Roger Cotes – natural philosopher

RONALD GOWING

CAMBRIDGE UNIVERSITY PRESS

Cambridge
London New York New Rochelle
Melbourne Sydney

PUBLISHED BY THE PRESS SYNDICATE OF THE UNIVERSITY OF CAMBRIDGE
The Pitt Building, Trumpington Street, Cambridge, United Kingdom

CAMBRIDGE UNIVERSITY PRESS
The Edinburgh Building, Cambridge CB2 2RU, UK
40 West 20th Street, New York NY 10011–4211, USA
477 Williamstown Road, Port Melbourne, VIC 3207, Australia
Ruiz de Alarcón 13, 28014 Madrid, Spain
Dock House, The Waterfront, Cape Town 8001, South Africa

http://www.cambridge.org

First published 1983
First paperback edition 2002

A catalogue record for this book is available from the British Library

Library of Congress catalogue card number: 82-1154

ISBN 0 521 23741 6 hardback
ISBN 0 521 52649 3 paperback

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Introduction

Roger Cotes, one of that impressive band of able and distinguished men and women who have emerged from English rectories, was a Fellow of Trinity College, Cambridge, at the age of twenty-five, and the first Plumian Professor of Astronomy and Experimental Philosophy at the age of twenty-six. He is best known for his meticulous and creative editing of the second edition of Newton's *Principia* (Cambridge, 1713) and for his notable Preface to that work. Contemporary opinion places him among the most able of British mathematicians of his day, and close study of his work supports that view. His influence on the subsequent development of mathematics was much less than his ability and achievement merit: his name survives in 'the Cotes property of the circle', 'the Cotes spiral', 'the Cotes–Newton formulae for approximate quadrature'; otherwise, the man and his work are relatively little known. It is the aim of this book, in this the tercentenary year of his birth, to make his work more widely known, and so to reveal something of the man.

There is little biographical information. The good article by Agnes Mary Clarke, in the *Dictionary of National Biography*, refers to most of what is available and is largely, but not entirely, correct. We do, however, have the remarkable, fascinating and almost complete correspondence between Cotes and Newton, written during the four years in which Cotes was editing *Principia*, from Cotes' first tentative letter in 1709, to the final rather chilly exchange about the errors after the publication of the second edition in 1713. The correspondence is preserved in the library of Trinity College, Cambridge, and in the Cambridge University Library. It has been superbly edited and published in *Correspondence of Sir Isaac Newton and Professor Cotes*, by J. Edleston, published by Frank Cass and Company in 1850 (and in a new impression as number 12 of the Cass Library of Science Classics, in 1969). Excellent and detailed commentaries have been provided

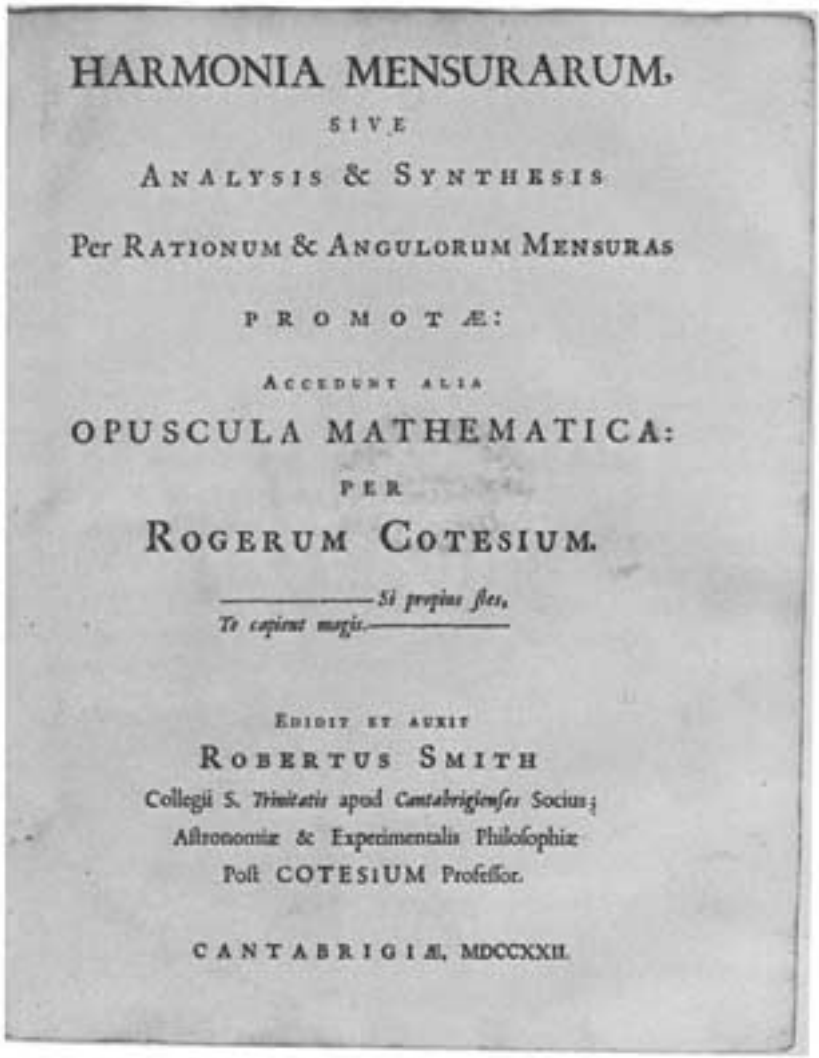
by I. B. Cohen in *Introduction to Newton's Principia* (Cambridge, 1971), and by Rupert Hall and Laura Tilling in *Correspondence of Sir Isaac Newton*, vol. 5 (7 vols., Cambridge, 1975). It would be presumptuous of me to attempt to recapitulate or add to these distinguished works. In Chapter 1 I have drawn together such parts of the correspondence as throw light on Cotes' personality, the circumstances surrounding his appointment as Plumian Professor, and his relationship with Newton. It was against the background here sketched that Cotes was preparing his own work for publication, and the one paper he submitted for publication in his lifetime, Logometria, is discussed in Chapter 2. The original Latin paper is fairly readily available in *Harmonia Mensurarum, Sive Analysis & Synthesis per Rationum & Angulorum Mensurae Promotae*, ed. R. Smith (Cambridge, 1722), pp. 4–41, but an English translation has not previously been published; the full translation is therefore given in Appendix 1. In Logometria, Cotes developed the theory of logarithms, following earlier work, particularly that of Halley and De Moivre. J. E. Hofmann says in his article 'Weiterbildung der Logarithmischen Reihe Mercators in England, III', *Deutsche Mathematik*, 5 (1940–1), 368–75: 'As a young man Cotes had that density of thought that occurs only rarely and is the hallmark of the genius who departs before his time.' The principal application of the theory was to the integration of certain rational and irrational algebraic forms, yielding logarithmic, trigonometric and hyperbolic functions, functions for the most part not then clearly recognised. The power of the new methods was demonstrated in the long Scholium Generale of Logometria, where the solutions of a number of problems of contemporary interest were given, without proof. The proofs depend upon the eighteen tables of integrals developed by Cotes. These integrals and their application form the subject of Chapter 3; the eighteen fluxional forms which Cotes integrated are shown in Appendix 3.

