• ~~Dato~~ l'insieme $S := \{x \in \mathbb{R} : x = \frac{1}{n}, n \in \mathbb{N}, n \geq 1\}$ verificare che $MAX(S) = 1 = sup(S)$ e $inf(S) = 0$.

– MAX (S): bisogna verificare due cose:

a) $1 \in S$

b) $\forall x \in S, x \leq 1$

a) basta porre $1 = \frac{1}{n}$ e risolvere. Subito si ha $n = 1$, quindi $1 \in S$.

b) basta porre $\frac{1}{n} \leq 1$ e risolvere: si ha allora $1 \leq n \iff n \geq 1$, ok!

Quindi 1 è max(S) e anche sup(S).

– INF(S): bisogna verificare due cose:

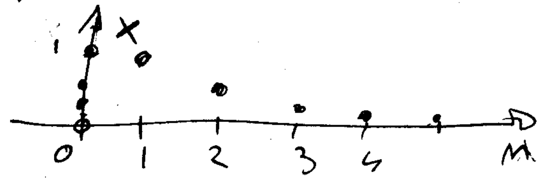
a) $0 \leq x \quad \forall x \in S$ (cioè che 0 è minorante)

b) $\forall \varepsilon > 0 \ \exists n : \frac{1}{n} < 0 + \varepsilon$

a) $0 \leq x \iff 0 \leq \frac{1}{n} \iff 0 \leq 1$ eq. vera $\forall n \in \mathbb{N}$ in particolare per $n \geq 1$

b) $\frac{1}{n} < \varepsilon \iff n > \frac{1}{\varepsilon}$ eq. vera per la proprietà archimedea di $\mathbb{R}$.

Quindi 0 è inf(S) ma non min(S) infatti ponendo $0 = \frac{1}{n}$ si ha $0 = 1$ che è impossibile.



• Dato l'insieme $S = \{ x \in \mathbb{R} : x = \frac{n-1}{n^2+1} , n \in \mathbb{N} \}$ trovare $\sup(S)$, $\min(S)$, $\inf(S)$, $\max(S)$ se esistono.

Calcoliamo i primi elementi di S:

