx = \frac{1}{2} x^2 \lg^2 x - \int \frac{1}{\cancel{2}} x^2 \cancel{2} \lg x \frac{1}{x} \, dx =$

$= \frac{1}{2} x^2 \lg^2 x - \int x \lg x \, dx =$

$= \frac{1}{2} x^2 \lg^2 x - \left(\frac{x^2}{2} \lg x - \int \frac{x^2}{2} \frac{1}{x} dx \right)$

$= \frac{1}{2} x^2 \lg^2 x - \frac{x^2}{2} \lg x + \frac{x^2}{4} + c$

- $\int x \operatorname{arctg} x \, dx = \frac{x^2}{2} \operatorname{arctg} x - \int \frac{x^2}{2} \frac{1}{1+x^2} dx =$

$= \frac{x^2}{2} \operatorname{arctg} x - \frac{1}{2} \int \frac{x^2+1-1}{x^2+1} dx$

$= \frac{x^2}{2} \operatorname{arctg} x - \frac{1}{2} \left(\int 1 \, dx - \int \frac{1}{1+x^2} dx \right)$

$= \frac{x^2}{2} \operatorname{arctg} x - \frac{1}{2} \left(x - \operatorname{arctg} x \right)$

$= \frac{x^2}{2} \operatorname{arctg} x + \frac{1}{2} \operatorname{arctg} x - \frac{1}{2} x + c$

$$\int_1^2 \frac{1}{x^3} \operatorname{arctg}\left(\frac{1}{x}\right) dx$$

$$\frac{1}{x} = t \qquad x = \frac{1}{t} \qquad dx = -\frac{1}{t^2} dt$$

$$x = 1 \rightarrow t = 1$$
$$x = 2 \rightarrow t = \frac{1}{2}$$

$$\int_1^{\frac{1}{2}} t^3 \operatorname{arctg} t \left(-\frac{1}{t^2}\right) dt = \int_{\frac{1}{2}}^1 t \operatorname{arctg} t \, dt$$

$$= \left[\frac{t^2}{2} \operatorname{arctg} t + \frac{1}{2} \operatorname{arctg} t - \frac{1}{2} t\right]_{\frac{1}{2}}^1$$

$$= \frac{\pi}{4} - \frac{1}{4} - \frac{5}{8} \operatorname{arctg} \frac{1}{2}$$

- $$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \operatorname{sen} 2x\, e^{\operatorname{sen} x}\, dx = 2\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \operatorname{sen} x \cos x\, e^{\operatorname{sen} x}\, dx$$

$$\operatorname{sen} x = t \qquad \cos x\, dx = dt \qquad \begin{array}{l} x = \frac{\pi}{4} \rightarrow t = \frac{\sqrt{2}}{2} \\ x = \frac{\pi}{2} \rightarrow t = 1 \end{array}$$

$$= 2\int_{\frac{\sqrt{2}}{2}}^{1} t e^{t} = 2\left(\left[e^{t} t\right]_{\frac{\sqrt{2}}{2}}^{1} - \int_{\frac{\sqrt{2}}{2}}^{1} e^{t}\, dt\right) =$$

$$= 2\left(e - \frac{\sqrt{2}}{2} e^{\frac{\sqrt{2}}{2}} - \left(e - e^{\frac{\sqrt{2}}{2}}\right)\right)$$

$$= 2 e^{\frac{1}{\sqrt{2}}}\left(1 - \frac{\sqrt{2}}{2}\right) = e^{\frac{1}{\sqrt{2}}}\left(2 - \sqrt{2}\right)$$

$$\int_0^1 \frac{\log(\operatorname{arctg} x+1)}{(1+x^2)(\operatorname{arctg} x+1)^2}\,dx$$

$t = \operatorname{arctg} x \qquad x = \operatorname{tg} t \qquad x=0 \Rightarrow t=0$