The tables of integrals were published posthumously in 1722, by Robert Smith, Cotes' cousin, assistant and eventual successor to the Plumian chair, in *Harmonia Mensurarum*. In this work, Smith collected together most of Cotes' surviving papers; I have translated and worked through the whole of this work. The first three parts of the book are often referred to as Logometria, Parts one, two and three. Part one is Logometria almost exactly as it appeared in the *Philosophical Transactions of the Royal Society*, vol. 29, no. 338 (1714), pp. 5–45. Part two consists of Cotes' eighteen tables of integrals and Part three is a collection of twelve illustrative problems in which the application of the integrals to the solutions is shown in detail. All this is discussed

in Chapter 3. Cotes was busy preparing his papers for publication at the time of his death in 1716. His discovery of a general method of factorising $a^n \pm x^n$ into linear and quadratic factors for integral values of n, enabled him, towards the end of his life, to extend his work on integrals. Smith edited and further developed this later work, arranging the results in ninety-three tables, with a good deal of needless elaboration. I refer to the forms here integrated as the Cotes–Smith Forms: they form Part four of *Harmonia Mensurarum*. The tables are discussed, together with the theorem on which they depend, in Chapter 4.

As Professor of Astronomy, Cotes carried a large share of the responsibility for the construction and equipping of the observatory which was constructed on top of the King's gate at Trinity College. He seems to have entered upon this work with great enthusiasm, but there is little record of practical work carried out. Such as there is I have gathered together in Chapter 5, in which I have also discussed the tract, Aestimatio Errorum in Mixta Mathesi, published in *Harmonia Mensurarum*. This last is a serious attempt to apply fluxional concepts to a study of the errors arising in astronomical observations. Cotes' work on *Principia* led him to consider ways of computing cometary orbits from a limited number of observations, and to an awareness of the need for more accurate lunar tables. Agnes Mary Clarke states in the *Dictionary of National Biography* that Cotes remodelled Cassini's tables, and had undertaken to prepare new tables of the moon. I have found no record of this work, but Cotes did write two tracts concerned with the construction of tables. These are De Methodo Differentiali Newtoniana, and Canonotechnia (or, The Construction of Tables by Differences). Both these papers are printed in *Harmonia Mensurarum*, and formed part of the public lectures which Cotes was required to give by the statutes of his appointment. The first was largely completed before the publication of Newton's tract on the *Method of Differences* (London, 1711) and, having seen Newton's paper, Cotes added the section which contains the so-called Cotes–Newton formulae for approximate quadrature (Simpson's rule, the 'three-eighths rule', and so on). Canonotechnia is one of the most interesting of Cotes' papers, and deals with more general methods of interpolation and sub-tabulation. Both are discussed under the heading Numerical methods, in Chapter 6.

Besides the papers appearing in *Harmonia Mensurarum*, there is one other published work by Cotes, namely, his *Hydrostatical and Pneumatical Lectures*. These were published in 1738 by Robert Smith (see second edition, Cambridge, 1747), as he says in his Preface, to forestall a



HARMONIA MENSURARUM,

SIVE

ANALYSIS & SYNTHESIS

Per RATIONUM & ANGULORUM MENSURAS

PROMOTÆ:

ACCEDUNT ALIA

OPUSCULA MATHEMATICA:

PER

ROGERUM COTESIUM.

Si propius ſtes, Te capient magis.

EDIDIT ET AUXIT

ROBERTUS SMITH

Collegii S. *Trinitatis* apud *Cantabrigienſes* Socius;

Aſtronomiæ & Experimentalis Philoſophiæ

Poſt COTESIUM Profeſſor.

CANTABRIGIÆ, MDCCXXII.

Fig. 2. Title page of *Harmonia Mensurarum*, Cambridge University Press, 1722.

pirated edition. The lectures were intended for undergraduates and are at a fairly elementary level, although full of interesting detail. My brief comments about them, together with some more general remarks, fit most readily into the final chapter.