$n = 0 \Rightarrow x = 1$

$n = 1 \Rightarrow x = 0$

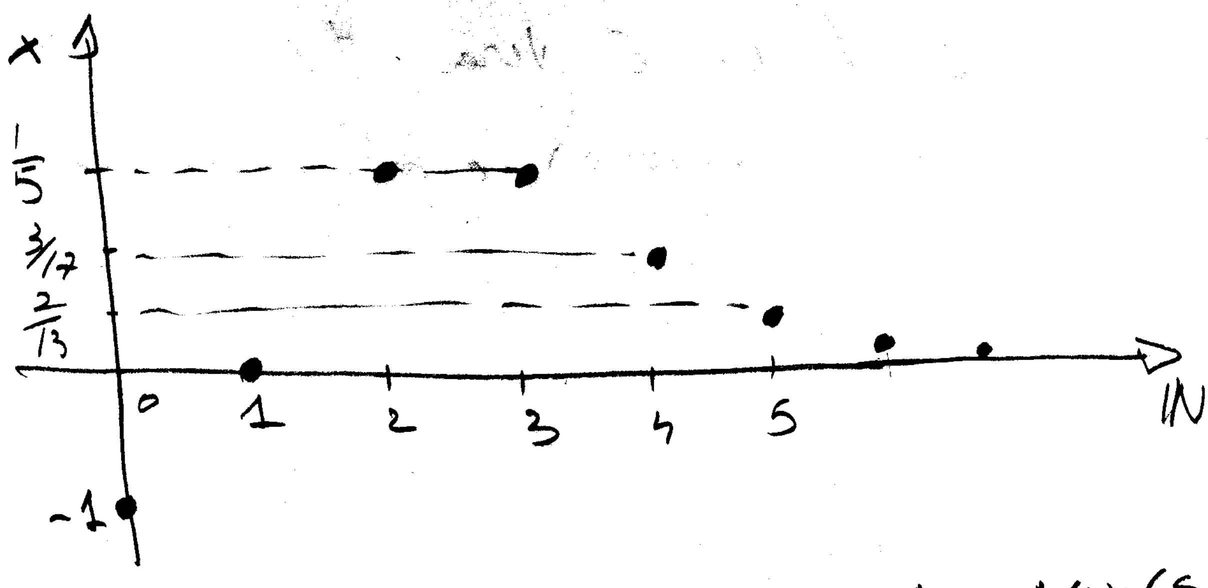
$n = 2 \Rightarrow x = \frac{1}{5}$

$n = 3 \Rightarrow x = \frac{1}{5}$

$n = 4 \Rightarrow x = \frac{3}{17}$

$n = 5 \Rightarrow x = \frac{2}{13}$

e abbozziamo un grafico



Ipotizziamo che $-1 = MIN(S)$ e $\frac{1}{5} = MAX(S)$

Verifichiamo che $MIN(S) = -1$:

a) $-1 = \frac{n-1}{n^2+1} \iff -n^2+1 = n-1 \iff -n^2-n=0$

$\iff n(n+1) = 0 \iff \underset{\text{OK}}{n=0} \lor \underset{\text{NON ACC.}}{n=-1}$

b) $\dfrac{n-1}{n^2+1} \geq -1 \iff n-1 \geq -n^2-1$

$n^2+n \geq 0 \qquad n(n+1) \geq 0 \Rightarrow n \leq -1 \vee n \geq 0$

ok!

quindi -1 è $MIN(S)$ e anche $INF(S)$.

$MAX(S) = \frac{1}{5}$, verifichiamolo:

a) $\dfrac{1}{5} = \dfrac{n-1}{n^2+1}$ è soddisfatta per $n=2$ e $n=3$.

b) $\dfrac{1}{5} \geq \dfrac{n-1}{n^2+1} \iff n^2+1 \geq 5n-5 \Rightarrow$

$n^2-5n+6 \geq 0 \qquad n_{1,2} = \dfrac{5 \pm \sqrt{25-24}}{2} = \begin{cases} 2 \\ 3 \end{cases}$

$\Rightarrow n \leq 2 \vee n \geq 3$ in particolare

quindi l'eq. è vera $\forall n \geq 0$, $n \in \mathbb{N}$.

Allora $\frac{1}{5}$ è $MAX(S)$ e allora anche $SUP(S)$.

• Dato l'insieme

$$S = \left\{ y \in \mathbb{R} : y = \frac{x-1}{x^2+1} , x > 0 , x \in \mathbb{R} \right\}$$

verificare che $\inf(S) = -1$ e $\max(S) = \frac{1}{2(1+\sqrt{2})}$.

$\operatorname{Inf}(S) = -1$:

a) $-1 \leq \frac{x-1}{x^2+1} \iff -x^2-1 \leq x-1 \iff$

$-x^2-x \leq 0 \iff x^2+x \geq 0 \iff x \leq -1 \vee x \geq 0$

ok, in particolare per $x > 0$.

b) $\forall \varepsilon > 0 , \exists x > 0 : \frac{x-1}{x^2+1} < -1+\varepsilon \iff$

$x-1 < (-1+\varepsilon)(x^2+1) \iff$

$x^2(1-\varepsilon) + x - \varepsilon < 0$ soluzioni:

$$x_{12} = \frac{-1 \pm \sqrt{1+4\varepsilon-4\varepsilon^2}}{2(1-\varepsilon)}$$

ma dobbiamo controllare che $\Delta \geq 0$

$\Delta = 1+4\varepsilon-4\varepsilon^2 \geq 0$ altrimenti le soluzioni

non esistono: $1+4\varepsilon-4\varepsilon^2 \geq 0 \rightarrow$

$\Delta' = 16-4(-4)1 = 32 \rightarrow \varepsilon_{12} = \frac{-4+\sqrt{32}}{8} = \frac{1 \pm \sqrt{2}}{2}$

$\rightarrow \frac{1-\sqrt{2}}{2} < \varepsilon < \frac{1+\sqrt{2}}{2}$ in particolare