$\qquad\qquad\qquad\qquad\qquad\qquad x=1 \Rightarrow t=\frac{\pi}{4}$

$$dt = \frac{1}{1+x^2}\,dx$$

$$= \int_0^{\frac{\pi}{4}} \frac{\log(t+1)}{(t+1)^2}\,dt =$$

$$= \left[-\frac{\log(t+1)}{(t+1)}\right]_0^{\frac{\pi}{4}} - \int -\frac{1}{(t+1)}\frac{1}{(t+1)}\,dt =$$

$$= -\frac{\log\left(\frac{\pi}{4}+1\right)}{\frac{\pi}{4}+1} - \left[\frac{1}{t+1}\right]_0^{\frac{\pi}{4}} =$$

$$= -\frac{4}{\pi+4}\log\left(\frac{\pi+4}{4}\right) - \frac{1}{\frac{\pi}{4}+4} + 1$$

$$= -\frac{4}{\pi+4}\log\left(\frac{\pi+4}{4}\right) - \frac{4}{\pi+4} + 1$$

$$\int_{-4}^{4} \sqrt{x^2 - |2x| + 2x + 4}\, dx =$$

$$= \int_{-4}^{0} \sqrt{x^2 + 2x + 2x + 4}\, dx + \int_{0}^{4} \sqrt{x^2 - 2x + 2x + 4}\, dx =$$

$$= \int_{-4}^{0} \sqrt{(x^2 + 4x + 4)}\, dx + \int_{0}^{4} \sqrt{x^2 + 4}\, dx =$$

$$= \int_{-4}^{0} |x + 2|\, dx + \int_{0}^{4} \sqrt{x^2 + 4}\, dx = 1 + 2$$

1 $$\int_{-4}^{-2} (-x - 2)\, dx + \int_{-2}^{0} (x + 2)\, dx = \left[-\frac{x^2}{2} - 2x\right]_{-4}^{-2} + \left[\frac{x^2}{2} + 2x\right]_{-2}^{0} = 4$$

2 $$\int_{0}^{4} \sqrt{x^2 + 4}\, dx = 2\int_{0}^{4} \sqrt{\frac{x^2}{4} + 1}\, dx =$$

$\frac{x}{2} = \operatorname{sh} t \quad dx = 2 \operatorname{ch} t\, dt$

$x = 0 \to t = 0$

$x = 4 \to t = \operatorname{settsh} 2$

$$= 2\int_{0}^{\operatorname{settsh} 2} \sqrt{\operatorname{sh}^2 t + 1}\; 2 \operatorname{ch} t\, dt =$$

$$= 2\int_{0}^{\operatorname{settsh} 2} 2 \operatorname{ch}^2 t\, dt = 4\int_{0}^{\operatorname{settsh} 2} \frac{1 + \operatorname{ch} 2t}{2}\, dt = 4\left[\frac{1}{2} t + \frac{1}{4} \operatorname{sh} 2t\right]_{0}^{\operatorname{settsh} 2} =$$

$$= 4\left(\frac{1}{2} \operatorname{settsh} 2 + \frac{1}{4}\, 2 \cdot 2 \sqrt{1 + 4} - 0\right) = 2 \operatorname{settsh} 2 + 4\sqrt{5}$$

quindi

$$\int_{-4}^{4} \sqrt{x^2 - |2x| + 2x + 4}\, dx = 4 + 2 \operatorname{settsenh} 2 + 4\sqrt{5}$$

- $\displaystyle\int_0^{\frac{\pi}{4}} \frac{\cos x}{(\operatorname{sen} x)^\alpha}\, dx \qquad \alpha > 0 \qquad$ IMPROPRIO

$x \to 0^+ \qquad \dfrac{\cos x}{\operatorname{sen}^\alpha x} \sim \dfrac{1}{x^\alpha} \qquad$ conv. $0 < \alpha < 1$

in tal caso

$$\int_0^{\frac{\pi}{4}} \frac{\cos x}{(\sin x)^\alpha}\, dx = \begin{cases} \dfrac{1}{1-\alpha} \operatorname{sen}^{1-\alpha} \Big|_0^{\frac{\pi}{4}} & \alpha \neq 1 \\ \log(\sin x) \Big|_0^{\frac{\pi}{4}} & \alpha = 1 \end{cases}$$

$$= \begin{cases} \dfrac{1}{1-\alpha}\left[\left(\dfrac{\sqrt{2}}{2}\right)^{1-\alpha} - \lim\limits_{x \to 0^+} (\operatorname{sen} x)^{1-\alpha}\right] & \alpha \neq 1 \\ \log \dfrac{\sqrt{2}}{2} - \lim\limits_{x \to 0^+} \log \sin x & \alpha = 1 \end{cases}$$

$$\Rightarrow \begin{cases} \dfrac{1}{1-\alpha}\, 2^{\frac{\alpha-1}{2}} & 0 < \alpha < 1 \\ +\infty & \alpha \geq 1 \end{cases}$$

