Soluzioni esercizi su funzioni più variabili.

1) • $f(x,y) = e^{x/y}$

$f_x = e^{x/y}\left(\frac{1}{y}\right) \qquad f_y = e^{x/y}\left(-\frac{x}{y^2}\right).$

• $f(x,y) = y^{\log x} = e^{\log x \log y}$

$f_x = e^{\log x \log y} \cdot \frac{\log y}{x} = y^{\log x} \cdot \frac{\log y}{x}$

$f_y = y^{\log x} \cdot \frac{\log x}{y}$

• $f(x,y) = tg\left(\frac{y}{x}\right)$

$f_x = \frac{1}{\cos^2\left(\frac{y}{x}\right)} \cdot \left(-\frac{y}{x^2}\right), \quad f_y = \frac{1}{\cos^2\left(\frac{y}{x}\right)} \cdot \frac{1}{x}$

2) $f(x,y) = e^{x^2+2y} + \log(x^2-y)$

a) dominio $\quad x^2 - y > 0 \qquad y < x^2$

b) è aperto e illimitato (def. da $g(x,y) < 0$)

c) $f_x = e^{x^2+2y} \cdot 2x + \frac{1}{x^2-y} \cdot 2x$

$f_y = e^{x^2+2y} \cdot 2 + \frac{1}{x^2-y}(-1)$ continue nel dominio di f

$\Rightarrow$ dal teorema del diff. totale $\Rightarrow$ f differenziabile nel suo dominio

$f_x(1,0) = 2e + 2$

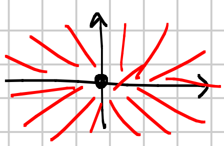
$f_y(1,0) = 2e - 1$

d) $\frac{\partial f}{\partial v} = (\nabla f(1,0)) \cdot \left(\frac{1}{3}, \frac{2\sqrt{2}}{3}\right) =$

$= (2e+2)\frac{1}{3} + (2e-1)\left(\frac{2\sqrt{2}}{3}\right) =$

$= \ldots$

3) $f(x,y) = \operatorname{arctg}\left(\frac{x}{\sqrt{x^2+y^2}}\right)$



a) dominio $D = \mathbb{R}^2 \setminus \{0,0\}$

b) $$f_x = \frac{1}{1+\frac{x^2}{x^2+y^2}} \cdot \frac{\sqrt{x^2+y^2} - x\,\frac{2x}{2\sqrt{x^2+y^2}}}{x^2+y^2}$$

$$f_x(1,1) = \frac{1}{1+\frac{1}{2}} \cdot \frac{\sqrt{2}-\frac{1}{\sqrt{2}}}{2} = \frac{1}{3}\,\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{6}$$

$$f_y = \frac{1}{1+\frac{x^2}{x^2+y^2}} \cdot x\left(-\frac{1}{2}\right)\frac{1 \cdot 2y}{(x^2+y^2)^{3/2}}$$

$$f_y(1,1) = \frac{1}{1+\frac{1}{2}}\left(-\frac{1}{2}\right)\frac{\cancel{2}}{\cancel{2}\sqrt{2}} = \frac{1}{3}\left(-\frac{1}{\sqrt{2}}\right) = -\frac{\sqrt{2}}{6}$$

c) $$\frac{\partial f}{\partial v}(1,1) = \frac{\sqrt{2}}{6}\cdot\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{6}\cdot\frac{\sqrt{2}}{2} = 0$$

4) $f(x,y) = x^2y + x^2 - 2y$

p.ti critici $\quad f_x = 2xy + 2x = 2x(y+1) = 0$

$\qquad f_y = x^2 - 2 = 0$

$y = -1, \quad x = \pm\sqrt{2} \qquad \boxed{(\pm\sqrt{2}, 1)}$

$f(x,y) = xy \qquad f_x = y = 0 \quad f_y = x = 0 \qquad \boxed{(0,0)}$

$f(x,y) = x^3 + y^3 - (1+x+y)^3$

$$\begin{cases} f_x = 3x^2 - 3(1+x+y)^2 = 0 \\ f_y = 3y^2 - 3(1+x+y)^2 = 0 \end{cases}$$

$x^2 = (1+x+y)^2$
$y^2 = (1+x+y)^2 \quad \Rightarrow \quad x^2 = y^2 \Rightarrow x = \pm y$

- $x = y \quad \Rightarrow \quad x^2 - (1+2x)^2 = x^2 - 1 - 4x - 4x^2 = 0$

$3x^2 + 4x + 1 = 0 \qquad \Delta = 4 - 3 = 1$

$x = \frac{-2 \pm 1}{3} = \begin{cases} -1 \\ -\frac{1}{3} \end{cases} \qquad \Rightarrow \boxed{\begin{matrix}(-1,-1) \\ (-\frac{1}{3}, -\frac{1}{3})\end{matrix}}$

- $x = -y \qquad x^2 = 1 \qquad x = \pm 1 \qquad \boxed{\begin{matrix}(1,-1) \\ (-1, 1)\end{matrix}}$

5) $f(x,y) = 2x^2 + e^{x^2+y} - y$

$f_x = 4x + e^{x^2+y}\cdot 2x = 2x(2 + e^{x^2+y}) = 0 \iff x=0$

$f_y = e^{x^2+y} - 1 = 0 \iff x^2+y=0$

$x = 0$
$y = 0 \qquad$ Unico p.to critico $(0,0)$

$f_{xx} = 4 + 2e^{x^2+y} + 2x\,e^{x^2+y}\cdot 2x$

$$f_{xy} = e^{x^2+y}\, 2x$$
$$f_{yy} = e^{x^2+y}$$

$$D^2_f(0,0) = \begin{pmatrix} 6 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\det\left(D^2_f(0,0)\right) = 6 > 0$$
$$a = 6 > 0$$

$(0,0)$ p.to di minimo locale.

10) distanza della retta $3x+2y+4=0$ dall'origine

$$\begin{cases} \min \; f(x,y) = x^2+y^2 \\ 3x+2y+4=0 \end{cases}$$

$$y = \frac{-4-3x}{2} \qquad f/_{\text{vincolo}} = x^2 + \frac{1}{4}(4+3x)^2 =: h(x)$$

$$h'(x) = 2x + \frac{1}{2}(4+3x)\cdot 3 = 2x + \frac{9}{2}x + 6 = 0$$

$$x\left(\frac{13}{2}\right) = -6 \qquad x = -\frac{12}{13} \qquad h'(x) > 0 \qquad x > -\frac{12}{13}$$

$-12/13$

$x = -\frac{12}{13}$ p.to di minimo

$$y = -2 + \frac{3}{\cancel{2}}\cdot\frac{\cancel{12}^{\,6}}{13} = \frac{18}{13} - 2 = -\frac{8}{13}$$

$\left(-\frac{12}{13}, -\frac{8}{13}\right)$ p.to di minimo.

distanza retta dall'origine $= \left(\frac{12}{13}\right)^2 + \left(\frac{8}{13}\right)^2$.