$0 < \varepsilon < \frac{1+\sqrt{2}}{2}$ quindi x_{12} sono soluzioni accettabili $\forall \varepsilon : 0 < \varepsilon < \frac{1+\sqrt{2}}{2} \sim 1{,}2$

Tenendo conto che per definizione di S $x > 0$, le soluzioni di $x^2(1-\varepsilon) + x - \varepsilon < 0$ sono
$$\frac{-1-\sqrt{1+4\varepsilon-4\varepsilon^2}}{2(1-\varepsilon)} < x < \frac{-1+\sqrt{1+4\varepsilon-4\varepsilon^2}}{2(1-\varepsilon)}.$$

Dobbiamo però verificare che
$\frac{-1+\sqrt{1+4\varepsilon-4\varepsilon^2}}{2(1-\varepsilon)}$ sia maggiore di zero;

ponendo $1-\varepsilon > 0$ cioè $\varepsilon < 1$ (non è restrittivo) si ha

$$\frac{-1+\sqrt{1+4\varepsilon-4\varepsilon^2}}{2(1-\varepsilon)} > 0 \iff \sqrt{1+4\varepsilon-4\varepsilon^2} > 1$$
$$\iff 4\varepsilon(1-\varepsilon) > 0 \iff 0 < \varepsilon < 1 \quad \text{ok!}$$

Quindi finalmente possiamo dire che
$\mathrm{INF}(S) = -1$ ma NON è $\mathrm{MIN}(S)$ (verificare!)

$-\ \mathrm{MAX}(S) = \frac{1}{2(1+\sqrt{2})}$ verifichiamolo:

b) $\frac{x-1}{x^2+1} \leq \frac{1}{2(1+\sqrt{2})} \iff$

$$x^2 - 2(1+\sqrt{2})x + 3 + 2\sqrt{2} \geq 0$$
$(x-(1+\sqrt{2}))^2 \geq 0 \quad \forall x \in \mathbb{R}$ in particolare per $x > 0$.
E per $x = 1+\sqrt{2}$ si ha $y = \frac{1}{2(1+\sqrt{2})}$.

RISOLVERE

$$\frac{x+3}{(x+1)^2\cdot\sqrt{x^2-3}} \geq 0 \qquad \text{C.E.} \begin{cases} x < -\sqrt{3} \vee x > \sqrt{3} \\ x \neq 1 \end{cases}$$

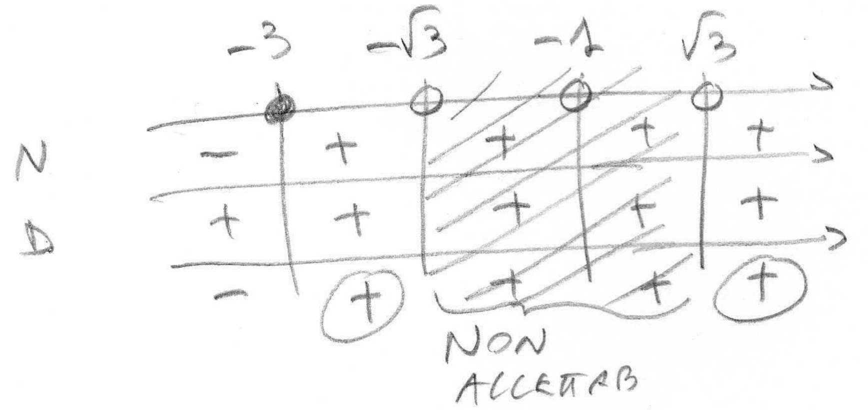
$N \geq 0 \qquad x+3 \geq 0 \Rightarrow x \geq -3$