①

- $$\int_0^{+\infty} \frac{\log(1+x^2)}{x^\alpha(1+x^3)}\,dx \qquad \alpha\in\mathbb{R}$$

D: $x\neq 0$ $\cancel{x\neq -1}$

$x\to 0^+$ $\quad f(x)\sim \dfrac{\log(1+x^3)}{x^\alpha} \sim \dfrac{x^3}{x^\alpha} = x^{3-\alpha} = \dfrac{1}{x^{\alpha-3}}$

$\alpha-3<1 \qquad \underline{\alpha<3}$

$x\to+\infty$ $\quad f(x)\sim \dfrac{2\log x}{x^{3+\alpha}} = 2\dfrac{1}{x^{3+\alpha}\log^{-1}x} \qquad 3+\alpha>1 \quad \underline{\alpha>-2}$

$\Rightarrow \quad -2<\alpha<3$

- $$\int_0^1 \frac{|\log x|\,\mathrm{sen}\,x}{|x^2-1|^\alpha}\,dx \qquad \alpha\in\mathbb{R}$$

$x\to 0^+ \quad x\to 1^-$

$x\to 0^+$ $\quad f\sim |\log x|\,x \to 0 \Rightarrow$ integr. $\forall\alpha$

$x\to 1^-$ $\quad f\sim \mathrm{sen}\,1 \cdot \dfrac{|\log(1+(x-1))|}{|x-1|^\alpha|x+1|^\alpha} \sim \dfrac{|x-1|}{|x-1|^\alpha}\dfrac{\mathrm{sen}\,1}{2^\alpha}$

$= \dfrac{\mathrm{sen}\,1}{2^\alpha}\dfrac{1}{|x-1|^{\alpha-1}}$

$\displaystyle\int_0^1 \frac{1}{|x-1|^\alpha}\,dx \qquad x-1=t \qquad \begin{matrix} x=0 & t=-1 \\ x=1 & t=0 \end{matrix}$

$\displaystyle\int_{-1}^0 \frac{1}{|t|^\alpha}\,dx$ $\quad$ per $t\to 0^-$ conv. $\alpha-1<1 \to \alpha<2$

②

• $\displaystyle\int_0^1 \frac{\operatorname{arctg} x^{\frac{1}{2\alpha}}}{\sin\sqrt[3]{x} + x^2}\,dx \qquad \alpha \in \mathbb{R}\setminus\{0\}$

$x \neq 0 \qquad \sin\sqrt[3]{x} + x^2 \neq 0$ sempre verificato in $x \in]0,1]$

in $x = 1$ ok.

$x \to 0^+ \qquad$ per $\alpha > 0 \quad f \sim \dfrac{x^{\frac{1}{2\alpha}}}{\sqrt[3]{x} + x^2} \sim \dfrac{x^{\frac{1}{2\alpha}}}{x^{\frac{1}{3}}} = \dfrac{1}{x^{\frac{1}{3} - \frac{1}{2\alpha}}}$

$\Rightarrow$ int. $\dfrac{1}{3} - \dfrac{1}{2\alpha} < 1$

$\dfrac{2\alpha - 3}{6\alpha} < 1 \qquad 2\alpha - 3 < 6\alpha \qquad -4\alpha < 3 \qquad \alpha > -\dfrac{3}{4}$

$\begin{cases} \alpha > -\frac{3}{4} \\ \alpha > 0 \end{cases} \Rightarrow \alpha > 0$

per $\alpha < 0 \quad f(x) \sim \dfrac{\frac{\pi}{2}}{\sqrt[3]{x} + x^2} \sim \dfrac{\frac{\pi}{2}}{\sqrt[3]{x}} = \dfrac{\pi}{2x^{\frac{1}{3}}}$

$\dfrac{1}{3} < 1 \Rightarrow$ int. $\forall \alpha < 0$

$\Rightarrow$ f è integr. $\forall \alpha \in \mathbb{R}\setminus\{0\}$

- $$\int_0^{+\infty} \frac{x^{\frac{1}{4}}+2}{x^{\frac{1}{2}}(x^{\frac{1}{2}}+1)^2}\,dx$$

$x \to 0^+$ $\quad f(x) \sim \frac{2}{\sqrt{x}} = \frac{1}{x^{1/2}}$ infatti

$$\lim_{x\to 0^+} \frac{f(x)}{\frac{2}{\sqrt{x}}} = \lim_{x\to 0^+} \frac{x^{\frac{1}{4}}+2}{x^{\frac{1}{2}}(x^{\frac{1}{2}}+1)} \cdot \frac{x^{1/2}}{2} = 1$$

quindi f è integrabile in 0^+. $\left(\frac{1}{2} < 1\right)$

$x \to +\infty$

$$f(x) \sim \frac{x^{\frac{1}{4}}}{x^{1/2} \cdot x} = \frac{1}{x^{5/4}} \qquad 5/4 > 1 \quad \Rightarrow$$

f è integrabile a $+\infty$.

