Es 1: $\displaystyle\int_0^{+\infty} \frac{(e^{\sqrt{x}}-1)\,dx}{x(4+\cos x)^x} = \underbrace{\int_0^1}_{I_1} + \underbrace{\int_1^{+\infty}}_{I_2}$ convergente perché I_1 e I_2 sono convergenti.

$I_1 \sim \displaystyle\int_0^1 \frac{\sqrt{x}}{x}\,dx = \int_0^1 \frac{1}{\sqrt{x}}\,dx$ convergente $\Rightarrow$ per confronto asintotico I_1 è convergente

$I_2 \underset{|}{\leq} \displaystyle\int_1^{+\infty} \frac{e^{\sqrt{x}}}{x\,3^x}\,dx \leq \int_1^{+\infty} \left(\frac{e}{3}\right)^x dx$, $\displaystyle\lim_{x\to+\infty} \frac{\left(\frac{e}{3}\right)^x}{\frac{1}{x^2}} = 0$

$((4+\cos x) \geq 3)$

$\displaystyle\int_1^{+\infty} \frac{1}{x^2}\,dx$ convergente $\Rightarrow I_2$ convergente per confronto asintotico

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Es 2 $x^4 \ln^2 x - 1 + \operatorname{senh}^2(x) + \cos\left(1-e^{\sqrt{2x}}\right) =$

$= o(x^3) - 1 + \left(x + \frac{x^3}{6} + o(x^3)\right)^2 + \cos\left(-\sqrt{2x} - x + o(x)\right) =$

$= -1 + x^2 + o(x^2) + 1 - x + o(x) = -x + o(x)$

$$\operatorname{senh}(x) - x^\alpha = \begin{cases} x + o(x) & \text{se } \alpha > 1 \\ \frac{x^3}{6} + o(x^3) & \text{se } \alpha = 1 \\ -x^\alpha + o(x^\alpha) & \text{se } \alpha < 1 \end{cases}$$

$$\ell_\alpha = \lim_{x\to 0^+} \frac{x^4 \ln^2 x - 1 + \operatorname{senh}^2(x) + \cos\left(1-e^{\sqrt{2x}}\right)}{\operatorname{senh}(x) - x^\alpha} = \begin{cases} -1 & \text{se } \alpha > 1 \\ -\infty & \text{se } \alpha = 1 \\ 0 & \text{se } \alpha < 1 \end{cases}$$

Es 3 $f(x) = \dfrac{|x^{3/2} - 8|}{\sqrt{x}}$

i) $\mathrm{Dom}(f) =]0, +\infty[$

ii) $\lim_{x\to 0} f(x) = +\infty \Rightarrow x = 0$ asintoto verticale

$\lim_{x\to+\infty} f(x) = +\infty$, $\lim_{x\to+\infty} \frac{f(x)}{x} = 1$, $\lim_{x\to+\infty} f(x) - x = 0$

$\Rightarrow y = x$ asintoto obliquo per $x \to +\infty$

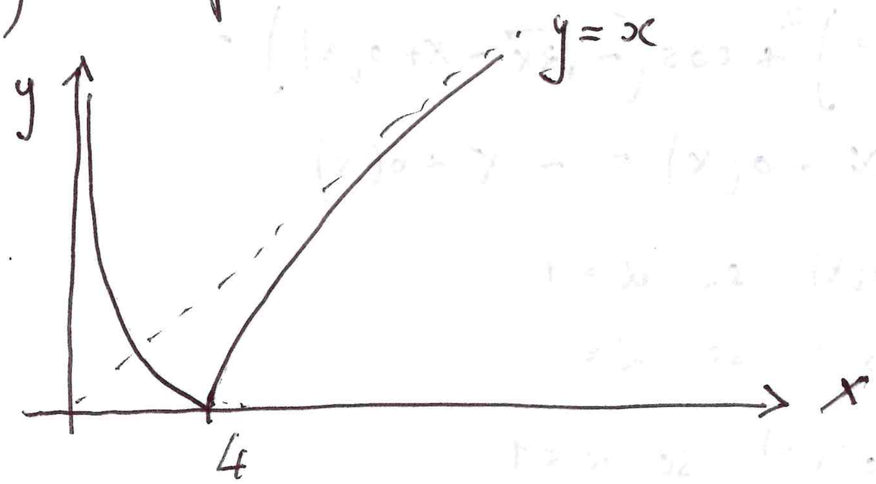
iii) $f'(x) = \dfrac{\mathrm{sgn}\,(x^{3/2} - 8)(x^{3/2} + 4)}{x^{3/2}} \geq 0 \iff x^{3/2} - 8 \geq 0$

$\iff x \geq 4 \Rightarrow f$ monotona crescente su $[4, +\infty[$

f " decrescente su $]0, 4]$

iv) $x = 4$ punto di minimo assoluto, $0 = f(4) = \min f$

v) $\mathrm{Im}(f) = [0, +\infty[$



Es. 4 $\quad f(x) = \displaystyle\int_{-\sqrt{\ln x}}^{\sqrt{\ln x}} e^{t^2}\, dt$

i) $\mathrm{Dom}(f) = [1, +\infty[$, $\{f \geq 0\} = [1, +\infty[$

ii) $f'(x) = \dfrac{1}{2\sqrt{\ln x}} \cdot \dfrac{1}{x}\left[e^{|\ln x|} + e^{|\ln x|}\right] = \dfrac{1}{\sqrt{\ln x}} > 0 \quad \forall x \in \mathrm{Dom}(f)$

$\Rightarrow f$ è crescente su $[1, +\infty[$

$\boxed{\text{Es. 5}}$ $y'(x^2+e) = x(y+2)^2$

i) Soluzione costante: $\boxed{\varphi_1(x) = -2 \quad \forall x \in \mathbb{R}}$

Soluzioni non costanti:

$$\int \frac{dy}{(y+2)^2} = \int \frac{x\,dx}{(x^2+e)} \Rightarrow -\frac{1}{(y+2)} = \frac{\ln(x^2+e)}{2} + c_1, \quad c_1 \in \mathbb{R}$$

$$\Rightarrow y+2 = -\frac{2}{\ln(x^2+2)+c_2}, \quad c_2 \in \mathbb{R}$$

$$\Rightarrow \boxed{\varphi_2(x;c) = -2\left[1 + \frac{1}{\ln(x^2+e)+c}\right], \quad c \in \mathbb{R}}$$

ii) Pb di Cauchy $y(0) = 1 \Rightarrow 1 = \varphi_2(0;c) = -2\left(1 + \frac{1}{1+c}\right)$

$\Rightarrow c = -\frac{5}{3} \Rightarrow$ Sol. del Pb di Cauchy è:

$$\boxed{\varphi(x) = -2\left[1 + \frac{1}{\ln(x^2+c) - 5/3}\right]}$$