$D > 0 \qquad (x+1)^2\cdot\sqrt{x^2-3} > 0$

$(x+1)^2 > 0 \qquad \forall x \neq -1$

$\sqrt{x^2-3} > 0 \qquad \forall x \neq \pm\sqrt{3}$

$\Rightarrow \qquad \forall\ x \neq -1 \wedge x \neq \pm\sqrt{3}$



$$\boxed{S: \quad x > -3 \ \wedge \ x \neq -1 \ \wedge \ x \neq \pm\sqrt{3}}$$

RISOLVERE

- $$\frac{\sqrt{x^2+4x+4}-|2x-1|}{1-\sqrt[3]{x^2-8}}<0$$

$N>0$

$$\sqrt{x^2+4x+4}>|2x-1|$$

$$\sqrt{(x+2)^2}>|2x-1|$$

$$|x+2|>|2x-1|$$

$$(x+2)^2>(2x-1)^2 \rightarrow 3x^2-8x-3<0$$

$$x_{12}=\frac{8\pm\sqrt{100}}{6} \begin{cases} 3 \\ -\frac{1}{3} \end{cases} \rightarrow -\frac{1}{3}<x<3$$

$D>0$

$$\sqrt[3]{x^2-8}<1 \rightarrow x^2-8<1$$

$$x^2<9 \rightarrow -3<x<3$$

		-3	$-\frac{1}{3}$	3
N	$-$	$-$	$+$	$-$
D	$-$	$+$	$+$	$-$
	$+$	$-$	$+$	$+$

$$\boxed{S: -3<x<-\frac{1}{3}}$$

RISOLVERE

- $$\frac{\left|\frac{3x-1}{x-2}\right|+4}{3-2x-\sqrt{x-1}}>0$$

C.E. NEI CALCOLI

$N>0 \quad \forall x \in \mathbb{R}\setminus\{2\}$

$D>0 \quad 3-2x-\sqrt{x-1}>0$

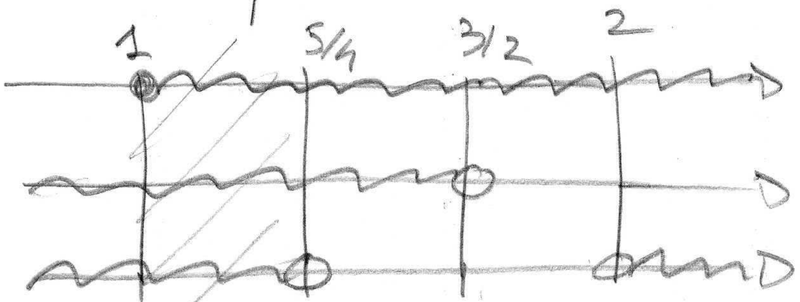
$\sqrt{x-1}<3-2x \quad \rightarrow$

$$\begin{cases} x-1\geq 0 \\ 3-2x>0 \\ x-1<(3-2x)^2 \end{cases} \rightarrow \begin{cases} x\geq 1 \\ x<\frac{3}{2} \\ -4x^2+13x-10<0 \quad ⓐ \end{cases}$$

ⓐ $4x^2-13x+10>0$

$$x_{12}=\frac{13\pm 3}{8} < \begin{matrix} 2 \\ \frac{5}{4} \end{matrix}$$

$$\rightarrow \begin{cases} x\geq 1 \\ x<\frac{3}{2} \\ x<\frac{5}{4} \vee x>2 \end{cases}$$



$D>0$

$1\leq x<\frac{5}{4}$

$\boxed{S: \ [1, 5/4[}$

RISOLVERE

$$\frac{x-1-\sqrt[3]{x^2(x-3)}}{\sqrt{4-2x}-1+x}\geq 0$$

C.E. NEI CALCOLI...

$N\geq 0$

$$x-1-\sqrt[3]{x^2(x-3)}\geq 0$$

$$\Rightarrow \sqrt[3]{x^2(x-3)}\leq x-1$$

$$\Rightarrow x^2(x-3)\leq (x-1)^3$$

$$\Rightarrow x^3-3x^2\leq x^3-1-3x^2+3x$$

$$\Rightarrow 0\leq -1+3x \Rightarrow x\geq \frac{1}{3}$$

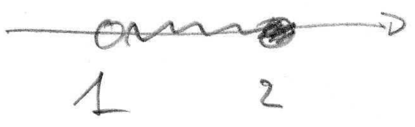
$D>0$

$$\sqrt{4-2x}-1+x>0$$

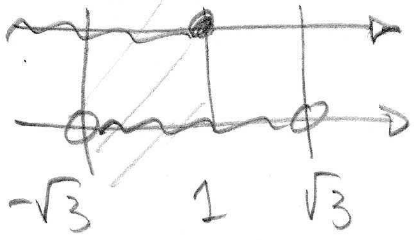
$$\sqrt{4-2x}>1-x \Rightarrow$$

$$\begin{cases}4-2x\geq 0\\ 1-x<0\end{cases} \cup \begin{cases}1-x\geq 0\\ 4-2x>(1-x)^2\end{cases}$$