Quindi $$\int_0^{+\infty} f(x)\,dx < +\infty$$

- $\displaystyle\int_2^3 \frac{x \,\text{sen}(x-2)^\alpha}{\sqrt{x^2-4}}\,dx$

$x \to 2^+ \qquad f(x) \sim \dfrac{2(x-2)^\alpha}{(x-2)^{\frac{1}{2}}\,(2+2)^{\frac{1}{2}}} =$

$$= \frac{1}{(x-2)^{\frac{1}{2}-\alpha}}$$

f è integrabile se $\frac{1}{2}-\alpha < 1$ cioè

$\alpha > -\frac{1}{2}$.

- $\displaystyle\int_0^2 \frac{e^x-1}{(\text{senh}\,x)^\alpha\,|x-1|^{\frac{\alpha}{3}}}\,dx$

$x \to 0^+ \qquad \text{senh}\,x = x + o(x)$

$(\text{senh}\,x)^\alpha \sim x^\alpha$

$e^x - 1 \sim x$

$f(x) \sim \dfrac{x}{x^\alpha \cdot 1} = \dfrac{1}{x^{\alpha-1}}$ allora f è

integrabile in 0^+ se $\alpha - 1 < 1 \Rightarrow \alpha < 2$

$$x \to 1^{\pm} \quad f(x) = \frac{e^{x}-1}{(\operatorname{senh} x)^{\alpha}} \frac{1}{|x-1|^{\alpha/3}}$$

$$\sim \frac{e-1}{(\operatorname{senh} 1)^{\alpha}} \frac{1}{|x-1|^{\alpha/3}}$$

per $\frac{\alpha}{3} < 1 \iff \alpha < 3$ f è integrabile in $x = 1$.

Quindi $\int_0^2 f(x)\,dx < +\infty \iff \alpha < 2$.

- $\int_{-\infty}^{+\infty} \frac{e^{-\frac{1}{|x|}}}{x^2}\,dx$. $f(x)$ è pari $\Rightarrow$

$$= 2\int_0^{+\infty} e^{-\frac{1}{x}} \frac{1}{x^2}\,dx = 2\left[e^{-\frac{1}{x}} \right]_0^{+\infty}$$

$$= 2\left(e^{-\frac{1}{+\infty}} - e^{-\frac{1}{0^+}} \right) = 2\left(1 - e^{-\infty}\right) = 2.$$

- $\int_2^3 x^3 e^{x^2}\,dx = \frac{1}{2}\int_2^3 x^2 (2x e^{x^2})\,dx =$

$$= \frac{1}{2}\left(\left[e^{x^2} x^2\right]_2^3 - \int_2^3 e^{x^2} 2x\,dx \right) =$$

$$= \frac{1}{2}\left(e^9 \cdot 9 - e^4 \cdot 4 - \left[e^{x^2}\right]_2^3 \right) =$$

$$= \frac{9}{2} e^9 - 2e^4 - \frac{e^9}{2} + \frac{e^4}{2}$$

$$= 4e^9 - \frac{3}{2} e^4$$

- $\int \lg\left(\sqrt{x+1} + \sqrt{x-1}\right) dx =$

$$= x \lg\left(\sqrt{x+1} + \sqrt{x-1}\right) - \int x \frac{1}{\sqrt{x+1} + \sqrt{x-1}} \left(\frac{1}{2\sqrt{1+x}} + \frac{1}{2\sqrt{x-1}} \right) dx$$

$$= x \lg\left(\sqrt{x+1} + \sqrt{x-1}\right) - \int \frac{x}{2\sqrt{x^2-1}}\,dx$$

$$= x \lg\left(\sqrt{x+1} + \sqrt{x-1}\right) - \frac{1}{2}\sqrt{x^2-1} + c$$