$\boxed{\text{Es 1:}}$ $$\int_0^{+\infty} \frac{\sqrt{x}\,(1+\sin(x))^{\sqrt{x}}}{e^x-1}\,dx = \underbrace{\int_0^1}_{I_1} + \underbrace{\int_1^{+\infty}}_{I_2}$$

$I_1 \sim \int_0^1 \frac{\sqrt{x}}{x}\,dx = \int_0^1 \frac{1}{\sqrt{x}}\,dx$ convergente $\Rightarrow$ per confronto asintotico I_1 è convergente

$I_2 \sim \int_1^{+\infty} \frac{\sqrt{x}\,(1+\sin(x))^{\sqrt{x}}}{e^x}\,dx \leq \int_1^{+\infty} \sqrt{x}\cdot\left(\frac{2}{e}\right)^x dx$, $\lim_{x\to+\infty} \frac{\sqrt{x}\left(\frac{2}{e}\right)^x}{\frac{1}{x^2}} = 0$

$\int_1^{+\infty} \frac{1}{x^2}\,dx$ convergente $\Rightarrow I_2$ convergente per confronto asintotico

$\Rightarrow I_1 + I_2$ è convergente

$\boxed{\text{Es.2}}$ $$x^4 \ln^3 x + 1 + \sin^2 x - \cosh\left(e^{\sqrt{2x}}-1\right) =$$

$$= o(x^3) + 1 + \left(x - \frac{x^3}{6} + o(x^3)\right)^2 - \cosh\left(\sqrt{2x} + x + o(x)\right) =$$

$$= 1 + x^2 + o(x^2) - 1 - x + o(x) = -x + o(x)$$

$$\operatorname{sen}(x) - x^\alpha = \begin{cases} x + o(x) & \text{se } \alpha > 1 \\ -\frac{x^3}{6} + o(x^3) & \text{se } \alpha = 1 \\ -x^\alpha + o(x^\alpha) & \text{se } \alpha < 1 \end{cases}$$

$$\ell_\alpha = \lim_{x\to 0} \frac{\operatorname{sen} x - x^\alpha}{x^4 \ln^3 x + 1 + \sin^2 x - \cosh\left(e^{\sqrt{2x}}-1\right)} = \begin{cases} -1 & \text{se } \alpha > 1 \\ 0 & \text{se } \alpha = 1 \\ +\infty & \text{se } \alpha < 1 \end{cases}$$

$\boxed{\text{Es 3}}$ $$f(x) = \frac{x^{3/2}}{|\sqrt{x} - 2|}$$

i) $\mathrm{Dom}(f) = [0,4[\, \cup\,]4,+\infty[$

ii) $\lim_{x \to 4} f(x) = +\infty \Rightarrow x = 4$ asintoto verticale

$\lim_{x \to +\infty} f(x) = +\infty$, $\lim_{x \to +\infty} \frac{f(x)}{x} = 1$, $\lim_{x \to +\infty} f(x) - x = +\infty$

$\Rightarrow \nexists$ asintoto obliquo

iii) $f'(x) = \dfrac{\operatorname{sgn}(\sqrt{x}-2)(x - 3\sqrt{x})}{(\sqrt{x}-2)^2} \geq 0 \iff x \in [0,4[\cup [9,+\infty[$

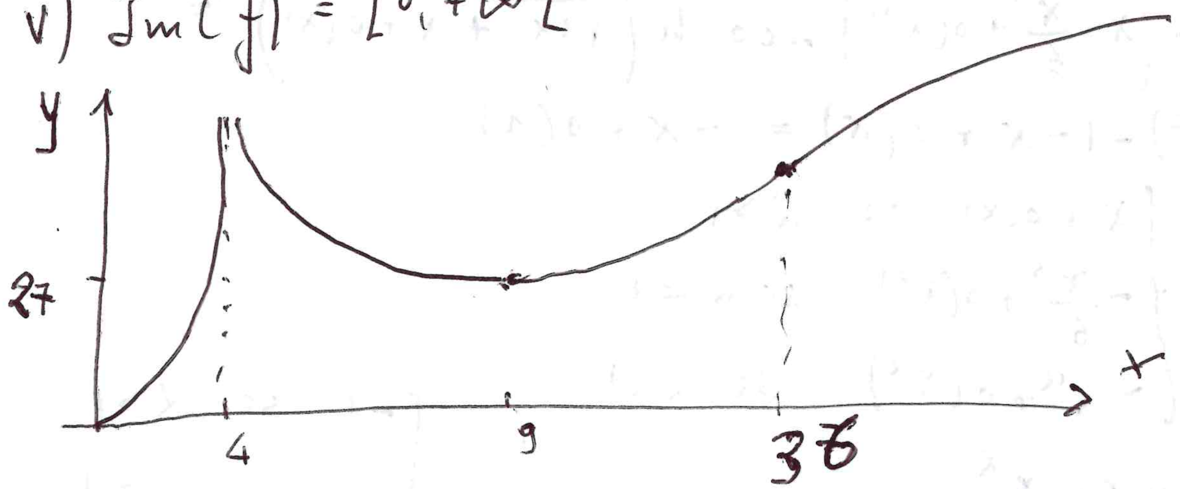
$\Rightarrow$ f è monotona crescente su $[0,4[$ e su $[9,+\infty[$

f è " decrescente su $]4,9]$

iv) $x = 0$ punto di minimo assoluto, $0 = f(0) = \min f$

$x = 9$ punto di minimo relativo, $f(9) = 27$

v) $\mathrm{Im}(f) = [0,+\infty[$



Es. 4

$$f(x) = \int_{-\sqrt{\ln x}}^{\sqrt{\ln x}} e^{t^2}\, dt$$

i) $\mathrm{Dom}(f) = [1,+\infty[$, $\lim_{x \to +\infty} f(x) = \int_{-\infty}^{+\infty} e^{t^2}\, dt = +\infty$

$\lim_{x \to +\infty} \frac{f(x)}{x} = \lim_{x \to +\infty} f'(x) = 0 \Rightarrow \nexists$ asintoti obliqui

$$f'(x) = \frac{1}{2\sqrt{\ln x}} \cdot \frac{1}{x}\left[e^{|\ln x|} + e^{|\ln x|}\right] = \frac{1}{\sqrt{\ln x}} \Rightarrow f \text{ crescente}$$

$x = 1$ punto di minimo assoluto

$\boxed{\text{Es } 5}$ $y'(x^2+1) + 2x(y-3)^2 = 0$

i) Soluzione costante: $\varphi_1(x) = 3 \quad \forall x \in \mathbb{R}$

Soluzioni non costanti

$$\int \frac{dy}{(y-3)^2} = \int \frac{-2x}{x^2+1}\,dx \Rightarrow -\frac{1}{y-3} = -\ln(x^2+1) + c_1, \quad c_1 \in \mathbb{R}$$

$$\Rightarrow y-3 = \frac{1}{\ln(x^2+1)+c_2}, \quad c_2 \in \mathbb{R}$$

$$\Rightarrow \boxed{\varphi_2(x;c) = 3 + \frac{1}{\ln(x^2+1)+c}, \quad c \in \mathbb{R}}$$

ii) Pb di Cauchy, $y(0) = -2 \Rightarrow -2 = \varphi_2(0;c) = 3 + \frac{1}{c}$

$\Rightarrow c = -\frac{1}{5} \Rightarrow$ Sol. del Pb di Cauchy è:

$$\boxed{\varphi(x) = 3 + \frac{1}{\ln(x^2+1) - 1/5}}$$