$$\Rightarrow \begin{cases}x\leq 2\\ x>1\end{cases} \cup \begin{cases}x\leq 1\\ x^2+1-4<0 \Rightarrow -\sqrt{3}<x<\sqrt{3}\end{cases}$$



$\cup$



$D > 0 \quad]1, 2] \cup]-\sqrt{3}, 1] =]-\sqrt{3}, 2]$

		$-\sqrt{3}$		$\frac{1}{3}$		2	
N	$-$		$-$		$+$		$+$
D	$-$		$+$		$+$		$-$
	$+$		$-$		$+$		$-$

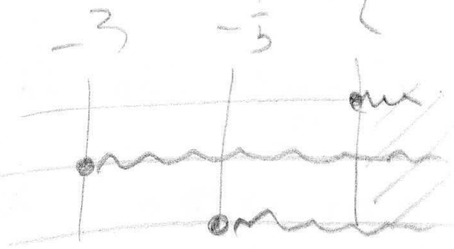
$$\boxed{S : \;]-\infty, -\sqrt{3}[\; \cup \; [\tfrac{1}{3}, 2] \; -}$$

RISOLVERE:

$$\sqrt{2x-4}+\sqrt{x+3}<\sqrt{3x+1}$$

Dom

$$\begin{cases}2x-4\geq 0\\ x+3\geq 0\\ 3x+1\geq 0\end{cases}\qquad \begin{cases}x\geq 2\\ x\geq -3\\ x\geq -\frac{1}{3}\end{cases}\quad\Rightarrow$$



$\underline{x\geq 2}$

Elevo al quadrato a dx e sx

$$\cancel{2x}-4+\cancel{x}+3+2\sqrt{(2x-4)(x+3)}<\cancel{3x}+1$$

$$\cancel{2}\sqrt{(2x-4)(x+3)}<\cancel{2}\qquad \text{elevo}$$

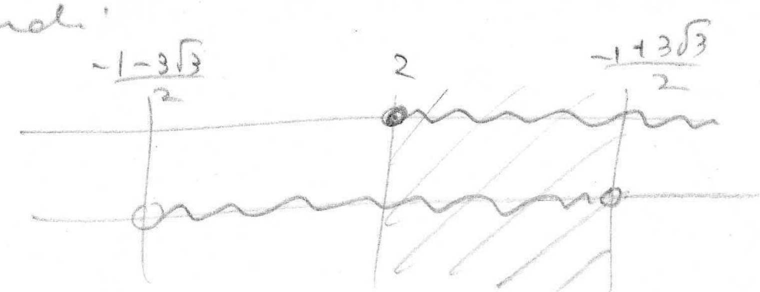
$$2x^2+6x-4x-12-1<0$$

$$2x^2+2x-13<0\qquad x_{12}=\frac{-2\pm\sqrt{4+104}}{4}$$

$$x_{12}=\frac{-2\pm 6\sqrt{3}}{4}=\frac{-1\pm 3\sqrt{3}}{2}$$

$$\frac{-1-3\sqrt{3}}{2}<x<\frac{-1+3\sqrt{3}}{2}$$

Quindi:

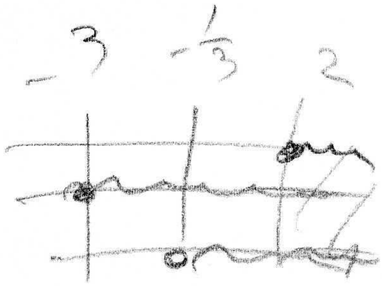


$$x\in\left[2,\frac{-1+3\sqrt{3}}{2}\right[$$

RISOLVERE:

$$\sqrt{2x-4}+\sqrt{x+3}<\sqrt{3x+1}$$

C.E. $\begin{cases} 2x-4\geq 0 \\ x+3\geq 0 \\ 3x+1\geq 0 \end{cases}$ $\begin{cases} x\geq 2 \\ x\geq -3 \\ x\geq -\frac{1}{3} \end{cases}$



$\Rightarrow x\geq 2 \Rightarrow$ eleviamo al quadrato a DX e SX

$$\begin{cases} x\geq 2 \\ \cancel{2x}-4+\cancel{x}+3+2\sqrt{(2x-4)(x+3)}<\cancel{3x}+1 \end{cases}$$

$$\begin{cases} x\geq 2 \\ 2\sqrt{}<2 \end{cases} \quad \begin{cases} x\geq 2 \\ \sqrt{(2x-4)(x+3)}<1 \end{cases}$$

eleviamo

$$2x^2+6x-4x-12<1$$

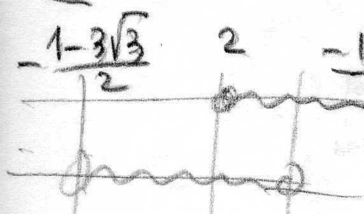
$$\begin{cases} x\geq 2 \\ 2x^2+2x-13<0 \end{cases}$$

$$x_{1,2}=\frac{-2\pm\sqrt{4-4(2)(-13)}}{4}=\frac{-2\pm\sqrt{4+104}}{4}=\frac{-2\pm 6\sqrt{3}}{4}=\frac{-1\pm 3\sqrt{3}}{2}$$

$\sim 2,09$

$\sim -3,09$

$$\begin{cases} x\geq 2 \\ \frac{-1-3\sqrt{3}}{2}<x<\frac{-1+3\sqrt{3}}{2} \end{cases}$$



$$\boxed{2\leq x<\frac{-1+3\sqrt{3}}{2}}$$

CALCOLARE IL DOMINIO DI

$$f(x) = \log\left(2x+1-\sqrt{4x^2-4x-15}\right)$$

C.E:

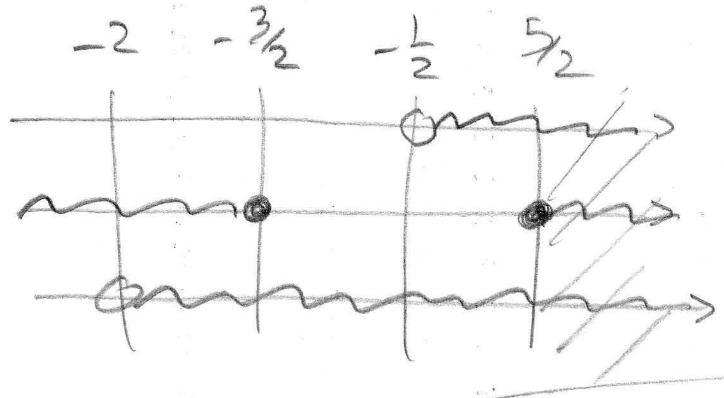
$$\begin{cases} 2x+1-\sqrt{4x^2-4x-15} > 0 \\ 4x^2-4x-15 \geq 0 \end{cases}$$

La seconda eq. sarà contenuta nella soluzione della prima:

$$\sqrt{4x^2-4x-15} < 2x+1$$

$$\begin{cases} 2x+1 > 0 \\ 4x^2-4x-15 \geq 0 \\ 4x^2-4x-15 < (2x+1)^2 \end{cases} \rightarrow$$

$$\begin{cases} x > -\frac{1}{2} \\ x \leq -\frac{3}{2} \quad \vee \quad x \geq \frac{5}{2} \\ x > -2 \end{cases}$$



DOMINIO DI $f = \boxed{\left[\frac{5}{2}, +\infty\right[}$

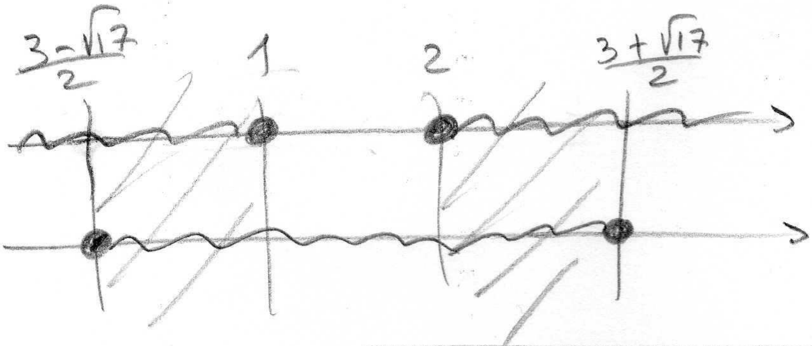
TROVARE IL DOMINIO

- $f(x) = \sqrt[4]{2 - |x^2 - 3x|}$

$2 - |x^2 - 3x| \geq 0$

$|x^2 - 3x| \geq 2 \rightarrow$

$$\begin{cases} x^2 - 3x \geq -2 \\ x^2 - 3x \leq 2 \end{cases} \Rightarrow \begin{cases} x \leq 1 \quad \vee \quad x \geq 2 \\ \frac{3-\sqrt{17}}{2} \leq x \leq \frac{3+\sqrt{17}}{2} \end{cases}$$



$$\boxed{S: \quad \frac{3-\sqrt{17}}{2} \leq x \leq 1 \quad \vee \quad 2 \leq x \leq \frac{3+\sqrt{17}}{2}}$$

CALCOLARE IL DOMINIO

$$f(x) = \frac{1}{\log(\cot g\,x - tg\,x)}$$

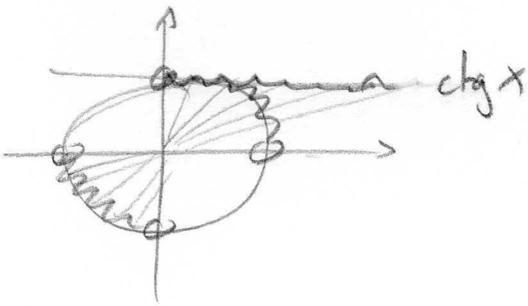
$$\cot g\,x \equiv \frac{\cos x}{\operatorname{sen} x} \qquad tg\,x \equiv \frac{\operatorname{sen} x}{\cos x}$$

$$\begin{cases} \text{①}\quad \cot g\,x - tg\,x > 0 \\ \text{②}\quad \log(\cot g\,x - tg\,x) \neq 0 \\ \text{③}\quad \cos x \neq 0 \wedge \operatorname{sen} x \neq 0 \end{cases}$$

① $ctg\,x > tg\,x \rightarrow \dfrac{\cos x}{\operatorname{sen} x} - \dfrac{\operatorname{sen} x}{\cos x} > 0$

$$\frac{\cos^2 x - \operatorname{sen}^2 x}{\operatorname{sen} x \cos x} > 0 \rightarrow \frac{\cos 2x}{\frac{\operatorname{sen} 2x}{2}} > 0$$

$\rightarrow \cot g\,2x > 0$

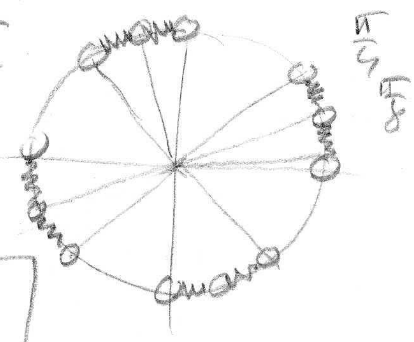


$$0 + k\pi < 2x < \frac{\pi}{2} + k\pi$$

$$\rightarrow k\frac{\pi}{2} < x < \frac{\pi}{4} + k\frac{\pi}{2}$$

② $ctg\,x - tg\,x \neq 1 \rightarrow ctg\,2x \neq 1$

$$2x \neq \frac{\pi}{4} + k\pi \rightarrow x \neq \frac{\pi}{8} + k\frac{\pi}{2}$$



③ $x \neq k\frac{\pi}{2}$

$$\boxed{S: \quad k\frac{\pi}{2} < x < \frac{\pi}{4} \wedge x \neq \frac{\pi}{8} + k\frac{\pi}{2}}$$

$$\left(\frac{\operatorname{sen} x}{x^2-1}\right)^{\log(6x^2+x-1)}$$

$$\begin{cases} \frac{\operatorname{sen} x}{x^2-1} > 0 & (1) \\ 6x^2+x-1>0 & (2) \end{cases}$$

(1) $N>0$ $\operatorname{sen} x > 0 \Rightarrow 2k\pi + 0 < x < \pi + 2k\pi$

$D>0$ $x^2-1>0 \Rightarrow x<-1 \cup x>1$

$-2\pi \quad -\pi \quad -1 \quad 0 \quad 1 \quad \pi \quad 2\pi \quad 3\pi$

−	+	−	−	+	+	−	+	−
+	+	+	−	−	+	+	+	+
−	+	−	+	−	+	−	+	−

$2k\pi < x < \pi + 2k\pi \quad k \in \mathbb{Z} \setminus \{0\} \cup -1<x<0$

$\cup \ 1<x<\pi$

(2)

$6x^2+x-1>0 \quad \Delta = 1-4(6)(-1) = 25$

$$x_{12} = \frac{-1\pm 5}{12} \begin{cases} \frac{4}{12} = \frac{1}{3} \\ -\frac{6}{12} = -\frac{1}{2} \end{cases} \Rightarrow x<-\frac{1}{2} \cup x>\frac{1}{3}$$

$\Rightarrow$

$$\boxed{S: \ 2k\pi < x < \pi + 2k\pi \quad k \in \mathbb{Z}_0 \ \cup \\ -1<x<-\frac{1}{2} \ \cup \ \frac{1}{3}<x<\pi}$$

DOMINIO:

- $f(x) = \operatorname{arctg}\left(\sqrt{4e^{2x} - 9e^{x} + 2} - 2e^{x}\right)$

$4e^{2x} - 9e^{x} + 2 \geq 0$

$e^{x} = t \Rightarrow 4t^{2} - 9t + 2 \geq 0$

$t_{12} = \dfrac{9 \pm \sqrt{81 - 4 \cdot 4 \cdot 2}}{8} = \dfrac{9 \pm 7}{8} \begin{matrix} 2 \\ \frac{1}{4} \end{matrix}$

$t \leq \frac{1}{4} \ \vee \ t \geq 2$

$e^{x} \leq \frac{1}{4} \ \vee \ e^{x} \geq 2$

$x \leq \log \frac{1}{4} \ \vee \ x \geq \log 2$

$\mathrm{Dom}(f): \ ]-\infty, \log \tfrac{1}{4}] \cup [\log 2, +\infty[$

• $f(x) = \log\left(4\,\mathrm{senh}^2 x - 5\,\mathrm{senh}\,x + 1\right)$

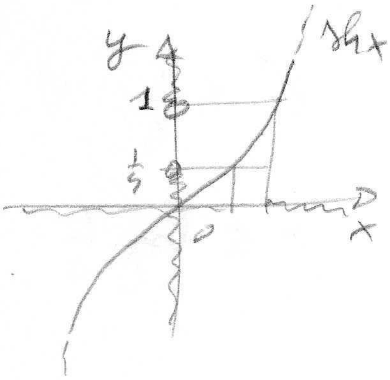
$4\,\mathrm{senh}^2 x - 5\,\mathrm{senh}\,x + 1 > 0$

$\mathrm{senh}\,x = t$

$4t^2 - 5t + 1 > 0$

$$t_{12} = \frac{5 \pm \sqrt{25 - 4\cdot 4\cdot 1}}{8} = \frac{5 \pm 3}{8} \begin{cases} 1 \\ \frac{1}{4} \end{cases}$$

$t < \frac{1}{4} \quad \vee \quad t > 1 \quad \Rightarrow$



$\mathrm{senh}\,x < \frac{1}{4} \quad \vee \quad \mathrm{senh}\,x > 1$

$x < \mathrm{settsh}\,\frac{1}{4} \quad \vee \quad x > \mathrm{settsh}\,1$